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Understanding of the relationship between polyhedrons and patterns among second-grade elementary students

1. Problems and objectives

In mathematical curriculums, geometry is as important as number and calculations. The Japanese school system is a homogeneous educational system, and therefore Japanese students learn the same material nationwide until at least the ninth grade. In the national curriculum, second-grade students learn the construction of polyhedrons, such as their shapes and the number of sides by tracing every side of a box onto a paper and reconstructing the traced shapes to a box(shown in the left of Figure 1). In the fourth grade, they learn the names of polyhedron, rectangular prism and cub, and drawing patterns of which each side is connected (shown in the middle of Figure 1). Furthermore, the same materials are taught in the fifth grade for a polygonal prism and a circular cylinder (shown in the right of Figure 1), and in the seventh grade for a pyramid and a circular cone.

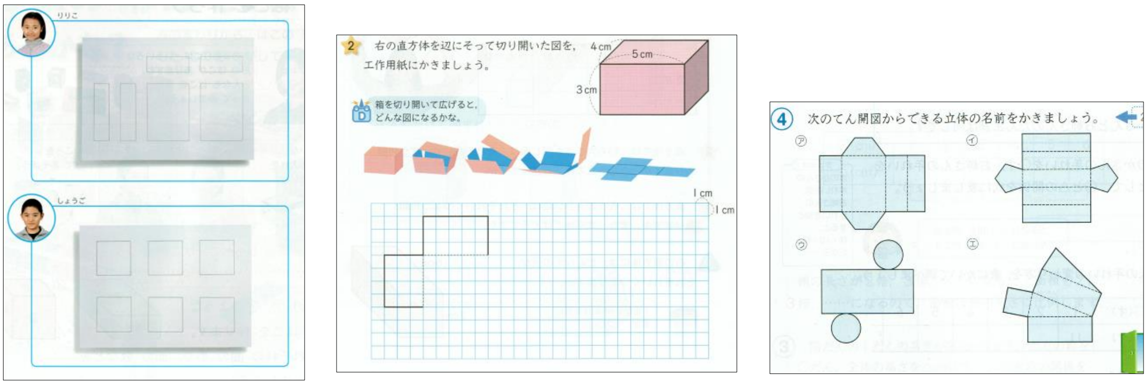

FIGURE 1. Contents on Textbook of Elementary school(Fujii(2015))

As opposed to the national curriculum, the studies conducted by Yokoichi (1979) and Suzuki (1979) in which they had students create a given cub using a paper which suggested that even first and second-grade students, with a certain degree of instructions, are capable of learning the contents taught in fourth grade. Therefore, the first purpose of the present study was to examine the possibility that second-grade students are capable of drawing patterns of boxes that have six sides.

As mentioned earlier, the mathematics curriculum for second-grade students includes learning the number and shapes of sides of rectangular prisms and cubes and learning that opposite sides are congruent. However, the curriculum does not include learning that a polyhedron is created from a pattern on a sheet of paper. Thus, to clarify the first objective, we examined the understandings of second-grade students about the relationships between pattern and polyhedron through the task of reconstructing polyhedrons from a plane sheet.

The second objective of the study was to examine effective teaching methods to teach second-grade students to draw patterns of rectangular prisms and cubes. It was hypothesized that when converting three-dimensional polyhedrons to a pattern, observing the object from the inside, as well as from the outside would enable students to understand relationships between the individual sides and improve the execution of the task (Hypothesis 1). It was also hypothesized that sides of a rectangular prism are orthogonal to each other and therefore, would facilitate students' understanding of the correlation of length of sides that are touching, whereas it would be difficult for students to understand the relationship when the sides are not orthogonal (Hypothesis 2).

2. Method

Second-grade students from three classes (N = 79), A, B and C, who had not learned the relationship between pattern and three-dimensional objects, participated in the study. The numbers of students from classes A, B, and C were 28, 25, and 26 respectively. Students from each class were divided into groups of 4-5 students. Groups in classes A and B received the rectangular prism shown on the left of Fig.2, and groups in class C received the model house shown on the right of Fig.2. All the students were provided with an A3 size paper. The tasks of the students were (1) to draw a pattern of the given polyhedron using a triangular ruler, (2) cut the pattern with scissors, and (3) create a polyhedron from the clipped pattern using tape. The difference between the rectangular prisms provided to class A and B was that class B was given a rectangular prism with one open side so that students could open the side and observe the inside of the rectangular prism (shown in the middle of Fig.2), whereas that provided to the class A could not be observed from the inside. The students had 60 minutes to complete the task.

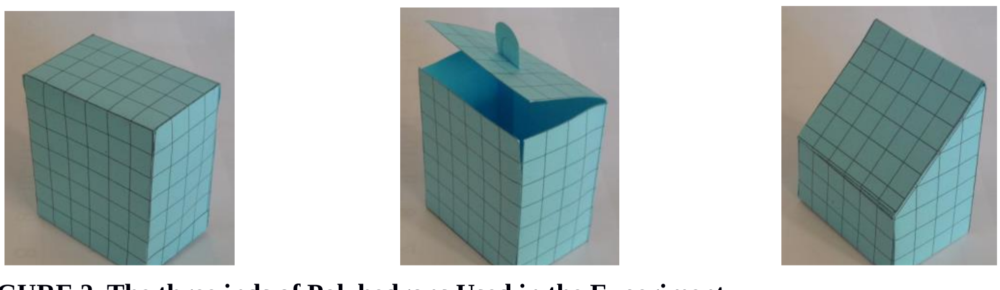

FIGURE 2. The three inds of Polyhedrons Used in the Experiment

Those that finished the task within the time limit were instructed to repeat the procedure to create multiple polyhedrons. In addition, the students were told not to talk with their group members during the task.

3. Results and Discussion

The polyhedrons created by students were evaluated from three perspectives; (1) whether all corners of sides were right angles, (2) whether all edges were the same lengths as the given sample, and (3) whether all six sides were connected. The results are shown in Table 1. The circle "o" in the table indicates that students satisfied (1) and (2), and the triangle "Δ" indicates that students satisfied either (1) or (2) whereas the other was partially inaccurate. The blank "x" indicates that the students satisfied either (1) or (2), or were severely inaccurate in both. However, everyone was accurate in (1), thus the length of the edges in (2) determined the grouping into the circle. With regard to connecting the sides in (3), when students connected all the sides it was labelled "all combined", whereas it was labelled "partially combined", when students connected some sides but not all, and named "all separate" when all sides were disconnected.

	Class A			Class B			Class C		
	○	△	X	○	△	X	○	△	X
all combined	7 (25%)	5 (18%)	0(0%)	3 (7%)	1 (4%)	1 (4%)	5 (19%)	3 (12%)	3 (12%)
partially combined	2 (7%)	3 (11%)	0(0%)	13 (52%)	3 (12%)	3 (12%)	4 (15%)	5 (19%)	1 (4%)
all separate	6 (21%)	5 (18%)	0(0%)	2 (8%)	0 (0%)	0 (0%)	2 (8%)	2 (8%)	1 (4%)
sum	15 (54%)	13 (46%)	0(0%)	18 (72%)	4 (16%)	3 (12%)	11 (42%)	10 (38%)	5 (19%)

TABLE 1. Results of Experiment

The proportion of students that created the polyhedron based on the accurate pattern with all the sides connected with correct edges and angles were 25%, 7%, and 19% in class A, B, and C, respectively. Moreover, students that created an accurate polyhedron from partially connected patterns were 7%, 52%, and 15% in class A, B, and C, respectively. These students were considered to be in the process of being able to draw a pattern with all sides connected. Therefore, 34 students were able to draw a correct pattern by considering the relationship of the sides that construct a polyhedron, and this was 43% out of 79 students. Moreover, students that drew a pattern with disconnected sides might have been able to draw a pattern with connected or partially connected sides. The students that participated in the present study engaged in the given task without any special instructions. Therefore, the results of the present study imply that second-grade students are capable of learning the contents taught in the fourth grade with the inclusion of certain instructions.

The number of students that drew the correct pattern with connected or partially connected sides was 9, 16, 9 students in class A, B, and C, respectively. We call this type of responders high-level achievers. To examine Hypothesis 2, we conducted a $\chi 2$ test for difference between the proportion of high-level achievers in classes A and B, which indicated significant differences ($\chi 2$ (1) = 4.68, $p < .05$), suggesting that the observation of the inside facilitated drawing the pattern. This finding supported Hypothesis 1. To test Hypothesis 2, we conducted a $\chi 2$ test for differences in the proportion of high-level achievers between groups A and C. The results indicated no significant difference, and therefore Hypothesis 2 was rejected.

The results of this study indicate the following. Firstly, second-grade students are capable of learning material that is currently taught in the fourth grade, as indicated by at least 43% of students drawing a pattern by considering connections of multiple sides and the length of the corresponding sides. Therefore, it is suggested that learning material in curriculums on relationships between polyhedrons and patterns should be reconsidered. Secondly, Hypothesis 1 was supported, indicating that observations made not only from the outside but also from the inside facilitated the task of converting polyhedrons to a pattern from a flat surface. Thirdly, there was a non-significant difference in the proportion of high achievers between groups A and C that utilized a house-shaped polyhedron with non-orthogonal sides, indicating that orthogonal sides did not affect task difficulty.

Support for Hypothesis 1 suggests the efficacy of facilitating observations of the inside of polyhedrons when teaching about polyhedrons and patterns to second-grade students. The rejection of Hypothesis 2 suggests the possibility of incorporating the use of house-shaped polyhedrons for second-grade students, in addition to the use of rectangular prisms and cubes.

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