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How much knowledge do students need for the high school final exams in mathematics? A comparison between Hungary and Germany

In this paper we compare the final exams in mathematics in Hungary and Germany (exemplified by the federal state Berlin). Both the high school curricula and the examination systems in these two countries vary considerably. Therefore we have to consider not only the "level" of mathematical knowledge which is needed to pass the exams but also the wideness of knowledge and skills which students need and the "predictability" of the examination tasks. We take these factors into account by analyzing Hungarian and German examination assignments during the last five years. Due to lack of space, we restrict to the fields of non-linear equations, functions and analysis. The results we get based on these topics are symptomatic also for other fields. Both in Hungary and in Berlin (as in the most federal states of Germany) there are two levels of exams. Here we compare the "lower level" exams (intermediate level in Hungary and basic courses in Berlin).

The curriculum of the intermediate level final exam in Hungary

The Hungarian math graduation requirements[1] cover the entire curriculum of the last four years of the high school. The following themes appeared in the intermediate exams in the last five years (2012-2016):

- Methods of mathematical reasoning (20% of the points): operations on sets, cardinality of sets, mathematical logic, combinatorics, graphs;
- Algebra (25%): divisibility, powers, roots, concept of logarithm, algebraic fractions, use of formulas, percentages, linear and quadratic equations, equations containing absolute value, exponential and trigonometric equations, linear and quadratic inequalities, means;
- Functions (15%): linear functions, quadratic functions, functions with absolute value, exponential functions, trigonometric functions, arithmetic and geometric progression;
- Geometry (25%): geometrical calculations in the plane and in the space, trigonometry, vector operations, equations of lines and circles;
- Statistics, probability (15%): representation of data, descriptive statistics, classical probability, binomial distribution.

[1] Central examination requirements in mathematics (Hungarian): http://www.oktatas.hu/pub_bin/dload/kozoktatas/erettsegi/vizsgakovetelmenyek2012/matematika_vk.pdf

In Table 1 it is marked with an X if a task from the given topic featured in the intermediate level final exam in the current year. Due to lack of space, not all the above mentioned topics are shown, just those that are comparable to Analysis tasks in Germany. It is clear, that in the maths final exams students meet rather various tasks (Csapodi & Koncz, 2016), even from the topics of Table 1. Tasks from the themes of arithmetic and geometric progressions occurred every year, but for other topics the themes of the problems were less predictable. In addition the questions on functions also varied: students should be able to interpret, to represent and to basically characterize them (derivation is not part of the curriculum on the intermediate level). The equations also appear in "pure" form and as a tool for the purpose of solving a text problem on the exam.

Topic group	**Topic**	**2012**	**2013**	**2014**	**2015**	**2016**
Algebra	Quadratic equations	X		X	X	
	Equations containing absolute value				X	
	Exponential equations	X	X		X	
	Trigonometric equation		X	X		X
	Linear inequalities		X			
	Quadratic inequalities					X
Functions	Linear functions				X	X
	Quadratic functions	X	X		X	
	Functions with absolute value		X	X		X
	Exponential functions			X		
	Trigonometric functions	X		X		
	Arithmetic progressions	X	X	X	X	X
	Geometric progressions	X	X	X	X	X

Table 1: Overview of the topics of the intermediate level final exams in Hungary, 2012-2016

Contents of the basic course exams in Berlin

In all federal states of Germany mathematics education at the upper secondary level (grades 11 and 12) consists of the domains Analysis, Analytic Geometry/ Linear Algebra and Stochastics. The high school final exams of all states contain tasks of these domains. The following table gives an overview of the mathematical knowledge which is required to solve the Analysis parts of the basic course exams in Berlin from 2012 to 2016. Every year students can choose between two assignments (1.1 and 1.2).

	2012		2013		2014		2015		2016	
Analysis	**1.1**	**1.2**	**1.1**	**1.2**	**1.1**	**1.2**	**1.1**	**1.2**	**1.1**	**1.2**
Polynomial functions	X	X		X	X			X	X	X
Exponential functions		X	X			X	X			X
Function graphs, zeros	X	X	X	X	X	X	X	X	X	X
Derivatives, rate of growth	X	X	X	X	X	X	X	X	X	X
Extremal values	X	X	X	X	X		X	X	X	X
Inflexion points		X	X	X	X			X	X	X
Integral, area between function graphs	X	X	X	X	X	X	X	X		X
Angle between tangents		X			X			X		
Solving quadratic equations	X	X		X	X				X	
Systems of linear equations (2x2)		X				X			X	

Table 2: Mathematical contents of the Analysis parts of the high school final exams for students in basic courses in Berlin, 2012-2016

From Table 2 follows that students are well prepared for the Analysis part of the exam, even if they know only one specific type of functions (polynomial functions, almost always 3rd degree). As a second type of functions exponential functions appear, but students have the choice between tasks with or without exponential functions. They have to create (or work with given) function graphs and in some cases to calculate zeros. In any case they have to calculate derivatives and use it to determine extremal values, also often inflexion points (extremal points of slope). Integrals and their utilization for calculating areas between function graphs are also standard parts of the exams, in very simple cases students have to calculate antiderivatives or they have to verify given antiderivatives by derivation. Some assignments contain calculations of angles between tangents on function graphs but students can avoid this by choosing alternative assignments. Generally, students need skills in elementary algebra to solve the tasks, it is often required to solve quadratic equations and in some cases simple systems of linear equations. All assignments in Berlin (and also in some other states of Germany) are "packed" into more or less "realistic" contexts, which is currently under critical discussion, see e. g. (Jahnke et al. 2014).

Conclusion

According to the German curricula the assignments (especially in basic courses) contain elements of "*higher mathematics*" (differential and integral calculus, vector algebra and stochastics). They only include contents which are taught in the last two years of high school (although, of course, basic skills, which should have been acquired in earlier years, are necessary). The variation of the tasks is low (see Table 2), the function classes which are to be considered are very limited on polynomial and exponential functions (and when students do the "right" choice then even polynomials are sufficient).

Hungarian exams cover topics of the last four years of the high school, they don't contain tasks which are based on differential or integral calculus and can be seen on a "lower level". But the variation of the mathematical content is much greater than in Berlin (and also in other federal states of Germany) so students need a *broader knowledge*. Especially they need to know more types of functions and have to be able to solve more kinds of equations to solve the tasks successfully.

Because Analysis and Linear Algebra are the primary mathematical fields in many study courses at universities (like in natural sciences, engineering and economy), it might seem obvious that the contents of the German exams provide a better preparation for university studies. But on the other hand, in German universities there are intensifying complaints about students lacking elementary knowledge and skills in mathematics – for this reason a growing number of universities provide preparatory ("bridge") courses, in which students learn (or refresh) topics especially in elementary algebra (which are parts of the Hungarian exams, see Table 1). For the same reasons these kind of preparatory courses are provided in Hungarian universities as well (Dékány et al. 2012). It isn't clear yet and should therefore be subject of further research, which approach is more successful regarding the preparation of students for university courses: doing "higher mathematics" on a narrow base (e.g. a very limited selection of functions) or a limitation on more "elementary mathematics" but with a wider variety of examples and applications. A promising way to get answers to this question is to compere the performance of Hungarian and German Students in mathematical courses for beginners at universities.

References

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