Fully coupled semi-analytical model for an electromagnetic-mechanical-thermal problem of a ring expansion test

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Abstract

A fully coupled semi-analytical model is proposed for a ring expansion test. In this model, we consider the electromagnetic, mechanical and thermal effects. During the development, an ideal case of electromagnetic ring expansion test is modelled using both semi-analytical and finite element methods. The analytical model also includes the changes in electrical resistance, mutual inductance and self-inductance as a function of radius during the ring expansion. The development procedure is divided into four separate calculation parts. Each individual part is validated before making the independent validation of the semi-analytical method to obtain a model with high accuracy and a robust calculation speed. The final prediction using this model closely resemble with the coupled finite element predictions, and it can be further extended to exploit for an inverse identification problem.

Keywords

Electromagnetic pulse forming, Analytical modelling, Material characterization, High strain rate, Multiphysics coupling

1 Introduction

Electromagnetic forming (EMF) processes involve multi-physics interactions resulting from electromagnetic, mechanical and thermal effects. Various research studies were considered to model EMF processes and they can be classified into three types according to the coupling level; (1) without coupling, (2) loosely coupled (i.e. only consider the coupling of electromagnetic and mechanical contributions), and (3) fully coupled (that includes electromagnetic-mechanical-thermal effects). All the existing models use weak coupling formulae, which contain separate governing equations to compute each physical quantity. In contrast, strong coupling formulae require combining all the governing equations, which must be computed simultaneously, and it has not been possible till date.

In general, multi-physics calculations are performed using finite element (FE) and semi-analytical methods. In some cases, FE methods are also combined with a meshless method or a boundary element method to improve the efficiency and accuracy. Using FE method, the influence of processing parameters was investigated and it provides better understanding of the EMF processes. FE simulation also provides an alternate option to predict the material parameters using an inverse identification procedure. Pandolfi *et al.*, investigated a case study of ring expansion test, conducted by Grady and Banson (Grady and Benson, 1983), for an aluminum AA1100 alloy using a 3D FE model (Pandolfi *et al.*, 1999). In another study, FE model was used to investigate a mild steel ring expansion test (Rusinek and Zaera, 2007). Recently, Rodríguez-Martínez *et al.* considered three different approaches including 3-D FE model, 2-D axisymmetric model and 1-D simplified model to investigate the ring expansion test (Rodríguez-Martínez *et al.*, 2013). Although these FE models help to identify the material properties, they require significantly high computational time. Semi-analytical models simplify the geometry and do not require spatial discretization. They use multiple approximations to simplify the governing partial differential equations. Hence, large matrix calculations are often not required for analytical models, thus they require less computational time.

Semi-analytical methods have been used for the purpose of process analysis since the early development of EMF technology in 1960s (Niordson, 1965). Later, semi-analytical methods were considered to be coupled with thermo-mechanical effect (Gourdin, 1989; Tamhane *et al.*, 1996). Due to the coupled problems' complexity, all the existing semi-analytical models have some limitations when considering their direct application for an electromagnetic ring expansion test. For instance, in the recent work of Altynova *et al.*, researchers did not consider the mutual inductance between the ring and coil and the self-inductance of ring (Altynova et al., 1996). In the thesis of Gupta, the eddy current in the ring was not directly computed, instead a proportionality constant was used to estimate the eddy current (Gupta, 2013). Moreover, recently Paese *et al.* (Paese et al., 2015; Paese et al., 2014) and Hahn *et al.* proposed semi-analytical methods for electromagnetic sheet metal forming (Hahn *et al.*, 2016). Those sheet metal forming models cannot directly be used for an electromagnetic ring expansion. In the model describing the sheet metal forming, Hahn *et al.* treated the magnetic field in the gap between sheet and coil as uniformly distributed and parallel to the surface of the sheet. Although the agility of a semi-analytical method is highly valued for preliminary calculations, there is further work required to obtain a complete analytical-model for a ring expansion test.

In this paper, a new fully coupled semi-analytical model is proposed. It requires the measured input current flow through the coil and calculates all other physical quantities in the process. It also considers the changes in electrical resistance, mutual inductance and self-inductance as a function of radius, and calculates eddy current based on the fundamental theory of electromagnetics. An ideal case of ring expansion test is investigated using the proposed semi-analytical model and a FEM coupled with boundary element method (BEM) using LS-DYNA software (LS-DYNA Keyword User's Manual R8.0, 2015). The comparison between results from these two methods is used to validate the calculation and to analyze the onset and propagation of errors. Errors in various calculation steps are carefully analyzed, thus it provides a robust semi-analytical method to analyze the ring expansion problem.

2. Electromagnetic ring expansion test and its application for material parameters’ calibration

A high frequency impulse current through a coil induces large eddy current and Lorentz forces in the ring during electromagnetic ring expansion test. During the test, ring reaches velocities in the range of 100 m/s and a strain rate of up to 10^4 s^{-1}. Thus, this technique is considered to identify the material properties under very high strain rate. Niordson pioneered in an electromagnetic forming (EMF) application that consists of a coil with 24 turns in 2 layers to expand an aluminum and a copper rings (Niordson, 1965). The study concluded that it can achieve a uniform strain field that is difficult to obtain using conventional methods due to inertia effect or “end effect” of both machine and workpiece (Niordson, 1965). Moreover, a semi-analytical model was used to describe the motion and it considers the approximation of ignoring the height and thickness of the ring (Niordson, 1965).

Jeanson *et al.* performed a series of work about identification of material properties, particularly parameters of Johnson-Cook model, using electromagnetic tube expansion test (Jeanson *et al.*, 2013; Jeanson *et al.*, 2014). The “tube” used in their model was approximated as a ring. Several pre-designed sets of Johnson-Cook parameters (i.e. A, B, n and C in Eq. 3 below in Section 4) were used in a numerical model. Computed velocities were compared with experimental measurements to inversely identify the suitable material parameters of Johnson-Cook model (Jeanson *et al.*, 2013), for instance for an annealed aluminum AA1050 (Jeanson *et al.*, 2016). There are two major difficulties of their identification method: the parameters B and n are interrelated and there is a lack of sensitivity to parameter C. Nevertheless sensitivity of each parameter and experimental uncertainties have been analyzed (Jeanson *et al.*, 2016).

Our model also shows the great potential of electromagnetic ring expansion test for the inverse identification of material parameters at high strain rate. An FE model is used in the development of the semi-analytical model and it enables to systematically handle the complex magnetic field for the helix coil and ring. This improves the analysis of errors for each calculation step so then a better analytical model can be obtained.

3 Procedure of the semi-analytical model

The analytical model includes mechanical, electromagnetic and thermal computations that involve various physical quantities viz. deformation (i.e. related with the expansion velocity), Lorentz force, eddy current, input current and temperature. Fig. 1 shows the calculation procedure which is divided into 4 parts.

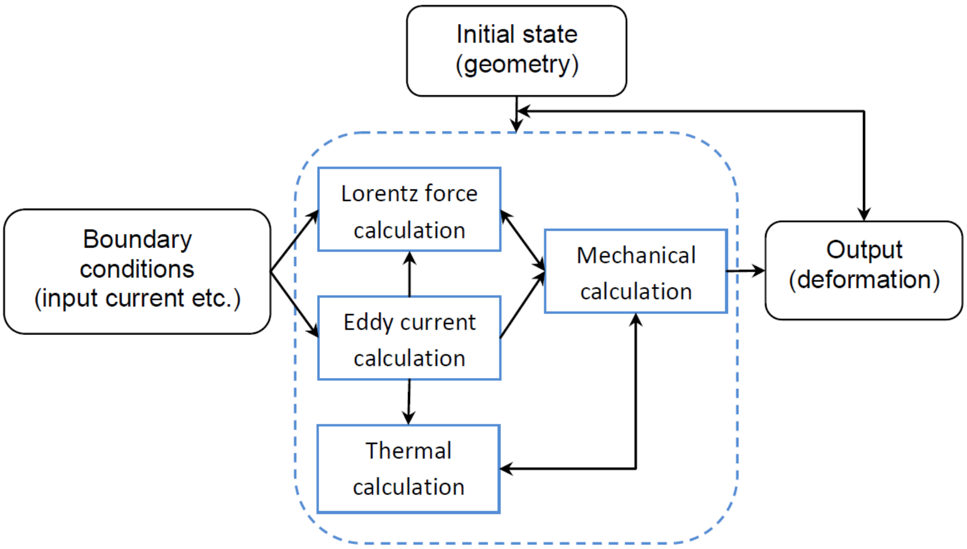

***Figure 1**: The flowchart showing the calculation steps and coupling stages. The output of previous calculation is used as the input of the next calculation step.*

Part 1: Mechanical calculation: Lorentz force is used as input physical quantity to compute velocity and deformation, which are output data.
Part 2: Lorentz force calculation: The Lorentz force is computed using workpiece deformation, eddy current and input current.
Part 3: Eddy current calculation: Workpiece deformation and input current are used to compute the eddy current. Usually, the part 2 combined with part 3 is considered as the electromagnetic calculation, but here they are separately treated for a better error analysis.
Part 4: Thermal calculation: The eddy current and plastic deformation cause the thermal effect in the workpiece during the test. In this part, the eddy current and heat dissipation due to plastic deformation are considered as inputs to compute a temperature field. Generally, a temperature change affects the mechanical calculation (Part 1) as well as the eddy current calculation (Part 3) due to temperature dependent material parameters, but this model does not include such an influence.

4 An ideal electromagnetic ring expansion test

An ideal electromagnetic ring expansion test is considered to validate the semi-analytical method. It simplifies the influence of some geometric parameters to avoid the complex distribution of magnetic field. This model has small ring thickness and height as compared to coil radius and height. This allows considering the ring as a simple line circle during the mechanical and electromagnetic calculations. The cross-section of the coil is also small enough to be considered as circle loops. The ideal model consists of a helix coil and a ring as shown in Fig. 2. A 7-turns helix coil is designed with a 2 mm × 2 mm cross-section and 1 mm winding pitch. The inner and outer radii of the coil are 21 mm and 23 mm, respectively. The ring is 4 mm wide, 2 mm thick and have an inner radius of 25 mm.

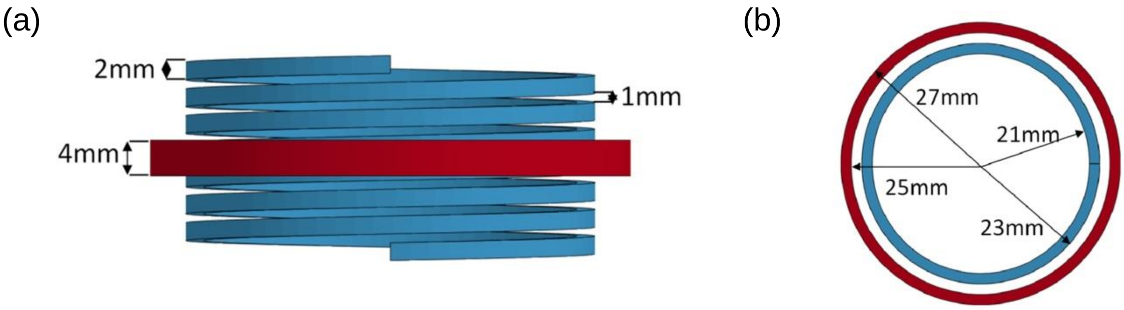

Figure 2: *Illustration showing the geometry of the ideal model (a) side view (b) top view.*

The input current flow through the coil is given by Eq. 1:

$$I_c = 27440 \cdot exp(-11294 \cdot t) \cdot sin(70598 \cdot t) \tag{1}$$

Johnson-Cook model describes the constitutive behavior with the influence of high strain rate and temperature as follows:

$$\bar{\sigma} = \left(A + B\bar{\varepsilon}_p^n\right)(1 + Cln\dot{\bar{\varepsilon}})[1 - \left(\frac{T - T_r}{T_m - T_r}\right)^m] \tag{2}$$

where, $\bar{\varepsilon}_p$ is the effective plastic strain and $\dot{\bar{\varepsilon}}$ is the effective strain rate. *A*, *B*, *C*, *n* and *m* are Johnson Cook material parameters. *T* represents the instantaneous temperature; T_r and T_m indicate the ambient temperature (25 °C) and melting temperature, respectively. The values of Johnson Cook parameters and other material properties are listed in Table 1.

Material parameters	*A* (MPa)	*B* (MPa)	*C*	*n*	m	Young's Modulus (GPa)	Poisson ratio	Density (kg/m^3)	Melting temperature(°C)
Numerical values	324	114	0.002	0.42	1	68.9	0.33	2700	660

Table 1: *Material properties used in the simulation and semi-analytical model.*

Note that the Johnson-Cook model does not include the effect of temperature at the very onset of a computation, alone. The J-C model is simplified into the following temporary model during this stage:

$$\bar{\sigma} = \left(A + B\bar{\varepsilon}_p^n\right)\left(1 + C\ln\dot{\bar{\varepsilon}}\right) \tag{3}$$

5 Calculation theory of the semi-analytical method

In this method, the discretization in time is used but the space is not discretized, thus it is called a semi-analytical method. The helix coil is treated as circles loops (Fig. 3).

5.1 Loosely coupled model

The simplified equation describing the motion of the electromagnetic ring expansion is given by (Johnson et al., 2010);

$$\rho\frac{dv}{dt} = f^r_{lorentz} - \frac{\sigma}{r} \tag{4}$$

where, v, t, σ, r and ρ represent the radial velocity, time, tensile stress, instantaneous radius and density of the ring material, respectively. $f^r_{lorentz}$ is the radial Lorentz force density. In this case, the axial Lorentz force is ignored. Thus, it gives $f^r_{lorentz} = f_{lorentz}$.

The magnetic field experienced by the ring comes from two components including the magnetic field from the coil (denoted by $\vec{B}_c$) and the self-magnetic field of the ring itself. (Davidson and Lawson, 1972). Thus, there are two contributions to Lorentz forces within the ring: the Lorentz force from coil denoted f_c and the self-magnetic force denoted f_{self} (Hugel, 1980). Besides, in the semi-analytical method, only the radial force component is considered ($f_{lorentz} = f^r_{lorentz}$), thus only the longitudinal magnetic field strength from the coil ($B_c^{\,z}$) and angular eddy current components are considered in the calculation. The Lorentz force can be written as:

$$f_{lorentz} = f_c + f_{self} = B_c^{\,z} \times j_{eddy} + \frac{1}{2V} I_{eddy}{}^2 \frac{dL}{dr} \tag{5}$$

where, *L*, *V*, *r* are the self-inductance, volume and instantaneous effective radius of the ring, respectively. I_{eddy} and j_{eddy} are the induced eddy current and eddy current density, respectively. For the ideal model, the ring thickness can be ignored as it is small compared to the ring radius so that the mean radius is the effective radius. In the semi-analytical method, the coil can be simplified into circle loops and each loop having the same current flow. Each circle loop can be further simplified with an effective radius *r*. The center of all the circle loops is considered to be located at the center point of the coil, and it is also set as the origin of the coordinate system.

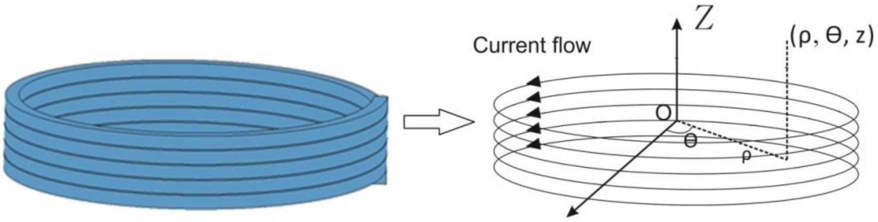

***Figure 3**: Simplification of a coil and the coordinate system into circle loops.*

In the case of multi-turn coil (n turns), at the point ($\rho,\theta,0$) the total longitudinal magnetic field generated by the coil is given by the summation of all those individual longitudinal magnetic fields generated by each circle loop. More detail about the magnetic field from electric current loop can be found in literature (Strom, 1928). Thus, the eddy current can be calculated from the input current, according to Faraday's law of induction;

$$\frac{d\emptyset}{dt} = -RI_{eddy} \qquad with \qquad \emptyset = MI_C + LI_{eddy} \tag{6}$$

where, *R* and $\emptyset$ are respectively the resistance of the ring and magnetic flux in the ring. *L* is the self inductance which is also used in Equation 5. *M* is the mutual inductance between the ring and coil. Using time discretization, the eddy current can be calculated numerically.

5.2 Fully coupled model

The temperature increases during the process due to Joule heating effect and plastic dissipation. The heat equation that derives from the first law of thermodynamics is then:

$$\rho C_m \frac{dT}{dt} = \bar{\sigma} \cdot \dot{\bar{\varepsilon}}_p + \frac{{j_{eddy}}^2}{\gamma} \tag{7}$$

where, C_m is the material specific heat capacity, ρ is the material density and γ is the electrical conductivity.

6 Results and discussion

The predicted velocity from the semi-analytical model is compared with the predicted velocity from LS-DYNA in Fig. 4. To check the influence of the error on the eddy current calculation, the predicted velocity from the semi-analytical model using the eddy current output from LS-DYNA is also included.

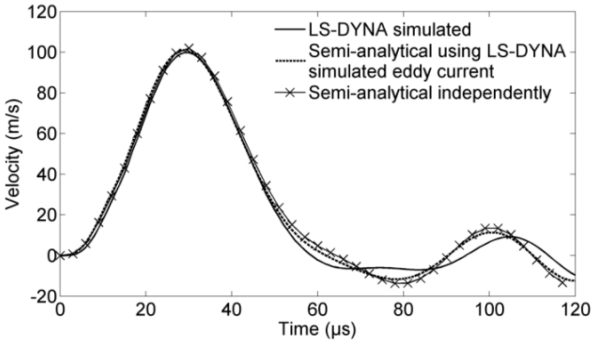

***Figure 4:** Comparison of expansion velocities from LS-DYNA simulation, semi-analytical calculation using simulated eddy current and fully semi-analytical calculation.*

Both calculated velocities from semi-analytical model (with and without LS-DYNA eddy current) show good agreement when compared to the LS-DYNA FE results. Relatively, small discrepancy between LS-DYNA results and those from semi-analytical calculations is observed at about 40μs. This discrepancy is due to the sensitivity of the velocity that mainly depends on the precision of the Lorentz force calculation. The Lorentz force is highly influenced by small changes in both displacement and velocity of the ring at the onset of the vibration stage. Because of this reason, the imperceptible error in Lorentz force calculation eventually leads to a noticeable error in velocity. However, the maximum error does not have a significant influence since the vibration stage is not as important as the pure expansion stage to determine the final dimension. More details about the vibration phenomenon can be found in our previous work [13].

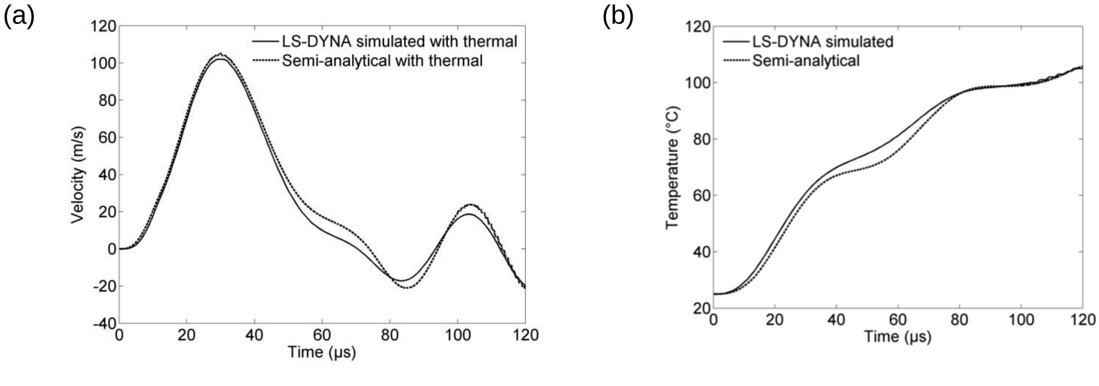

***Figure 5:** Comparison of result from LS-DYNA simulation and semi-analytical calculation (a) velocity (b) temperature.*

The time dependent temperature can be computed from the Mechanical, Electromagnetic and Thermal coupled calculations (MET). Computed average temperature of this calculation (average temperature of the whole ring) is compared with temperature given by the semi-analytical method (Fig. 5b). The analytical model is in good agreement with the MET computation using LS-Dyna, in terms of temperature and velocity.

7 Conclusions

A semi-analytical method was developed to predict the velocity of the ring against time during an electromagnetic ring expansion test and the method has been validated using a coupled finite element model. The method includes the changes in electrical resistance, mutual inductance, self-inductance during the expansion of the ring. And this method only requires input current to perform a calculation. The procedure involves four major physical quantities in various parts of the calculation which are mechanical quantities, Lorentz force, eddy current and thermal quantities. Each individual part is separately validated before making the independent validation of the semi-analytical method. The results indicate that the semi-analytical method provides an accurate fast prediction with a very few computational resources whereas coupled multiphysics finite element methods generally require long time and heavy computational resources.

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References

Altynova, M., Hu, X., Daehn, G.S., 1996. Increased ductility in high velocity electromagnetic ring expansion. Metall and Mat Trans A 27, 1837-1844.

Davidson, R., Lawson, J., 1972. Self-consistent Vlasov Description Of Relativistic Electron ring equilibria. Part. Accel. 4, 1-18.

Gourdin, W.H., 1989. Analysis and assessment of electromagnetic ring expansion as a high-strain-rate test. Journal of Applied Physics 65, 411-422.

Grady, D., Benson, D., 1983. Fragmentation of metal rings by electromagnetic loading. Experimental Mechanics 23, 393-400.

Gupta, S., 2013. The Constitutive Behavior of Copper at High Strain Rate as Determined by the Free Expansion Ring Test. The Ohio State University.

Hahn, M., Weddeling, C., Lueg-Althoff, J., Tekkaya, A.E., 2016. Analytical approach for magnetic pulse welding of sheet connections. Journal of Materials Processing Technology 230, 131-142.

Hugel, H., 1980. Effect of self-magnetic forces on the anode mechanism of a high current discharge. IEEE Transactions on Plasma Science 8, 437-442.

Jeanson, A.C., Avrillaud, G., Mazars, G., Bay, F., Massoni, E., Jacques, N., Arrigoni, M., 2013. Identification du comportement mécanique dynamique de tubes d'aluminium par un essai d'expansion électromagnétique, CSMA 2013-11ème colloque national en calcul des structures.

Jeanson, A.C., Avrillaud, G., Mazars, G., TABER, G., Daehn, G.S., Bay, F., Jacques, N., Arrigoni, M., 2014. Identification of material constitutive parameters for dynamic applications: Magnetic Pulse Forming and electrohydraulic forming, 6th International Conference on High Speed Forming. Institut für Umformtechnik - Technische Universität Dortmund, pp. 31-36.

Jeanson, A.C., Bay, F., Jacques, N., Avrillaud, G., Arrigoni, M., Mazars, G., 2016. A coupled experimental/numerical approach for the characterization of material behaviour at high strain-rate using electromagnetic tube expansion testing. International Journal of Impact Engineering 98, 75-87.

Johnson, J., Taber, G., Daehn, G., 2010. Constitutive relation development through the FIRE test, 4th International Conference on High Speed Forming, pp. 295-306.

LS-DYNA Keyword User's Manual R8.0, 2015. Livermore Software Technology Corporation (LSTC), Livermore, CA, USA.

Niordson, F.I., 1965. A unit for testing materials at high strain rates. Experimental Mechanics 5, 29-32.

Paese, E., Geier, M., Homrich, R., Rossi, R., 2015. A coupled electric–magnetic numerical procedure for determining the electromagnetic force from the interaction of thin metal sheets and spiral coils in the electromagnetic forming process. Applied Mathematical Modelling 39, 309-321.

Paese, E., Rosa, P.A., Geier, M., Homrich, R.P., Rossi, R., 2014. An Analysis of Electromagnetic Sheet Metal Forming Process, Applied Mechanics and Materials. Trans Tech Publ, pp. 9-14.

Pandolfi, A., Krysl, P., Ortiz, M., 1999. Finite element simulation of ring expansion and fragmentation: the capturing of length and time scales through cohesive models of fracture. International Journal of Fracture 95, 279-297.

Rodríguez-Martínez, J.A., Vadillo, G., Fernández-Sáez, J., Molinari, A., 2013. Identification of the critical wavelength responsible for the fragmentation of ductile rings expanding at very high strain rates. Journal of the Mechanics and Physics of Solids 61, 1357-1376.

Rusinek, A., Zaera, R., 2007. Finite element simulation of steel ring fragmentation under radial expansion. International Journal of Impact Engineering 34, 799-822.

Strom, A.P., 1928. The magnetic field intensity at any point in the plane of a circular loop. Journal of the Franklin Institute 206, 339-344.

Tamhane, A.A., Altynova, M.M., Daehn, G.S., 1996. Effect of sample size on ductility in electromagnetic ring expansion. Scripta materialia 34, 1345-1350.