New Methods to Speed-up the Boundary Element Method in LS-DYNA

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Abstract

LS-DYNA is a general purpose explicit and implicit finite element program used to analyse the non linear dynamic response of three-dimensional solids and fluids. It is developed by Livermore Software Technology Corporation (LSTC). An electromagnetism (EM) module has been added to LS-DYNA for coupled mechanical/thermal/electromagnetic simulations, which have been extensively performed and benchmarked against experimental results for Magnetic Metal Forming (MMF) and Welding (MMW) applications. These simulations are done using a Finite Element Method (FEM) for the conductors coupled with a Boundary Element Method (BEM) for the surrounding air.

The BEM has the advantage that it does not require an air mesh, which can be difficult to build when the gaps between conductors are very small, and to adapt when the conductors are moving, with contact possibly arising. Besides, the BEM does not require the introduction of infinite boundary conditions which are somehow artificial and can create discrepancies. On another hand, it generates dense matrices which take time to assemble and solve, and require a lot of memory. In LS-DYNA, the memory issue is handled by using low rank approximations on the off diagonal sub-blocks of the BEM matrices, creating a so-called Block Low Rank (BLR) matrix structure.

The issue of the assembly and solve time is now being studied, and we present the so called "Multi-Center" (MC) method where the computation of the far-field submatrices is greatly reduced and the solve time somehow reduced.

We will present the method as well as some first results.

Keywords

Simulation, Boundary Element Method (BEM), Electroforming

1 Introduction

The Boundary Element Method (BEM) has been widely used for the simulation of many physical problems, and in particular for electromagnetics simulations (L'Eplattenier 2009, L'Eplattenier 2010, L'Eplattenier 2015, Çaldichoury 2012, Duhovic 2012, Ulacia 2008). It presents many advantages over the Finite Element Method (FEM), notably by avoiding an air mesh between the conductors, which can be cumbersome to build when the inter-conductor gaps are very small or when the conductors are deforming. The inter-conductor volume also often proves more complicated to mesh than the conductor geometry itself, requiring tetrahedral meshes which often give less accurate results than hexahedral ones. Also, when using the BEM, no artificial approximations at the boundary are needed.
The drawback of the BEM is that it generates dense matrices which need large amounts of memory to store, and take a long time to assemble and to solve. In order to reduce the memory needed to store the matrices, as well as the matrix vector product time, hence the solve time, a Block Low Rank (BLR) method has been introduced (L'Eplattenier 2015). In this method, low rank approximations are performed on blocks of the BEM matrices corresponding to far away domains, which can considerably reduce the storage of the block. The only drawback of this method is that it first goes through the assembly of the full dense block which takes a lot of computation time. The multicentre (MC) method that will be presented in this paper allows computing directly the low rank approximation of the block without going through the lengthy dense block assembly.

2 The multicentre (MC) method

2.1 Domain decomposition and block matrix

The BEM mesh is decomposed into domains using an octree. This creates a block structure of the BEM matrix, where each block corresponds to a domain versus another one. In the BLR method, the diagonal blocks of the BEM matrix are kept dense, whereas low rank approximations are performed on the non-diagonal ones.

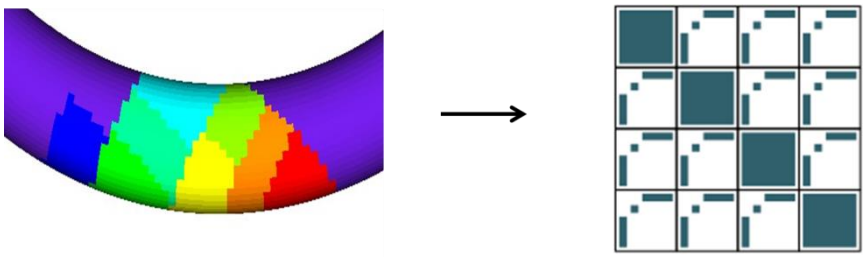

***Figure 1**: Domain decomposition (left) and corresponding BLR structure for a BEM matrix (right), where the diagonal blocks are kept dense, whereas low rank approximations are performed on the non-diagonal ones.*

2.2 A few definitions

We call:

- B the set of basis functions (or degrees of freedom).
- I the set of integration points used to compute the BEM integrals, using gaussian integration.
- The octree presented in section 2.1 gives a domain decomposition on the integration points which we write as:

$$I = \bigcup_{i \epsilon D} I_i \tag{1}$$

- In each domain I_i, we define a set of nodes $C_i \; c \; I_i$ which are well separated. We can find them using the min-max algorithm for example. We suppose moreover that C_i is ordered such that each subset of C_i built by taking the first k nodes of C_i also gives some well separated points.
- We define a distance between 2 domains (typically, equal to the distance between the centers over the max of the radii of the domains). This distance allows to partition the couples of domains into near couples N , and far couples F:

$$D \times D = N \oplus F \tag{2}$$

  This decomposition is such that the integrations between 2 far domains can be done using standard gaussian integration and does not involve any singular integrals. In the rest of this paper, we will be interested in the far domains.
- For each domain $i \in D$, we define the near and far domains of i as:

$$N_i = \{j \in D \; such \; that \; (i,j) \in N\} \tag{3}$$

and

$$F_i = \{j \in D \; such \; that \; (i,j) \in F\} \tag{4}$$

- For each domain $i \in D$, we define a set of "target" points $T_i \; c \; I$ which is supposed to be a good sampling of the points in the far domains F_i. These target points are supposed to be well separated. T_i can be built for example as

$$T_i = RoundRobin \, \{C_j, j \in F_i\}$$

2.3 BEM system

We consider a BEM matrix P on B. We can write, using the domain decomposition, and the separation between near and far domains,

$$P_{B,B} = P^N_{B,B} + P^F_{B,B} \tag{5}$$

$$= P^N_{B,B} + \sum_{(i,j) \in F} V^t_{B,I_i} \, K_{I_i,I_j} V_{I_j,B} \tag{6}$$

$$:= P^N_{B,B} + V^t_{B,I} K^F_{I,I} V_{I,B} \tag{7}$$

where the last term in the second equation represents, in matrix form, the standard gaussian integration:
The matrix $V_{I,B}$ is the matrix of the evaluation of the basis functions at the integration points multiplied by the integration weights and face jacobian determinants.
The matrix $K_{I,I}^{F}$ is the restriction of the kernel matrix, evaluated only between integration points in far domains (and equal to zero elsewhere).

Note: in the future, $K_{a,b}$ will always represent the evaluation of the kernel, between a set a and a set b. We also suppose that the evaluation of the kernel is symmetric i.e.
$k(x,y) = k(y,x), \forall\ (x,y)$.

2.4 Presentation of the multi-center method

In order to save memory and CPU time, we are looking for a low rank approximation for the submatrices

$$K_{I_i,I_j}\ , (i,j) \in F \tag{8}$$

We could get one by doing a Singular Value or QR decomposition on K_{I_i,I_j}.
However, we would like to get such a low rank approximation without having to assemble K_{I_i,I_j}, in order to save some assembly time.
In order to do so, we start from the intuition that:

- Most (up to a certain tolerance) of the information contained in K_{I_i,I_j} is actually contained only in the intersection K_{c_i,c_j} of a given subset $c_i\ c\ I_i$ of the rows with a given subset $c_j\ c\ I_j$ of the columns of the original matrix. We call these subsets of the integration points the "centers" of the domains.
- We also would like to find the subset $c_i\ c\ I_i$ independently of the domains j.

2.4.1 Finding the centers

In order to find the best possible centers c_i of a domain i in an automatic way, taking into account a given tolerance ε, we try to optimize them using the target points T_i of domain i, which should be a good representation of all the integration points in the far domains of i.
We thus form the kernel matrix K_{T_i,I_i} and do a rank revealing QR factorization with column pivoting on this matrix:

$$K_{T_i,I_i} P_{I_i,I_i} = Q_{T_i,T_i} R_{T_i,I_i} \tag{9}$$

We then check the successive norms R_k of the lines of R_{T_i,I_i}, and decide to stop when $||R\_k|| < \varepsilon\ ||R\_1||$. The permutation matrix P_{I_i,I_i} then gives the subset $c_i\ c\ I_i$ of size k.

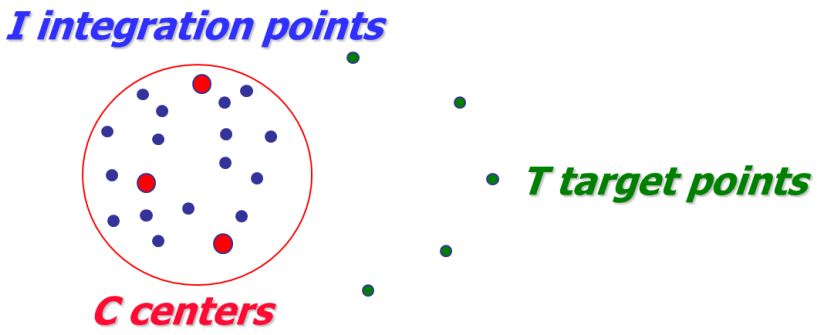

***Figure 2**: Schematics of the construction of the centers of a domain using the target points*

2.4.2 Construction of the barycentric matrices

We now try to find the best linear correlations between the centers c_i and the integration points I_i. In order to do so, we form the matrix K_{T_i,c_i} (kernel between the target points and centers) and solve (in the least square sense):

$$K_{T_i,I_i} = K_{T_i,c_i} B_{c_i,I_i} \tag{10}$$

which gives the barycentric matrix B_{c_i,I_i}.

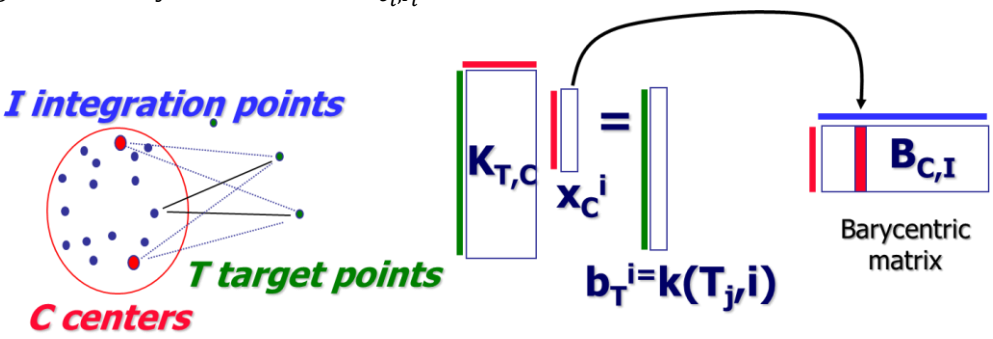

***Figure 3**: Schematics of the construction of the barycentric matrices.*

2.4.3 Low rank approximation of a subblock

The low rank approximation of a subblock K_{I_j,I_i} can be found in 2 steps:

- We first consider I_i as the far away points from domain i (similar to the previous target points of domain i), and using the centers and barycentric matrix of domain i, and can write:

$$K_{I_j,I_i} = K_{I_j,c_i} B_{c_i,I_i} \tag{11}$$

- We then consider the centers c_i as the far away points from domain j and use the centers c_j and barycentric matrix of domain j to write:

$$K_{c_i,I_j} = K_{c_i,c_j} B_{c_j,I_j} \tag{12}$$

- By combining equations (11) and (12), and using the fact that the evaluation of the kernel is symmetric, we find

$$K_{I_j,I_i} = B^t_{I_j,c_j} K_{c_j,c_i} B_{c_i,I_i} \tag{13}$$

which represents the low rank approximation we were looking for.

2.4.4 Rewriting the BEM matrix

Using equations (7) and (13), we can now write

$$P_{B,B} = P^N_{B,B} + P^F_{B,B} \tag{14}$$

$$= P^N_{B,B} + \sum_{(i,j)\in F} V^t_{B,I_i} B^t_{I_i,c_i} K_{c_i,c_j} B_{c_j,I_j} V_{I_j,B} \tag{15}$$

$$= P^N_{B,B} + \sum_{(i,j)\in F} M^t_{B,c_i} K_{c_i,c_j} M_{c_j,B} \tag{16}$$

$$\coloneqq P^N_{B,B} + M^t_{B,c} K^F_{c,c} M_{c,B} \tag{17}$$

with, as before, $K^F_{c_i,c_j} = 0$ if $(i,j) \in N$(near domains), and where

$$M_{c,B} \coloneqq B_{c,I} V_{I,B} \tag{18}$$

combines the basis evaluation (+integration weights + face jacobian determinants) with the barycentric matrix. It represents the basis evaluation directly at the centers. The matrix $M_{c,B}$ is sparse, block diagonal and is assembled using (18). The matrix $K^F_{c,c}$ is dense and stored in a BLR format (further low rank approximations are performed on the different blocks), but the size of c (total number of centers) is much smaller than the size of B (number of basis functions).

3 Examples

We will now show the benefit of the multicentre method on an example. We consider a torus with a circular cross section, and will consider different mesh densities. The first 3 mesh densities are shown of figure (4), and table (1) details the sizes of the BEM mesh.

case	1	2	3	4	5	6
# elem	900	7,200	57,600	230,400	460,800	921,600
nrow P	1,202	4,802	19,202	38,402	76,802	153,602
#domain	32	64	128	128	256	256

Table 1: *Sizes of the different meshes*

***Figure 4**: The first 3 torus meshes used in the comparisons*

For the timings, we consider one BEM matrices assembly, followed by 10 time steps advances using the coupled FEM-BEM algorithm used for the electromagnetics in LS-DYNA (L'Eplattenier 2009). All the runs were performed on the same (Xeon E5506, 2.13 GHz) machine using 8 CPU's.

Figure (5) shows a comparison between the memory needed by the BLR method and the MC one, for the assembly of one of the BEM matrices used in the solver, so called "P" matrix (L'Eplattenier 2009). One can see that as the models get larger, the savings of the MC method get larger and larger.

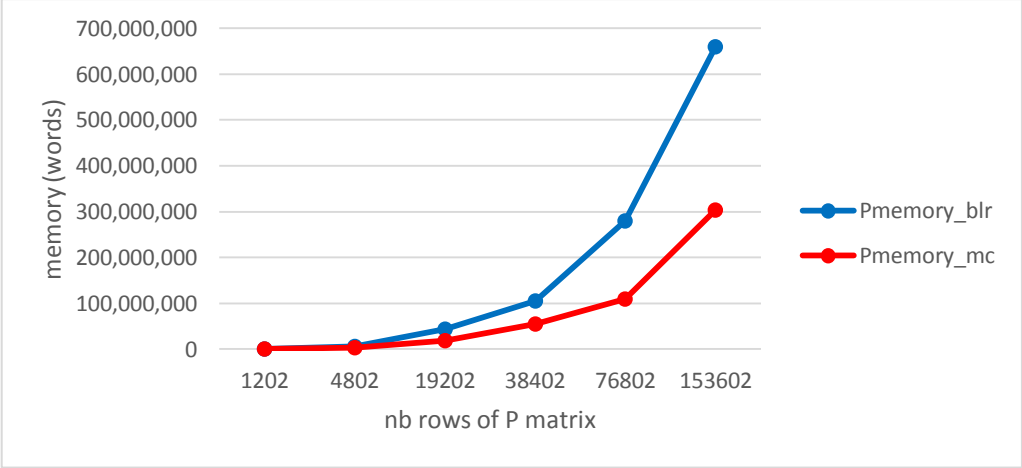

***Figure 5**: Comparison of the memory needed to store the BEM P matrix, between BLR (blue) and MC (red)*

Figure (6) shows a comparison between the BLR and MC methods for the timing of the matrix assembly, system solves, and total computation time. It shows a fairly large gain in the matrix assembly time, but not that much in the solve time. Further work in under way to also improve the solve time.

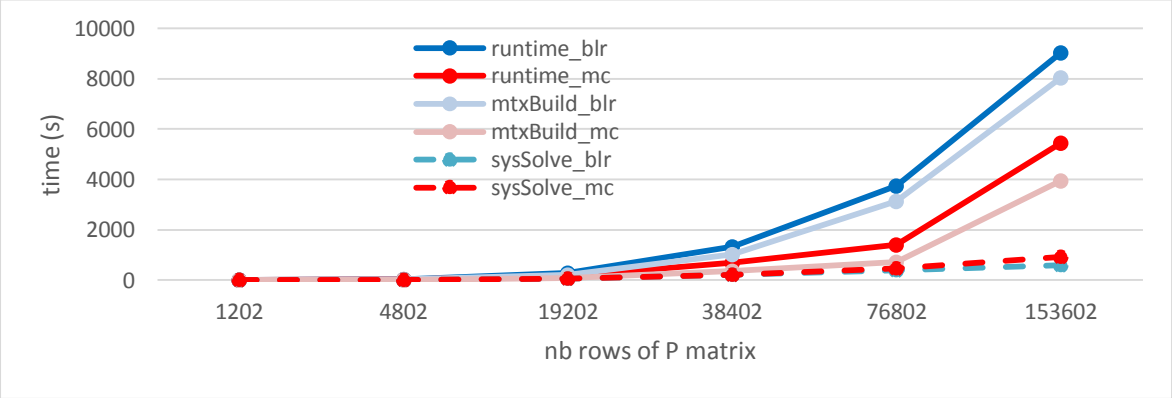

***Figure 6**: Comparison of the run time between the BLR (blue) and MC (red) methods. The total run time is decomposed in matrix build time and system solve time (using a Preconditioned Gradient Method (PCG)).*

4 Conclusion

The BEM method is attractive for many reasons, and most notably because it does not need an air mesh which can be cumbersome to build when the geometry of the conductors is complicated, or when the gaps between conductors are small. It also dispenses with all the remeshing issues when the conductors are moving. Its main drawback, though, is the generation of dense matrices which take a lot of memory to store, and a long time to assemble and to solve. We already worked in the past on the memory issue by introducing the BLR representation. In this paper, we presented a method to shorten the assembly time, the multicentre (or MC) method. We showed on a test case that this method allows significant computation time reductions without losses in the accuracy of the solution. Further studies will be done on this method in the future, and other method will be introduced to lower the solve time, notably a MPP factorization of the BEM matrices.

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