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How mathematics education became a ritual

The following essay, which is based on the first half of my talk at the GDM conference in Potsdam 2017, should be read as a *conjecture* concerning mathematics education, intending to spur discussion and open up new pathways for research. I start by trying to show that present day mathematics education can be interpreted as a ritual, after which I indicate five distinct origins of this ritual. The reason why the second half of my talk is not included in this essay, is that an account of the explanatory framework presented there can be found in texts published elsewhere (Lundin, 2013; Christensen & Lundin, 2017).

A definition of mathematics education

The purpose of this first section is to show how present day mathematics education can be understood as a *ritual*. Following the American anthropologist Roy Rappaport (1999, pp. 23–49), I thus want to draw attention to how mathematics education is a *specific* practice, making it easy to recognize (Dowling, 1998). This becomes particularly clear when considering the fact that the activity is taking place in schools, i.e. at a location *geographically separated* and thus insulated from non-school activities in society. What takes place in mathematics education is to a large extent *determined by others than its participants*, and how it should be performed is *encoded* in laws, curricula, textbooks and formal schedules. In mathematics education, participants have pre-established *roles*, while at the same time meaning is attributed to participants according to variations in performance. Mathematics education follows a certain *rhythm*, where training is followed by assessment, in a movement towards increasingly challenging courses and it no doubt constitutes an "ordeal" for the participating pupils (cf. Fontaine, 1986). The performance of mathematics education is *geographically invariant*, i.e. it is performed in similar (if not identical) ways in many different places, and it is – despite claims to the contrary – quite *stable*.

Furthermore, as is characteristic of rituals, it is on the one hand connected to *expectations of outcome*, articulated in claims such that mathematics education should lead to self-confidence, democracy, economic growth, critical thinking, etc. But on the other hand it is very *well known* (cf. Pfaller, 2002), at a certain level of understanding, *that these outcomes are not attained*. This insight is articulated in persistent critique of mathematics education, and in articulations of "other social functions" of mathematics education, such as keeping children busy so that their parents can work and sorting them between stations in society.

Five origins of mathematics education

(1) A first origin of mathematics education is the practical arithmetic as it was represented in textbooks before the 17^{th} century. The chronology is important because it was in the 17^{th} century that mathematics rose to prominence in western culture and became intertwined with metaphysics and theology. Drawing on Stephen Toulmin (1992), my point is that the practical arithmetic concerned *the local, particular and specific*, in sharp contrast to the later mathematical science, which as we know concerns *the timeless, universal and general*. The practical arithmetic was an art of computing, a craft to be mastered. Textbooks of practical arithmetic typically started with a presentation of the number system. Then followed an account of "the four species", in whole numbers, in fractions and "with units". These elementary techniques were then employed in an often large number of more or less complex (some would say convoluted) applications of the "rule of three" (see Lundin, 2008).

The practical arithmetic was essentially a set of techniques for efficient manipulation of digits. It was thus in a sense *superficial* and *mechanical*. But this did not prevent it from functioning. The practical arithmetic was difficult because of units of measurement. Not only was volume, weight, time and money measured differently; types of goods often had their own specific units of measurements, as well as specific conventions for how measures were to be handled practically. Therefore, manipulation of fractions was central to the practical arithmetic. Mastery required memorization of tables of multiplication and conversion. An important role was played by "shortcuts" that made computations manageable.

From the practical arithmetic mathematics education has inherited most of its content. Two examples: Textbooks in practical arithmetic typically contained a few examples of addition of whole numbers so that adult readers could check if they had understood the algorithm (the right answer was always printed next to the exercise). In the course of the history of mathematics education these few examples were multiplied by a factor of several thousand, and were removed from the presentation of the algorithm and from the presentation of the right answer, to constitute a time-consuming *activity for children*. Textbooks in arithmetic typically explained how to compute the cost of a certain quantity of some goods, given the cost of some other quantity of the same goods. Such examples were presented *realistically*, in that they involved many different types of coins and units of measurement, necessitating knowledge of these coins and units as well as skill in manipulation of fractions. In the course of the history of mathematics education, such examples were stripped of the complications of coins and units of measurement, thus making them suitable as subject matter of an activity for children. Worth pointing out is that the "reality" that made the tables and shortcuts of the practical arithmetic useful was long gone already in the late 19th century, and in many respects even a century earlier.

(2) A second origin of mathematics education is the discourse and practices of scientific mathematics that became increasingly central to the self-understanding of the West in the 17th century (Dear, 1995; Funkenstein, 1986; Gaukroger, 2006; Gillespie, 2008; Toulmin, 1992). In this discourse, concerns of science were intertwined with those of metaphysics and theology. Crucial for the history of mathematics education was the identification of mathematics with abstract *ideas* or *concepts* as distinct from the concrete, visible, superficial *signs* of the practical arithmetic. Mathematics was considered the highest form of knowledge that humans could attain, by some philosophers even considered identical with the knowledge of God. To know mathematics was to be a certain kind of person: not only rational, logical and disciplined, but also moral. Mathematics became an invisible presence, often identified with the presence of God, in nature as well as in the mind, and to know mathematics thus meant to be attuned to reality.

Within this discourse of metaphysics, philosophy and theology, mathematics was not understood as a set of useful techniques that anybody could master through hard work, but as a power of transformation, of the self and of the world. To learn mathematics was in this context understood as a moral exercise. It was primarily work with Euclid's *Elements* that was interpreted in this way, but more important for the history of mathematics education was what happened to the practical arithmetic when it was *displaced* by this set of ideas and practices. Focus was shifted from acquisition of skill to manipulate digits efficiently by means of algorithms, to conceptual understanding of why algorithms worked. The many "recipes" for how to handle specific problems were replaced by general principles, presented with the help of algebra. Difficulties external to mathematics, for instance those deriving from units of measurement, were replaced by what we today recognize as "realistic" word problems. Practical shortcuts were considered irrelevant.

(3) A third origin is the materialization of these ideas and practices in more or less local meritocracies, towards the end of the 18th century. I think here mainly of military education in France as described by Ken Alder (1997, 1999) and the Cambridge "Tripos" as described by John Gascoigne (1984). These local settings functioned as paradigms for the somewhat later introduction of mathematical subjects (geometry, arithmetic, algebra, etc.) into public education.

It is easy to forget the difference between mathematics as a set of ideas, and mathematics as a formalized, time-consuming activity of increasingly more challenging tasks. Mathematics was *made* into such an activity, encoded in curricula, textbooks and examinations. It acquired this form for the particular purpose of the establishment of hierarchy. The associations of scientific mathematics were in these settings conferred to participants in mathematics education according to their formally assessed performance. In ritual-theoretical terms, it is important to note that these settings were largely "self-referential", in that they could determine autonomously what was to be counted as a valuable performance, interpreted as "mathematical knowledge". The very complicated problems that were constructed in these settings, were *specific* to these particular settings. Furthermore, this was well known (and sometimes subject of critique).

(4) A fourth origin of mathematics is the intertwinement of practical arithmetic with pedagogy as this tradition emerged in Germany in the second half of the 18th century (Braun, 1979; Hartmann, 1904). While the discourse of scientific mathematics was focused on the benefits of possessing mathematical knowledge, the discourse of education was focused on the difficulties associated with the acquisition of such knowledge. This difference in focus can be related to the fact that the discourse of scientific mathematics pertained to elites for which mathematical knowledge functioned as a new source of legitimation, while the discourse of mathematical education filled the function of legitimating strategies of improvement of children belonging to lower social strata.

This is the origin of the rich language of mathematics education for talking about learning, the prerequisites of learning and the causes of failed attempts at learning. Very interesting, in this language, is how this language introduces a certain *precarious possibility*, connected to certain metaphors and practical arrangements, always together with a potential explanation of failure. And not only that. The practical arrangements often, if not always, have *in themselves*, some "dangerous" element, which could actually work *against* learning, if it is not handled appropriately.

For instance, according to this line of thinking, knowledge must be the result of inner self-movement, what in German was called *Selbsttätigkeit* (probably drawing on the idealist philosopher Johann Gottlieb Fichte). As a consequence, instruction for the purpose of "transfer" of knowledge, memorization, and the acquisition of skills, became seen as *threats* to the kind of learning that was sought for.

The focus on self-movement was connected to the idea that the goal of teaching was not the mastery of some particular subject matter or the acquisition of some particular skills, but the total transformation of the child. It was seen as *not yet human*, but with a potential for becoming human, what in German was called *Bildsamkeit*. The purpose of the self-movement that teachers were to induce in children, was to make the very capacity for self-movement develop, with the goal of making the children “self-determined”, because to be self-determined, i.e. free, was equated with being fully human.

These ideas were connected to the introduction of activities in which the teacher led the children to produce certain answers by means of leading questions. This method had many names: the “catechetic” method, the “maieutic” method and perhaps most often the “Socratic” method. The self-activity of children was to be trained by having them “discover” their new knowledge by themselves.

The idea of the not yet human child, in combination with the idea that the child should discover everything new by herself, was furthermore connected to the idea that teaching should *start with the very simple*, and *progress slowly and without gaps* towards the more complex. Curiosity and initiative on behalf of children to learn new stuff, i.e. moving “too quickly”, was seen as a threat to learning.

According to this theory of learning, knowledge must be introduced “through the senses”, in particular by means of visualization but also by means of manipulation of concrete materials such as sticks. The term used to talk about this in Germany was *Anschauung*. The general purpose of visualization was that children should build, in their minds, proper concepts, most importantly of *numbers*. Visualization was employed to avoid superficiality. The very possibility of superficial but efficient manipulation of digits, i.e. the rationale of the practical arithmetic, was seen as a threat to learning. It is crucial to understand that the goal of mathematics education was not the acquisition of useful skills, but to transform children into fully human beings.

With these educational ideas, the goal became more ambitious, and thus more difficult to attain. Focus was shifted from the visible, concrete, particular and directly useful manipulation of digits, to the invisible formation of abstract and universal concepts. Learning became, one could say, *referring* to a different realm, where everything important was supposed to take place. But this displacement of the “scene” of learning, made it precarious. And not only that: the very techniques by which invisible, benign processes were to be induced and kept in motion in this other scene, were hostile to the directly visible, to instrumentality, to knowledge of facts, to speed, precision, efficiency, mastery.

(5) As a fifth and final origin of mathematics education I count the mix-up of these educational ideas with practical concerns connected to public schooling in the 19th century (Lundin, 2012). The number of children increased and they stayed increasingly longer in school. At the same time public education became subject to increased state control, with more detailed curricula, inspections of classroom activities as well as of textbooks, and with the establishment of formal teacher education seminars, even for the *Volksschule*. It becomes clear, at this point, that the activity of the mathematics education classroom is a compromise: on the one hand, teachers try to follow the tenets of educational theory, on the other hand they adapt to *externally imposed necessities* of the teaching situation.

For instance, in Sweden, until the 1860's it had been common practice in the *Volksschule* to use more able pupils as help-teachers. However, in 1862 this became forbidden by law and then it became an urgent topic of discussion in teacher magazines how a single teacher could teach a heterogeneous group of children all by herself. The solution that was to become hegemonic was called "silent practice" (*tyst övning*). Pupils were put to work with a series of exercises, intending to keep them busy and thus quiet, not bothering the teacher with questions. While exercises for silent practice could be constructed in any school subject, it turned out that arithmetic was especially well suited for this purpose. The construction of these sequences of exercises in arithmetic was no trivial task however. If they were too easy, the children would get bored and unruly. If they were too difficult, they would seek the assistance of the teacher. Usefulness for silent practice thus became a selling point for textbooks, together with their compliance with the latest curricula, and (if possible) a positive review of the latest textbook inspection.

As the number of exercises increased, other content was removed, in particular explicit presentation and explanation of algorithms. Furthermore, it became crucial that children found the exercises compelling, and they therefore often referred to the everyday life of themselves and their parents. A characteristic of the books that "worked" was a meticulously designed progression, in small steps, together with hints that helped the pupils overcome new obstacles by themselves. In Sweden, it was at this point, in the 1880's, that mathematics education textbooks got the form that they to a large extent still have.

This transformation of the mathematics education textbook and the classroom activity that it supported was accompanied by arguments on two different levels. On the one hand it was argued that silent practice was *necessary* for the teacher to be able to teach. On the other hand it was argued that silent practice was good for learning. On the one hand, silent practice was criticised for being detrimental to learning, because it could lead to superficial and "mechanical" computations doing nothing but killing time. On the other hand, silent practice was celebrated as an improvement of teaching, with reference to exactly the same educational theory. Focus in these positive evaluations was put on the slowness of progression, on how the pupils were "discovering" their new knowledge independently by means of what we today would call "problem solving", and on how the exercises connected to the everyday life of the pupils and aroused their interest in finding solutions.

Educational theory made it possible to see a *potential* for learning in the silent practice that was imposed as an external necessity. But not all educational theory could do that. For instance, in the Swedish discussion, one teacher argued that work with exercises should end, when the pupil had "understood" (see Lundin, 2012). He thought in terms of enlightenment, with learning being like turning on the light. This metaphor was not suited to "make sense" out of the silent practice, which needed to commence, continue, pause, recommence and eventually end according to a rhythm disconnected from the state of mind of the pupil. I argue that this was the reason why it was excluded from the theory of learning dominant in mathematics education. Instead, metaphors of growth, building and development became hegemonic. They made it possible to see silent practice as the laying of a "stable foundation", firmer the more time that was spent on it.

A first point is thus that which theory of learning that became dominant in the history of mathematics education was determined partly by external forces, such as the need to keep children silent. A second point, more directly related to ritual theory, is that this "solution" was encoded into curricula and textbooks and supported by the methodological discussion that was later to become mathematics education research. The activity of mathematics education thus to an increasing extent became determined by others than the participants themselves. In Sweden, before 1890, many teachers wrote their own textbooks and participated as independent voices in the emerging discussion among teachers on how to teach elementary arithmetic. These activities effectively came to an end in the 1890's, and were replaced by a social structure where individual teachers had a subordinated position within a partly formalized social structure, with teacher educators, researchers and public officials setting the agenda.

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