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Combining uncertainty with uncertainty to get certainty? Efficiency analysis for regulation purposes

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Abstract

Data envelopment analysis (DEA) and stochastic frontier analysis (SFA), as well as combinations thereof, are widely applied in incentive regulation practice, where the assessment of efficiency plays a major role in regulation design and benchmarking. Using a Monte Carlo simulation experiment, this paper compares the performance of six alternative methods commonly applied by regulators. Our results demonstrate that combination approaches, such as taking the maximum or the mean over DEA and SFA efficiency scores, have certain practical merits and might offer an useful alternative to strict reliance on a singular method. In particular, the results highlight that taking the maximum not only minimizes the risk of underestimation, but can also improve the precision of efficiency estimation. Based on our results, we give recommendations for the estimation of individual efficiencies for regulation purposes and beyond.

JEL codes: C10, C50, D24, L50.

Keywords: Data Envelopment Analysis; Stochastic Frontier Analysis; Efficiency Analysis; Regulation; Network operators

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1 Introduction

Over the past three decades, many countries have introduced regulatory reforms and incentive regulation in network sectors like electricity, gas, telecommunications, and water. Although concrete regulation designs may vary fundamentally, in many incentive-based regulation schemes, efficiency estimation methods play a major role (see, for example, Bogetoft & Otto 2010, Coelli & Lawrence 2006, Haney & Pollitt 2009). These methods are used to benchmark regulated firms and account for firm-specific efficiency estimates in the regulatory design. If the current efficiency level of a firm was not considered in the regulation process, incentive-based regulation schemes would favor firms that are less efficient at the beginning of the regulation period. After estimating the efficiency of the firms, each regulated firm obtains an individual efficiency improvement target, the so-called X-factor, with the aim of decreasing its inefficiency through the end of the regulation period (e.g. Nykamp, Andor & Hurink 2012). Usually, the firm-specific X-factors enable firms to earn a fair rate of return on capital if they achieve the efficient cost level that is defined by the regulatory authority (Coelli, Estache, Perelman & Trujillo 2003, p. 8).

Thus, estimated efficiency scores have a substantial financial impact on regulated firms and the choice of the estimation method is of major relevance as it often heavily influences the estimated level of (in)efficiency. Nevertheless, both the theoretical literature as well as regulation practice have not yet found the "best" strategy for determining individual efficiency targets. Consequently, regulators have used a broad array of concepts to determine the individual X-factor. In the energy sector for example, many regulating authorities estimate firm level efficiency by adopting either (I) data envelopment analysis (DEA, Charnes, Cooper & Rhodes 1978), which is quite flexible and whose frontier is only restricted via its axiomatic foundation[1], but estimates efficiency without considering statistical

[1]The axioms are convexity, inefficiency ("free disposability"), ray unboundedness, and minimum extrapolation (Banker, Charnes & Cooper 1984, Fried, Lovell & Schmidt 2008).

noise[2], (II) parametric stochastic frontier analysis (SFA, Aigner, Lovell & Schmidt 1977, Meeusen & van Den Broeck 1977), which takes statistical noise into account, but typically requires assumptions concerning the functional form of the frontier as well as the distribution of inefficiency, or (III) a combination of the estimates of these two (see also Banker, Førsund & Zhang 2017).

Aside from the direct application of individual estimation methods, for example DEA being deployed to benchmark in Norway, regulation authorities have recently begun to apply combination approaches; see the studies listed in Table 1 that have taken this route for just the electricity regulation arena. One reason that these combination approaches may be appealing to regulators is that the uncertainty underlying various assumptions with the use of either DEA (assumptions on the presence of noise) or SFA (functional form/distributional assumptions) are potentially mitigated when estimates from the two approaches are combined. Moreover, it may be that the precision of an individual efficiency estimate stemming from either DEA or SFA is improved, using some combination of methods. Overall, it is possible that regulators adopt combination approaches to reduce various aspects of uncertainty that are likely to impact their insights and the firms under regulation.

Table 1: Overview of Electricity Regulation Approaches to Determine Efficiency Objectives

Method	Country	Source
DEA	Belgium	Agrell & Bogetoft (2013)
	Brazil	Haney & Pollitt (2013)
	Norway	Miguéis, Camanho, Bjørndal & Bjørndal (2012)
DEA + SFA	Austria	Schweinsberg, Stronzik & Wissner (2011)
	Finland (2008-2011)	Kuosmanen (2012)
	Germany	Schweinsberg et al. (2011)

To be more specific, for example, both the Austrian and Finnish regulators determined efficiency improvement targets using the arithmetic mean of the firm-

[2]Several extensions to the orginal DEA model have been proposed in the literature to account for statistical noise, such as stochastic DEA (Simar & Zelenyuk 2011) or extended versions of the Benefit-of-the-Doubt-method (see, for an introduction, Cherchye, Moesen, Rogge & Van Puyenbroeck 2007), for example the robust order-m model (Cazals, Florens & Simar 2002).

specific DEA and SFA efficiency estimates in their respective regulation periods. A different approach is taken in Germany, where the regulator uses the highest firm-specific efficiency estimates from DEA and SFA to avoid (to some degree) underestimating the efficiency of regulated firms (Andor 2009). Although this approach is primarily applied to minimize the risk of underestimation, it might also improve the accuracy of efficiency estimation as existing evaluations of DEA and SFA estimators have come to the conclusion that both methods underestimate firm specific inefficiency, except for the most efficient firms (Badunenko, Henderson & Kumbhakar 2012, Andor & Hesse 2014).

Combination approaches appear to be viewed as best practice for benchmark regulation. For instance, Jamasb & Pollitt (2003, p. 1621) state that "…a practical approach in the absence of consensus on the most appropriate technique, model specification, and variables is to combine the results from different models." Furthermore, Nillesen & Pollitt (2007, p. 284) conclude in their analysis of the 2001-2003 electricity distribution price control review in the Netherlands that a "…more balanced approach whereby the benchmarking results were based on several techniques would have potentially made the results more acceptable." Lastly, Haney & Pollitt (2009) construct a best practice index for determining individual X-factors, whereby the use of more than one benchmarking technique is seen as best practice. Combination approaches may be used to parry the thrusts of firms criticizing one efficiency method over another, potentially insulating the regulator from aggressive litigation due to regulatory action.

Aside from the regulatory literature advocating on behalf of combination approaches, some scholars in the efficiency analysis milieu also have posited that combination approaches might be useful. For example, Banker, Cooper, Grifell-Tajté, Pastor, Wilson, Ley & Lovell (1994, p. 261) state that the "…use of more than one methodology can help to avoid the possible occurrence of 'methodological bias'." This is echoed by Sickles (2005, p. 331) who states that "[o]ne way out of this dilemma is to combine the various stochastic frontier and DEA models of efficiency in a portmanteau estimator that would weight the various efficiency measures ...". In addition, Coelli & Perelman (1999, p. 335) use the geometric mean of three efficiency estimation methods estimates as final results for the purpose of discussing the relative performance of European railways: "This idea is borrowed from the time-series forecasting literature where many authors contend that the average of the predictions from a number of models will often outperform any one particular predictive model."

However, to date, combination approaches might be best characterized as ad-hoc and without a proper theoretical foundation to rest on. As an example, combination estimators, which average, do so in a naïve fashion, treating all estimates equally. Furthermore, to the authors' knowledge, there currently does not exist theoretical or empirical evidence that combining various efficiency estimation methods improves the accuracy of efficiency estimates. A notable exception is Kuosmanen, Saastamoinen & Sipiläinen (2013) who explore the potential benefits of an average of SFA and DEA efficiency estimates in a brief Monte Carlo setting.[3] Moreover, given that combination approaches can be a reasonable alternative to estimate the efficiency of firms (as they moderate potential model misspecification), the question arises how the efficiency estimates of several techniques should be transformed into a single efficiency objective for the individual regulated firm, i.e. if combination approaches are useful, which approach should be chosen.

The aim of this paper is to compare two alternative combination approaches that have appeared in the regulation literature (the arithmetic mean and maximum), to standard individual estimates stemming from DEA and SFA using Monte Carlo (MC) experiments. MC studies are widely recognized as a "statistical referee" (Perelman & Santín 2009, p. 303), used to verify the potential strengths and weaknesses of competing efficiency estimation methods and have been widely deployed for comparing the performance of different efficiency esti-

[3] Another exception is the working paper of Andor & Hesse (2011) whose ideas we develop and extend here.

mators.[4] The MC study performed here will enable us to identify factors that determine the relative performance of the six approaches (SFA-CD, SFA-TL, DEA-CRS, DEA-VRS, AM, MAX) under scrutiny in a wide range of scenarios, varying the sample size, the amount of statistical noise, the amount and distribution of inefficiency, and the shape of the cost frontier itself.

Our results show that combination approaches can be a valuable alternative to the application of a single estimation method. We find that taking the maximum of DEA and SFA estimates exhibits the best precision in estimating individual efficiency scores on average across our simulations. Moreover, this result seems relatively robust to an array of assumptions regarding the data generating process, for example whether we consider scenarios with or without noise. In contrast, taking the mean of DEA and SFA estimates does not perform particularly well. As taking the maximum not only minimizes the risk of underestimation of regulated firms, but also maximizes the accuracy of estimating the level of inefficiency, we recommend using the maximum as a combination approach for regulation purposes.

The remainder of this paper is organized as follows. In Section 2, we describe the methods used in our simulations, while in Section 3 the data, which underlies our simulations, is detailed. Section 4 describes the simulation design of the MC experiment and Section 5 presents and discusses the results. Finally, Section 6 summarizes the most important findings, gives recommendations for the estimation of efficiencies, and provides some directions for further research.

[4]Without aiming to present a complete list, we are aware of the following studies that use MC simulations to evaluate efficiency estimators: Adler & Yazhemsky 2010, Andor & Hesse 2014, Andor & Parmeter 2017, Badunenko et al. 2012, Banker, Gadh & Gorr 1993, Banker, Cooper, Seiford, Thrall & Zhu 2004, Cook & Seiford 2009, Cordero, Pedraja & Santín 2009, Cordero, Polo & Santín 2018, Nieswand & Seifert 2018, Olson, Schmidt & Waldman 1980, Ondrich & Ruggiero 2001, Perelman & Santín 2009, Ruggiero 1999, Resti 2000, Yu 1998.

2 Methodology

Since the groundbreaking work of Farrell (1957), several methods for measuring efficiency have been developed. Despite an array of research on competing efficiency analysis approaches, there is no single superior method, which is why regulators face the problem of choosing between several estimation methods. In general, the efficiency analysis literature can be divided into parametric and nonparametric methods, which in addition can be categorized into models that account for statistical noise and those that ignore it; see Simar & Wilson (2013) and Parmeter & Kumbhakar (2014) for more thorough reviews, which cover the theoretical underpinnings of these approaches.

For our analysis, we employ DEA and SFA, which are without doubt the most widely used nonparametric and parametric approaches, respectively. DEA is a linear programming technique that is generally attributed to Charnes et al. (1978), although previous papers also proposed mathematical programming methods (see, for example Afriat 1972). The main disadvantage of DEA is that – in its most commonly implemented form – it ignores the presence of statistical noise. This is widely viewed as a detriment as no ability to account for measurement errors or omitted variables exists. In contrast, SFA, simultaneously developed by Aigner et al. (1977) and Meeusen & van Den Broeck (1977), is able to measure efficiency, while considering the presence of statistical noise. Yet, the main disadvantage of SFA is that assumptions concerning the functional form of the frontier and the distributions of the two error terms are required. Since DEA and SFA are widely applied in the benchmarking literature, we only briefly review them. For a detailed empirical illustration and discussion of the methods, we refer to Andor & Hesse (2014) and Andor & Parmeter (2017).

In our MC study, we use the standard CCR (Charnes, Cooper & Rhodes 1978) and BBC (Banker, Charnes & Cooper 1984) models that allow for constant returns to scale (CRS) and variable returns to scale (VRS), respectively, and apply them to estimate a cost frontier. Accordingly, for the production of s outputs $y_{rj}(r = 1, \ldots, s)$ each firm $j = 1, \ldots, n$ exhibits costs x_j. In order to determine the individual efficiency of firm j, the following input-oriented two-stage model (cf. Banker et al. 2004) must be minimized

$$\begin{aligned} \text{minimize}_{\theta,\lambda} \quad & \theta \\ \text{subject to} \quad & \theta x \geq \sum_j \lambda_j x_j, \\ & y_r \leq \sum_j \lambda_j y_{rj} \;\; \forall\, r = 1, ..., s, \\ & \sum_{j=1} \lambda_j = 1, \\ & \lambda_j \geq 0 \end{aligned} \tag{1}$$

where λ_j are weights and θ denotes the estimated technical efficiency $\widehat{TE}_j = \theta_j$ with $0 \leq \widehat{TE} \leq 1$. A value of one indicates a point on the efficient frontier and thus a fully efficient firm, according to Farrell (1957). The difference between the CCR and BBC model is the assumption on $\sum_{j=1} \lambda_j$. When we set the additional convexity constraint on the λ_j (Cook & Seiford 2009), i.e. $\sum_{j=1} \lambda_j = 1$, we assume variable returns to scale and apply the BBC model. In order to obtain efficiency values for all n firms, the linear programming model must be solved for each firm, i.e. n times.[5]

For parametric SFA, the model of costs across n firms is

$$x_j = m(\boldsymbol{y}_j; \beta) + u_j + v_j = m(\boldsymbol{y}_j; \beta) + \varepsilon_j,\ \varepsilon_j = u_j + v_j, \tag{2}$$

where x_j represents total cost of firm j, $m(\mathbf{y}_j; \beta)$ is the parametric cost frontier, and u_j and v_j capture inefficiency and stochastic noise, respectively. We assume that $u_j \sim N_+(0, \sigma_u^2)$ and $v_j \sim N(0, \sigma_v^2)$. In this case, maximum likelihood (ML) es-

[5] As noted by one referee, by construction, outputs' cost shadow shares cannot be negative, but they could take extreme values: 0% or 100%. In other words, without imposing restrictions on output shares weights, it is possible that some outputs will not be considered as cost drivers at all.

timation proceeds by optimizing $\mathcal{L} = \prod_{j=1}^{n} f(\varepsilon_j)$, where $\varepsilon_j = x_j - m(\boldsymbol{y}_j; \boldsymbol{\beta})$, yielding

$$\ln \mathcal{L}(\beta, \lambda, \sigma) = -n \ln \sigma + \sum_{j=1}^{n} \ln \Phi(-\varepsilon_j \lambda/\sigma) - \frac{1}{2\sigma^2} \sum_{j=1}^{n} \varepsilon_j^2, \tag{3}$$

with $\Phi(\cdot)$ representing the cumulative distribution function of a standard normal random variable and the parameterization $\sigma = \sigma_u + \sigma_v$ and $\lambda = \frac{\sigma_u}{\sigma_v}$ has been used. An estimator for expected firm level inefficiency can be obtained through the conditional expectation of u given ε following the approach of Jondrow, Lovell, Materov & Schmidt (1982). We apply the Battese & Coelli (1988) point estimator of technical efficiency

$$\widehat{TE}_j = \hat{E}(e^{-u_j}|\hat{\varepsilon}_j) = \frac{\Phi(\hat{\mu}_{*j}/\hat{\sigma}_* - \hat{\sigma}_*)}{\Phi(\hat{\mu}_{*j}/\hat{\sigma}_*)} \cdot e^{\left(\frac{1}{2}\hat{\sigma}_*^2 - \hat{\mu}_{*j}\right)}, \tag{4}$$

where $\hat{\mu}_* = -\hat{\varepsilon}\hat{\sigma}_u^2/\hat{\sigma}^2$ and $\hat{\sigma}_*^2 = \hat{\sigma}_u^2\hat{\sigma}_v^2/\hat{\sigma}^2$.

As parametric expressions of our cost function $m(\boldsymbol{y}_j; \beta)$, we analyze two different functional forms. First, we use the following log-linear Cobb-Douglas (CD) specification:

$$lnx_j = \beta_0 + \sum_{r=1}^{s} \beta_r \ln y_{jr}, \tag{5}$$

where x_j denotes the cost and y_{jr} the rth output of firm j and β are the coefficients to be estimated. In addition, we estimate the following Translog model:

$$lnx_j = \gamma_0 + \sum_{r=1}^{s} \beta_r \ln y_{jr} + \frac{1}{2} \sum_{r=1}^{s} \beta_r r(\ln y_{jr})^2 + \sum_{k=1}^{s} \sum_{l=1}^{s} \ln y_{jk} \ln y_{jl}, \tag{6}$$

which is more flexible than CD, but might be problematic in small sample sizes due to the larger amount of parameters that need to be estimated.

Owing to the lack of evidence on the superiority of either benchmarking method, regulation practitioners often desire to combine the virtues of DEA and SFA to reduce the uncertainty connected with their estimates of TE. For instance, the

German regulator of distribution system operators determines the individual TE score by the maximum of the DEA and SFA estimates (MAX), while the Finnish regulator used the arithmetic mean (AM) during the incentive regulation regime from 2008 to 2011 (Kuosmanen & Kortelainen 2012). Moreover, regulators have to make further assumptions: for SFA an essential decision is the a priori specification of the functional form of the cost frontier, whereas for DEA the linear program must be specified as CRS or VRS and the regulator has to be willing to assume the lack of statistical noise.[6]

In our analysis, we therefore compute TE based on two methods for each estimator. For SFA, we assume either a Cobb-Douglas (CD) or a Translog (TL) cost function, while for DEA we assume either CRS or VRS. Furthermore, we apply the two combination approaches, MAX and AM. Accordingly, the individual TE levels are based on:

$$AM_j = \frac{1}{4}\left(\widehat{TE}_{j,DEA-CRS} + \widehat{TE}_{j,DEA-VRS} + \widehat{TE}_{j,SFA-CD} + \widehat{TE}_{j,SFA-TL}\right) \quad (7)$$

$$MAX_j = max\left\{\widehat{TE}_{j,DEA-CRS}, \widehat{TE}_{j,DEA-VRS}, \widehat{TE}_{j,SFA-CD}, \widehat{TE}_{j,SFA-TL}\right\}, \quad (8)$$

where $\widehat{TE}_{j,DEA}$ and $\widehat{TE}_{j,SFA}$ denote the estimated technical efficiency of DEA and SFA for firm j, respectively. Our explicit goal here is to compare the results of these six approaches (SFA-CD, SFA-TL, DEA-CRS, DEA-VRS, MAX, AM) in an MC experiment.

3 Data

For the purpose of illustration, we use a data set from the Finnish distribution system operators (DSO). Finland provides an interesting case for an efficiency analysis of the DSOs since its Energy Market Authority (Energiamarkkinavirasto,

[6]These are just a few of the modeling decisions that need to be made. The researcher also has to decide which variables to include as inputs and outputs, whether, and if so, which contextual variables should be accounted for.

EMV) was among the first to employ benchmark regulation (Kuosmanen et al. 2013). For this reason, the Finnish DSOs have been extensively scrutinized in the literature (see for instance, Edvardsen & Førsund 2003, Kopsakangas-Savolainen & Svento 2008, Korhonen & Syrjänen 2003, Kuosmanen 2012, and Kuosmanen et al. 2013).

The data set comprises 89 DSOs that cover all regions of Finland and provides information on total cost and three output variables. Total cost denotes the sum of controllable operational cost, capital expenditures, and interruption cost (Table 2). The output variables are the amount of energy transmitted, the total length of the network, and the total number of customers. The amount of energy transmitted is weighted according to the average cost of transmission.[7] Thus, high-voltage transmission is weighted lower than low-voltage transmission (Kuosmanen 2012). All variables are defined as yearly averages over the period 2005-2008.

Table 2: Descriptive Statistics of Finnish Electricity Distribution Companies

Variable	Mean	Std. Dev.	Minimum	Median	Maximum
Entire Data Set					
Total cost in EUR 1000	8,418.9	18,047.8	267.8	3,102.2	117,554.1
Energy transmission in GWh	480.4	971.5	14.8	171.3	6,599.7
Length of network in km	4,135.3	10,223.3	50.8	988.6	67,611.1
No. of customers	35,448.7	71,870.7	24.3	11,081.3	420,473.0
Final Data Set Used for the Simulation					
Total cost in EUR 1000	12,750.5	23,243.2	1,588.8	5,458.5	117,554.1
Energy transmission in GWh	683.4	1,180.0	94.9	305.3	6,599.7
Length of network in km	6,881.6	13,673.9	340.1	2,656.3	67,611.1
No. of customers	50,107.3	82,521.9	5,146.3	21,987.4	420,473.0

Source: Finnish Energy Market Authority (Energiamarkkinavirasto, EMV), www.emvi.fi

Due to its vast geographic variation, Finland's DSOs exhibit a large degree of heterogeneity. For instance, the shortest distribution grid is only 51 km long,

[7] As two referees pointed out, this weighting could cause an endogeneity bias. However, our definition stems directly from the definitions provided by the Finnish regulator for this dataset and while there may exist an endogeneity bias in the original data, given that we are generating our data with a prespecified data generating process, there is no concern that this bias is transferred in the Monte Carlo simulations. Yet, we agree that it is questionable why the regulator does not consider the physical measure without a weigthing.

whereas the longest grid measures more than 67,000 km and the number of customers ranges from 24 to 420,000. Additionally, the total cost averages about EUR 8.5 million and varies between EUR 0.268 and 118 million. For further information on the Finnish regulation and the data set, we refer the reader to Kuosmanen et al. (2013).

To maintain fidelity with axioms of production, prior to engaging in the simulations we first estimated a generic Translog cost function and checked for monotonicity. Accordingly, we restrict the data set to those firms that pass this basic check (lower panel of Table 2). On average, the 46 firms in our final data set exhibit higher costs, reflecting the fact that they are larger with respect to all three outputs. Removing these smaller firms from our initial setup reduces the likelihood of poor draws in the Monte Carlo analysis, where an extreme observation may have undue influence on all of the estimates.

4 Simulation Design

Owing to the fact that the performance evaluation of efficiency estimation methods is not possible using empirical data as the “true” level of efficiency is not known, researchers use MC simulations to evaluate competing methods (see for instance Andor & Hesse 2014, Andor & Parmeter 2017, and Badunenko et al. 2012). MC studies enable researchers to generate their own data set and compare the “true” efficiency with the estimated efficiency values. The problem is that the results depend on the underlying data generating process (DGP). Therefore, it is important to vary the assumptions in the DGP to derive a reasonable set of scenarios. By analyzing different scenarios, the influence of specific factors can be measured, for instance varying the number of firms or the relation of inefficiency to noise. Furthermore, it is useful to replicate the DGP with the same assumptions several times in order to obtain reliable results.

In our study, we analyze 16 different cases that combine the underlying true cost function, the distribution of the inefficiency term u, and the amount of noise in the data σ_v. Each of these cases include several scenarios where we vary the sample size n and the amount of inefficiency σ_u (Table 3). To analyze the scenarios, we first estimate the true cost frontier based on the actual data and set these estimates as the true parameter values. We analyze four different true cost functions, whereby the first two provide an advantage for the parametric SFA estimation methods and the last two an advantage for the nonparametric DEA approaches. The first cost function (Cases 1-4, see Table 3) is a Cobb-Douglas function equivalent to Equation (5) and the estimated parameters for β_r are equal to 0.39, 0.36, and 0.23, resulting in returns of scale of 0.98. In the Cases 5-8, we estimate a Translog function (Equation 6) and find that the estimated returns to scale are on average a bit lower compared to the Cobb-Douglas case, 0.95. To provide an advantage for the DEA estimator, in the Cases 9-12, we estimate the true cost frontier based on Translog, but then monotonize and convexify the estimates by performing DEA-VRS on the fitted values. This approach is consistent with the stochastic DEA advocated in Simar & Zelenyuk (2011) and allows us to generate costs that are consistent with the axioms of producer theory while also not favoring any particular parametric model. In this setting, the nonparametric cost frontier that is constructed is most favorable to application of the DEA estimator. Lastly, in the Cases 13-16, we determine the cost frontier by just applying DEA-VRS on the original data set and using the frontier estimates to generate the data. This procedure also ensures that the assumptions of monotonicity and concavity hold on the generated frontier.

In a second step, we take smooth samples from the three outputs and add a random component to the outputs based on a standard normal distribution, following the approach of Andor & Parmeter (2017). We vary the sample size n from $\{50, 100, 200, 400, 800\}$ and the variation of both inefficiency σ_u and noise σ_v from $\{0.01, 0.05\}$, following Badunenko et al. (2012). This results in three different signal-to-noise ratios $\lambda = \frac{\sigma_u}{\sigma_v} \in \{0.2, 1, 5\}$ that are equal to, greater than, and

Table 3: Overview of Cases and Scenarios

Case	σ_v	Distribution of u	True cost function	σ_u	Sample size n
1 "SFA-CD"	$0.01, 0.05$	$N_+(0, \sigma_u^2)$	CD	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
2	0	$N_+(0, \sigma_u^2)$	CD	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
3	$0.01, 0.05$	$\Gamma(1, \sigma_u)$	CD	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
4	0	$\Gamma(1, \sigma_u)$	CD	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
5	$0.01, 0.05$	$N_+(0, \sigma_u^2)$	TL	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
6	0	$N_+(0, \sigma_u^2)$	TL	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
7 "SFA-TL"	$0.01, 0.05$	$\Gamma(1, \sigma_u)$	TL	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
8	0	$\Gamma(1, \sigma_u)$	TL	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
9	$0.01, 0.05$	$N_+(0, \sigma_u^2)$	TL + DEA	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
10	0	$N_+(0, \sigma_u^2)$	TL + DEA	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
11 "TL-DEA"	$0.01, 0.05$	$\Gamma(1, \sigma_u)$	TL + DEA	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
12	0	$\Gamma(1, \sigma_u)$	TL + DEA	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
13	$0.01, 0.05$	$N_+(0, \sigma_u^2)$	DEA	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
14	0	$N_+(0, \sigma_u^2$	DEA	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
15	$0.01, 0.05$	$\Gamma(1, \sigma_u)$	DEA	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$
16 "DEA"	0	$\Gamma(1, \sigma_u)$	DEA	$0.01, 0.05, 0.15$	$50, 100, 200, 400, 800$

less than 1. In addition, we consider $\sigma_u = 0.15$ to analyze scenarios with heavy inefficiency, resulting in $\lambda \in \{3, 15\}$. We study this wide range of signal-to-noise ratios as several studies verify that it has a substantial bearing on the performance of efficiency estimation methods (Olson et al. 1980, Ondrich & Ruggiero 2001, Ruggiero 1999). In total, we consider 16 cases, 360 different scenarios, and each scenario is replicated $R = 10,000$ times. We thus consider 3.6 million simulated data sets.

In the first case (1 "SFA-CD"), we generate the data by assuming a CD function as the true underlying cost function, a half normally distributed inefficiency term, and noise. As SFA – in contrast to DEA – generally considers noise and we estimate the SFA assuming a CD as well as a half normal distributed inefficiency term, the assumptions in the DGP are aligned with SFA-CD and we expect the SFA-CD estimator to perform particularly well in this case. In all other cases, we introduce various types of misspecifications designed to expose various modeling assumptions within the stochastic frontier model. The case with the largest contrast is case (16 "DEA") where we consider no noise, a gamma distributed inefficiency term, and a cost frontier that is determined by applying DEA to the underlying data set. Accordingly, these misspecifications should impact negatively the standard maximum likelihood estimator of the stochastic frontier model and therefore we expect that the DEA estimators should perform particularly well by comparison.

Given that regulators aim to determine individual X-factors for each DSO, we focus on measuring individual technical efficiency ($\widehat{TE}_j$) and evaluate the performance of the methods based on this measure. For the comparison of the performance of our estimators, our main criterion is the root mean square error (RMSE) between the estimated and true value of technical efficiency:

$$RMSE_r = \left(\frac{1}{n} \sum_{j=1}^{n} (\widehat{TE}_{j,r} - TE_{j,r})^2 \right)^{\frac{1}{2}},$$

where $TE_{j,r}$ is the true technical efficiency value for the jth firm in the rth simulation and $\widehat{TE}_{j,r}$ the estimated value. Given that each simulation provides a RMSE, to easily distill this information, we present the median RMSE, defined as $\underset{r=1,\ldots,R}{\text{median}}\, RMSE_r$, across the $R = 10,000$ simulations. Using the median RMSE avoids influence from a small percentage of poor simulations for any given scenario, which could happen if we were to use the mean RMSE instead.

In order to gain additional insights on the performance of the methods, we calculate the percentage of underestimated firms (PU) because it is a measure that is easy to interpret and of major relevance from a regulator's perspective.[8] Regulators usually prefer to overestimate the performance of regulated firms. As this might be seen as suboptimal at first glance, the reason is that regulated firms are natural monopolists. If the efficiency of such a firm is underestimated, the consequence is a cost saving requirement, which is too strict. In the long run this

[8]This measure was originally proposed by Andor & Hesse (2011) and was applied, for instance, by Henningsen, Henningsen & Jensen (2015).

could lead to insolvency through no fault of the firm. The PU is given by

$$PU_r = \frac{1}{n}\sum_{j=1}^{n} D_j, D_j = \begin{cases} 1\ if\ \widehat{TE}_{j,r} < TE_{j,r} \\ 0\ otherwise. \end{cases}$$

For example, a PU value of 0.70 implies that the method in question leads to estimated efficiency scores that are lower than the actual efficiency scores for 70% of the firms.

Additionally, we calculate the bias for each of the R simulations:

$$bias_r = \frac{1}{n}\sum_{j=1}^{n} \widehat{TE}_{j,r} - TE_{j,r}.$$

A positive bias shows that the method overestimates on average, and vice versa. As for the RMSE, to compare the methods we calculate the median of both the PU and the bias over all simulations for each scenario (i.e. $\underset{r=1,\ldots,R}{\text{median}}\, PU_r$ and $\underset{r=1,\ldots,R}{\text{median}}\, bias_r$ across the $R = 10,000$ simulations).[9]

5 Results

Table 4 displays the performance of our six estimation methods to estimate TE in terms of the RMSE, bias, and PU across all 360 scenarios (first block I All), the scenarios without noise (second block II No Noise), and with noise (third block III With Noise). The method that performs best is a combination approach: the MAX method has the lowest average RMSE regardless of whether we consider

[9]While we consider the RMSE, the PU, and the bias in regard to the estimation of TE as performance criteria, one could use several other statistical metrics to provide insights into how well these different estimators work, for example the mean absolute deviation (MAD) or the rank correlation (see, for instance, Andor & Hesse 2014). Furthermore, one can aim to evaluate the ability to estimate shadow costs or returns to scale (cf. Andor & Parmeter 2017). We decided to consider the RMSE, the PU, and the bias in regard to the estimation of TE as performance criteria because we are convinced that they deliver the most important information for regulators, but suggest that different metrics could be used by researchers and practitioners depending upon their aims and needs from the analysis. The Monte Carlo experiments are conducted in R (version 3.3.0) and all code is available upon request.

Table 4: Overall Performance of the Six Methods

Scenarios	Performance Measure	Estimation Method					
		SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I All	RMSE	0.0795	0.0303	0.0821	0.0503	0.0519	0.0215
	Bias	−0.0531	−0.0064	−0.0553	−0.0057	−0.0337	0.0052
	PU	0.8500	0.7000	0.8350	0.5950	0.9300	0.3150
II No Noise	RMSE	0.0793	0.0195	0.0626	0.0203	0.0383	0.0142
	Bias	−0.0526	−0.0032	−0.0399	0.0032	−0.0276	0.0084
	PU	0.8400	0.6700	0.7012	0.2150	0.9250	0.0775
III With Noise	RMSE	0.0796	0.0348	0.0942	0.0586	0.0585	0.0267
	Bias	−0.0534	−0.0081	−0.0668	−0.0180	−0.0399	0.0025
	PU	0.8550	0.7200	0.8800	0.7200	0.9325	0.4650

Note: RMSE = Root Mean Square Error, PU = Percentage of Underestimated Firms.

all scenarios or scenarios with or without noise. The second best method is the SFA-TL with RMSEs between 2% and 3.5%, followed either by the DEA-VRS or the AM method. The methods with the highest RMSE are the least flexible ones, the SFA-CD and the DEA-CRS.

While it might surprise that the MAX method outperforms the other methods, the reason lies in the fact that all individual methods usually, on average, underestimate the efficiency of the firms as can be seen from the bias and the PU (a finding that is in line with the existing literature, for instance, Andor & Hesse 2014). For example, both SFA variants underestimate inefficiency for between 67% and 86% of the firms. The only exception is DEA-VRS, which underestimates only 21.5% of the firms in the scenarios without noise. In the following, we focus on four specific cases to have a closer look how these aggregate outcomes emerge, but present the results for all other cases in the appendix, enabling the interested reader to investigate the influence of specific types of misspecification on the relative performance of the competing methods.[10]

[10] As one referee was interested to learn about the precision to estimate the returns to scale (RTS) as well as the frequency of monotonicity violations for the SFA models, we have calculated the RMSE of the returns to scale and the percentage of monotonicity violations for the Cases 1, 7, 11 and 16. The interested reader can find the results in the online appendix, Tables A45 to A48 for the RTS and Tables A49 to A52 for the monotonicity violations. In all cases, a large percentage of our estimates satisfy the assumption of monotonicity.

5.1 Case 1 ("SFA-CD"): Correctly Specified SFA-CD

Table 5 shows the performance of the six alternatives to estimate TE, measured via RMSE, for Case 1. As expected, both estimators for the stochastic frontier model (SFA-CD and SFA-TL) perform best in this case as indicated by the smaller RMSE (and bias, see Table A13 in the online appendix). The median deviation is below 2% from the true values. Since the underlying assumption of DEA – the absence of noise – is violated in every setting and the estimated SFA methods are not misspecified, the superiority of SFA is not surprising. Yet, the MAX approach also performs relatively well in this case. As can be seen in the last row of Table 5, the MAX combination method is the best method in terms of minimizing the mean and maximum RMSE, i.e. when we consider the average or the highest RMSE for each method over all 30 scenarios, MAX possesses the lowest values. In contrast, the AM method has a relatively high RMSE that, while lower than the DEA estimators, is considerably higher than the RMSEs of the SFA and MAX estimators.

Considering the different scenarios, the stochastic frontier estimators consistently perform well, and in a similar fashion, to our baseline discussion, whereas DEA (as one might expect) exhibits a particularly poor performance when there is more noise in the data ($\sigma_v = 0.05$). Moreover, as observed by Badunenko et al. (2012) and Andor & Hesse (2014), the RMSE of SFA tends to decline with the sample size for most values of σ_v and σ_u, which is not the case for DEA. When there is a sizable amount of inefficiency in the data ($\sigma_u = 0.15$), the MAX approach worsens because all of the methods are less biased and underestimate efficiency levels for fewer firms (see Tables A13 and A29, respectively, in the online appendix).

As the underestimation of regulated firms is a particular concern for regulators, in Table A29 we present the percentage of firms whose technical efficiency level is underestimated (PU). The approach that has the lowest percentage of underestimated firms is, by construction, the MAX, followed by SFA-CD. The differences between the six methods are remarkable. While the MAX underestimates

Table 5: Performance of the Six Methods for Case 1 ("SFA-CD") in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.0117	0.0125	0.0757	0.0608	0.0383	* 0.0097
	n=100	0.01	0.01	1.0	0.0287	0.0115	0.0852	0.0698	0.0464	* 0.0096
	n=200	0.01	0.01	1.0	0.0615	0.0114	0.0909	0.0751	0.0560	* 0.0101
	n=400	0.01	0.01	1.0	0.0843	0.0124	0.0948	0.0784	0.0641	* 0.0115
	n=800	0.01	0.01	1.0	0.0905	0.0126	0.0978	0.0812	0.0675	* 0.0121
II	n=50	0.05	0.01	0.2	0.0106	0.0105	0.1117	0.0923	0.0579	* 0.0100
	n=100	0.05	0.01	0.2	0.0102	0.0101	0.1283	0.1102	0.0657	* 0.0100
	n=200	0.05	0.01	0.2	0.0097	0.0097	0.1424	0.1253	0.0724	0.0098
	n=400	0.05	0.01	0.2	0.0090	0.0095	0.1548	0.1384	0.0787	0.0097
	n=800	0.05	0.01	0.2	0.0073	0.0093	0.1658	0.1501	0.0837	0.0095
III	n=50	0.01	0.05	5.0	0.0113	0.0153	0.0613	0.0498	0.0278	0.0221
	n=100	0.01	0.05	5.0	0.0100	0.0122	0.0711	0.0583	0.0329	0.0163
	n=200	0.01	0.05	5.0	0.0094	0.0104	0.0787	0.0649	0.0365	0.0124
	n=400	0.01	0.05	5.0	0.0090	0.0094	0.0842	0.0695	0.0389	0.0103
	n=800	0.01	0.05	5.0	0.0089	0.0090	0.0884	0.0729	0.0408	0.0093
IV	n=50	0.05	0.05	1.0	0.0395	0.0411	0.0927	0.0766	0.0458	0.0411
	n=100	0.05	0.05	1.0	0.0339	0.0358	0.1088	0.0924	0.0531	0.0365
	n=200	0.05	0.05	1.0	0.0286	0.0293	0.1228	0.1067	0.0600	0.0303
	n=400	0.05	0.05	1.0	0.0264	0.0266	0.1356	0.1199	0.0666	0.0273
	n=800	0.05	0.05	1.0	0.0254	0.0254	0.1469	0.1315	0.0724	0.0258
V	n=50	0.01	0.15	15.0	0.0182	0.0348	0.0498	0.0555	0.0301	0.0535
	n=100	0.01	0.15	15.0	0.0138	0.0224	0.0551	0.0520	0.0275	0.0369
	n=200	0.01	0.15	15.0	0.0118	0.0170	0.0630	0.0541	0.0305	0.0249
	n=400	0.01	0.15	15.0	0.0110	0.0683	0.0700	0.0585	0.0444	0.0171
	n=800	0.01	0.15	15.0	0.0106	0.0693	0.0757	0.0627	0.0484	0.0132
VI	n=50	0.05	0.15	3.0	0.0452	0.0540	0.0720	0.0710	0.0505	0.0640
	n=100	0.05	0.15	3.0	0.0407	0.0457	0.0841	0.0748	0.0517	0.0508
	n=200	0.05	0.15	3.0	0.0385	0.0403	0.0971	0.0844	0.0555	0.0433
	n=400	0.05	0.15	3.0	0.0375	0.0383	0.1097	0.0957	0.0602	0.0396
	n=800	0.05	0.15	3.0	0.0369	0.0373	0.1214	0.1072	0.0654	0.0379
	min	–	–	–	0.0073	0.0090	0.0498	0.0498	0.0275	0.0093
	med	–	–	–	0.0160	0.0162	0.0918	0.0758	0.0524	0.0167
	mean	–	–	–	0.0263	0.0251	0.0979	0.0847	0.0523	* 0.0238
	max	–	–	–	0.0905	0.0693	0.1658	0.1501	0.0837	* 0.0640

Note: The data generating process is based on a Cobb-Douglas cost function. Cells highlighted in dark gray indicate the best performing estimator among SFA-CD, SFA-TL, DEA-CRS and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

on average 45% of the firms, the other methods underestimate at least 60%. In particular, DEA-CRS underestimates 89% of the firms on average. Regarding the different scenarios, it is obvious that the percentage of firms whose technical efficiency scores are underestimated falls with increasing inefficiency in the data.

5.2 Case 16 ("DEA"): Misspecified SFA

The picture changes considerably, when we consider the Case 16 ("DEA"), in which the stochastic frontier estimators are deployed on models that possess several misspecifications. On average, the best method over all 15 scenarios is DEA-VRS, followed by MAX (see Table 6). The worst method in this case is SFA-CD, but DEA-CRS and SFA-TL have also high RMSEs. Consequently, the AM method also performs poorly. Although the DEA-VRS results are worse in the scenarios with heavy inefficiency in the data (Block III), they are relatively decent through the entire set of scenarios. Regarding PU (Table A44), the stochastic frontier estimators and DEA-CRS substantially underestimate the efficiency level of firms (between 75% and 90% on average), whereas DEA-VRS and MAX, on average, underestimate only 8.5% and 7.5% of the firms, respectively.

5.3 Case 7 ("SFA-TL") and Case 11 ("TL-DEA")

The two previous cases have shown that SFA methods are superior when the estimated SFA models are correctly specified and the DEA-VRS method outperforms the other estimators when the data is generated based on a nonparametric function and without noise. In both scenarios, the MAX method yields lower RMSE than the AM combination approach, as at least one set of estimators performs relatively poorly, exerting greater influence on AM than on MAX. We now focus on two additional cases, in which SFA and DEA are partly misspecified, the relative superiority of the methods depends on the specific scenario and the cases can be viewed as more balanced between the competing estimators.

Table 6: Performance of the Six Methods for Case 16 ("DEA") in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.1445	0.0290	0.1036	0.0072	0.0657	0.0086
	n=100	0	0.01	Inf	0.1475	0.0502	0.1095	0.0046	0.0706	0.0058
	n=200	0	0.01	Inf	0.1488	0.0594	0.1125	0.0029	0.0742	0.0037
	n=400	0	0.01	Inf	0.1489	0.0653	0.1135	0.0022	0.0778	0.0029
	n=800	0	0.01	Inf	0.1493	0.0682	0.1141	0.0022	0.0794	0.0027
II	n=50	0	0.05	Inf	0.1260	0.0545	0.0942	0.0273	0.0563	0.0337
	n=100	0	0.05	Inf	0.1286	0.0556	0.0998	0.0179	0.0625	0.0226
	n=200	0	0.05	Inf	0.1290	0.0575	0.1039	0.0109	0.0671	0.0162
	n=400	0	0.05	Inf	0.1294	0.0591	0.1068	0.0063	0.0697	0.0131
	n=800	0	0.05	Inf	0.1297	0.0602	0.1085	0.0039	0.0716	0.0120
III	n=50	0	0.15	Inf	0.1084	0.0689	0.0832	0.0581	* 0.0541	0.0631
	n=100	0	0.15	Inf	0.1099	0.0650	0.0865	0.0399	0.0569	0.0433
	n=200	0	0.15	Inf	0.1112	0.0686	0.0910	0.0254	0.0637	0.0295
	n=400	0	0.15	Inf	0.1117	0.1179	0.0948	0.0155	0.0724	0.0209
	n=800	0	0.15	Inf	0.1118	0.1192	0.0975	0.0095	0.0749	0.0170
	min	–	–	–	0.1084	0.0290	0.0832	0.0022	0.0541	0.0027
	med	–	–	–	0.1290	0.0602	0.1036	0.0095	0.0697	0.0162
	mean	–	–	–	0.1290	0.0666	0.1013	0.0156	0.0678	0.0197
	max	–	–	–	0.1493	0.1192	0.1141	0.0581	0.0794	0.0631

Note: The data generating process is based on DEA. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table 7: Performance of the Six Methods for Case 7 (“SFA-TL”) in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.0910	0.0116	0.0584	0.0133	0.0359	* 0.0105
	n=100	0.01	0.01	1.0	0.0949	0.0099	0.0647	0.0165	0.0398	* 0.0096
	n=200	0.01	0.01	1.0	0.0965	0.0086	0.0691	0.0197	0.0423	0.0087
	n=400	0.01	0.01	1.0	0.0972	0.0083	0.0720	0.0225	0.0442	0.0083
	n=800	0.01	0.01	1.0	0.0974	0.0082	0.0747	0.0252	0.0457	0.0082
II	n=50	0.05	0.01	0.2	0.0859	0.0149	0.0955	0.0592	0.0586	* 0.0143
	n=100	0.05	0.01	0.2	0.0884	0.0139	0.1099	0.0723	0.0667	0.0140
	n=200	0.05	0.01	0.2	0.0889	0.0133	0.1229	0.0852	0.0739	0.0138
	n=400	0.05	0.01	0.2	0.0890	0.0130	0.1344	0.0967	0.0798	0.0137
	n=800	0.05	0.01	0.2	0.0893	0.0121	0.1451	0.1078	0.0852	0.0133
III	n=50	0.01	0.05	5.0	0.0741	0.0179	0.0493	0.0256	0.0268	0.0272
	n=100	0.01	0.05	5.0	0.0778	0.0208	0.0552	0.0197	0.0379	0.0199
	n=200	0.01	0.05	5.0	0.0793	0.0910	0.0604	0.0173	0.0506	* 0.0170
	n=400	0.01	0.05	5.0	0.0799	0.0920	0.0641	0.0180	0.0542	* 0.0174
	n=800	0.01	0.05	5.0	0.0801	0.0924	0.0672	0.0199	0.0565	* 0.0189
IV	n=50	0.05	0.05	1.0	0.0722	0.0487	0.0792	0.0515	0.0502	* 0.0449
	n=100	0.05	0.05	1.0	0.0727	0.0416	0.0926	0.0596	0.0588	* 0.0400
	n=200	0.05	0.05	1.0	0.0731	0.0389	0.1051	0.0698	0.0655	* 0.0377
	n=400	0.05	0.05	1.0	0.0736	0.0384	0.1164	0.0806	0.0724	* 0.0373
	n=800	0.05	0.05	1.0	0.0736	0.0827	0.1272	0.0911	0.0869	* 0.0535
V	n=50	0.01	0.15	15.0	0.0675	0.0420	0.0450	0.0531	* 0.0295	0.0573
	n=100	0.01	0.15	15.0	0.0691	0.0271	0.0461	0.0367	* 0.0265	0.0389
	n=200	0.01	0.15	15.0	0.0702	0.0951	0.0500	0.0250	0.0421	0.0265
	n=400	0.01	0.15	15.0	0.0705	0.0986	0.0540	0.0186	0.0476	0.0200
	n=800	0.01	0.15	15.0	0.0707	0.0195	0.0576	0.0170	0.0318	0.0170
VI	n=50	0.05	0.15	3.0	0.0778	0.0603	0.0644	0.0613	* 0.0502	0.0638
	n=100	0.05	0.15	3.0	0.0771	0.0554	0.0729	0.0552	0.0541	* 0.0517
	n=200	0.05	0.15	3.0	0.0767	0.1044	0.0831	0.0566	0.0677	* 0.0483
	n=400	0.05	0.15	3.0	0.0764	0.1111	0.0942	0.0634	0.0785	* 0.0503
	n=800	0.05	0.15	3.0	0.0764	0.1097	0.1048	0.0723	0.0822	* 0.0516
	min	–	–	–	0.0675	0.0082	0.0450	0.0133	0.0265	0.0082
	med	–	–	–	0.0775	0.0387	0.0725	0.0523	0.0523	* 0.0199
	mean	–	–	–	0.0802	0.0467	0.0812	0.0477	0.0547	* 0.0285
	max	–	–	–	0.0974	0.1111	0.1451	0.1078	0.0869	* 0.0638

Note: The data generating process is based on a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

In Case 7, the data generating process is based on a Translog cost function. In this case and if we ignore for a moment the combination approaches, SFA-TL performs best, but DEA-VRS estimates on average also relatively well and has the lowest RMSE in 11 of 30 scenarios (Table 7). However, the method with the lowest RMSE is the MAX approach, again. In this case, the difference to the second best approach, the SFA-TL, is relatively large. While SFA-TL exhibits an average RMSE of around 5%, the RMSE of MAX is around 3%. Again, the reason for this outcome is that all other methods underestimate the efficiency of the firms (Tables A19 and A35).

In Case 11, the data is generated through a Translog cost function that is subsequently monotonized and convexified through DEA with VRS. The results are very similar to the Case 7. The MAX approach displays the lowest RMSE on average (with 2.8%), the SFA-TL has the lowest average RMSE of all individual methods, but DEA-VRS is superior in nearly half of the scenarios (13 out of 30, Table 8). Once more, all individual methods underestimate the efficiency of the firms, some methods with a very high percentage of over 80%, for example SFA-CD underestimates 86% (see Table A39).

5.4 Graphical Analysis

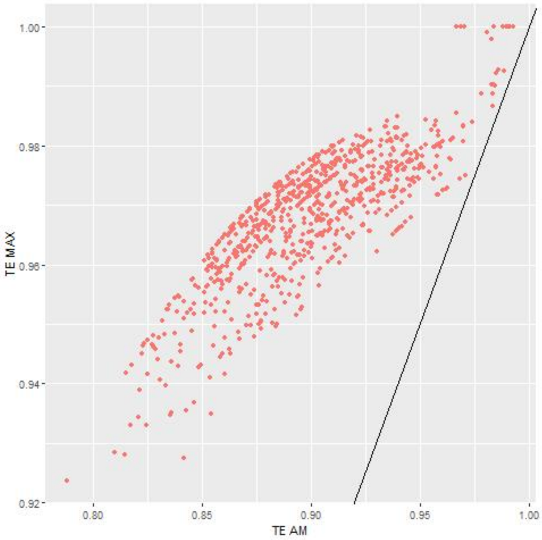
In addition to comparing the relative performance as expressed by the RMSE and PU, we compare the estimated levels of TE between the estimators themselves. Figure A1 contrasts random draws of the estimates of technical efficiency of DEA-VRS and SFA-CD when σ_u =0.05, n =800, and σ_v =0.05 or σ_v =0, respectively. Points along the diagonal line exhibit the same value for the two methods, while points above it indicate that the SFA-CD estimator yields a higher estimate and vice versa. For Case 1 ("SFA-CD") it is shown that for a great deal of observations the estimates of DEA-VRS are below those of SFA-CD. On the other hand, for Case 16 ("DEA") Figure A1 reveals that DEA-VRS yields higher efficiency scores for almost all firms. When contrasting the estimates of technical efficiency of our

Table 8: Performance of the Six Methods for Case 11 ("TL-DEA") in terms of the Root Mean Square Error (RMSE)

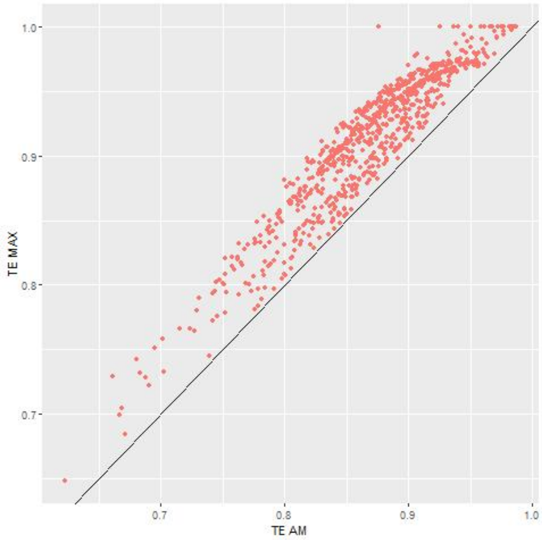
	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.0902	0.0133	0.0535	0.0105	0.0336	* 0.0104
	n=100	0.01	0.01	1.0	0.0934	0.0119	0.0589	0.0114	0.0370	* 0.0096
	n=200	0.01	0.01	1.0	0.0945	0.0108	0.0627	0.0133	0.0394	* 0.0092
	n=400	0.01	0.01	1.0	0.0951	0.0103	0.0656	0.0157	0.0412	* 0.0093
	n=800	0.01	0.01	1.0	0.0954	0.0101	0.0681	0.0182	0.0427	* 0.0095
II	n=50	0.05	0.01	0.2	0.0878	0.0142	0.0926	0.0565	0.0575	* 0.0141
	n=100	0.05	0.01	0.2	0.0899	0.0137	0.1063	0.0696	0.0656	0.0138
	n=200	0.05	0.01	0.2	0.0904	0.0133	0.1187	0.0821	0.0725	0.0139
	n=400	0.05	0.01	0.2	0.0906	0.0132	0.1301	0.0937	0.0784	0.0139
	n=800	0.05	0.01	0.2	0.0908	0.0120	0.1406	0.1047	0.0837	0.0133
III	n=50	0.01	0.05	5.0	0.0739	0.0200	0.0456	0.0259	0.0260	0.0277
	n=100	0.01	0.05	5.0	0.0772	0.0831	0.0505	0.0186	0.0398	0.0194
	n=200	0.01	0.05	5.0	0.0781	0.0921	0.0546	0.0140	0.0483	0.0147
	n=400	0.01	0.05	5.0	0.0787	0.0931	0.0581	0.0128	0.0515	0.0131
	n=800	0.01	0.05	5.0	0.0790	0.0934	0.0610	0.0137	0.0537	* 0.0136
IV	n=50	0.05	0.05	1.0	0.0733	0.0494	0.0769	0.0499	0.0495	* 0.0445
	n=100	0.05	0.05	1.0	0.0734	0.0419	0.0894	0.0573	0.0577	* 0.0397
	n=200	0.05	0.05	1.0	0.0742	0.0389	0.1012	0.0672	0.0642	* 0.0377
	n=400	0.05	0.05	1.0	0.0743	0.0388	0.1123	0.0775	0.0707	* 0.0375
	n=800	0.05	0.05	1.0	0.0745	0.0410	0.1229	0.0881	0.0825	0.0419
V	n=50	0.01	0.15	15.0	0.0669	0.0430	0.0434	0.0543	* 0.0293	0.0582
	n=100	0.01	0.15	15.0	0.0683	0.0290	0.0431	0.0373	* 0.0258	0.0393
	n=200	0.01	0.15	15.0	0.0692	0.0956	0.0457	0.0249	0.0404	0.0265
	n=400	0.01	0.15	15.0	0.0695	0.0993	0.0490	0.0169	0.0454	0.0190
	n=800	0.01	0.15	15.0	0.0697	0.0197	0.0522	0.0131	0.0300	0.0165
VI	n=50	0.05	0.15	3.0	0.0780	0.0611	0.0629	0.0614	* 0.0499	0.0640
	n=100	0.05	0.15	3.0	0.0773	0.0559	0.0707	0.0540	0.0532	* 0.0513
	n=200	0.05	0.15	3.0	0.0766	0.1040	0.0804	0.0549	0.0663	* 0.0475
	n=400	0.05	0.15	3.0	0.0764	0.1114	0.0907	0.0610	0.0772	* 0.0491
	n=800	0.05	0.15	3.0	0.0764	0.1103	0.1009	0.0697	0.0811	* 0.0505
	min	0.01	0.01	0.2	0.0669	0.0101	0.0431	0.0105	0.0258	* 0.0092
	med	0.03	0.05	2.0	0.0772	0.0400	0.0668	0.0519	0.0507	* 0.0192
	mean	0.03	0.07	4.2	0.0801	0.0481	0.0770	0.0449	0.0531	* 0.0276
	max	0.05	0.15	15.0	0.0954	0.1114	0.1406	0.1047	0.0837	* 0.0640

Note: The data generating process is based on DEA on the fitted values of a Translog cost function. Cells highlighted in dark gray indicate the best performing method among the two types of SFA and DEA. Cells highlighted in lighter gray indicate the better performing method among AM and MAX. If these are attached with an asterisk the corresponding method performs best across all six estimators.

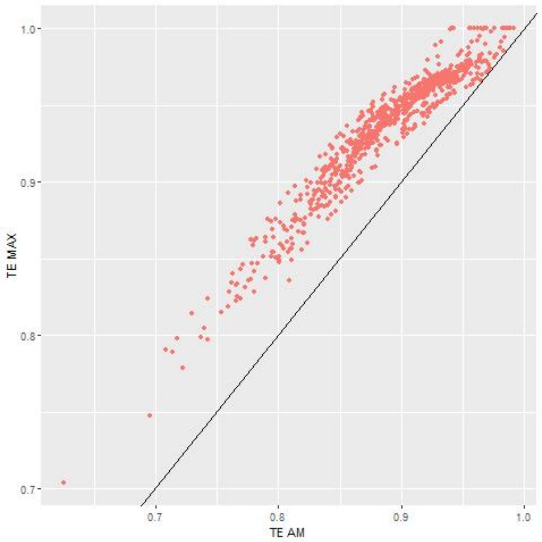
two combination approaches (Figure 1), we detect that MAX, by construction, yields higher efficiency scores, especially in Case 16 (“DEA”).



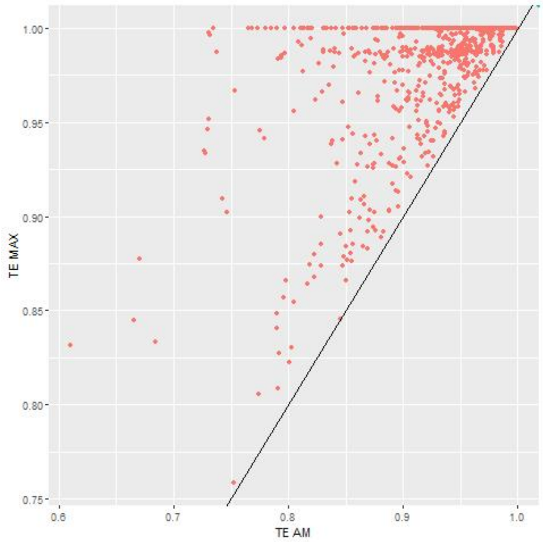
(a) Case 1 (“SFA-CD”)



(b) Case 7 (“SFA-TL”)



(c) Case 11 (“TL-DEA”)



(d) Case 16 (“DEA”)

Figure 1: Comparison of AM and MAX estimates

In Cases 1, 7, and 11, we assume that $N = 800$ and $\sigma_u = \sigma_v = 0.05$, while in Case 16, we assume that $\sigma_v = 0$.

The usefulness of graphs of this nature is that they very quickly highlight how different (or similar) competing methods are at predicting efficiency levels. This ability to distinguish differences in predictions may offer insights to regulators when deciding on X-factors for specific firms as well as in the process of justifying why one method or estimator was used as opposed to another.

Additionally, we look at three percentiles of the estimated efficiency distribution (Figure A2), the 5th, 50th, and 95th percentiles of the true and estimated efficiency scores. To keep the analysis simple we only show the distribution for the case when n =800 and $\sigma_u = \sigma_v$ =0.05 or σ_v =0, respectively. The upper panel of Figure A2 shows a high degree of efficiency for the case when we assume a CD cost function in the data generating process. Moreover, it confirms that the DEA approaches notably underestimate true efficiency scores in the 5th and 50th percentiles, while all other methods yield scores very close to the true value.

In addition, the range of DEA-CRS is considerably larger with respect to the other estimators, indicating that it confuses noise with inefficiency (cf. Badunenko et al. 2012). The lowest panel of Figure A2 illustrates that the overall efficiency of the SFA methods is considerably lower when the data generating process is based on DEA, compared to assuming a CD cost function. Consequently, the distance to the true efficiency scores is also larger. On the other hand, DEA-VRS slightly overestimates the true efficiency scores at the 95th percentile.

The kernel density plots in Figure A2 allow us to easily see how different the competing estimators behave across the simulations at various points of the technical efficiency distribution. The RMSEs that are presented in Tables 5 to 8 only capture central behavior, but the analysis of efficiency is more aptly characterized by looking at extremes, either those firms that are heavily efficient or heavily inefficient. The 5th and 95th percentiles across the MC simulations quickly showcase which estimator is adequately modeling this part of the distribution.

Moreover, plotting the percentiles of the efficiency scores of the two combination approaches reveals that the distribution pattern of MAX is closer to the distribution of the true efficiency scores compared to AM, irrespective of the underlying cost function (Figure 2). In particular, the distribution of the efficiency scores estimated by the MAX method closely resembles the distribution of the true values when we assume a DEA-VRS cost function in the data generating process.

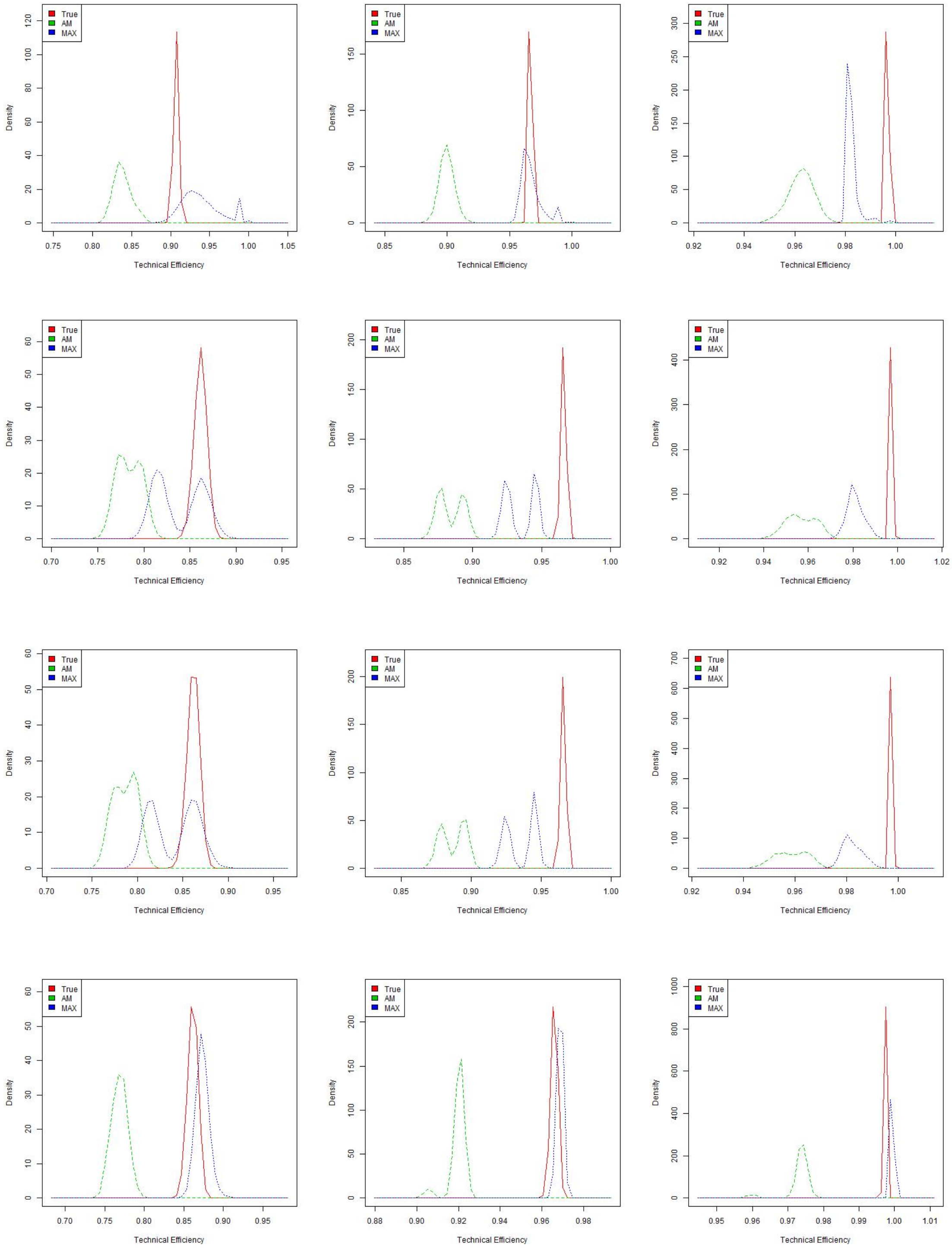


Figure 2: Distributions of 5th, 50th, and 95th Percentiles of Technical Efficiency Estimates in Case 1 ("SFA-CD"), Case 7 ("SFA-TL"), Case 11 ("TL-DEA"), and Case 16 ("DEA") for the True Efficiency Scores, AM, and MAX

In Cases 1, 7, and 11, we assume that $N = 800$ and $\sigma_u = \sigma_v = 0.05$, while in Case 16, we assume that $\sigma_v = 0$.

6 Conclusion

In this paper, we have investigated the performance of estimated technical efficiency (TE) scores stemming from the application of DEA and SFA in a setting similar to what a regulator commonly faces. Additionally, we have analyzed the performance of two ad-hoc combination approaches that have recently curried favor with various European regulators: taking the arithmetic mean (AM) or the maximum (MAX) of the technical efficiency scores across a variety of methods. On their own each method has relative benefits and costs; DEA is fully nonparametric in the construction of the frontier (and the subsequent measurement of efficiency), but does not allow for the (likely) presence of statistical noise, whereas SFA overcomes the hurdle of omitting statistical noise, but requires assumptions on the functional form and distribution of the error components. Using a Monte Carlo simulation based around a data set of Finnish distribution system operators, we have compared the performance of six methods (SFA-CD, SFA-TL, DEA-CRS, DEA-VRS, AM, MAX), varying the sample size, the amount of statistical noise, the amount and distribution of inefficiency, and the shape of the cost frontier itself. In particular, a novelty of our MC simulation is that we consider inter alia a nonparametric data generating process, which we see as another contribution of this paper.

As expected, the SFA variants estimate firm level technical efficiency relatively well when they are not misspecified. Likewise, DEA-VRS performs relatively well when the data does not contain noise and is generated nonparametrically. Yet, the method that performs the best, on average, is the combination approach involving taking the maximum of the DEA and SFA estimates. Furthermore, this result is relatively robust. It does not appear to depend on whether we consider only scenarios with or without noise for instance. The reason behind this result is that DEA and SFA are well known to underestimate efficiency scores for the majority of firms (Andor & Hesse 2014, Badunenko et al. 2012, for example). In contrast, the averaging combination approach (AM) does not perform well in general. This is linked to the fact that as soon as one of the methods performs poorly, perhaps due to model misspecification, the naïvety of the weighting scheme hurts the averaging estimator, distorting the inefficiency estimates.

Furthermore, we demonstrate that the methods can substantially differ in the degree to which firm level efficiency is underestimated. By construction, MAX is the method that underestimates efficiency considerably less often than the other methods. Yet, striking is the size of the underestimation for all other methods. While MAX underestimates on average 32%, the other methods underestimate between 60% and 93% of the firms. These results are also consistent with the findings of Andor & Hesse (2014) and Badunenko et al. (2012), who also consider both DEA and SFA, but not combination approaches.

Our findings have important implications for regulatory practice and beyond. First, combination approaches should be viewed as a viable tool for regulation, as opposed to strict reliance on individual estimates stemming from classic DEA or SFA. Second, combination approaches, though lacking proper theoretical foundation, can improve on the estimation of efficiency in many settings. Moreover, these methods could easily be applied to more advanced efficiency analysis approaches, such as the nonparametric SFA estimators of Fan, Li & Weersink (1996), Kuosmanen & Kortelainen (2012), and Parmeter & Racine (2012), the nonparametric bias corrected DEA estimator of Kneip, Simar & Wilson (2008), and the stochastic DEA approach by Simar & Zelenyuk (2011).

Consequently, we would like to advise that for practical regulatory applications, Monte Carlo simulations should be conducted under concrete real-world conditions and considering a variety of estimation methods before deciding on the final estimation method. Given that regulators have important knowledge on key factors of the data generating process (such as sample size, the number of inputs and outputs, the likely presence of endogeneity, etc.), this allows them to perform their own "tailored" MC studies and select the most appropriate efficiency estimation method as suggested by both Andor & Hesse (2014) and Kuosmanen et al. (2013). While benchmarking for regulation purposes has crucial financial consequences for the regulated companies, the cost to conduct "tailored" MC simulations are rather low. All in all, we consider "tailored" MC simulations as a valuable part of benchmarking for regulation purposes. In this respect, our paper shows a simple approach how to generate the data nonparametrically to allow for a (more) balanced comparison between parametric, semi-, and nonparametric approaches.

Finally, our results are very useful as they show both the limits and merits of combination approaches beyond the use of a singular estimation approach. More work on the benefits of combination approaches still remains. For example, while the arithmetic mean did not perform well in our simulations, this is a direct result of the naïve weighting scheme that was deployed. An interesting extension would be to consider alternative weighting schemes for taking the average or a theoretical criterion, perhaps based on model fit, to develop weights. Future research could also focus on alternative metrics of combination, such as a median, or another quantile of interest.

Appendix

Due to page limitations, we present all results for the remaining cases in an additional appendix that will be published online. Please see the separate document.

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Online Appendix

Combining Uncertainty with Uncertainty to Get Certainty? Efficiency Estimation for Regulatory Oversight

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In this online appendix, we display the performance of our the six estimation methods in terms of the root square mean error (Section A.1), the bias (Section A.2), and the percentage of underestimation (Section A.3) for all cases that are not discussed in the main text. Moreover, we show how estimated scale elasticities (under SFA-CD and SFA-TL methods) deviate from true model scale elasticities (Section A.4) and the percentage of violations of monotonicity (Section A.5). Lastly, we display the graphical analysis with respect to the estimated levels of technical efficiency as well their distribution for the SFA and DEA estimators (Section B).

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A Tables

A.1 Root Square Mean Error

A.1.1 Cases 2-4: "SFA-CD"

Table A1: Performance of the Six Methods for Case 2 in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.0465	0.0038	0.0734	0.0585	0.0409	0.0051
	n=100	0	0.01	Inf	0.0171	0.0021	0.0818	0.0659	0.0404	0.0035
	n=200	0	0.01	Inf	0.0625	0.0015	0.0855	0.0689	0.0516	0.0024
	n=400	0	0.01	Inf	0.0704	0.0012	0.0868	0.0702	0.0545	0.0017
	n=800	0	0.01	Inf	0.0733	0.0011	0.0874	0.0707	0.0559	0.0013
II	n=50	0	0.05	Inf	0.0061	0.0125	0.0594	0.0484	0.0254	0.0208
	n=100	0	0.05	Inf	0.0042	0.0075	0.0688	0.0563	0.0298	0.0144
	n=200	0	0.05	Inf	0.0033	0.0046	0.0759	0.0619	0.0331	0.0097
	n=400	0	0.05	Inf	0.0031	0.0030	0.0804	0.0650	0.0350	0.0069
	n=800	0	0.05	Inf	0.0033	0.0028	0.0828	0.0667	0.0360	0.0058
III	n=50	0	0.15	Inf	0.0162	0.0339	0.0487	0.0546	0.0287	0.0528
	n=100	0	0.15	Inf	0.0113	0.0208	0.0538	0.0509	0.0256	0.0363
	n=200	0	0.15	Inf	0.0093	0.0157	0.0612	0.0526	0.0282	0.0241
	n=400	0	0.15	Inf	0.0083	0.0676	0.0678	0.0563	0.0423	0.0158
	n=800	0	0.15	Inf	0.0070	0.0684	0.0728	0.0594	0.0459	0.0107
	min	–	–	–	0.0031	0.0011	0.0487	0.0484	0.0254	0.0013
	med	–	–	–	0.0093	0.0046	0.0734	0.0594	0.0360	0.0097
	mean	–	–	–	0.0228	0.0164	0.0724	0.0604	0.0382	* 0.0141
	max	–	–	–	0.0733	0.0684	0.0874	0.0707	0.0559	* 0.0528

Note: The data generating process is based on a Cobb-Douglas cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A2: Performance of the Six Methods for Case 3 in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.0105	0.0116	0.0749	0.0600	0.0363	* 0.0101
	n=100	0.01	0.01	1.0	0.0101	0.0099	0.0844	0.0693	0.0414	* 0.0091
	n=200	0.01	0.01	1.0	0.0186	0.0087	0.0906	0.0747	0.0462	* 0.0086
	n=400	0.01	0.01	1.0	0.0450	0.0083	0.0944	0.0782	0.0543	0.0084
	n=800	0.01	0.01	1.0	0.0355	0.0082	0.0976	0.0810	0.0539	0.0083
II	n=50	0.05	0.01	0.2	0.0154	0.0146	0.1103	0.0912	0.0570	* 0.0142
	n=100	0.05	0.01	0.2	0.0145	0.0141	0.1269	0.1090	0.0649	0.0141
	n=200	0.05	0.01	0.2	0.0140	0.0139	0.1411	0.1242	0.0720	0.0141
	n=400	0.05	0.01	0.2	0.0134	0.0135	0.1535	0.1370	0.0781	0.0139
	n=800	0.05	0.01	0.2	0.0120	0.0131	0.1650	0.1492	0.0835	0.0135
III	n=50	0.01	0.05	5.0	0.0122	0.0177	0.0607	0.0504	0.0295	0.0240
	n=100	0.01	0.05	5.0	0.0115	0.0226	0.0707	0.0585	0.0437	0.0175
	n=200	0.01	0.05	5.0	0.0104	0.0910	0.0785	0.0649	0.0553	0.0132
	n=400	0.01	0.05	5.0	0.0106	0.0921	0.0841	0.0695	0.0596	0.0115
	n=800	0.01	0.05	5.0	0.0110	0.0924	0.0882	0.0729	0.0622	0.0111
IV	n=50	0.05	0.05	1.0	0.0441	0.0493	0.0917	0.0763	0.0559	0.0450
	n=100	0.05	0.05	1.0	0.0397	0.0420	0.1076	0.0917	0.0648	0.0399
	n=200	0.05	0.05	1.0	0.0385	0.0391	0.1217	0.1057	0.0720	* 0.0379
	n=400	0.05	0.05	1.0	0.0382	0.0385	0.1343	0.1186	0.0793	* 0.0378
	n=800	0.05	0.05	1.0	0.0382	0.0819	0.1458	0.1303	0.0930	0.0383
V	n=50	0.01	0.15	15.0	0.0201	0.0415	0.0497	0.0573	0.0314	0.0545
	n=100	0.01	0.15	15.0	0.0158	0.0272	0.0555	0.0530	0.0299	0.0367
	n=200	0.01	0.15	15.0	0.0136	0.0945	0.0632	0.0546	0.0470	0.0246
	n=400	0.01	0.15	15.0	0.0123	0.0989	0.0700	0.0587	0.0540	0.0175
	n=800	0.01	0.15	15.0	0.0114	0.0195	0.0754	0.0626	0.0381	0.0148
VI	n=50	0.05	0.15	3.0	0.0499	0.0602	0.0720	0.0729	0.0541	0.0638
	n=100	0.05	0.15	3.0	0.0485	0.0552	0.0838	0.0757	0.0597	0.0535
	n=200	0.05	0.15	3.0	0.0477	0.1039	0.0970	0.0845	0.0745	0.0492
	n=400	0.05	0.15	3.0	0.0472	0.1111	0.1095	0.0956	0.0863	0.0476
	n=800	0.05	0.15	3.0	0.0469	0.1100	0.1209	0.1068	0.0895	0.0469
	min	–	–	–	0.0101	0.0082	0.0497	0.0504	0.0295	0.0083
	med	–	–	–	0.0156	0.0388	0.0912	0.0760	0.0565	0.0175
	mean	–	–	–	0.0252	0.0468	0.0973	0.0845	0.0589	0.0267
	max	–	–	–	0.0499	0.1111	0.1650	0.1492	0.0930	0.0638

Note: The data generating process is based on a Cobb-Douglas cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A3: Performance of the Six Methods for Case 4 in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.0037	0.0035	0.0728	0.0580	0.0335	0.0054
	n=100	0	0.01	Inf	0.0110	0.0023	0.0813	0.0656	0.0383	0.0037
	n=200	0	0.01	Inf	0.0208	0.0018	0.0854	0.0689	0.0426	0.0025
	n=400	0	0.01	Inf	0.0116	0.0015	0.0868	0.0701	0.0414	0.0017
	n=800	0	0.01	Inf	0.0096	0.0015	0.0874	0.0707	0.0414	* 0.0013
I	n=50	0	0.05	Inf	0.0060	0.0151	0.0593	0.0493	0.0270	0.0229
	n=100	0	0.05	Inf	0.0048	0.0128	0.0687	0.0566	0.0346	0.0151
	n=200	0	0.05	Inf	0.0044	0.0895	0.0759	0.0619	0.0515	0.0097
	n=400	0	0.05	Inf	0.0040	0.0907	0.0803	0.0649	0.0550	0.0066
	n=800	0	0.05	Inf	0.0037	0.0911	0.0825	0.0665	0.0565	0.0049
I	n=50	0	0.15	Inf	0.0182	0.0410	0.0487	0.0569	0.0302	0.0547
	n=100	0	0.15	Inf	0.0130	0.0259	0.0540	0.0517	0.0279	0.0363
	n=200	0	0.15	Inf	0.0100	0.0943	0.0614	0.0530	0.0450	0.0233
	n=400	0	0.15	Inf	0.0077	0.0981	0.0679	0.0564	0.0514	0.0153
	n=800	0	0.15	Inf	0.0060	0.0182	0.0725	0.0590	0.0341	0.0127
	min	–	–	–	0.0037	0.0015	0.0487	0.0493	0.0270	* 0.0013
	med	–	–	–	0.0077	0.0182	0.0728	0.0590	0.0414	0.0097
	mean	–	–	–	0.0090	0.0391	0.0723	0.0606	0.0407	0.0144
	max	–	–	–	0.0208	0.0981	0.0874	0.0707	0.0565	0.0547

Note: The data generating process is based on a Cobb-Douglas cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

A.1.2 Cases 5,6, and 8: "SFA-TL"

Table A4: Performance of the Six Methods for Case 5 in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.0926	0.0126	0.0590	0.0129	0.0370	* 0.0098
	n=100	0.01	0.01	1.0	0.0961	0.0110	0.0650	0.0164	0.0404	* 0.0100
	n=200	0.01	0.01	1.0	0.0974	0.0104	0.0694	0.0198	0.0429	* 0.0101
	n=400	0.01	0.01	1.0	0.0980	0.0120	0.0723	0.0227	0.0451	* 0.0110
	n=800	0.01	0.01	1.0	0.0983	0.0126	0.0748	0.0252	0.0469	* 0.0117
II	n=50	0.05	0.01	0.2	0.0883	0.0102	0.0967	0.0596	0.0598	* 0.0099
	n=100	0.05	0.01	0.2	0.0895	0.0098	0.1109	0.0732	0.0677	0.0099
	n=200	0.05	0.01	0.2	0.0904	0.0095	0.1241	0.0862	0.0748	0.0098
	n=400	0.05	0.01	0.2	0.0907	0.0091	0.1354	0.0979	0.0809	0.0096
	n=800	0.05	0.01	0.2	0.0908	0.0074	0.1460	0.1087	0.0860	0.0093
III	n=50	0.01	0.05	5.0	0.0726	0.0154	0.0493	0.0239	0.0251	0.0255
	n=100	0.01	0.05	5.0	0.0760	0.0122	0.0554	0.0191	0.0290	0.0188
	n=200	0.01	0.05	5.0	0.0771	0.0104	0.0603	0.0171	0.0321	0.0144
	n=400	0.01	0.05	5.0	0.0777	0.0094	0.0641	0.0176	0.0343	0.0119
	n=800	0.01	0.05	5.0	0.0779	0.0090	0.0672	0.0195	0.0361	0.0108
IV	n=50	0.05	0.05	1.0	0.0658	0.0405	0.0796	0.0502	0.0439	0.0406
	n=100	0.05	0.05	1.0	0.0671	0.0354	0.0934	0.0596	0.0513	0.0363
	n=200	0.05	0.05	1.0	0.0678	0.0293	0.1059	0.0702	0.0580	0.0305
	n=400	0.05	0.05	1.0	0.0681	0.0266	0.1176	0.0813	0.0641	0.0274
	n=800	0.05	0.05	1.0	0.0680	0.0254	0.1280	0.0916	0.0697	0.0258
V	n=50	0.01	0.15	15.0	0.0603	0.0351	0.0462	0.0533	* 0.0304	0.0604
	n=100	0.01	0.15	15.0	0.0576	0.0225	0.0466	0.0379	0.0246	0.0437
	n=200	0.01	0.15	15.0	0.0568	0.0172	0.0500	0.0263	0.0246	0.0327
	n=400	0.01	0.15	15.0	0.0563	0.0683	0.0542	0.0193	0.0350	0.0270
	n=800	0.01	0.15	15.0	0.0563	0.0693	0.0579	0.0170	0.0384	0.0251
VI	n=50	0.05	0.15	3.0	0.0681	0.0543	0.0648	0.0605	* 0.0476	0.0686
	n=100	0.05	0.15	3.0	0.0613	0.0459	0.0729	0.0549	0.0464	0.0544
	n=200	0.05	0.15	3.0	0.0586	0.0404	0.0835	0.0563	0.0486	0.0469
	n=400	0.05	0.15	3.0	0.0578	0.0383	0.0945	0.0631	0.0527	0.0432
	n=800	0.05	0.15	3.0	0.0576	0.0373	0.1049	0.0719	0.0571	0.0416
	min	–	–	–	0.0563	0.0074	0.0462	0.0129	0.0246	0.0093
	med	–	–	–	0.0703	0.0163	0.0726	0.0517	0.0457	0.0253
	mean	–	–	–	0.0747	0.0249	0.0817	0.0478	0.0477	0.0262
	max	–	–	–	0.0983	0.0693	0.1460	0.1087	0.0860	* 0.0686

Note: The data generating process is based on a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A5: Performance of the Six Methods for Case 6 in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.0916	0.0039	0.0564	0.0089	0.0336	0.0060
	n=100	0	0.01	Inf	0.0949	0.0021	0.0612	0.0111	0.0362	0.0043
	n=200	0	0.01	Inf	0.0962	0.0015	0.0635	0.0124	0.0376	0.0029
	n=400	0	0.01	Inf	0.0963	0.0013	0.0645	0.0130	0.0382	0.0020
	n=800	0	0.01	Inf	0.0955	0.0012	0.0649	0.0133	0.0382	0.0015
II	n=50	0	0.05	Inf	0.0727	0.0125	0.0479	0.0233	0.0232	0.0246
	n=100	0	0.05	Inf	0.0765	0.0075	0.0532	0.0173	0.0264	0.0176
	n=200	0	0.05	Inf	0.0783	0.0046	0.0573	0.0136	0.0291	0.0124
	n=400	0	0.05	Inf	0.0788	0.0030	0.0599	0.0123	0.0308	0.0093
	n=800	0	0.05	Inf	0.0791	0.0027	0.0615	0.0122	0.0317	0.0079
III	n=50	0	0.15	Inf	0.0599	0.0341	0.0453	0.0528	* 0.0293	0.0601
	n=100	0	0.15	Inf	0.0575	0.0210	0.0453	0.0375	0.0232	0.0432
	n=200	0	0.15	Inf	0.0568	0.0159	0.0483	0.0254	0.0229	0.0320
	n=400	0	0.15	Inf	0.0567	0.0676	0.0518	0.0173	0.0331	0.0258
	n=800	0	0.15	Inf	0.0565	0.0684	0.0546	0.0131	0.0358	0.0233
	min	–	–	–	0.0565	0.0012	0.0453	0.0089	0.0229	0.0015
	med	–	–	–	0.0783	0.0046	0.0564	0.0133	0.0317	0.0124
	mean	–	–	–	0.0765	0.0165	0.0557	0.0189	0.0313	0.0182
	max	–	–	–	0.0963	0.0684	0.0649	0.0528	* 0.0382	0.0601

Note: The data generating process is based on a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A6: Performance of the Six Methods for Case 8 in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.0904	0.0035	0.0560	0.0098	0.0326	0.0069
	n=100	0	0.01	Inf	0.0942	0.0023	0.0610	0.0113	0.0357	0.0047
	n=200	0	0.01	Inf	0.0954	0.0017	0.0635	0.0124	0.0373	0.0031
	n=400	0	0.01	Inf	0.0957	0.0014	0.0644	0.0130	0.0380	0.0020
	n=800	0	0.01	Inf	0.0951	0.0012	0.0649	0.0133	0.0381	0.0016
II	n=50	0	0.05	Inf	0.0744	0.0152	0.0478	0.0249	0.0250	0.0266
	n=100	0	0.05	Inf	0.0780	0.0128	0.0531	0.0178	0.0315	0.0183
	n=200	0	0.05	Inf	0.0801	0.0894	0.0571	0.0136	0.0477	0.0139
	n=400	0	0.05	Inf	0.0810	0.0906	0.0598	0.0123	0.0508	0.0127
	n=800	0	0.05	Inf	0.0814	0.0911	0.0612	0.0123	0.0522	0.0128
III	n=50	0	0.15	Inf	0.0668	0.0410	0.0442	0.0531	* 0.0285	0.0573
	n=100	0	0.15	Inf	0.0687	0.0260	0.0450	0.0366	* 0.0247	0.0387
	n=200	0	0.15	Inf	0.0699	0.0943	0.0482	0.0241	0.0404	0.0258
	n=400	0	0.15	Inf	0.0706	0.0981	0.0515	0.0161	0.0455	0.0182
	n=800	0	0.15	Inf	0.0709	0.0182	0.0542	0.0127	0.0286	0.0151
	min	–	–	–	0.0668	0.0012	0.0442	0.0098	0.0247	0.0016
	med	–	–	–	0.0801	0.0182	0.0560	0.0133	0.0373	0.0139
	mean	–	–	–	0.0808	0.0391	0.0555	0.0189	0.0371	* 0.0172
	max	–	–	–	0.0957	0.0981	0.0649	0.0531	* 0.0522	0.0573

Note: The data generating process is based on a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

A.1.3 Cases 9, 10, and 12: "TL-DEA"

Table A7: Performance of the Six Methods for Case 9 in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.0917	0.0116	0.0539	0.0096	0.0340	* 0.0090
	n=100	0.01	0.01	1.0	0.0944	0.0097	0.0593	0.0111	0.0364	* 0.0091
	n=200	0.01	0.01	1.0	0.0957	0.0094	0.0630	0.0133	0.0386	* 0.0092
	n=400	0.01	0.01	1.0	0.0961	0.0092	0.0658	0.0158	0.0403	0.0093
	n=800	0.01	0.01	1.0	0.0962	0.0082	0.0682	0.0182	0.0419	0.0092
II	n=50	0.05	0.01	0.2	0.0902	0.0102	0.0937	0.0571	0.0588	* 0.0100
	n=100	0.05	0.01	0.2	0.0911	0.0098	0.1074	0.0704	0.0665	0.0099
	n=200	0.05	0.01	0.2	0.0920	0.0095	0.1199	0.0830	0.0735	0.0099
	n=400	0.05	0.01	0.2	0.0923	0.0091	0.1311	0.0947	0.0794	0.0097
	n=800	0.05	0.01	0.2	0.0925	0.0072	0.1414	0.1055	0.0846	0.0093
III	n=50	0.01	0.05	5.0	0.0726	0.0177	0.0458	0.0243	0.0244	0.0265
	n=100	0.01	0.05	5.0	0.0752	0.0148	0.0506	0.0181	0.0275	0.0194
	n=200	0.01	0.05	5.0	0.0764	0.0129	0.0546	0.0140	0.0299	0.0152
	n=400	0.01	0.05	5.0	0.0768	0.0121	0.0582	0.0127	0.0318	0.0130
	n=800	0.01	0.05	5.0	0.0770	0.0117	0.0611	0.0134	0.0335	0.0122
IV	n=50	0.05	0.05	1.0	0.0668	0.0405	0.0779	0.0485	0.0432	* 0.0403
	n=100	0.05	0.05	1.0	0.0686	0.0359	0.0904	0.0573	0.0501	0.0365
	n=200	0.05	0.05	1.0	0.0695	0.0294	0.1020	0.0675	0.0569	0.0305
	n=400	0.05	0.05	1.0	0.0695	0.0268	0.1134	0.0784	0.0628	0.0276
	n=800	0.05	0.05	1.0	0.0697	0.0256	0.1237	0.0887	0.0684	0.0261
V	n=50	0.01	0.15	15.0	0.0601	0.0358	0.0448	0.0538	* 0.0304	0.0608
	n=100	0.01	0.15	15.0	0.0574	0.0238	0.0438	0.0383	0.0242	0.0439
	n=200	0.01	0.15	15.0	0.0568	0.0191	0.0459	0.0264	0.0237	0.0328
	n=400	0.01	0.15	15.0	0.0568	0.0688	0.0492	0.0181	0.0331	0.0260
	n=800	0.01	0.15	15.0	0.0568	0.0700	0.0525	0.0136	0.0364	0.0230
VI	n=50	0.05	0.15	3.0	0.0678	0.0546	0.0638	0.0600	* 0.0471	0.0683
	n=100	0.05	0.15	3.0	0.0613	0.0460	0.0711	0.0537	* 0.0457	0.0541
	n=200	0.05	0.15	3.0	0.0588	0.0408	0.0805	0.0545	0.0478	0.0468
	n=400	0.05	0.15	3.0	0.0580	0.0387	0.0910	0.0606	0.0515	0.0431
	n=800	0.05	0.15	3.0	0.0579	0.0378	0.1010	0.0694	0.0559	0.0416
	min	–	–	–	0.0568	0.0072	0.0438	0.0096	0.0237	0.0090
	med	–	–	–	0.0712	0.0184	0.0670	0.0511	0.0426	0.0245
	mean	–	–	–	0.0749	0.0252	0.0775	0.0450	0.0459	0.0261
	max	–	–	–	0.0962	0.0700	0.1414	0.1055	0.0846	* 0.0683

Note: The data generating process is based on DEA on the fitted values of a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A8: Performance of the Six Methods for Case 10 in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.0911	0.0123	0.0514	0.0063	0.0328	0.0064
	n=100	0	0.01	Inf	0.0938	0.0125	0.0552	0.0047	0.0350	0.0049
	n=200	0	0.01	Inf	0.0946	0.0133	0.0570	0.0035	0.0363	0.0037
	n=400	0	0.01	Inf	0.0952	0.0142	0.0580	0.0031	0.0373	0.0031
	n=800	0	0.01	Inf	0.0931	0.0147	0.0584	0.0032	0.0374	0.0032
II	n=50	0	0.05	Inf	0.0726	0.0150	0.0441	0.0242	0.0227	0.0258
	n=100	0	0.05	Inf	0.0758	0.0118	0.0483	0.0172	0.0253	0.0185
	n=200	0	0.05	Inf	0.0770	0.0104	0.0515	0.0114	0.0277	0.0132
	n=400	0	0.05	Inf	0.0774	0.0095	0.0538	0.0072	0.0290	0.0100
	n=800	0	0.05	Inf	0.0777	0.0091	0.0553	0.0047	0.0299	0.0085
III	n=50	0	0.15	Inf	0.0596	0.0347	0.0438	0.0538	* 0.0295	0.0606
	n=100	0	0.15	Inf	0.0575	0.0221	0.0425	0.0382	0.0228	0.0432
	n=200	0	0.15	Inf	0.0568	0.0171	0.0442	0.0259	0.0220	0.0321
	n=400	0	0.15	Inf	0.0569	0.0680	0.0467	0.0166	0.0310	0.0249
	n=800	0	0.15	Inf	0.0570	0.0691	0.0491	0.0104	0.0336	0.0213
	min	–	–	–	0.0568	0.0091	0.0425	0.0031	0.0220	0.0031
	med	–	–	–	0.0770	0.0142	0.0514	0.0104	0.0299	0.0132
	mean	–	–	–	0.0757	0.0222	0.0506	0.0154	0.0302	0.0186
	max	–	–	–	0.0952	0.0691	0.0584	0.0538	* 0.0374	0.0606

Note: The data generating process is based on DEA on the fitted values of a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A9: Performance of the Six Methods for Case 12 in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.0896	0.0108	0.0509	0.0073	0.0318	0.0075
	n=100	0	0.01	Inf	0.0929	0.0109	0.0549	0.0051	0.0344	0.0053
	n=200	0	0.01	Inf	0.0939	0.0107	0.0571	0.0036	0.0358	0.0039
	n=400	0	0.01	Inf	0.0944	0.0106	0.0579	0.0031	0.0364	0.0033
	n=800	0	0.01	Inf	0.0927	0.0108	0.0584	0.0033	0.0365	0.0033
II	n=50	0	0.05	Inf	0.0739	0.0174	0.0441	0.0255	0.0242	0.0272
	n=100	0	0.05	Inf	0.0771	0.0204	0.0482	0.0174	0.0339	0.0184
	n=200	0	0.05	Inf	0.0785	0.0907	0.0516	0.0111	0.0453	0.0123
	n=400	0	0.05	Inf	0.0791	0.0918	0.0537	0.0068	0.0478	0.0086
	n=800	0	0.05	Inf	0.0794	0.0921	0.0550	0.0043	0.0490	0.0068
III	n=50	0	0.15	Inf	0.0665	0.0421	0.0427	0.0541	* 0.0282	0.0578
	n=100	0	0.15	Inf	0.0678	0.0276	0.0419	0.0369	* 0.0240	0.0391
	n=200	0	0.15	Inf	0.0690	0.0947	0.0439	0.0243	0.0384	0.0260
	n=400	0	0.15	Inf	0.0694	0.0985	0.0466	0.0153	0.0430	0.0178
	n=800	0	0.15	Inf	0.0696	0.0180	0.0486	0.0093	0.0267	0.0149
	min	–	–	–	0.0665	0.0106	0.0419	0.0031	0.0240	0.0033
	med	–	–	–	0.0785	0.0204	0.0509	0.0093	0.0358	0.0123
	mean	–	–	–	0.0796	0.0431	0.0504	0.0151	0.0357	0.0168
	max	–	–	–	0.0944	0.0985	0.0584	0.0541	* 0.0490	0.0578

Note: The data generating process is based on DEA on the fitted values of a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

A.1.4 Cases 13-15: "DEA"

Table A10: Performance of the Six Methods for Case 13 in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.1465	0.0260	0.1061	0.0097	0.0683	* 0.0088
	n=100	0.01	0.01	1.0	0.1495	0.0495	0.1125	0.0112	0.0734	* 0.0101
	n=200	0.01	0.01	1.0	0.1501	0.0606	0.1165	0.0135	0.0776	* 0.0125
	n=400	0.01	0.01	1.0	0.1506	0.0658	0.1193	0.0160	0.0817	* 0.0151
	n=800	0.01	0.01	1.0	0.1510	0.0691	0.1216	0.0186	0.0846	* 0.0176
II	n=50	0.05	0.01	0.2	0.1559	0.0136	0.1338	0.0534	0.0878	0.0137
	n=100	0.05	0.01	0.2	0.1578	0.0353	0.1470	0.0671	0.0967	* 0.0321
	n=200	0.05	0.01	0.2	0.1579	0.0433	0.1587	0.0803	0.1045	* 0.0414
	n=400	0.05	0.01	0.2	0.1583	0.0461	0.1689	0.0924	0.1113	* 0.0460
	n=800	0.05	0.01	0.2	0.1583	0.0458	0.1786	0.1038	0.1170	0.0464
III	n=50	0.01	0.05	5.0	0.1278	0.0422	0.0958	0.0256	0.0548	0.0329
	n=100	0.01	0.05	5.0	0.1297	0.0417	0.1018	0.0185	0.0595	0.0238
	n=200	0.01	0.05	5.0	0.1304	0.0437	0.1070	0.0139	0.0640	0.0180
	n=400	0.01	0.05	5.0	0.1307	0.0472	0.1105	0.0126	0.0679	0.0158
	n=800	0.01	0.05	5.0	0.1309	0.0494	0.1134	0.0136	0.0708	0.0158
IV	n=50	0.05	0.05	1.0	0.1342	0.0409	0.1185	0.0465	0.0698	* 0.0399
	n=100	0.05	0.05	1.0	0.1352	0.0395	0.1301	0.0546	0.0777	* 0.0385
	n=200	0.05	0.05	1.0	0.1354	0.0382	0.1414	0.0651	0.0848	* 0.0375
	n=400	0.05	0.05	1.0	0.1356	0.0373	0.1516	0.0762	0.0916	* 0.0370
	n=800	0.05	0.05	1.0	0.1356	0.0364	0.1611	0.0870	0.0974	0.0364
V	n=50	0.01	0.15	15.0	0.1018	0.0627	0.0851	0.0582	* 0.0521	0.0682
	n=100	0.01	0.15	15.0	0.1021	0.0557	0.0889	0.0410	0.0521	0.0500
	n=200	0.01	0.15	15.0	0.1023	0.0516	0.0933	0.0275	0.0542	0.0390
	n=400	0.01	0.15	15.0	0.1021	0.0499	0.0975	0.0182	0.0562	0.0341
	n=800	0.01	0.15	15.0	0.1024	0.0493	0.1011	0.0136	0.0581	0.0321
VI	n=50	0.05	0.15	3.0	0.1044	0.0754	0.0993	0.0628	* 0.0605	0.0739
	n=100	0.05	0.15	3.0	0.1019	0.0632	0.1084	0.0541	0.0634	0.0580
	n=200	0.05	0.15	3.0	0.1017	0.0571	0.1181	0.0532	0.0683	* 0.0510
	n=400	0.05	0.15	3.0	0.1021	0.0546	0.1276	0.0585	0.0733	* 0.0488
	n=800	0.05	0.15	3.0	0.1021	0.0534	0.1371	0.0673	0.0781	* 0.0487
	min	–	–	–	0.1017	0.0136	0.0851	0.0097	0.0521	* 0.0088
	med	–	–	–	0.1326	0.0482	0.1173	0.0499	0.0721	* 0.0367
	mean	–	–	–	0.1295	0.0482	0.1217	0.0445	0.0753	* 0.0348
	max	–	–	–	0.1583	0.0754	0.1786	0.1038	0.1170	* 0.0739

Note: The data generating process is based on DEA. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A11: Performance of the Six Methods for Case 14 in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.1457	0.0284	0.1039	0.0061	0.0663	0.0065
	n=100	0	0.01	Inf	0.1483	0.0521	0.1096	0.0042	0.0709	0.0045
	n=200	0	0.01	Inf	0.1498	0.0605	0.1124	0.0028	0.0745	0.0031
	n=400	0	0.01	Inf	0.1503	0.0659	0.1136	0.0022	0.0779	0.0024
	n=800	0	0.01	Inf	0.1505	0.0694	0.1141	0.0022	0.0798	0.0022
II	n=50	0	0.05	Inf	0.1274	0.0424	0.0952	0.0253	0.0539	0.0326
	n=100	0	0.05	Inf	0.1294	0.0421	0.1002	0.0173	0.0583	0.0229
	n=200	0	0.05	Inf	0.1301	0.0441	0.1049	0.0110	0.0625	0.0167
	n=400	0	0.05	Inf	0.1304	0.0480	0.1075	0.0067	0.0663	0.0134
	n=800	0	0.05	Inf	0.1306	0.0502	0.1091	0.0042	0.0684	0.0122
III	n=50	0	0.15	Inf	0.1010	0.0621	0.0845	0.0583	$*$ 0.0515	0.0684
	n=100	0	0.15	Inf	0.1020	0.0552	0.0881	0.0409	0.0515	0.0498
	n=200	0	0.15	Inf	0.1020	0.0513	0.0919	0.0269	0.0533	0.0387
	n=400	0	0.15	Inf	0.1025	0.0497	0.0960	0.0167	0.0555	0.0337
	n=800	0	0.15	Inf	0.1025	0.0492	0.0989	0.0102	0.0570	0.0315
	min	–	–	–	0.1010	0.0284	0.0845	0.0022	0.0515	0.0022
	med	–	–	–	0.1301	0.0502	0.1039	0.0102	0.0625	0.0167
	mean	–	–	–	0.1268	0.0514	0.1020	0.0157	0.0632	0.0226
	max	–	–	–	0.1505	0.0694	0.1141	0.0583	0.0798	0.0684

Note: The data generating process is based on DEA. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A12: Performance of the Six Methods for Case 15 in terms of the Root Mean Square Error (RMSE)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.1453	0.0275	0.1056	0.0106	0.0672	0.0106
	n=100	0.01	0.01	1.0	0.1482	0.0484	0.1121	0.0114	0.0726	* 0.0107
	n=200	0.01	0.01	1.0	0.1492	0.0583	0.1164	0.0135	0.0768	* 0.0125
	n=400	0.01	0.01	1.0	0.1493	0.0644	0.1189	0.0160	0.0809	* 0.0150
	n=800	0.01	0.01	1.0	0.1497	0.0677	0.1213	0.0186	0.0839	* 0.0175
II	n=50	0.05	0.01	0.2	0.1551	0.0166	0.1331	0.0526	0.0869	0.0166
	n=100	0.05	0.01	0.2	0.1562	0.0338	0.1462	0.0664	0.0954	* 0.0311
	n=200	0.05	0.01	0.2	0.1563	0.0420	0.1574	0.0793	0.1032	* 0.0404
	n=400	0.05	0.01	0.2	0.1569	0.0444	0.1683	0.0915	0.1099	* 0.0442
	n=800	0.05	0.01	0.2	0.1569	0.0445	0.1776	0.1029	0.1158	0.0454
III	n=50	0.01	0.05	5.0	0.1264	0.0551	0.0956	0.0276	0.0573	0.0338
	n=100	0.01	0.05	5.0	0.1286	0.0558	0.1013	0.0193	0.0636	0.0236
	n=200	0.01	0.05	5.0	0.1293	0.0576	0.1062	0.0140	0.0685	0.0179
	n=400	0.01	0.05	5.0	0.1297	0.0593	0.1099	0.0127	0.0717	0.0163
	n=800	0.01	0.05	5.0	0.1300	0.0603	0.1128	0.0139	0.0741	0.0164
IV	n=50	0.05	0.05	1.0	0.1316	0.0596	0.1168	0.0481	0.0719	* 0.0466
	n=100	0.05	0.05	1.0	0.1333	0.0571	0.1291	0.0548	0.0815	* 0.0449
	n=200	0.05	0.05	1.0	0.1334	0.0564	0.1401	0.0646	0.0896	* 0.0449
	n=400	0.05	0.05	1.0	0.1336	0.0566	0.1501	0.0753	0.0963	* 0.0466
	n=800	0.05	0.05	1.0	0.1337	0.0566	0.1600	0.0863	0.1021	* 0.0488
V	n=50	0.01	0.15	15.0	0.1086	0.0693	0.0836	0.0581	* 0.0547	0.0626
	n=100	0.01	0.15	15.0	0.1105	0.0657	0.0877	0.0400	0.0578	0.0433
	n=200	0.01	0.15	15.0	0.1113	0.0688	0.0923	0.0261	0.0643	0.0302
	n=400	0.01	0.15	15.0	0.1120	0.1181	0.0967	0.0171	0.0737	0.0219
	n=800	0.01	0.15	15.0	0.1121	0.1196	0.0998	0.0131	0.0767	0.0188
VI	n=50	0.05	0.15	3.0	0.1142	0.0815	0.0980	0.0638	0.0674	0.0680
	n=100	0.05	0.15	3.0	0.1151	0.0778	0.1074	0.0539	0.0741	* 0.0537
	n=200	0.05	0.15	3.0	0.1154	0.0752	0.1172	0.0533	0.0807	* 0.0483
	n=400	0.05	0.15	3.0	0.1153	0.0776	0.1264	0.0588	0.0905	* 0.0489
	n=800	0.05	0.15	3.0	0.1154	0.1234	0.1359	0.0676	0.0998	* 0.0528
	min	–	–	–	0.1086	0.0166	0.0836	0.0106	0.0547	0.0106
	med	–	–	–	0.1308	0.0588	0.1166	0.0503	0.0768	* 0.0371
	mean	–	–	–	0.1321	0.0633	0.1208	0.0444	0.0803	* 0.0344
	max	–	–	–	0.1569	0.1234	0.1776	0.1029	0.1158	* 0.0680

Note: The data generating process is based on DEA. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

A.2 Bias

A.2.1 Cases 1-4: "SFA-CD"

Table A13: Performance of the Six Methods for Case 1 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	-0.0078	−0.0085	−0.0621	−0.0434	−0.0313	* -0.0020
	n=100	0.01	0.01	1.0	−0.0260	-0.0081	−0.0724	−0.0535	−0.0393	* -0.0025
	n=200	0.01	0.01	1.0	−0.0560	-0.0089	−0.0789	−0.0597	−0.0493	* -0.0030
	n=400	0.01	0.01	1.0	−0.0785	-0.0099	−0.0833	−0.0640	−0.0575	* -0.0078
	n=800	0.01	0.01	1.0	−0.0871	-0.0101	−0.0868	−0.0674	−0.0611	* -0.0090
II	n=50	0.05	0.01	0.2	−0.0022	-0.0021	−0.0936	−0.0689	−0.0475	* 0.0015
	n=100	0.05	0.01	0.2	−0.0024	-0.0022	−0.1124	−0.0895	−0.0562	* 0.0000
	n=200	0.05	0.01	0.2	−0.0021	-0.0020	−0.1285	−0.1067	−0.0636	* -0.0008
	n=400	0.05	0.01	0.2	−0.0019	-0.0018	−0.1424	−0.1215	−0.0704	* -0.0011
	n=800	0.05	0.01	0.2	-0.0017	-0.0017	−0.1547	−0.1347	−0.0761	* -0.0012
III	n=50	0.01	0.05	5.0	0.0012	0.0043	−0.0426	−0.0226	−0.0146	0.0125
	n=100	0.01	0.05	5.0	0.0003	0.0002	−0.0555	−0.0363	−0.0226	0.0069
	n=200	0.01	0.05	5.0	0.0000	−0.0004	−0.0650	−0.0460	−0.0277	0.0038
	n=400	0.01	0.05	5.0	0.0000	−0.0001	−0.0716	−0.0527	−0.0310	0.0021
	n=800	0.01	0.05	5.0	0.0000	0.0000	−0.0765	−0.0576	−0.0334	0.0012
IV	n=50	0.05	0.05	1.0	0.0018	0.0063	−0.0693	−0.0441	−0.0257	0.0253
	n=100	0.05	0.05	1.0	0.0010	0.0015	−0.0897	−0.0659	−0.0367	0.0121
	n=200	0.05	0.05	1.0	0.0005	0.0007	−0.1065	−0.0841	−0.0456	0.0064
	n=400	0.05	0.05	1.0	0.0004	0.0006	−0.1216	−0.1004	−0.0539	0.0040
	n=800	0.05	0.05	1.0	0.0001	0.0003	−0.1344	−0.1142	−0.0611	0.0025
V	n=50	0.01	0.15	15.0	0.0069	0.0148	−0.0147	0.0109	* 0.0050	0.0361
	n=100	0.01	0.15	15.0	0.0044	0.0069	−0.0322	−0.0101	-0.0075	0.0223
	n=200	0.01	0.15	15.0	0.0037	0.0006	−0.0457	−0.0260	−0.0180	0.0134
	n=400	0.01	0.15	15.0	0.0039	−0.0656	−0.0558	−0.0375	−0.0374	0.0083
	n=800	0.01	0.15	15.0	0.0038	−0.0672	−0.0630	−0.0452	−0.0426	0.0059
VI	n=50	0.05	0.15	3.0	0.0005	0.0049	−0.0347	−0.0042	-0.0062	0.0319
	n=100	0.05	0.15	3.0	-0.0002	−0.0014	−0.0574	−0.0306	−0.0218	0.0168
	n=200	0.05	0.15	3.0	-0.0001	−0.0004	−0.0764	−0.0524	−0.0320	0.0096
	n=400	0.05	0.15	3.0	0.0000	0.0001	−0.0926	−0.0709	−0.0405	0.0057
	n=800	0.05	0.15	3.0	0.0000	0.0000	−0.1070	−0.0866	−0.0482	0.0034
	min	–	–	–	−0.0871	-0.0672	−0.1547	−0.1347	−0.0761	* -0.0090
	med	–	–	–	0.0000	−0.0002	−0.0765	−0.0556	−0.0383	0.0039
	mean	–	–	–	−0.0079	-0.0050	−0.0809	−0.0595	−0.0385	0.0068
	max	–	–	–	0.0069	0.0148	−0.0147	0.0109	* 0.0050	0.0361

Note: The data generating process is based on a Cobb-Douglas cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A14: Performance of the Six Methods for Case 2 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	−0.0398	-0.0018	−0.0597	−0.0408	−0.0354	* 0.0009
	n=100	0	0.01	Inf	−0.0149	-0.0007	−0.0687	−0.0492	−0.0341	0.0007
	n=200	0	0.01	Inf	−0.0569	-0.0005	−0.0728	−0.0528	−0.0446	* 0.0004
	n=400	0	0.01	Inf	−0.0638	-0.0005	−0.0744	−0.0543	−0.0470	* 0.0002
	n=800	0	0.01	Inf	−0.0662	-0.0005	−0.0752	−0.0551	−0.0477	* 0.0000
II	n=50	0	0.05	Inf	0.0037	0.0069	−0.0410	−0.0212	-0.0127	0.0139
	n=100	0	0.05	Inf	0.0028	0.0040	−0.0533	−0.0344	−0.0201	0.0091
	n=200	0	0.05	Inf	0.0024	0.0023	−0.0620	−0.0430	−0.0248	0.0064
	n=400	0	0.05	Inf	0.0025	0.0014	−0.0674	−0.0480	−0.0275	0.0053
	n=800	0	0.05	Inf	0.0027	0.0019	−0.0703	−0.0508	−0.0290	0.0050
III	n=50	0	0.15	Inf	0.0082	0.0155	−0.0138	0.0112	* 0.0057	0.0364
	n=100	0	0.15	Inf	0.0059	0.0084	−0.0310	−0.0092	-0.0062	0.0233
	n=200	0	0.15	Inf	0.0054	0.0031	−0.0441	−0.0246	−0.0164	0.0150
	n=400	0	0.15	Inf	0.0054	−0.0653	−0.0536	−0.0352	−0.0359	0.0099
	n=800	0	0.15	Inf	0.0048	−0.0667	−0.0601	−0.0420	−0.0408	0.0070
	min	–	–	–	−0.0662	−0.0667	−0.0752	-0.0551	−0.0477	* 0.0000
	med	–	–	–	0.0027	0.0014	−0.0601	−0.0420	−0.0290	0.0064
	mean	–	–	–	−0.0132	-0.0062	−0.0565	−0.0366	−0.0278	0.0089
	max	–	–	–	0.0082	0.0155	−0.0138	0.0112	* 0.0057	0.0364

Note: The data generating process is based on a Cobb-Douglas cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A15: Performance of the Six Methods for Case 3 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	-0.0062	−0.0064	−0.0610	−0.0422	−0.0288	∗ -0.0011
	n=100	0.01	0.01	1.0	−0.0066	-0.0058	−0.0715	−0.0527	−0.0343	∗ -0.0027
	n=200	0.01	0.01	1.0	−0.0161	-0.0050	−0.0785	−0.0594	−0.0396	∗ -0.0037
	n=400	0.01	0.01	1.0	−0.0425	-0.0046	−0.0829	−0.0637	−0.0485	∗ -0.0041
	n=800	0.01	0.01	1.0	−0.0336	-0.0046	−0.0866	−0.0672	−0.0480	∗ -0.0043
II	n=50	0.05	0.01	0.2	−0.0012	-0.0009	−0.0917	−0.0677	−0.0460	0.0033
	n=100	0.05	0.01	0.2	−0.0015	-0.0013	−0.1108	−0.0879	−0.0549	0.0016
	n=200	0.05	0.01	0.2	-0.0051	−0.0051	−0.1271	−0.1053	−0.0627	∗ 0.0008
	n=400	0.05	0.01	0.2	−0.0065	-0.0059	−0.1410	−0.1202	−0.0696	∗ 0.0003
	n=800	0.05	0.01	0.2	−0.0055	-0.0054	−0.1538	−0.1339	−0.0755	∗ 0.0004
III	n=50	0.01	0.05	5.0	−0.0041	-0.0003	−0.0416	−0.0210	−0.0168	0.0106
	n=100	0.01	0.05	5.0	-0.0053	−0.0119	−0.0551	−0.0359	−0.0324	∗ 0.0031
	n=200	0.01	0.05	5.0	-0.0043	−0.0895	−0.0650	−0.0462	−0.0496	∗ 0.0004
	n=400	0.01	0.05	5.0	-0.0055	−0.0909	−0.0717	−0.0529	−0.0546	∗ -0.0027
	n=800	0.01	0.05	5.0	-0.0062	−0.0912	−0.0765	−0.0578	−0.0576	∗ -0.0048
IV	n=50	0.05	0.05	1.0	−0.0209	-0.0200	−0.0677	−0.0416	−0.0354	∗ -0.0002
	n=100	0.05	0.05	1.0	-0.0210	−0.0212	−0.0882	−0.0641	−0.0479	∗ -0.0099
	n=200	0.05	0.05	1.0	-0.0213	-0.0213	−0.1055	−0.0829	−0.0573	∗ -0.0150
	n=400	0.05	0.05	1.0	-0.0215	−0.0216	−0.1201	−0.0990	−0.0661	∗ -0.0178
	n=800	0.05	0.05	1.0	-0.0216	−0.0737	−0.1332	−0.1132	−0.0793	∗ -0.0198
V	n=50	0.01	0.15	15.0	−0.0017	0.0001	−0.0164	0.0099	-0.0023	0.0311
	n=100	0.01	0.15	15.0	−0.0044	-0.0030	−0.0336	−0.0118	-0.0134	0.0172
	n=200	0.01	0.15	15.0	-0.0058	−0.0890	−0.0468	−0.0276	−0.0385	0.0062
	n=400	0.01	0.15	15.0	-0.0060	−0.0950	−0.0563	−0.0383	−0.0477	∗ 0.0008
	n=800	0.01	0.15	15.0	−0.0060	-0.0059	−0.0630	−0.0457	−0.0302	∗ 0.0027
VI	n=50	0.05	0.15	3.0	−0.0220	−0.0145	−0.0354	-0.0044	-0.0190	0.0190
	n=100	0.05	0.15	3.0	−0.0261	-0.0251	−0.0578	−0.0311	−0.0351	∗ -0.0011
	n=200	0.05	0.15	3.0	-0.0270	−0.0930	−0.0768	−0.0529	−0.0571	∗ -0.0146
	n=400	0.05	0.15	3.0	-0.0271	−0.1018	−0.0927	−0.0713	−0.0727	∗ -0.0213
	n=800	0.05	0.15	3.0	-0.0271	−0.1013	−0.1066	−0.0866	−0.0751	∗ -0.0238
	min	–	–	–	-0.0425	−0.1018	−0.1538	−0.1339	−0.0793	∗ -0.0238
	med	–	–	–	-0.0064	−0.0132	−0.0767	−0.0554	−0.0480	∗ -0.0006
	mean	–	–	–	-0.0137	−0.0338	−0.0805	−0.0591	−0.0465	∗ -0.0016
	max	–	–	–	−0.0012	0.0001	−0.0164	0.0099	-0.0023	0.0311

Note: The data generating process is based on a Cobb-Douglas cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A16: Performance of the Six Methods for Case 4 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	−0.0027	-0.0005	−0.0589	−0.0401	−0.0262	0.0021
	n=100	0	0.01	Inf	−0.0096	-0.0004	−0.0681	−0.0488	−0.0316	0.0012
	n=200	0	0.01	Inf	−0.0187	-0.0004	−0.0728	−0.0529	−0.0362	0.0006
	n=400	0	0.01	Inf	−0.0106	-0.0004	−0.0744	−0.0543	−0.0349	* 0.0003
	n=800	0	0.01	Inf	−0.0089	-0.0004	−0.0751	−0.0550	−0.0347	* 0.0002
II	n=50	0	0.05	Inf	0.0005	0.0025	−0.0405	−0.0201	−0.0145	0.0129
	n=100	0	0.05	Inf	0.0008	−0.0011	−0.0533	−0.0342	−0.0255	0.0075
	n=200	0	0.05	Inf	0.0016	−0.0882	−0.0622	−0.0432	−0.0463	0.0052
	n=400	0	0.05	Inf	0.0015	−0.0897	−0.0676	−0.0482	−0.0505	0.0037
	n=800	0	0.05	Inf	0.0012	−0.0903	−0.0702	−0.0508	−0.0523	0.0026
III	n=50	0	0.15	Inf	0.0003	0.0013	−0.0150	0.0109	-0.0010	0.0322
	n=100	0	0.15	Inf	−0.0016	-0.0014	−0.0325	−0.0107	−0.0119	0.0187
	n=200	0	0.15	Inf	-0.0024	−0.0890	−0.0452	−0.0259	−0.0370	0.0088
	n=400	0	0.15	Inf	-0.0019	−0.0945	−0.0541	−0.0361	−0.0456	0.0042
	n=800	0	0.15	Inf	-0.0015	−0.0028	−0.0600	−0.0421	−0.0267	0.0060
	min	–	–	–	-0.0187	−0.0945	−0.0751	−0.0550	−0.0523	* 0.0002
	med	–	–	–	−0.0016	-0.0011	−0.0600	−0.0421	−0.0347	0.0042
	mean	–	–	–	-0.0035	−0.0304	−0.0567	−0.0368	−0.0317	0.0071
	max	–	–	–	0.0016	0.0025	−0.0150	0.0109	* -0.0010	0.0322

Note: The data generating process is based on a Cobb-Douglas cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

A.2.2 Cases 5-8: "SFA-TL"

Table A17: Performance of the Six Methods for Case 5 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	−0.0711	−0.0086	−0.0385	-0.0030	−0.0301	* -0.0004
	n=100	0.01	0.01	1.0	−0.0747	-0.0075	−0.0455	−0.0079	−0.0335	* -0.0026
	n=200	0.01	0.01	1.0	−0.0762	-0.0068	−0.0508	−0.0125	−0.0361	* -0.0035
	n=400	0.01	0.01	1.0	−0.0767	-0.0097	−0.0547	−0.0162	−0.0385	* -0.0066
	n=800	0.01	0.01	1.0	−0.0769	-0.0101	−0.0580	−0.0195	−0.0406	* -0.0081
II	n=50	0.05	0.01	0.2	−0.0730	-0.0020	−0.0753	−0.0388	−0.0495	* 0.0011
	n=100	0.05	0.01	0.2	−0.0747	-0.0020	−0.0922	−0.0546	−0.0581	* -0.0003
	n=200	0.05	0.01	0.2	−0.0757	-0.0019	−0.1076	−0.0698	−0.0657	* -0.0009
	n=400	0.05	0.01	0.2	−0.0761	-0.0018	−0.1206	−0.0834	−0.0724	* -0.0012
	n=800	0.05	0.01	0.2	−0.0762	-0.0016	−0.1327	−0.0960	−0.0783	* -0.0013
III	n=50	0.01	0.05	5.0	−0.0461	0.0045	−0.0212	0.0140	-0.0116	0.0177
	n=100	0.01	0.05	5.0	−0.0502	0.0003	−0.0311	0.0061	−0.0184	0.0104
	n=200	0.01	0.05	5.0	−0.0516	-0.0004	−0.0387	−0.0010	−0.0228	0.0061
	n=400	0.01	0.05	5.0	−0.0524	-0.0001	−0.0443	−0.0066	−0.0257	0.0038
	n=800	0.01	0.05	5.0	−0.0526	0.0000	−0.0488	−0.0112	−0.0281	0.0026
IV	n=50	0.05	0.05	1.0	−0.0444	0.0082	−0.0519	−0.0152	−0.0249	0.0250
	n=100	0.05	0.05	1.0	−0.0463	0.0035	−0.0703	−0.0326	−0.0354	0.0112
	n=200	0.05	0.05	1.0	−0.0473	0.0018	−0.0861	−0.0485	−0.0440	0.0057
	n=400	0.05	0.05	1.0	−0.0479	0.0011	−0.1004	−0.0632	−0.0517	0.0035
	n=800	0.05	0.05	1.0	−0.0479	0.0006	−0.1129	−0.0764	−0.0586	0.0021
V	n=50	0.01	0.15	15.0	−0.0147	0.0149	0.0027	0.0392	0.0117	0.0470
	n=100	0.01	0.15	15.0	−0.0181	0.0070	−0.0125	0.0247	* 0.0008	0.0325
	n=200	0.01	0.15	15.0	−0.0196	0.0005	−0.0234	0.0131	-0.0084	0.0225
	n=400	0.01	0.15	15.0	−0.0198	−0.0656	−0.0320	0.0041	−0.0270	0.0151
	n=800	0.01	0.15	15.0	−0.0200	−0.0672	−0.0382	-0.0027	−0.0317	0.0101
VI	n=50	0.05	0.15	3.0	−0.0113	0.0055	−0.0194	0.0199	* 0.0018	0.0427
	n=100	0.05	0.15	3.0	−0.0130	-0.0008	−0.0402	−0.0018	-0.0131	0.0256
	n=200	0.05	0.15	3.0	−0.0141	0.0001	−0.0580	−0.0206	−0.0229	0.0172
	n=400	0.05	0.15	3.0	−0.0146	0.0004	−0.0739	−0.0375	−0.0312	0.0130
	n=800	0.05	0.15	3.0	−0.0148	0.0001	−0.0873	−0.0520	−0.0386	0.0107
	min	–	–	–	−0.0769	-0.0672	−0.1327	−0.0960	−0.0783	* -0.0081
	med	–	–	–	−0.0479	0.0000	−0.0513	−0.0138	−0.0314	0.0059
	mean	–	–	–	−0.0466	-0.0046	−0.0588	−0.0217	−0.0328	0.0100
	max	–	–	–	−0.0113	0.0149	0.0027	0.0392	0.0117	0.0470

Note: The data generating process is based on a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A18: Performance of the Six Methods for Case 6 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	−0.0697	−0.0018	−0.0349	0.0011	−0.0267	0.0035
	n=100	0	0.01	Inf	−0.0733	-0.0007	−0.0402	−0.0021	−0.0292	0.0023
	n=200	0	0.01	Inf	−0.0747	-0.0005	−0.0429	−0.0042	−0.0305	0.0014
	n=400	0	0.01	Inf	−0.0749	-0.0005	−0.0441	−0.0054	−0.0311	0.0009
	n=800	0	0.01	Inf	−0.0741	-0.0005	−0.0448	−0.0062	−0.0312	0.0005
II	n=50	0	0.05	Inf	−0.0461	0.0069	−0.0193	0.0160	-0.0102	0.0187
	n=100	0	0.05	Inf	−0.0505	0.0040	−0.0282	0.0090	−0.0163	0.0127
	n=200	0	0.05	Inf	−0.0524	0.0023	−0.0347	0.0033	−0.0201	0.0087
	n=400	0	0.05	Inf	−0.0531	0.0015	−0.0387	-0.0008	−0.0224	0.0064
	n=800	0	0.05	Inf	−0.0535	0.0018	−0.0411	−0.0033	−0.0237	0.0057
III	n=50	0	0.15	Inf	−0.0142	0.0154	0.0032	0.0398	0.0121	0.0473
	n=100	0	0.15	Inf	−0.0187	0.0084	−0.0107	0.0261	* 0.0016	0.0332
	n=200	0	0.15	Inf	−0.0201	0.0031	−0.0212	0.0153	-0.0070	0.0236
	n=400	0	0.15	Inf	−0.0205	−0.0653	−0.0288	0.0072	−0.0258	0.0165
	n=800	0	0.15	Inf	−0.0206	−0.0667	−0.0339	0.0017	−0.0296	0.0121
	min	–	–	–	−0.0749	−0.0667	−0.0448	-0.0062	−0.0312	* 0.0005
	med	–	–	–	−0.0524	0.0015	−0.0347	0.0017	−0.0237	0.0087
	mean	–	–	–	−0.0478	-0.0062	−0.0307	0.0065	−0.0193	0.0129
	max	–	–	–	−0.0142	0.0154	0.0032	0.0398	0.0121	0.0473

Note: The data generating process is based on a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A19: Performance of the Six Methods for Case 7 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	−0.0694	−0.0064	−0.0377	-0.0022	−0.0287	* 0.0006
	n=100	0.01	0.01	1.0	−0.0735	-0.0057	−0.0451	−0.0075	−0.0329	* -0.0018
	n=200	0.01	0.01	1.0	−0.0749	-0.0049	−0.0504	−0.0121	−0.0355	* -0.0030
	n=400	0.01	0.01	1.0	−0.0756	-0.0046	−0.0544	−0.0161	−0.0376	* -0.0036
	n=800	0.01	0.01	1.0	−0.0758	-0.0046	−0.0579	−0.0195	−0.0394	* -0.0041
II	n=50	0.05	0.01	0.2	−0.0706	-0.0013	−0.0740	−0.0377	−0.0478	0.0025
	n=100	0.05	0.01	0.2	−0.0732	-0.0014	−0.0909	−0.0534	−0.0567	* 0.0009
	n=200	0.05	0.01	0.2	−0.0740	-0.0035	−0.1061	−0.0686	−0.0646	* 0.0002
	n=400	0.05	0.01	0.2	−0.0743	-0.0056	−0.1194	−0.0822	−0.0711	* -0.0008
	n=800	0.05	0.01	0.2	−0.0745	-0.0056	−0.1317	−0.0950	−0.0773	* -0.0011
III	n=50	0.01	0.05	5.0	−0.0479	-0.0002	−0.0211	0.0141	-0.0136	0.0171
	n=100	0.01	0.05	5.0	−0.0525	−0.0108	−0.0314	0.0057	−0.0266	0.0084
	n=200	0.01	0.05	5.0	−0.0543	−0.0894	−0.0392	-0.0017	−0.0451	* 0.0013
	n=400	0.01	0.05	5.0	−0.0551	−0.0907	−0.0448	-0.0074	−0.0491	* -0.0041
	n=800	0.01	0.05	5.0	−0.0553	−0.0912	−0.0491	-0.0119	−0.0518	* -0.0080
IV	n=50	0.05	0.05	1.0	−0.0475	−0.0191	−0.0509	-0.0136	−0.0299	* 0.0066
	n=100	0.05	0.05	1.0	−0.0495	-0.0211	−0.0693	−0.0314	−0.0421	* -0.0065
	n=200	0.05	0.05	1.0	−0.0506	-0.0212	−0.0852	−0.0477	−0.0509	* -0.0124
	n=400	0.05	0.05	1.0	−0.0510	-0.0214	−0.0994	−0.0625	−0.0592	* -0.0158
	n=800	0.05	0.05	1.0	-0.0512	−0.0746	−0.1121	−0.0758	−0.0723	* -0.0210
V	n=50	0.01	0.15	15.0	−0.0334	0.0001	−0.0003	0.0364	0.0007	0.0407
	n=100	0.01	0.15	15.0	−0.0391	-0.0030	−0.0142	0.0219	-0.0086	0.0254
	n=200	0.01	0.15	15.0	−0.0420	−0.0895	−0.0251	0.0107	−0.0337	0.0143
	n=400	0.01	0.15	15.0	−0.0430	−0.0947	−0.0327	0.0023	−0.0413	0.0066
	n=800	0.01	0.15	15.0	−0.0434	−0.0058	−0.0386	-0.0040	−0.0229	0.0071
VI	n=50	0.05	0.15	3.0	−0.0400	-0.0146	−0.0210	0.0186	* -0.0137	0.0287
	n=100	0.05	0.15	3.0	−0.0441	−0.0255	−0.0414	-0.0034	−0.0286	0.0080
	n=200	0.05	0.15	3.0	−0.0450	−0.0935	−0.0588	-0.0223	−0.0503	* -0.0064
	n=400	0.05	0.15	3.0	−0.0451	−0.1019	−0.0742	-0.0388	−0.0646	* -0.0174
	n=800	0.05	0.15	3.0	-0.0453	−0.1011	−0.0875	−0.0530	−0.0677	* -0.0208
	min	–	–	–	-0.0758	−0.1019	−0.1317	−0.0950	−0.0773	* -0.0210
	med	–	–	–	−0.0511	-0.0127	−0.0507	−0.0129	−0.0417	* -0.0003
	mean	–	–	–	−0.0557	−0.0338	−0.0588	-0.0219	−0.0421	* 0.0014
	max	–	–	–	−0.0334	0.0001	−0.0003	0.0364	0.0007	0.0407

Note: The data generating process is based on a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A20: Performance of the Six Methods for Case 8 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	-0.0684	-0.0005	-0.0345	0.0015	-0.0256	0.0041
	n=100	0	0.01	Inf	-0.0723	-0.0004	-0.0400	-0.0020	-0.0287	0.0026
	n=200	0	0.01	Inf	-0.0737	-0.0004	-0.0429	-0.0043	-0.0302	0.0015
	n=400	0	0.01	Inf	-0.0740	-0.0003	-0.0442	-0.0056	-0.0309	0.0010
	n=800	0	0.01	Inf	-0.0735	-0.0003	-0.0448	-0.0063	-0.0311	0.0006
II	n=50	0	0.05	Inf	-0.0481	0.0025	-0.0193	0.0161	-0.0121	0.0186
	n=100	0	0.05	Inf	-0.0527	-0.0013	-0.0286	0.0085	-0.0217	0.0111
	n=200	0	0.05	Inf	-0.0549	-0.0882	-0.0349	0.0026	-0.0426	0.0052
	n=400	0	0.05	Inf	-0.0559	-0.0897	-0.0390	-0.0014	-0.0461	* 0.0007
	n=800	0	0.05	Inf	-0.0564	-0.0902	-0.0412	-0.0038	-0.0478	* -0.0017
III	n=50	0	0.15	Inf	-0.0328	0.0012	0.0011	0.0378	0.0019	0.0417
	n=100	0	0.15	Inf	-0.0390	-0.0015	-0.0128	0.0236	-0.0075	0.0268
	n=200	0	0.15	Inf	-0.0420	-0.0889	-0.0228	0.0131	-0.0324	0.0163
	n=400	0	0.15	Inf	-0.0432	-0.0944	-0.0296	0.0056	-0.0396	0.0091
	n=800	0	0.15	Inf	-0.0439	-0.0028	-0.0341	0.0006	-0.0201	0.0099
	min	–	–	–	-0.0740	-0.0944	-0.0448	-0.0063	-0.0478	* -0.0017
	med	–	–	–	-0.0549	-0.0013	-0.0345	0.0015	-0.0302	0.0052
	mean	–	–	–	-0.0554	-0.0304	-0.0312	0.0057	-0.0276	0.0098
	max	–	–	–	-0.0328	0.0025	0.0011	0.0378	0.0019	0.0417

Note: The data generating process is based on a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

A.2.3 Cases 9-12: "TL-DEA"

Table A21: Performance of the Six Methods for Case 9 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	−0.0703	−0.0069	−0.0333	0.0003	−0.0267	0.0025
	n=100	0.01	0.01	1.0	−0.0732	0.0003	−0.0398	−0.0036	−0.0292	0.0030
	n=200	0.01	0.01	1.0	−0.0745	-0.0003	−0.0446	−0.0075	−0.0316	0.0019
	n=400	0.01	0.01	1.0	−0.0750	-0.0010	−0.0484	−0.0110	−0.0335	* 0.0009
	n=800	0.01	0.01	1.0	−0.0750	-0.0017	−0.0517	−0.0142	−0.0353	* -0.0002
II	n=50	0.05	0.01	0.2	−0.0743	-0.0021	−0.0725	−0.0365	−0.0485	* 0.0011
	n=100	0.05	0.01	0.2	−0.0757	-0.0022	−0.0884	−0.0519	−0.0568	* -0.0005
	n=200	0.05	0.01	0.2	−0.0768	-0.0021	−0.1033	−0.0666	−0.0643	* -0.0010
	n=400	0.05	0.01	0.2	−0.0772	-0.0020	−0.1164	−0.0802	−0.0708	* -0.0013
	n=800	0.05	0.01	0.2	−0.0774	-0.0018	−0.1282	−0.0929	−0.0767	* -0.0014
III	n=50	0.01	0.05	5.0	−0.0461	0.0035	−0.0164	0.0168	-0.0099	0.0195
	n=100	0.01	0.05	5.0	−0.0495	-0.0004	−0.0258	0.0096	−0.0163	0.0124
	n=200	0.01	0.05	5.0	−0.0508	-0.0004	−0.0328	0.0034	−0.0200	0.0080
	n=400	0.01	0.05	5.0	−0.0513	-0.0003	−0.0384	−0.0018	−0.0228	0.0052
	n=800	0.01	0.05	5.0	−0.0516	-0.0003	−0.0428	−0.0062	−0.0251	0.0035
IV	n=50	0.05	0.05	1.0	−0.0453	0.0098	−0.0497	−0.0130	-0.0242	0.0252
	n=100	0.05	0.05	1.0	−0.0475	0.0048	−0.0670	−0.0301	−0.0342	0.0125
	n=200	0.05	0.05	1.0	−0.0486	0.0011	−0.0820	−0.0456	−0.0428	0.0053
	n=400	0.05	0.05	1.0	−0.0489	0.0009	−0.0962	−0.0603	−0.0503	0.0034
	n=800	0.05	0.05	1.0	−0.0491	0.0005	−0.1086	−0.0733	−0.0570	0.0022
V	n=50	0.01	0.15	15.0	−0.0144	0.0141	0.0054	0.0409	0.0126	0.0481
	n=100	0.01	0.15	15.0	−0.0186	0.0056	−0.0083	0.0271	* 0.0019	0.0337
	n=200	0.01	0.15	15.0	−0.0201	-0.0008	−0.0186	0.0163	-0.0063	0.0238
	n=400	0.01	0.15	15.0	−0.0206	−0.0658	−0.0266	0.0079	−0.0249	0.0168
	n=800	0.01	0.15	15.0	−0.0208	−0.0675	−0.0327	0.0017	−0.0296	0.0122
VI	n=50	0.05	0.15	3.0	−0.0122	0.0052	−0.0181	0.0219	* 0.0022	0.0430
	n=100	0.05	0.15	3.0	−0.0139	-0.0006	−0.0381	0.0006	-0.0122	0.0261
	n=200	0.05	0.15	3.0	−0.0147	0.0002	−0.0549	−0.0183	−0.0218	0.0174
	n=400	0.05	0.15	3.0	−0.0152	0.0001	−0.0701	−0.0347	−0.0298	0.0128
	n=800	0.05	0.15	3.0	−0.0156	0.0002	−0.0834	−0.0493	−0.0369	0.0105
	min	–	–	–	−0.0774	-0.0675	−0.1282	−0.0929	−0.0767	* -0.0014
	med	–	–	–	−0.0490	-0.0003	−0.0465	−0.0092	−0.0294	0.0067
	mean	–	–	–	−0.0468	-0.0037	−0.0544	−0.0184	−0.0307	0.0116
	max	–	–	–	−0.0122	0.0141	0.0054	0.0409	0.0126	0.0481

Note: The data generating process is based on DEA on the fitted values of a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A22: Performance of the Six Methods for Case 10 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	−0.0692	−0.0082	−0.0296	0.0045	−0.0255	0.0047
	n=100	0	0.01	Inf	−0.0720	−0.0087	−0.0341	0.0028	−0.0278	0.0031
	n=200	0	0.01	Inf	−0.0731	−0.0104	−0.0364	0.0013	−0.0292	0.0017
	n=400	0	0.01	Inf	−0.0737	−0.0115	−0.0377	0.0002	−0.0303	0.0005
	n=800	0	0.01	Inf	−0.0715	−0.0121	−0.0384	-0.0007	−0.0305	* -0.0003
II	n=50	0	0.05	Inf	−0.0459	0.0047	−0.0143	0.0188	-0.0088	0.0204
	n=100	0	0.05	Inf	−0.0498	0.0004	−0.0228	0.0127	−0.0146	0.0142
	n=200	0	0.05	Inf	−0.0512	-0.0013	−0.0286	0.0080	−0.0181	0.0099
	n=400	0	0.05	Inf	−0.0517	-0.0013	−0.0325	0.0045	−0.0201	0.0071
	n=800	0	0.05	Inf	−0.0521	-0.0013	−0.0349	0.0022	−0.0214	0.0056
III	n=50	0	0.15	Inf	−0.0142	0.0148	0.0065	0.0419	0.0132	0.0487
	n=100	0	0.15	Inf	−0.0194	0.0069	-0.0066	0.0286	* 0.0027	0.0343
	n=200	0	0.15	Inf	−0.0204	0.0008	−0.0164	0.0186	-0.0052	0.0250
	n=400	0	0.15	Inf	−0.0211	−0.0654	−0.0234	0.0114	−0.0235	0.0185
	n=800	0	0.15	Inf	−0.0216	−0.0670	−0.0282	0.0065	−0.0273	0.0146
	min	–	–	–	−0.0737	−0.0670	−0.0384	-0.0007	−0.0305	* -0.0003
	med	–	–	–	−0.0512	−0.0013	−0.0286	0.0065	−0.0214	0.0099
	mean	–	–	–	−0.0471	-0.0106	−0.0252	0.0108	−0.0178	0.0139
	max	–	–	–	−0.0142	0.0148	0.0065	0.0419	0.0132	0.0487

Note: The data generating process is based on DEA on the fitted values of a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A23: Performance of the Six Methods for Case 11 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	−0.0687	−0.0070	−0.0325	0.0009	−0.0261	0.0028
	n=100	0.01	0.01	1.0	−0.0721	−0.0064	−0.0393	-0.0032	−0.0298	∗ -0.0002
	n=200	0.01	0.01	1.0	−0.0732	-0.0062	−0.0443	−0.0072	−0.0325	∗ -0.0024
	n=400	0.01	0.01	1.0	−0.0738	-0.0061	−0.0482	−0.0109	−0.0346	∗ -0.0038
	n=800	0.01	0.01	1.0	−0.0740	-0.0060	−0.0516	−0.0142	−0.0363	∗ -0.0046
II	n=50	0.05	0.01	0.2	−0.0718	-0.0007	−0.0709	−0.0351	−0.0468	0.0027
	n=100	0.05	0.01	0.2	−0.0742	-0.0011	−0.0873	−0.0507	−0.0556	∗ 0.0008
	n=200	0.05	0.01	0.2	−0.0749	-0.0027	−0.1019	−0.0654	−0.0630	∗ 0.0002
	n=400	0.05	0.01	0.2	−0.0753	-0.0056	−0.1152	−0.0792	−0.0697	∗ -0.0003
	n=800	0.05	0.01	0.2	−0.0755	-0.0052	−0.1273	−0.0919	−0.0757	∗ -0.0003
III	n=50	0.01	0.05	5.0	−0.0477	-0.0022	−0.0166	0.0167	-0.0123	0.0186
	n=100	0.01	0.05	5.0	−0.0518	−0.0798	−0.0262	0.0090	−0.0307	0.0105
	n=200	0.01	0.05	5.0	−0.0532	−0.0903	−0.0334	0.0026	−0.0429	0.0045
	n=400	0.01	0.05	5.0	−0.0539	−0.0915	−0.0388	-0.0026	−0.0466	∗ -0.0003
	n=800	0.01	0.05	5.0	−0.0542	−0.0919	−0.0431	-0.0069	−0.0490	∗ -0.0040
IV	n=50	0.05	0.05	1.0	−0.0483	−0.0192	−0.0483	-0.0113	−0.0290	∗ 0.0069
	n=100	0.05	0.05	1.0	−0.0504	-0.0213	−0.0659	−0.0289	−0.0408	∗ -0.0058
	n=200	0.05	0.05	1.0	−0.0515	-0.0210	−0.0814	−0.0450	−0.0494	∗ -0.0121
	n=400	0.05	0.05	1.0	−0.0516	-0.0216	−0.0952	−0.0593	−0.0574	∗ -0.0158
	n=800	0.05	0.05	1.0	−0.0519	-0.0247	−0.1079	−0.0728	−0.0679	∗ -0.0191
V	n=50	0.01	0.15	15.0	−0.0331	-0.0006	0.0029	0.0387	0.0020	0.0423
	n=100	0.01	0.15	15.0	−0.0388	-0.0048	−0.0102	0.0245	-0.0074	0.0272
	n=200	0.01	0.15	15.0	−0.0413	−0.0898	−0.0202	0.0140	−0.0320	0.0165
	n=400	0.01	0.15	15.0	−0.0422	−0.0952	−0.0276	0.0061	−0.0390	0.0093
	n=800	0.01	0.15	15.0	−0.0426	−0.0075	−0.0332	0.0002	−0.0207	0.0071
VI	n=50	0.05	0.15	3.0	−0.0404	-0.0154	−0.0193	0.0205	∗ -0.0132	0.0296
	n=100	0.05	0.15	3.0	−0.0444	−0.0257	−0.0393	-0.0013	−0.0277	0.0092
	n=200	0.05	0.15	3.0	−0.0450	−0.0929	−0.0558	-0.0198	−0.0486	∗ -0.0051
	n=400	0.05	0.15	3.0	−0.0452	−0.1020	−0.0705	-0.0359	−0.0631	∗ -0.0161
	n=800	0.05	0.15	3.0	-0.0454	−0.1016	−0.0837	−0.0503	−0.0670	∗ -0.0204
	min	–	–	–	-0.0755	−0.1020	−0.1273	−0.0919	−0.0757	∗ -0.0204
	med	–	–	–	−0.0517	−0.0173	−0.0463	-0.0090	−0.0399	∗ 0.0000
	mean	–	–	–	−0.0555	−0.0349	−0.0544	-0.0186	−0.0404	∗ 0.0026
	max	–	–	–	−0.0331	-0.0006	0.0029	0.0387	0.0020	0.0423

Note: The data generating process is based on DEA on the fitted values of a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A24: Performance of the Six Methods for Case 12 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	−0.0676	−0.0057	−0.0290	0.0048	−0.0244	0.0051
	n=100	0	0.01	Inf	−0.0711	−0.0064	−0.0339	0.0028	−0.0271	0.0032
	n=200	0	0.01	Inf	−0.0722	−0.0065	−0.0366	0.0012	−0.0284	0.0017
	n=400	0	0.01	Inf	−0.0728	−0.0065	−0.0378	0.0000	−0.0291	0.0008
	n=800	0	0.01	Inf	−0.0710	−0.0067	−0.0384	-0.0008	−0.0291	* 0.0002
II	n=50	0	0.05	Inf	−0.0476	0.0002	−0.0146	0.0185	-0.0108	0.0201
	n=100	0	0.05	Inf	−0.0517	-0.0093	−0.0232	0.0121	−0.0217	0.0131
	n=200	0	0.05	Inf	−0.0534	−0.0892	−0.0292	0.0072	−0.0403	0.0083
	n=400	0	0.05	Inf	−0.0542	−0.0905	−0.0328	0.0039	−0.0432	0.0051
	n=800	0	0.05	Inf	−0.0545	−0.0910	−0.0350	0.0017	−0.0446	0.0031
III	n=50	0	0.15	Inf	−0.0325	0.0003	0.0038	0.0395	0.0028	0.0430
	n=100	0	0.15	Inf	−0.0384	-0.0032	−0.0084	0.0261	-0.0060	0.0285
	n=200	0	0.15	Inf	−0.0413	−0.0892	−0.0178	0.0164	−0.0305	0.0185
	n=400	0	0.15	Inf	−0.0422	−0.0946	−0.0244	0.0097	−0.0371	0.0122
	n=800	0	0.15	Inf	−0.0428	-0.0050	−0.0285	0.0054	−0.0176	0.0104
	min	–	–	–	−0.0728	−0.0946	−0.0384	-0.0008	−0.0446	* 0.0002
	med	–	–	–	−0.0534	−0.0065	−0.0290	0.0054	−0.0284	0.0083
	mean	–	–	–	−0.0542	−0.0336	−0.0257	0.0099	−0.0258	0.0116
	max	–	–	–	−0.0325	0.0003	0.0038	0.0395	0.0028	0.0430

Note: The data generating process is based on DEA on the fitted values of a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

A.2.4 Cases 13-16: "DEA"

Table A25: Performance of the Six Methods for Case 13 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	−0.1132	−0.0229	−0.0614	0.0004	−0.0499	0.0021
	n=100	0.01	0.01	1.0	−0.1166	−0.0416	−0.0702	-0.0037	−0.0553	* -0.0021
	n=200	0.01	0.01	1.0	−0.1177	−0.0500	−0.0764	-0.0076	−0.0594	* -0.0064
	n=400	0.01	0.01	1.0	−0.1184	−0.0541	−0.0807	-0.0112	−0.0627	* -0.0102
	n=800	0.01	0.01	1.0	−0.1188	−0.0567	−0.0842	-0.0144	−0.0655	* -0.0135
II	n=50	0.05	0.01	0.2	−0.1255	-0.0119	−0.0955	−0.0316	−0.0710	* -0.0043
	n=100	0.05	0.01	0.2	−0.1284	-0.0326	−0.1131	−0.0479	−0.0809	* -0.0206
	n=200	0.05	0.01	0.2	−0.1291	-0.0396	−0.1282	−0.0635	−0.0894	* -0.0297
	n=400	0.05	0.01	0.2	−0.1294	-0.0421	−0.1415	−0.0776	−0.0969	* -0.0348
	n=800	0.05	0.01	0.2	−0.1296	-0.0419	−0.1536	−0.0907	−0.1033	* -0.0369
III	n=50	0.01	0.05	5.0	−0.0889	-0.0008	−0.0430	0.0178	−0.0284	0.0250
	n=100	0.01	0.05	5.0	−0.0919	−0.0168	−0.0541	0.0100	−0.0356	0.0156
	n=200	0.01	0.05	5.0	−0.0930	−0.0243	−0.0631	0.0033	−0.0412	0.0085
	n=400	0.01	0.05	5.0	−0.0935	−0.0283	−0.0692	-0.0020	−0.0458	0.0035
	n=800	0.01	0.05	5.0	−0.0938	−0.0303	−0.0742	-0.0064	−0.0490	* -0.0001
IV	n=50	0.05	0.05	1.0	−0.0986	0.0122	−0.0727	-0.0077	−0.0457	0.0215
	n=100	0.05	0.05	1.0	−0.1004	-0.0095	−0.0906	−0.0254	−0.0564	* 0.0039
	n=200	0.05	0.05	1.0	−0.1013	-0.0156	−0.1067	−0.0420	−0.0652	* -0.0053
	n=400	0.05	0.05	1.0	−0.1017	-0.0176	−0.1208	−0.0574	−0.0733	* -0.0105
	n=800	0.05	0.05	1.0	−0.1018	-0.0182	−0.1335	−0.0712	−0.0805	* -0.0134
V	n=50	0.01	0.15	15.0	−0.0491	0.0037	−0.0180	0.0452	* -0.0013	0.0549
	n=100	0.01	0.15	15.0	−0.0532	-0.0057	−0.0329	0.0295	-0.0143	0.0383
	n=200	0.01	0.15	15.0	−0.0540	-0.0075	−0.0446	0.0173	-0.0213	0.0286
	n=400	0.01	0.15	15.0	−0.0542	-0.0082	−0.0541	0.0082	−0.0262	0.0228
	n=800	0.01	0.15	15.0	−0.0545	−0.0089	−0.0611	0.0016	−0.0299	0.0192
VI	n=50	0.05	0.15	3.0	−0.0499	0.0027	−0.0393	0.0291	-0.0099	0.0492
	n=100	0.05	0.15	3.0	−0.0528	-0.0034	−0.0595	0.0066	-0.0257	0.0291
	n=200	0.05	0.15	3.0	−0.0536	-0.0042	−0.0773	−0.0135	−0.0365	0.0179
	n=400	0.05	0.15	3.0	−0.0545	-0.0051	−0.0927	−0.0310	−0.0453	0.0110
	n=800	0.05	0.15	3.0	−0.0547	-0.0054	−0.1065	−0.0464	−0.0529	0.0067
	min	–	–	–	−0.1296	-0.0567	−0.1536	−0.0907	−0.1033	* -0.0369
	med	–	–	–	−0.0962	−0.0162	−0.0753	-0.0077	−0.0495	* 0.0037
	mean	–	–	–	−0.0907	−0.0195	−0.0806	-0.0161	−0.0506	* 0.0057
	max	–	–	–	−0.0491	0.0122	−0.0180	0.0452	* -0.0013	0.0549

Note: The data generating process is based on DEA. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A26: Performance of the Six Methods for Case 14 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	−0.1119	−0.0246	−0.0579	0.0045	−0.0474	0.0049
	n=100	0	0.01	Inf	−0.1154	−0.0428	−0.0656	0.0028	−0.0520	0.0031
	n=200	0	0.01	Inf	−0.1170	−0.0497	−0.0696	0.0014	−0.0547	0.0017
	n=400	0	0.01	Inf	−0.1178	−0.0540	−0.0713	0.0005	−0.0565	0.0007
	n=800	0	0.01	Inf	−0.1180	−0.0566	−0.0722	-0.0001	−0.0580	∗ 0.0000
II	n=50	0	0.05	Inf	−0.0885	-0.0026	−0.0410	0.0195	−0.0273	0.0257
	n=100	0	0.05	Inf	−0.0914	−0.0170	−0.0513	0.0128	−0.0340	0.0173
	n=200	0	0.05	Inf	−0.0924	−0.0244	−0.0595	0.0078	−0.0386	0.0116
	n=400	0	0.05	Inf	−0.0929	−0.0285	−0.0643	0.0044	−0.0423	0.0081
	n=800	0	0.05	Inf	−0.0933	−0.0308	−0.0672	0.0023	−0.0445	0.0062
III	n=50	0	0.15	Inf	−0.0479	0.0043	−0.0166	0.0461	∗ -0.0005	0.0555
	n=100	0	0.15	Inf	−0.0531	-0.0051	−0.0314	0.0309	-0.0133	0.0392
	n=200	0	0.15	Inf	−0.0537	-0.0075	−0.0426	0.0194	-0.0200	0.0297
	n=400	0	0.15	Inf	−0.0544	-0.0084	−0.0515	0.0114	−0.0247	0.0245
	n=800	0	0.15	Inf	−0.0547	−0.0090	−0.0575	0.0063	−0.0276	0.0216
	min	–	–	–	−0.1180	−0.0566	−0.0722	-0.0001	−0.0580	∗ 0.0000
	med	–	–	–	−0.0924	−0.0244	−0.0579	0.0063	−0.0386	0.0116
	mean	–	–	–	−0.0868	−0.0238	−0.0546	0.0113	−0.0361	0.0167
	max	–	–	–	−0.0479	0.0043	−0.0166	0.0461	∗ -0.0005	0.0555

Note: The data generating process is based on DEA. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A27: Performance of the Six Methods for Case 15 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	−0.1117	−0.0232	−0.0606	0.0011	−0.0487	0.0033
	n=100	0.01	0.01	1.0	−0.1152	−0.0400	−0.0698	-0.0033	−0.0542	∗ -0.0014
	n=200	0.01	0.01	1.0	−0.1166	−0.0477	−0.0761	-0.0075	−0.0584	∗ -0.0060
	n=400	0.01	0.01	1.0	−0.1168	−0.0525	−0.0803	-0.0111	−0.0619	∗ -0.0098
	n=800	0.01	0.01	1.0	−0.1173	−0.0550	−0.0840	-0.0143	−0.0647	∗ -0.0131
II	n=50	0.05	0.01	0.2	−0.1243	-0.0113	−0.0946	−0.0302	−0.0696	∗ -0.0031
	n=100	0.05	0.01	0.2	−0.1266	-0.0303	−0.1120	−0.0468	−0.0794	∗ -0.0185
	n=200	0.05	0.01	0.2	−0.1271	-0.0377	−0.1269	−0.0621	−0.0879	∗ -0.0281
	n=400	0.05	0.01	0.2	−0.1278	-0.0400	−0.1408	−0.0766	−0.0952	∗ -0.0330
	n=800	0.05	0.01	0.2	−0.1279	-0.0401	−0.1526	−0.0896	−0.1018	∗ -0.0350
III	n=50	0.01	0.05	5.0	−0.0872	−0.0223	−0.0428	0.0181	−0.0307	0.0230
	n=100	0.01	0.05	5.0	−0.0908	−0.0313	−0.0543	0.0095	−0.0397	0.0132
	n=200	0.01	0.05	5.0	−0.0920	−0.0355	−0.0629	0.0025	−0.0456	0.0064
	n=400	0.01	0.05	5.0	−0.0925	−0.0376	−0.0694	-0.0028	−0.0496	∗ 0.0017
	n=800	0.01	0.05	5.0	−0.0928	−0.0387	−0.0742	-0.0071	−0.0526	∗ -0.0019
IV	n=50	0.05	0.05	1.0	−0.0950	−0.0242	−0.0708	-0.0056	−0.0464	0.0103
	n=100	0.05	0.05	1.0	−0.0980	−0.0321	−0.0894	-0.0241	−0.0589	∗ -0.0057
	n=200	0.05	0.05	1.0	−0.0987	-0.0348	−0.1056	−0.0412	−0.0688	∗ -0.0158
	n=400	0.05	0.05	1.0	−0.0990	-0.0363	−0.1195	−0.0564	−0.0771	∗ -0.0227
	n=800	0.05	0.05	1.0	−0.0993	-0.0366	−0.1326	−0.0706	−0.0842	∗ -0.0267
V	n=50	0.01	0.15	15.0	−0.0608	-0.0195	−0.0198	0.0427	∗ -0.0138	0.0467
	n=100	0.01	0.15	15.0	−0.0658	−0.0294	−0.0344	0.0268	∗ -0.0254	0.0302
	n=200	0.01	0.15	15.0	−0.0678	−0.0387	−0.0458	0.0149	−0.0362	0.0188
	n=400	0.01	0.15	15.0	−0.0689	−0.1052	−0.0546	0.0063	−0.0541	0.0103
	n=800	0.01	0.15	15.0	−0.0692	−0.1072	−0.0609	0.0003	−0.0581	0.0050
VI	n=50	0.05	0.15	3.0	−0.0657	−0.0289	−0.0400	0.0276	∗ -0.0256	0.0359
	n=100	0.05	0.15	3.0	−0.0705	−0.0395	−0.0604	0.0046	−0.0413	0.0140
	n=200	0.05	0.15	3.0	−0.0717	−0.0420	−0.0777	-0.0152	−0.0524	∗ -0.0003
	n=400	0.05	0.15	3.0	−0.0721	−0.0469	−0.0924	-0.0324	-0.0659	−0.0116
	n=800	0.05	0.15	3.0	−0.0722	−0.1070	−0.1060	-0.0474	-0.0781	−0.0212
	min	–	–	–	−0.1279	−0.1072	−0.1526	-0.0896	−0.1018	∗ -0.0350
	med	–	–	–	−0.0939	−0.0377	−0.0751	-0.0073	−0.0562	∗ -0.0017
	mean	–	–	–	−0.0947	−0.0424	−0.0804	-0.0163	−0.0576	∗ -0.0012
	max	–	–	–	−0.0608	-0.0113	−0.0198	0.0427	-0.0138	0.0467

Note: The data generating process is based on DEA. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A28: Performance of the Six Methods for Case 16 in terms of the Bias

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	−0.1106	−0.0244	−0.0573	0.0048	−0.0463	0.0058
	n=100	0	0.01	Inf	−0.1142	−0.0409	−0.0652	0.0028	−0.0512	0.0035
	n=200	0	0.01	Inf	−0.1158	−0.0482	−0.0695	0.0013	−0.0539	0.0018
	n=400	0	0.01	Inf	−0.1161	−0.0528	−0.0713	0.0004	−0.0559	0.0007
	n=800	0	0.01	Inf	−0.1166	−0.0551	−0.0723	-0.0002	−0.0570	∗ 0.0000
II	n=50	0	0.05	Inf	−0.0868	−0.0214	−0.0409	0.0197	−0.0295	0.0242
	n=100	0	0.05	Inf	−0.0905	−0.0311	−0.0516	0.0123	−0.0380	0.0154
	n=200	0	0.05	Inf	−0.0914	−0.0353	−0.0594	0.0071	−0.0431	0.0100
	n=400	0	0.05	Inf	−0.0919	−0.0374	−0.0644	0.0037	−0.0462	0.0069
	n=800	0	0.05	Inf	−0.0924	−0.0387	−0.0671	0.0019	−0.0480	0.0052
III	n=50	0	0.15	Inf	−0.0605	-0.0187	−0.0188	0.0434	∗ -0.0131	0.0474
	n=100	0	0.15	Inf	−0.0651	−0.0286	−0.0323	0.0284	∗ -0.0241	0.0315
	n=200	0	0.15	Inf	−0.0675	−0.0382	−0.0438	0.0170	−0.0350	0.0205
	n=400	0	0.15	Inf	−0.0686	−0.1050	−0.0518	0.0097	−0.0523	0.0131
	n=800	0	0.15	Inf	−0.0689	−0.1070	−0.0571	0.0053	−0.0555	0.0091
	min	–	–	–	−0.1166	−0.1070	−0.0723	-0.0002	−0.0570	∗ 0.0000
	med	–	–	–	−0.0914	−0.0382	−0.0573	0.0053	−0.0463	0.0091
	mean	–	–	–	−0.0905	−0.0455	−0.0549	0.0105	−0.0433	0.0130
	max	–	–	–	−0.0605	-0.0187	−0.0188	0.0434	∗ -0.0131	0.0474

Note: The data generating process is based on DEA. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

A.3 Percentage of Underestimation

A.3.1 Cases 1-4: "SFA-CD"

Table A29: Performance of the Six Methods for Case 1 in terms of the Percentage of Underestimated Firms (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.8400	0.8200	0.8800	0.7400	0.9400	* 0.5600
	n=100	0.01	0.01	1.0	0.9800	0.8400	0.9200	0.8300	0.9700	* 0.6400
	n=200	0.01	0.01	1.0	0.9950	0.8900	0.9500	0.8850	0.9900	* 0.7000
	n=400	0.01	0.01	1.0	1.0000	0.9200	0.9700	0.9275	0.9975	* 0.8525
	n=800	0.01	0.01	1.0	1.0000	0.9262	0.9825	0.9562	0.9988	* 0.8900
II	n=50	0.05	0.01	0.2	0.7200	0.7200	0.9000	0.7800	0.9400	* 0.4600
	n=100	0.05	0.01	0.2	0.7200	0.7100	0.9400	0.8700	0.9600	* 0.5700
	n=200	0.05	0.01	0.2	0.7000	0.6950	0.9700	0.9250	0.9800	* 0.6200
	n=400	0.05	0.01	0.2	0.6900	0.6850	0.9825	0.9600	0.9875	* 0.6425
	n=800	0.05	0.01	0.2	0.6750	0.6738	0.9912	0.9788	0.9938	* 0.6488
III	n=50	0.01	0.05	5.0	0.4600	0.3800	0.7800	0.6200	0.7200	* 0.2200
	n=100	0.01	0.05	5.0	0.5000	0.5000	0.8500	0.7200	0.8100	* 0.3200
	n=200	0.01	0.05	5.0	0.5100	0.5300	0.8950	0.7850	0.8650	* 0.3950
	n=400	0.01	0.05	5.0	0.5150	0.5175	0.9250	0.8400	0.9000	* 0.4350
	n=800	0.01	0.05	5.0	0.5138	0.5138	0.9500	0.8875	0.9250	* 0.4650
IV	n=50	0.05	0.05	1.0	0.5200	0.4600	0.8400	0.7000	0.7400	* 0.1800
	n=100	0.05	0.05	1.0	0.5300	0.5200	0.9000	0.8000	0.8300	* 0.3700
	n=200	0.05	0.05	1.0	0.5450	0.5350	0.9400	0.8800	0.8850	* 0.4600
	n=400	0.05	0.05	1.0	0.5425	0.5400	0.9675	0.9275	0.9250	* 0.5000
	n=800	0.05	0.05	1.0	0.5475	0.5462	0.9812	0.9575	0.9512	* 0.5238
V	n=50	0.01	0.15	15.0	0.3400	0.3000	0.6400	0.4600	0.5000	0.0800
	n=100	0.01	0.15	15.0	0.3700	0.3700	0.7500	0.5900	0.6500	* 0.1500
	n=200	0.01	0.15	15.0	0.3700	0.4950	0.8300	0.6800	0.7700	* 0.2200
	n=400	0.01	0.15	15.0	0.3600	1.0000	0.8725	0.7525	0.9500	* 0.2775
	n=800	0.01	0.15	15.0	0.3525	1.0000	0.9075	0.8100	0.9812	* 0.3038
VI	n=50	0.05	0.15	3.0	0.5000	0.4600	0.7000	0.5200	0.5600	* 0.2600
	n=100	0.05	0.15	3.0	0.5200	0.5300	0.8100	0.6800	0.6900	* 0.3700
	n=200	0.05	0.15	3.0	0.5150	0.5200	0.8850	0.7800	0.7700	* 0.4250
	n=400	0.05	0.15	3.0	0.5150	0.5150	0.9300	0.8600	0.8275	* 0.4600
	n=800	0.05	0.15	3.0	0.5175	0.5175	0.9588	0.9138	0.8712	* 0.4825
	min	–	–	–	0.3400	0.3000	0.6400	0.4600	0.5000	* 0.0800
	med	–	–	–	0.5200	0.5325	0.9138	0.8200	0.9125	* 0.4600
	mean	–	–	–	0.5955	0.6210	0.8933	0.8005	0.8626	* 0.4494
	max	–	–	–	1.0000	1.0000	0.9912	0.9788	0.9988	* 0.8900

Note: The data generating process is based on a Cobb-Douglas cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A30: Performance of the Six Methods for Case 2 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.9800	0.7400	0.8400	0.6600	0.9800	* 0.5200
	n=100	0	0.01	Inf	0.9800	0.6800	0.8700	0.7300	0.9700	* 0.5200
	n=200	0	0.01	Inf	1.0000	0.6600	0.8950	0.7500	0.9900	* 0.5150
	n=400	0	0.01	Inf	1.0000	0.6725	0.9050	0.7625	1.0000	* 0.5325
	n=800	0	0.01	Inf	1.0000	0.6838	0.9150	0.7750	1.0000	* 0.5512
II	n=50	0	0.05	Inf	0.2000	0.2200	0.7800	0.6000	0.6800	* 0.0400
	n=100	0	0.05	Inf	0.1700	0.2200	0.8300	0.6700	0.7800	* 0.0400
	n=200	0	0.05	Inf	0.1400	0.2100	0.8550	0.7200	0.8300	* 0.0350
	n=400	0	0.05	Inf	0.1125	0.1775	0.8775	0.7400	0.8550	* 0.0325
	n=800	0	0.05	Inf	0.0988	0.1013	0.8938	0.7500	0.8675	* 0.0312
III	n=50	0	0.15	Inf	0.2600	0.2800	0.6200	0.4400	0.5000	* 0.0400
	n=100	0	0.15	Inf	0.2400	0.3000	0.7400	0.5700	0.6300	* 0.0500
	n=200	0	0.15	Inf	0.2050	0.3800	0.8100	0.6500	0.7600	* 0.0600
	n=400	0	0.15	Inf	0.1725	1.0000	0.8450	0.7000	0.9625	* 0.0775
	n=800	0	0.15	Inf	0.1575	1.0000	0.8675	0.7338	0.9975	* 0.0825
	min	–	–	–	0.0988	0.1013	0.6200	0.4400	0.5000	* 0.0312
	med	–	–	–	0.2050	0.3800	0.8550	0.7200	0.8675	* 0.0600
	mean	–	–	–	0.4478	0.4883	0.8362	0.6834	0.8535	* 0.2085
	max	–	–	–	1.0000	1.0000	0.9150	0.7750	1.0000	* 0.5512

Note: The data generating process is based on a Cobb-Douglas cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A31: Performance of the Six Methods for Case 3 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.8000	0.7600	0.8600	0.7400	0.9000	* 0.5400
	n=100	0.01	0.01	1.0	0.8300	0.7900	0.9200	0.8200	0.9400	* 0.6400
	n=200	0.01	0.01	1.0	0.9600	0.7850	0.9500	0.8800	0.9700	* 0.7050
	n=400	0.01	0.01	1.0	1.0000	0.7800	0.9700	0.9250	0.9950	* 0.7400
	n=800	0.01	0.01	1.0	0.9988	0.7788	0.9825	0.9550	0.9950	* 0.7575
II	n=50	0.05	0.01	0.2	0.7000	0.6800	0.8800	0.7600	0.9200	* 0.4400
	n=100	0.05	0.01	0.2	0.7000	0.6900	0.9400	0.8600	0.9600	* 0.5400
	n=200	0.05	0.01	0.2	0.7700	0.7750	0.9650	0.9250	0.9750	* 0.6000
	n=400	0.05	0.01	0.2	0.8050	0.7925	0.9825	0.9575	0.9850	* 0.6500
	n=800	0.05	0.01	0.2	0.7850	0.7831	0.9912	0.9775	0.9925	* 0.7062
III	n=50	0.01	0.05	5.0	0.6400	0.5400	0.7800	0.6200	0.7600	* 0.3200
	n=100	0.01	0.05	5.0	0.7000	0.8300	0.8500	0.7200	0.9250	* 0.4800
	n=200	0.01	0.05	5.0	0.6800	1.0000	0.8950	0.7900	0.9900	* 0.5400
	n=400	0.01	0.05	5.0	0.7300	1.0000	0.9300	0.8450	0.9975	* 0.6375
	n=800	0.01	0.05	5.0	0.7588	1.0000	0.9538	0.8938	1.0000	* 0.7050
IV	n=50	0.05	0.05	1.0	0.7400	0.7000	0.8200	0.6800	0.8000	* 0.4800
	n=100	0.05	0.05	1.0	0.7600	0.7500	0.9000	0.8000	0.8700	* 0.6100
	n=200	0.05	0.05	1.0	0.7700	0.7650	0.9400	0.8750	0.9150	* 0.6850
	n=400	0.05	0.05	1.0	0.7750	0.7750	0.9650	0.9250	0.9475	* 0.7275
	n=800	0.05	0.05	1.0	0.7775	0.9788	0.9812	0.9575	0.9738	* 0.7550
V	n=50	0.01	0.15	15.0	0.5400	0.5000	0.6400	0.4600	0.6000	* 0.1800
	n=100	0.01	0.15	15.0	0.6200	0.5500	0.7700	0.6000	0.7300	* 0.2800
	n=200	0.01	0.15	15.0	0.6800	0.9900	0.8400	0.6950	0.9300	* 0.4500
	n=400	0.01	0.15	15.0	0.7100	1.0000	0.8825	0.7650	0.9825	* 0.5475
	n=800	0.01	0.15	15.0	0.7331	0.6188	0.9162	0.8238	0.9112	* 0.4575
VI	n=50	0.05	0.15	3.0	0.7000	0.6000	0.7200	0.5400	0.6600	* 0.3800
	n=100	0.05	0.15	3.0	0.7400	0.7000	0.8200	0.6800	0.7800	* 0.5200
	n=200	0.05	0.15	3.0	0.7600	0.9700	0.8900	0.7900	0.9000	* 0.6400
	n=400	0.05	0.15	3.0	0.7650	0.9925	0.9350	0.8650	0.9500	* 0.7075
	n=800	0.05	0.15	3.0	0.7675	0.9925	0.9612	0.9188	0.9600	* 0.7338
	min	–	–	–	0.5400	0.5000	0.6400	0.4600	0.6000	* 0.1800
	med	–	–	–	0.7594	0.7794	0.9181	0.8219	0.9438	* 0.6050
	mean	–	–	–	0.7565	0.7956	0.8944	0.8015	0.9072	* 0.5718
	max	–	–	–	1.0000	1.0000	0.9912	0.9775	1.0000	* 0.7575

Note: The data generating process is based on a Cobb-Douglas cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A32: Performance of the Six Methods for Case 4 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.8800	0.6000	0.8400	0.6600	0.8800	$*$ 0.3600
	n=100	0	0.01	Inf	0.9600	0.5800	0.8700	0.7300	0.9400	$*$ 0.4000
	n=200	0	0.01	Inf	0.9950	0.6000	0.8950	0.7500	0.9800	$*$ 0.4450
	n=400	0	0.01	Inf	1.0000	0.5975	0.9075	0.7650	0.9775	$*$ 0.4525
	n=800	0	0.01	Inf	1.0000	0.5962	0.9162	0.7788	0.9825	$*$ 0.4600
II	n=50	0	0.05	Inf	0.4800	0.4400	0.7800	0.5800	0.7400	$*$ 0.1400
	n=100	0	0.05	Inf	0.4100	0.6000	0.8300	0.6700	0.8700	$*$ 0.1800
	n=200	0	0.05	Inf	0.3150	1.0000	0.8600	0.7200	0.9900	$*$ 0.1550
	n=400	0	0.05	Inf	0.2950	1.0000	0.8825	0.7425	1.0000	$*$ 0.1625
	n=800	0	0.05	Inf	0.3075	1.0000	0.8962	0.7525	1.0000	$*$ 0.1812
III	n=50	0	0.15	Inf	0.4800	0.5000	0.6400	0.4600	0.6000	$*$ 0.1400
	n=100	0	0.15	Inf	0.5500	0.5200	0.7500	0.5800	0.7200	$*$ 0.1900
	n=200	0	0.15	Inf	0.6050	0.9950	0.8200	0.6550	0.9350	$*$ 0.3350
	n=400	0	0.15	Inf	0.6325	1.0000	0.8500	0.7100	0.9825	$*$ 0.4150
	n=800	0	0.15	Inf	0.6512	0.5500	0.8725	0.7375	0.8888	$*$ 0.2738
	min	–	–	–	0.2950	0.4400	0.6400	0.4600	0.6000	$*$ 0.1400
	med	–	–	–	0.6050	0.6000	0.8600	0.7200	0.9400	$*$ 0.2738
	mean	–	–	–	0.6374	0.7052	0.8407	0.6861	0.8991	$*$ 0.2860
	max	–	–	–	1.0000	1.0000	0.9162	0.7788	1.0000	$*$ 0.4600

Note: The data generating process is based on a Cobb-Douglas cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

A.3.2 Cases 5-8: "SFA-TL"

Table A33: Performance of the Six Methods for Case 5 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.9600	0.8200	0.7400	0.4600	0.9400	* 0.4200
	n=100	0.01	0.01	1.0	0.9800	0.8200	0.8200	0.6300	0.9600	* 0.5600
	n=200	0.01	0.01	1.0	0.9800	0.8300	0.8850	0.7550	0.9750	* 0.6650
	n=400	0.01	0.01	1.0	0.9800	0.9150	0.9300	0.8500	0.9825	* 0.7850
	n=800	0.01	0.01	1.0	0.9788	0.9288	0.9600	0.9100	0.9888	* 0.8538
II	n=50	0.05	0.01	0.2	1.0000	0.7000	0.8400	0.6800	0.9600	* 0.4600
	n=100	0.05	0.01	0.2	1.0000	0.7000	0.9100	0.8000	0.9800	* 0.5800
	n=200	0.05	0.01	0.2	1.0000	0.6900	0.9500	0.8850	0.9900	* 0.6300
	n=400	0.05	0.01	0.2	1.0000	0.6825	0.9725	0.9375	0.9925	* 0.6450
	n=800	0.05	0.01	0.2	1.0000	0.6700	0.9850	0.9662	0.9962	* 0.6512
III	n=50	0.01	0.05	5.0	0.8000	0.3800	0.5800	0.2000	0.7000	* 0.1000
	n=100	0.01	0.05	5.0	0.8300	0.5000	0.6700	0.3500	0.7900	* 0.2300
	n=200	0.01	0.05	5.0	0.8350	0.5300	0.7500	0.5000	0.8500	* 0.3250
	n=400	0.01	0.05	5.0	0.8400	0.5150	0.8225	0.6400	0.8825	* 0.3800
	n=800	0.01	0.05	5.0	0.8412	0.5150	0.8825	0.7562	0.9088	* 0.4225
IV	n=50	0.05	0.05	1.0	0.8200	0.4400	0.7600	0.5400	0.7400	* 0.1600
	n=100	0.05	0.05	1.0	0.8500	0.5000	0.8500	0.6900	0.8300	* 0.3600
	n=200	0.05	0.05	1.0	0.8650	0.5250	0.9100	0.8050	0.8900	* 0.4500
	n=400	0.05	0.05	1.0	0.8700	0.5350	0.9475	0.8825	0.9250	* 0.4950
	n=800	0.05	0.05	1.0	0.8712	0.5412	0.9688	0.9312	0.9500	* 0.5200
V	n=50	0.01	0.15	15.0	0.5600	0.3000	0.4200	0.0400	0.3400	* 0.0200
	n=100	0.01	0.15	15.0	0.5800	0.3700	0.5300	0.1300	0.4900	* 0.0600
	n=200	0.01	0.15	15.0	0.5900	0.4950	0.6250	0.2700	0.6500	* 0.1300
	n=400	0.01	0.15	15.0	0.5900	1.0000	0.7075	0.4175	0.9050	* 0.2425
	n=800	0.01	0.15	15.0	0.5888	1.0000	0.7837	0.5656	0.9475	* 0.3338
VI	n=50	0.05	0.15	3.0	0.5600	0.4600	0.6000	0.3400	0.5000	* 0.1800
	n=100	0.05	0.15	3.0	0.5800	0.5200	0.7300	0.5100	0.6200	* 0.2800
	n=200	0.05	0.15	3.0	0.5950	0.5150	0.8250	0.6500	0.7100	* 0.3500
	n=400	0.05	0.15	3.0	0.6000	0.5150	0.8900	0.7725	0.7775	* 0.3900
	n=800	0.05	0.15	3.0	0.6025	0.5162	0.9338	0.8562	0.8275	* 0.4150
	min	–	–	–	0.5600	0.3000	0.4200	0.0400	0.3400	* 0.0200
	med	–	–	–	0.8406	0.5275	0.8325	0.6650	0.8975	* 0.4025
	mean	–	–	–	0.8049	0.6143	0.8060	0.6240	0.8333	* 0.4031
	max	–	–	–	1.0000	1.0000	0.9850	0.9662	0.9962	* 0.8538

Note: The data generating process is based on a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A34: Performance of the Six Methods for Case 6 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.9600	0.7600	0.6200	0.2400	0.9400	* 0.1600
	n=100	0	0.01	Inf	0.9700	0.6900	0.6700	0.3400	0.9500	* 0.2400
	n=200	0	0.01	Inf	0.9750	0.6700	0.6950	0.4000	0.9550	* 0.2700
	n=400	0	0.01	Inf	0.9750	0.6875	0.7200	0.4350	0.9650	* 0.3000
	n=800	0	0.01	Inf	0.9738	0.7000	0.7450	0.4688	0.9750	* 0.3300
II	n=50	0	0.05	Inf	0.8000	0.2200	0.5200	0.0800	0.6800	* 0.0000
	n=100	0	0.05	Inf	0.8200	0.2300	0.5900	0.1800	0.7700	* 0.0200
	n=200	0	0.05	Inf	0.8300	0.2150	0.6350	0.2650	0.8200	* 0.0300
	n=400	0	0.05	Inf	0.8350	0.1750	0.6650	0.3400	0.8475	* 0.0325
	n=800	0	0.05	Inf	0.8375	0.1031	0.6900	0.3912	0.8650	* 0.0312
III	n=50	0	0.15	Inf	0.5400	0.2800	0.3800	0.0000	0.3200	0.0000
	n=100	0	0.15	Inf	0.5800	0.3000	0.4900	0.0400	0.4700	* 0.0000
	n=200	0	0.15	Inf	0.5850	0.3850	0.5650	0.1300	0.6300	* 0.0300
	n=400	0	0.15	Inf	0.5850	1.0000	0.6125	0.2200	0.9125	* 0.1250
	n=800	0	0.15	Inf	0.5812	1.0000	0.6475	0.3025	0.9550	* 0.1612
	min	–	–	–	0.5400	0.1031	0.3800	0.0000	0.3200	* 0.0000
	med	–	–	–	0.8300	0.3850	0.6350	0.2650	0.8650	* 0.0325
	mean	–	–	–	0.7898	0.4944	0.6163	0.2555	0.8037	* 0.1153
	max	–	–	–	0.9750	1.0000	0.7450	0.4688	0.9750	* 0.3300

Note: The data generating process is based on a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A35: Performance of the Six Methods for Case 7 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.9600	0.7600	0.7200	0.4400	0.9400	* 0.4000
	n=100	0.01	0.01	1.0	0.9700	0.7900	0.8100	0.6200	0.9600	* 0.5400
	n=200	0.01	0.01	1.0	0.9750	0.7800	0.8800	0.7500	0.9700	* 0.6350
	n=400	0.01	0.01	1.0	0.9750	0.7775	0.9300	0.8475	0.9775	* 0.7000
	n=800	0.01	0.01	1.0	0.9750	0.7788	0.9588	0.9100	0.9838	* 0.7375
II	n=50	0.05	0.01	0.2	0.9800	0.7000	0.8400	0.6800	0.9600	* 0.4600
	n=100	0.05	0.01	0.2	0.9900	0.7000	0.9100	0.8000	0.9700	* 0.5600
	n=200	0.05	0.01	0.2	0.9950	0.7350	0.9500	0.8850	0.9850	* 0.6500
	n=400	0.05	0.01	0.2	0.9950	0.7888	0.9725	0.9350	0.9900	* 0.7400
	n=800	0.05	0.01	0.2	0.9950	0.7875	0.9850	0.9650	0.9950	* 0.7638
III	n=50	0.01	0.05	5.0	0.8200	0.5200	0.5800	0.2000	0.7200	* 0.1400
	n=100	0.01	0.05	5.0	0.8400	0.8100	0.6800	0.3600	0.9100	* 0.3000
	n=200	0.01	0.05	5.0	0.8550	1.0000	0.7600	0.5250	0.9900	* 0.4550
	n=400	0.01	0.05	5.0	0.8575	1.0000	0.8350	0.6600	0.9975	* 0.5775
	n=800	0.01	0.05	5.0	0.8588	1.0000	0.8925	0.7750	0.9988	* 0.6762
IV	n=50	0.05	0.05	1.0	0.8200	0.7000	0.7600	0.5400	0.7800	* 0.4000
	n=100	0.05	0.05	1.0	0.8500	0.7500	0.8500	0.6900	0.8600	* 0.5500
	n=200	0.05	0.05	1.0	0.8600	0.7650	0.9100	0.8000	0.9050	* 0.6400
	n=400	0.05	0.05	1.0	0.8650	0.7750	0.9450	0.8800	0.9375	* 0.6975
	n=800	0.05	0.05	1.0	0.8675	0.9800	0.9688	0.9300	0.9700	* 0.7850
V	n=50	0.01	0.15	15.0	0.6800	0.5200	0.4400	0.0600	0.5000	* 0.0400
	n=100	0.01	0.15	15.0	0.7300	0.5500	0.5500	0.1600	0.6500	* 0.1100
	n=200	0.01	0.15	15.0	0.7600	0.9950	0.6450	0.3050	0.9300	* 0.2300
	n=400	0.01	0.15	15.0	0.7700	1.0000	0.7250	0.4550	0.9800	* 0.3600
	n=800	0.01	0.15	15.0	0.7738	0.6112	0.8025	0.6000	0.8688	* 0.3262
VI	n=50	0.05	0.15	3.0	0.7200	0.6000	0.6000	0.3600	0.6200	* 0.2800
	n=100	0.05	0.15	3.0	0.7600	0.7000	0.7400	0.5200	0.7400	* 0.4300
	n=200	0.05	0.15	3.0	0.7700	0.9750	0.8350	0.6700	0.8800	* 0.5500
	n=400	0.05	0.15	3.0	0.7700	0.9925	0.9000	0.7850	0.9375	* 0.6400
	n=800	0.05	0.15	3.0	0.7738	0.9912	0.9388	0.8675	0.9475	* 0.6788
	min	–	–	–	0.6800	0.5200	0.4400	0.0600	0.5000	* 0.0400
	med	–	–	–	0.8562	0.7781	0.8375	0.6750	0.9438	* 0.5500
	mean	–	–	–	0.8604	0.7944	0.8105	0.6325	0.8951	* 0.5018
	max	–	–	–	0.9950	1.0000	0.9850	0.9650	0.9988	* 0.7850

Note: The data generating process is based on a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A36: Performance of the Six Methods for Case 8 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.9600	0.6000	0.6200	0.2400	0.9200	* 0.1200
	n=100	0	0.01	Inf	0.9600	0.5800	0.6700	0.3400	0.9400	* 0.1800
	n=200	0	0.01	Inf	0.9650	0.6000	0.7000	0.4050	0.9550	* 0.2250
	n=400	0	0.01	Inf	0.9675	0.6050	0.7250	0.4425	0.9625	* 0.2525
	n=800	0	0.01	Inf	0.9662	0.6162	0.7512	0.4800	0.9675	* 0.2825
II	n=50	0	0.05	Inf	0.8000	0.4400	0.5200	0.0800	0.7200	* 0.0200
	n=100	0	0.05	Inf	0.8400	0.6000	0.6000	0.1800	0.8700	* 0.1100
	n=200	0	0.05	Inf	0.8450	1.0000	0.6400	0.2800	0.9950	* 0.2150
	n=400	0	0.05	Inf	0.8500	1.0000	0.6700	0.3525	1.0000	* 0.2975
	n=800	0	0.05	Inf	0.8500	1.0000	0.6962	0.4025	1.0000	* 0.3462
III	n=50	0	0.15	Inf	0.6800	0.5000	0.4000	0.0000	0.4800	0.0000
	n=100	0	0.15	Inf	0.7300	0.5200	0.5000	0.0500	0.6300	* 0.0200
	n=200	0	0.15	Inf	0.7600	0.9950	0.5800	0.1500	0.9400	* 0.1000
	n=400	0	0.15	Inf	0.7750	1.0000	0.6250	0.2425	0.9900	* 0.1800
	n=800	0	0.15	Inf	0.7850	0.5438	0.6562	0.3219	0.8375	* 0.0975
	min	–	–	–	0.6800	0.4400	0.4000	0.0000	0.4800	0.0000
	med	–	–	–	0.8450	0.6000	0.6400	0.2800	0.9400	* 0.1800
	mean	–	–	–	0.8489	0.7067	0.6236	0.2645	0.8805	* 0.1631
	max	–	–	–	0.9675	1.0000	0.7512	0.4800	1.0000	* 0.3462

Note: The data generating process is based on a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

A.3.3 Cases 9-12: "TL-DEA"

Table A37: Performance of the Six Methods for Case 9 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.9600	0.7400	0.7200	0.3600	0.9400	* 0.2800
	n=100	0.01	0.01	1.0	0.9800	0.5400	0.8100	0.5500	0.9500	* 0.3200
	n=200	0.01	0.01	1.0	0.9800	0.5750	0.8800	0.7050	0.9700	* 0.4300
	n=400	0.01	0.01	1.0	0.9825	0.6175	0.9275	0.8175	0.9800	* 0.5300
	n=800	0.01	0.01	1.0	0.9800	0.6594	0.9588	0.8938	0.9862	* 0.6075
II	n=50	0.05	0.01	0.2	1.0000	0.7000	0.8400	0.6600	0.9600	* 0.4800
	n=100	0.05	0.01	0.2	1.0000	0.7100	0.9100	0.7900	0.9800	* 0.5800
	n=200	0.05	0.01	0.2	1.0000	0.7000	0.9500	0.8800	0.9900	* 0.6300
	n=400	0.05	0.01	0.2	1.0000	0.6950	0.9725	0.9325	0.9925	* 0.6575
	n=800	0.05	0.01	0.2	1.0000	0.6800	0.9850	0.9638	0.9962	* 0.6600
III	n=50	0.01	0.05	5.0	0.8000	0.4200	0.5400	0.1200	0.6600	* 0.0800
	n=100	0.01	0.05	5.0	0.8300	0.5100	0.6500	0.2400	0.7800	* 0.1700
	n=200	0.01	0.05	5.0	0.8350	0.5200	0.7350	0.3950	0.8350	* 0.2550
	n=400	0.01	0.05	5.0	0.8400	0.5175	0.8150	0.5600	0.8725	* 0.3300
	n=800	0.01	0.05	5.0	0.8412	0.5200	0.8788	0.7050	0.9025	* 0.3900
IV	n=50	0.05	0.05	1.0	0.8200	0.4000	0.7600	0.5200	0.7400	* 0.1600
	n=100	0.05	0.05	1.0	0.8600	0.4800	0.8400	0.6800	0.8300	* 0.3400
	n=200	0.05	0.05	1.0	0.8650	0.5300	0.9050	0.7950	0.8850	* 0.4550
	n=400	0.05	0.05	1.0	0.8700	0.5375	0.9450	0.8750	0.9225	* 0.4950
	n=800	0.05	0.05	1.0	0.8738	0.5438	0.9688	0.9262	0.9475	* 0.5200
V	n=50	0.01	0.15	15.0	0.5400	0.3200	0.3800	0.0200	0.3000	0.0200
	n=100	0.01	0.15	15.0	0.5800	0.4000	0.4900	0.0800	0.4500	* 0.0400
	n=200	0.01	0.15	15.0	0.5900	0.5150	0.5950	0.1650	0.6050	* 0.0900
	n=400	0.01	0.15	15.0	0.5900	1.0000	0.6875	0.3050	0.9025	* 0.1775
	n=800	0.01	0.15	15.0	0.5913	1.0000	0.7725	0.4688	0.9500	* 0.2800
VI	n=50	0.05	0.15	3.0	0.5600	0.4600	0.5800	0.3200	0.4800	* 0.1600
	n=100	0.05	0.15	3.0	0.5900	0.5200	0.7200	0.4900	0.6200	* 0.2700
	n=200	0.05	0.15	3.0	0.5950	0.5150	0.8150	0.6350	0.7000	* 0.3450
	n=400	0.05	0.15	3.0	0.6000	0.5150	0.8850	0.7600	0.7675	* 0.3900
	n=800	0.05	0.15	3.0	0.6038	0.5162	0.9312	0.8488	0.8188	* 0.4162
	min	–	–	–	0.5400	0.3200	0.3800	0.0200	0.3000	0.0200
	med	–	–	–	0.8406	0.5250	0.8275	0.6475	0.8938	* 0.3425
	mean	–	–	–	0.8052	0.5786	0.7949	0.5820	0.8238	* 0.3520
	max	–	–	–	1.0000	1.0000	0.9850	0.9638	0.9962	* 0.6600

Note: The data generating process is based on DEA on the fitted values of a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A38: Performance of the Six Methods for Case 10 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.9600	0.8400	0.6000	0.0600	0.9600	* 0.0400
	n=100	0	0.01	Inf	0.9700	0.8700	0.6600	0.1300	0.9700	* 0.1100
	n=200	0	0.01	Inf	0.9700	0.9200	0.7000	0.2400	0.9900	* 0.2050
	n=400	0	0.01	Inf	0.9750	0.9425	0.7350	0.3525	0.9975	* 0.3100
	n=800	0	0.01	Inf	0.9738	0.9500	0.7625	0.4562	1.0000	* 0.4138
II	n=50	0	0.05	Inf	0.8000	0.3400	0.4800	0.0000	0.6400	0.0000
	n=100	0	0.05	Inf	0.8200	0.4500	0.5600	0.0200	0.7600	* 0.0100
	n=200	0	0.05	Inf	0.8350	0.5000	0.6150	0.0500	0.8150	* 0.0300
	n=400	0	0.05	Inf	0.8375	0.5050	0.6525	0.1150	0.8425	* 0.0600
	n=800	0	0.05	Inf	0.8400	0.5038	0.6912	0.2100	0.8588	* 0.0975
III	n=50	0	0.15	Inf	0.5400	0.3000	0.3400	0.0000	0.2800	0.0000
	n=100	0	0.15	Inf	0.5800	0.3400	0.4400	0.0000	0.4200	0.0000
	n=200	0	0.15	Inf	0.5850	0.4550	0.5300	0.0100	0.5850	* 0.0050
	n=400	0	0.15	Inf	0.5875	1.0000	0.5900	0.0300	0.9100	* 0.0175
	n=800	0	0.15	Inf	0.5875	1.0000	0.6312	0.0750	0.9625	* 0.0438
	min	–	–	–	0.5400	0.3000	0.3400	0.0000	0.2800	0.0000
	med	–	–	–	0.8350	0.5050	0.6150	0.0600	0.8588	* 0.0400
	mean	–	–	–	0.7907	0.6611	0.5992	0.1166	0.7994	* 0.0895
	max	–	–	–	0.9750	1.0000	0.7625	0.4562	1.0000	* 0.4138

Note: The data generating process is based on DEA on the fitted values of a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A39: Performance of the Six Methods for Case 11 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.9600	0.7400	0.7200	0.3600	0.9200	* 0.3000
	n=100	0.01	0.01	1.0	0.9700	0.7900	0.8100	0.5400	0.9600	* 0.4600
	n=200	0.01	0.01	1.0	0.9750	0.8000	0.8750	0.7000	0.9700	* 0.5900
	n=400	0.01	0.01	1.0	0.9750	0.8025	0.9250	0.8150	0.9800	* 0.6800
	n=800	0.01	0.01	1.0	0.9750	0.8012	0.9575	0.8925	0.9862	* 0.7312
II	n=50	0.05	0.01	0.2	0.9800	0.6800	0.8400	0.6600	0.9400	* 0.4400
	n=100	0.05	0.01	0.2	0.9900	0.6800	0.9100	0.7900	0.9700	* 0.5600
	n=200	0.05	0.01	0.2	0.9950	0.7150	0.9500	0.8750	0.9850	* 0.6350
	n=400	0.05	0.01	0.2	0.9950	0.7875	0.9725	0.9300	0.9900	* 0.7375
	n=800	0.05	0.01	0.2	0.9950	0.7775	0.9850	0.9625	0.9950	* 0.7512
III	n=50	0.01	0.05	5.0	0.8200	0.5600	0.5400	0.1200	0.7200	* 0.1000
	n=100	0.01	0.05	5.0	0.8400	1.0000	0.6600	0.2600	0.9400	* 0.2200
	n=200	0.01	0.05	5.0	0.8550	1.0000	0.7500	0.4200	0.9950	* 0.3650
	n=400	0.01	0.05	5.0	0.8575	1.0000	0.8275	0.5875	0.9975	* 0.5150
	n=800	0.01	0.05	5.0	0.8600	1.0000	0.8888	0.7275	0.9988	* 0.6388
IV	n=50	0.05	0.05	1.0	0.8300	0.7000	0.7400	0.5200	0.7800	* 0.3800
	n=100	0.05	0.05	1.0	0.8500	0.7500	0.8400	0.6700	0.8500	* 0.5400
	n=200	0.05	0.05	1.0	0.8650	0.7650	0.9050	0.7900	0.9000	* 0.6350
	n=400	0.05	0.05	1.0	0.8650	0.7725	0.9450	0.8725	0.9325	* 0.6950
	n=800	0.05	0.05	1.0	0.8675	0.8025	0.9688	0.9275	0.9650	* 0.7525
V	n=50	0.01	0.15	15.0	0.6800	0.5200	0.4000	0.0400	0.4800	* 0.0200
	n=100	0.01	0.15	15.0	0.7300	0.5700	0.5100	0.0900	0.6200	* 0.0700
	n=200	0.01	0.15	15.0	0.7600	0.9950	0.6200	0.1950	0.9350	* 0.1550
	n=400	0.01	0.15	15.0	0.7675	1.0000	0.7100	0.3450	0.9825	* 0.2750
	n=800	0.01	0.15	15.0	0.7712	0.6612	0.7925	0.5125	0.8600	* 0.3262
VI	n=50	0.05	0.15	3.0	0.7200	0.6200	0.6000	0.3400	0.6200	* 0.2600
	n=100	0.05	0.15	3.0	0.7600	0.7000	0.7300	0.5000	0.7400	* 0.4200
	n=200	0.05	0.15	3.0	0.7700	0.9750	0.8250	0.6500	0.8750	* 0.5400
	n=400	0.05	0.15	3.0	0.7725	0.9925	0.8925	0.7725	0.9350	* 0.6325
	n=800	0.05	0.15	3.0	0.7738	0.9912	0.9375	0.8588	0.9462	* 0.6775
	min	–	–	–	0.6800	0.5000	0.4000	0.0400	0.4800	* 0.0200
	med	–	–	–	0.8562	0.7825	0.8338	0.6550	0.9400	* 0.5275
	mean	–	–	–	0.8608	0.7983	0.8009	0.5909	0.8923	* 0.4701
	max	–	–	–	0.9950	1.0000	1.0000	0.9625	0.9988	* 0.7525

Note: The data generating process is based on DEA on the fitted values of a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A40: Performance of the Six Methods for Case 12 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.9600	0.7600	0.6000	0.0600	0.9400	* 0.0400
	n=100	0	0.01	Inf	0.9600	0.7900	0.6600	0.1400	0.9600	* 0.1100
	n=200	0	0.01	Inf	0.9650	0.8100	0.7050	0.2550	0.9800	* 0.2050
	n=400	0	0.01	Inf	0.9675	0.8150	0.7400	0.3700	0.9875	* 0.2900
	n=800	0	0.01	Inf	0.9650	0.8162	0.7688	0.4788	0.9900	* 0.3713
II	n=50	0	0.05	Inf	0.8200	0.5000	0.4800	0.0000	0.7000	0.0000
	n=100	0	0.05	Inf	0.8400	0.7900	0.5700	0.0200	0.9000	0.0200
	n=200	0	0.05	Inf	0.8500	1.0000	0.6250	0.0650	1.0000	* 0.0550
	n=400	0	0.05	Inf	0.8550	1.0000	0.6625	0.1375	1.0000	* 0.1225
	n=800	0	0.05	Inf	0.8562	1.0000	0.7025	0.2400	1.0000	* 0.2150
III	n=50	0	0.15	Inf	0.6800	0.5000	0.3600	0.0000	0.4400	0.0000
	n=100	0	0.15	Inf	0.7300	0.5300	0.4600	0.0000	0.6000	0.0000
	n=200	0	0.15	Inf	0.7600	0.9950	0.5450	0.0150	0.9450	* 0.0100
	n=400	0	0.15	Inf	0.7725	1.0000	0.6025	0.0400	0.9925	* 0.0300
	n=800	0	0.15	Inf	0.7825	0.6175	0.6425	0.0938	0.8250	* 0.0462
	min	–	–	–	0.6800	0.5000	0.3600	0.0000	0.4400	0.0000
	med	–	–	–	0.8500	0.8100	0.6250	0.0650	0.9600	* 0.0462
	mean	–	–	–	0.8509	0.7949	0.6082	0.1277	0.8840	* 0.1010
	max	–	–	–	0.9675	1.0000	0.7688	0.4788	1.0000	* 0.3713

Note: The data generating process is based on DEA on the fitted values of a Translog cost function. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

A.3.4 Cases 13-16: "DEA"

Table A41: Performance of the Six Methods for Case 13 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	1.0000	0.9000	0.8000	0.3600	0.9800	* 0.3000
	n=100	0.01	0.01	1.0	1.0000	0.9900	0.8900	0.5600	0.9900	* 0.5100
	n=200	0.01	0.01	1.0	1.0000	0.9950	0.9400	0.7250	0.9950	* 0.6950
	n=400	0.01	0.01	1.0	0.9975	0.9950	0.9675	0.8350	0.9975	* 0.8225
	n=800	0.01	0.01	1.0	0.9975	0.9962	0.9812	0.9050	0.9975	* 0.8988
II	n=50	0.05	0.01	0.2	1.0000	0.9200	0.8600	0.6200	0.9800	* 0.5600
	n=100	0.05	0.01	0.2	1.0000	0.9900	0.9200	0.7700	0.9900	* 0.7300
	n=200	0.05	0.01	0.2	1.0000	0.9950	0.9550	0.8700	0.9950	* 0.8450
	n=400	0.05	0.01	0.2	1.0000	0.9975	0.9775	0.9275	0.9975	* 0.9200
	n=800	0.05	0.01	0.2	1.0000	0.9988	0.9888	0.9612	0.9988	* 0.9575
III	n=50	0.01	0.05	5.0	0.9000	0.5400	0.6000	0.1000	0.7600	* 0.0400
	n=100	0.01	0.05	5.0	0.9300	0.7100	0.7400	0.2300	0.8400	* 0.1400
	n=200	0.01	0.05	5.0	0.9350	0.7700	0.8450	0.4000	0.9050	* 0.2850
	n=400	0.01	0.05	5.0	0.9375	0.7975	0.9050	0.5750	0.9375	* 0.4400
	n=800	0.01	0.05	5.0	0.9388	0.8100	0.9425	0.7238	0.9550	* 0.5650
IV	n=50	0.05	0.05	1.0	0.9200	0.4000	0.7600	0.4800	0.8200	* 0.2200
	n=100	0.05	0.05	1.0	0.9300	0.6600	0.8600	0.6400	0.8900	* 0.4400
	n=200	0.05	0.05	1.0	0.9400	0.7250	0.9150	0.7750	0.9250	* 0.5800
	n=400	0.05	0.05	1.0	0.9425	0.7425	0.9525	0.8650	0.9525	* 0.6600
	n=800	0.05	0.05	1.0	0.9425	0.7488	0.9738	0.9212	0.9688	* 0.7012
V	n=50	0.01	0.15	15.0	0.6600	0.4400	0.4000	0.0200	0.4400	* 0.0000
	n=100	0.01	0.15	15.0	0.7000	0.5200	0.5400	0.0700	0.5600	* 0.0400
	n=200	0.01	0.15	15.0	0.7050	0.5400	0.6750	0.1600	0.6500	* 0.0850
	n=400	0.01	0.15	15.0	0.7050	0.5475	0.7950	0.3000	0.7150	* 0.1500
	n=800	0.01	0.15	15.0	0.7088	0.5525	0.8750	0.4775	0.7638	* 0.2312
VI	n=50	0.05	0.15	3.0	0.7000	0.4800	0.6000	0.2600	0.5400	* 0.1400
	n=100	0.05	0.15	3.0	0.7200	0.5300	0.7300	0.4300	0.6700	* 0.2600
	n=200	0.05	0.15	3.0	0.7250	0.5450	0.8300	0.6000	0.7450	* 0.3500
	n=400	0.05	0.15	3.0	0.7325	0.5525	0.8975	0.7350	0.8050	* 0.4150
	n=800	0.05	0.15	3.0	0.7338	0.5550	0.9412	0.8350	0.8512	* 0.4562
	min	–	–	–	0.6600	0.4000	0.4000	0.0200	0.4400	* 0.0000
	med	–	–	–	0.9362	0.7338	0.8825	0.6100	0.9150	* 0.4400
	mean	–	–	–	0.8800	0.7315	0.8352	0.5710	0.8538	* 0.4479
	max	–	–	–	1.0000	0.9988	0.9888	0.9612	0.9988	* 0.9575

Note: The data generating process is based on DEA. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A42: Performance of the Six Methods for Case 14 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	1.0000	0.9000	0.7400	0.0200	0.9800	0.0200
	n=100	0	0.01	Inf	1.0000	0.9900	0.8400	0.0800	0.9900	∗ 0.0700
	n=200	0	0.01	Inf	0.9950	0.9950	0.8800	0.1500	0.9900	∗ 0.1400
	n=400	0	0.01	Inf	0.9975	0.9950	0.9050	0.2500	0.9950	∗ 0.2400
	n=800	0	0.01	Inf	0.9975	0.9962	0.9188	0.3550	0.9950	∗ 0.3500
II	n=50	0	0.05	Inf	0.9000	0.5400	0.5600	0.0000	0.7400	0.0000
	n=100	0	0.05	Inf	0.9300	0.7100	0.6900	0.0100	0.8400	∗ 0.0000
	n=200	0	0.05	Inf	0.9350	0.7700	0.7950	0.0250	0.9050	∗ 0.0150
	n=400	0	0.05	Inf	0.9375	0.8000	0.8450	0.0650	0.9375	∗ 0.0400
	n=800	0	0.05	Inf	0.9388	0.8125	0.8750	0.1300	0.9525	∗ 0.0875
III	n=50	0	0.15	Inf	0.6600	0.4400	0.3800	0.0000	0.4200	0.0000
	n=100	0	0.15	Inf	0.6900	0.5200	0.5000	0.0000	0.5400	0.0000
	n=200	0	0.15	Inf	0.7000	0.5400	0.6250	0.0000	0.6300	0.0000
	n=400	0	0.15	Inf	0.7025	0.5450	0.7475	0.0150	0.6950	∗ 0.0050
	n=800	0	0.15	Inf	0.7062	0.5500	0.8212	0.0400	0.7362	∗ 0.0138
	min	–	–	–	0.6600	0.4400	0.3800	0.0000	0.4200	0.0000
	med	–	–	–	0.9350	0.7700	0.7950	0.0250	0.9050	∗ 0.0150
	mean	–	–	–	0.8727	0.7402	0.7415	0.0760	0.8231	∗ 0.0654
	max	–	–	–	1.0000	0.9962	0.9188	0.3550	0.9950	∗ 0.3500

Note: The data generating process is based on DEA. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A43: Performance of the Six Methods for Case 15 in terms of the Percentage of Underestimation (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0.01	0.01	1.0	0.9800	0.8800	0.7800	0.3400	0.9800	* 0.2800
	n=100	0.01	0.01	1.0	0.9900	0.9700	0.8900	0.5500	0.9900	* 0.4900
	n=200	0.01	0.01	1.0	0.9950	0.9800	0.9400	0.7150	0.9900	* 0.6800
	n=400	0.01	0.01	1.0	0.9950	0.9875	0.9650	0.8325	0.9950	* 0.8125
	n=800	0.01	0.01	1.0	0.9950	0.9888	0.9812	0.9050	0.9962	* 0.8888
II	n=50	0.05	0.01	0.2	1.0000	0.8600	0.8400	0.6200	0.9800	* 0.5400
	n=100	0.05	0.01	0.2	1.0000	0.9600	0.9200	0.7600	0.9900	* 0.7000
	n=200	0.05	0.01	0.2	1.0000	0.9850	0.9550	0.8650	0.9900	* 0.8300
	n=400	0.05	0.01	0.2	0.9975	0.9875	0.9750	0.9250	0.9950	* 0.9075
	n=800	0.05	0.01	0.2	0.9975	0.9888	0.9875	0.9600	0.9975	* 0.9462
III	n=50	0.01	0.05	5.0	0.9000	0.7000	0.6000	0.1200	0.7800	* 0.0800
	n=100	0.01	0.05	5.0	0.9200	0.7900	0.7500	0.2500	0.8800	* 0.1900
	n=200	0.01	0.05	5.0	0.9300	0.8250	0.8500	0.4250	0.9250	* 0.3500
	n=400	0.01	0.05	5.0	0.9325	0.8375	0.9100	0.6025	0.9475	* 0.5000
	n=800	0.01	0.05	5.0	0.9338	0.8438	0.9462	0.7462	0.9600	* 0.6200
IV	n=50	0.05	0.05	1.0	0.9000	0.7200	0.7600	0.4600	0.8200	* 0.3400
	n=100	0.05	0.05	1.0	0.9200	0.7900	0.8500	0.6300	0.8900	* 0.5200
	n=200	0.05	0.05	1.0	0.9300	0.8150	0.9150	0.7700	0.9250	* 0.6400
	n=400	0.05	0.05	1.0	0.9300	0.8250	0.9525	0.8625	0.9500	* 0.7225
	n=800	0.05	0.05	1.0	0.9325	0.8288	0.9738	0.9212	0.9662	* 0.7650
V	n=50	0.01	0.15	15.0	0.7400	0.6200	0.4200	0.0400	0.5600	* 0.0200
	n=100	0.01	0.15	15.0	0.7700	0.7100	0.5600	0.0900	0.7000	* 0.0700
	n=200	0.01	0.15	15.0	0.7950	0.7950	0.7050	0.1850	0.8350	* 0.1450
	n=400	0.01	0.15	15.0	0.8000	0.9800	0.8175	0.3450	0.9600	* 0.2825
	n=800	0.01	0.15	15.0	0.8025	0.9838	0.8888	0.5225	0.9725	* 0.4250
VI	n=50	0.05	0.15	3.0	0.7600	0.6600	0.6200	0.2800	0.6600	* 0.2200
	n=100	0.05	0.15	3.0	0.7900	0.7300	0.7500	0.4500	0.7600	* 0.3700
	n=200	0.05	0.15	3.0	0.8050	0.7650	0.8400	0.6150	0.8300	* 0.4950
	n=400	0.05	0.15	3.0	0.8075	0.8000	0.9050	0.7500	0.9000	* 0.5975
	n=800	0.05	0.15	3.0	0.8088	0.9625	0.9462	0.8462	0.9388	* 0.6819
	min	–	–	–	0.7400	0.6200	0.4200	0.0400	0.5600	* 0.0200
	med	–	–	–	0.9300	0.8331	0.8894	0.6175	0.9488	* 0.5100
	mean	–	–	–	0.9019	0.8523	0.8398	0.5795	0.9021	* 0.5036
	max	–	–	–	1.0000	0.9888	0.9875	0.9600	0.9975	* 0.9462

Note: The data generating process is based on DEA. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

Table A44: Performance of the Six Methods for Case 16 in terms of the Percentage of Underestimated Firms (PU)

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL	DEA-CRS	DEA-VRS	AM	MAX
I	n=50	0	0.01	Inf	0.9800	0.8800	0.7400	0.0200	0.9800	0.0200
	n=100	0	0.01	Inf	0.9900	0.9700	0.8400	0.0800	0.9900	* 0.0700
	n=200	0	0.01	Inf	0.9950	0.9850	0.8850	0.1650	0.9900	* 0.1500
	n=400	0	0.01	Inf	0.9950	0.9875	0.9075	0.2700	0.9925	* 0.2550
	n=800	0	0.01	Inf	0.9950	0.9888	0.9212	0.3800	0.9938	* 0.3700
II	n=50	0	0.05	Inf	0.9000	0.7000	0.5600	0.0000	0.7800	0.0000
	n=100	0	0.05	Inf	0.9200	0.7900	0.6900	0.0100	0.8800	0.0100
	n=200	0	0.05	Inf	0.9300	0.8250	0.8000	0.0350	0.9250	* 0.0250
	n=400	0	0.05	Inf	0.9325	0.8375	0.8525	0.0800	0.9475	* 0.0600
	n=800	0	0.05	Inf	0.9350	0.8450	0.8812	0.1538	0.9588	* 0.1175
III	n=50	0	0.15	Inf	0.7400	0.6200	0.4000	0.0000	0.5600	0.0000
	n=100	0	0.15	Inf	0.7700	0.7000	0.5200	0.0000	0.6800	0.0000
	n=200	0	0.15	Inf	0.7900	0.8000	0.6500	0.0050	0.8400	* 0.0000
	n=400	0	0.15	Inf	0.7975	0.9800	0.7700	0.0200	0.9675	* 0.0125
	n=800	0	0.15	Inf	0.7975	0.9838	0.8338	0.0512	0.9762	* 0.0350
	min	–	–	Inf	0.7400	0.6200	0.4000	0.0000	0.5600	0.0000
	med	–	–	Inf	0.9300	0.8450	0.8000	0.0350	0.9588	* 0.0250
	mean	–	–	Inf	0.8978	0.8595	0.7501	0.0847	0.8974	* 0.0750
	max	–	–	Inf	0.9950	0.9888	0.9212	0.3800	0.9938	* 0.3700

Note: The data generating process is based on DEA. Cells highlighted in dark gray indicate the best performing method among SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS. Cells highlighted in lighter gray indicate the better performing method among the combination approaches AM and MAX. A lighter gray cell attached with an asterisk indicates that the corresponding combination approach performs best across all six estimators.

A.4 Returns to Scale

Table A45: Performance of the SFA-CD and SFA-TL for Case 1 (“SFA-CD”) in terms of the Root Mean Square Error (RMSE) of the Returns to Scale

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL
I	n=50	0.01	0.01	1.0	0.0024	0.0071
	n=100	0.01	0.01	1.0	0.0038	0.0048
	n=200	0.01	0.01	1.0	0.0090	0.0036
	n=400	0.01	0.01	1.0	0.0140	0.0034
	n=800	0.01	0.01	1.0	0.0189	0.0037
II	n=50	0.05	0.01	0.2	0.0057	0.0241
	n=100	0.05	0.01	0.2	0.0038	0.0155
	n=200	0.05	0.01	0.2	0.0027	0.0103
	n=400	0.05	0.01	0.2	0.0019	0.0072
	n=800	0.05	0.01	0.2	0.0013	0.0050
III	n=50	0.01	0.05	5.0	0.0030	0.0138
	n=100	0.01	0.05	5.0	0.0019	0.0086
	n=200	0.01	0.05	5.0	0.0013	0.0055
	n=400	0.01	0.05	5.0	0.0009	0.0035
	n=800	0.01	0.05	5.0	0.0006	0.0024
IV	n=50	0.05	0.05	1.0	0.0066	0.0282
	n=100	0.05	0.05	1.0	0.0045	0.0178
	n=200	0.05	0.05	1.0	0.0030	0.0118
	n=400	0.05	0.05	1.0	0.0022	0.0083
	n=800	0.05	0.05	1.0	0.0015	0.0059
V	n=50	0.01	0.15	15.0	0.0069	0.0373
	n=100	0.01	0.15	15.0	0.0044	0.0220
	n=200	0.01	0.15	15.0	0.0029	0.0154
	n=400	0.01	0.15	15.0	0.0022	0.0150
	n=800	0.01	0.15	15.0	0.0017	0.0135
VI	n=50	0.05	0.15	3.0	0.0109	0.0482
	n=100	0.05	0.15	3.0	0.0071	0.0307
	n=200	0.05	0.15	3.0	0.0049	0.0195
	n=400	0.05	0.15	3.0	0.0034	0.0133
	n=800	0.05	0.15	3.0	0.0024	0.0094
	min	–	–	–	0.0006	0.0024
	med	–	–	–	0.0030	0.0111
	mean	–	–	–	0.0045	0.0138
	max	–	–	–	0.0189	0.0482

Note: The data generating process is based on a Cobb-Douglas cost function.

Table A46: Performance of the SFA-CD and SFA-TL for Case 7 (“SFA-TL”) in terms of the Root Mean Square Error (RMSE) of the Returns to Scale

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL
I	n=50	0.01	0.01	1.0	0.0473	0.0072
	n=100	0.01	0.01	1.0	0.0467	0.0044
	n=200	0.01	0.01	1.0	0.0462	0.0028
	n=400	0.01	0.01	1.0	0.0460	0.0019
	n=800	0.01	0.01	1.0	0.0457	0.0013
II	n=50	0.05	0.01	0.2	0.0500	0.0242
	n=100	0.05	0.01	0.2	0.0499	0.0155
	n=200	0.05	0.01	0.2	0.0501	0.0105
	n=400	0.05	0.01	0.2	0.0501	0.0073
	n=800	0.05	0.01	0.2	0.0501	0.0051
III	n=50	0.01	0.05	5.0	0.0483	0.0187
	n=100	0.01	0.05	5.0	0.0479	0.0141
	n=200	0.01	0.05	5.0	0.0475	0.0129
	n=400	0.01	0.05	5.0	0.0474	0.0120
	n=800	0.01	0.05	5.0	0.0473	0.0113
IV	n=50	0.05	0.05	1.0	0.0503	0.0334
	n=100	0.05	0.05	1.0	0.0500	0.0209
	n=200	0.05	0.05	1.0	0.0501	0.0140
	n=400	0.05	0.05	1.0	0.0501	0.0103
	n=800	0.05	0.05	1.0	0.0501	0.0098
V	n=50	0.01	0.15	15.0	0.0492	0.0551
	n=100	0.01	0.15	15.0	0.0482	0.0306
	n=200	0.01	0.15	15.0	0.0474	0.0268
	n=400	0.01	0.15	15.0	0.0473	0.0217
	n=800	0.01	0.15	15.0	0.0472	0.0140
VI	n=50	0.05	0.15	3.0	0.0506	0.0635
	n=100	0.05	0.15	3.0	0.0499	0.0395
	n=200	0.05	0.15	3.0	0.0496	0.0292
	n=400	0.05	0.15	3.0	0.0496	0.0238
	n=800	0.05	0.15	3.0	0.0496	0.0149
	min	–	–	–	0.0457	0.0013
	med	–	–	–	0.0494	0.0140
	mean	–	–	–	0.0487	0.0186
	max	–	–	–	0.0506	0.0635

Note: The data generating process is based on a Translog cost function.

Table A47: Performance of the SFA-CD and SFA-TL for Case 11 ("TL-DEA") in terms of the Root Mean Square Error (RMSE) of the Returns to Scale

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL
I	n=50	0.01	0.01	1.0	0.0435	0.0396
	n=100	0.01	0.01	1.0	0.0445	0.0377
	n=200	0.01	0.01	1.0	0.0450	0.0368
	n=400	0.01	0.01	1.0	0.0451	0.0364
	n=800	0.01	0.01	1.0	0.0450	0.0360
II	n=50	0.05	0.01	0.2	0.0494	0.0530
	n=100	0.05	0.01	0.2	0.0495	0.0491
	n=200	0.05	0.01	0.2	0.0491	0.0470
	n=400	0.05	0.01	0.2	0.0490	0.0454
	n=800	0.05	0.01	0.2	0.0487	0.0441
III	n=50	0.01	0.05	5.0	0.0454	0.0461
	n=100	0.01	0.05	5.0	0.0454	0.0444
	n=200	0.01	0.05	5.0	0.0456	0.0438
	n=400	0.01	0.05	5.0	0.0456	0.0431
	n=800	0.01	0.05	5.0	0.0456	0.0424
IV	n=50	0.05	0.05	1.0	0.0522	0.0598
	n=100	0.05	0.05	1.0	0.0511	0.0524
	n=200	0.05	0.05	1.0	0.0503	0.0490
	n=400	0.05	0.05	1.0	0.0496	0.0473
	n=800	0.05	0.05	1.0	0.0490	0.0473
V	n=50	0.01	0.15	15.0	0.0552	0.0778
	n=100	0.01	0.15	15.0	0.0504	0.0573
	n=200	0.01	0.15	15.0	0.0478	0.0525
	n=400	0.01	0.15	15.0	0.0466	0.0480
	n=800	0.01	0.15	15.0	0.0461	0.0409
VI	n=50	0.05	0.15	3.0	0.0628	0.0878
	n=100	0.05	0.15	3.0	0.0575	0.0675
	n=200	0.05	0.15	3.0	0.0539	0.0600
	n=400	0.05	0.15	3.0	0.0519	0.0562
	n=800	0.05	0.15	3.0	0.0505	0.0506
	min	–	–	–	0.0435	0.0360
	med	–	–	–	0.0490	0.0473
	mean	–	–	–	0.0490	0.0500
	max	–	–	–	0.0628	0.0878

Note: The data generating process is based on DEA on the fitted values of a Translog cost function.

Table A48: Performance of the SFA-CD and SFA-TL for Case 16 ("DEA") in terms of the Root Mean Square Error (RMSE) of the Returns to Scale

	N	σ_v	σ_u	λ	SFA-CD	SFA-TL
I	n=50	0	0.01	Inf	0.1169	0.1391
	n=100	0	0.01	Inf	0.1230	0.1339
	n=200	0	0.01	Inf	0.1262	0.1330
	n=400	0	0.01	Inf	0.1267	0.1316
	n=800	0	0.01	Inf	0.1265	0.1304
II	n=50	0	0.05	Inf	0.1161	0.1407
	n=100	0	0.05	Inf	0.1197	0.1328
	n=200	0	0.05	Inf	0.1227	0.1303
	n=400	0	0.05	Inf	0.1241	0.1290
	n=800	0	0.05	Inf	0.1241	0.1280
III	n=50	0	0.15	Inf	0.1235	0.1597
	n=100	0	0.15	Inf	0.1230	0.1385
	n=200	0	0.15	Inf	0.1238	0.1338
	n=400	0	0.15	Inf	0.1241	0.1379
	n=800	0	0.15	Inf	0.1243	0.1355
	min	–	–	–	0.1161	0.1280
	med	–	–	–	0.1238	0.1338
	mean	–	–	–	0.1230	0.1356
	max	–	–	–	0.1267	0.1597

Note: The data generating process is based on DEA.

A.5 Violations of the Monotonicity Assumption

Table A49: Percentage of Violations of the Monotonicity Assumption for Case 1 (“SFA-CD”)

	N	σ_v	σ_u	λ	Y1-CD	Y2-CD	Y3-CD	Y1-TL	Y2-TL	Y3-TL
I	n=50	0.01	0.01	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=100	0.01	0.01	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=200	0.01	0.01	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=400	0.01	0.01	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=800	0.01	0.01	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
II	n=50	0.05	0.01	0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=100	0.05	0.01	0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=200	0.05	0.01	0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=400	0.05	0.01	0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=800	0.05	0.01	0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
III	n=50	0.01	0.05	5.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=100	0.01	0.05	5.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=200	0.01	0.05	5.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=400	0.01	0.05	5.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=800	0.01	0.05	5.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
IV	n=50	0.05	0.05	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=100	0.05	0.05	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=200	0.05	0.05	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=400	0.05	0.05	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=800	0.05	0.05	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
V	n=50	0.01	0.15	15.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0400
	n=100	0.01	0.15	15.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=200	0.01	0.15	15.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=400	0.01	0.15	15.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=800	0.01	0.15	15.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
VI	n=50	0.05	0.15	3.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0800
	n=100	0.05	0.15	3.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=200	0.05	0.15	3.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=400	0.05	0.15	3.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	n=800	0.05	0.15	3.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	min	–	–	–	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	med	–	–	–	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	mean	–	–	–	0.0000	0.0000	0.0000	0.0000	0.0000	0.0004
	max	–	–	–	0.0000	0.0000	0.0000	0.0000	0.0000	0.0800

Note: The data generating process is based on Cobb-Douglas cost function.

Table A50: Percentage of Violations of the Monotonicity Assumption for Case 7 ("SFA-TL")

	N	σ_v	σ_u	λ	Y1-CD	Y2-CD	Y3-CD	Y1-TL	Y2-TL	Y3-TL
I	n=50	0.01	0.01	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0400
	n=100	0.01	0.01	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0300
	n=200	0.01	0.01	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0200
	n=400	0.01	0.01	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0125
	n=800	0.01	0.01	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0088
II	n=50	0.05	0.01	0.2	0.0000	0.0000	0.0000	0.0200	0.0200	0.1400
	n=100	0.05	0.01	0.2	0.0000	0.0000	0.0000	0.0100	0.0100	0.0800
	n=200	0.05	0.01	0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0500
	n=400	0.05	0.01	0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0400
	n=800	0.05	0.01	0.2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0325
III	n=50	0.01	0.05	5.0	0.0000	0.0000	0.0000	0.0200	0.0200	0.1200
	n=100	0.01	0.05	5.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0700
	n=200	0.01	0.05	5.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0500
	n=400	0.01	0.05	5.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0375
	n=800	0.01	0.05	5.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0325
IV	n=50	0.05	0.05	1.0	0.0000	0.0000	0.0000	0.0400	0.0400	0.2200
	n=100	0.05	0.05	1.0	0.0000	0.0000	0.0000	0.0200	0.0200	0.1200
	n=200	0.05	0.05	1.0	0.0000	0.0000	0.0000	0.0050	0.0050	0.0700
	n=400	0.05	0.05	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0500
	n=800	0.05	0.05	1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0388
V	n=50	0.01	0.15	15.0	0.0000	0.0000	0.0000	0.0800	0.0800	0.3200
	n=100	0.01	0.15	15.0	0.0000	0.0000	0.0000	0.0400	0.0400	0.2000
	n=200	0.01	0.15	15.0	0.0000	0.0000	0.0000	0.0300	0.0300	0.1450
	n=400	0.01	0.15	15.0	0.0000	0.0000	0.0000	0.0200	0.0200	0.1025
	n=800	0.01	0.15	15.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0550
VI	n=50	0.05	0.15	3.0	0.0000	0.0000	0.0000	0.0800	0.0800	0.3400
	n=100	0.05	0.15	3.0	0.0000	0.0000	0.0000	0.0500	0.0500	0.2500
	n=200	0.05	0.15	3.0	0.0000	0.0000	0.0000	0.0350	0.0350	0.1750
	n=400	0.05	0.15	3.0	0.0000	0.0000	0.0000	0.0250	0.0250	0.1200
	n=800	0.05	0.15	3.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0662
	min	–	–	–	0.0000	0.0000	0.0000	0.0000	0.0000	0.0088
	med	–	–	–	0.0000	0.0000	0.0000	0.0000	0.0000	0.0681
	mean	–	–	–	0.0000	0.0000	0.0000	0.0158	0.0158	0.1012
	max	–	–	–	0.0000	0.0000	0.0000	0.0800	0.0800	0.3400

Note: The data generating process is based on a Translog cost function.

Table A51: Percentage of Violations of the Monotonicity Assumption for Case 11 ("TL-DEA")

	N	σ_v	σ_u	λ	Y1-CD	Y2-CD	Y3-CD	Y1-TL	Y2-TL	Y3-TL
I	n=50	0.01	0.01	1.0	0	0	0	0.0200	0.0200	0.0600
	n=100	0.01	0.01	1.0	0	0	0	0.0200	0.0200	0.0500
	n=200	0.01	0.01	1.0	0	0	0	0.0200	0.0200	0.0500
	n=400	0.01	0.01	1.0	0	0	0	0.0200	0.0200	0.0475
	n=800	0.01	0.01	1.0	0	0	0	0.0213	0.0213	0.0450
II	n=50	0.05	0.01	0.2	0	0	0	0.0400	0.0400	0.1200
	n=100	0.05	0.01	0.2	0	0	0	0.0300	0.0300	0.0800
	n=200	0.05	0.01	0.2	0	0	0	0.0250	0.0250	0.0600
	n=400	0.05	0.01	0.2	0	0	0	0.0200	0.0200	0.0500
	n=800	0.05	0.01	0.2	0	0	0	0.0200	0.0200	0.0462
III	n=50	0.01	0.05	5.0	0	0	0	0.0200	0.0200	0.1200
	n=100	0.01	0.05	5.0	0	0	0	0.0300	0.0300	0.0700
	n=200	0.01	0.05	5.0	0	0	0	0.0250	0.0250	0.0600
	n=400	0.01	0.05	5.0	0	0	0	0.0200	0.0200	0.0500
	n=800	0.01	0.05	5.0	0	0	0	0.0200	0.0200	0.0462
IV	n=50	0.05	0.05	1.0	0	0	0	0.0600	0.0600	0.1800
	n=100	0.05	0.05	1.0	0	0	0	0.0400	0.0400	0.1100
	n=200	0.05	0.05	1.0	0	0	0	0.0300	0.0300	0.0700
	n=400	0.05	0.05	1.0	0	0	0	0.0225	0.0225	0.0575
	n=800	0.05	0.05	1.0	0	0	0	0.0213	0.0213	0.0488
V	n=50	0.01	0.15	15.0	0	0	0	0.0800	0.0800	0.3000
	n=100	0.01	0.15	15.0	0	0	0	0.0500	0.0500	0.1700
	n=200	0.01	0.15	15.0	0	0	0	0.0450	0.0450	0.1250
	n=400	0.01	0.15	15.0	0	0	0	0.0350	0.0350	0.0925
	n=800	0.01	0.15	15.0	0	0	0	0.0238	0.0238	0.0625
VI	n=50	0.05	0.15	3.0	0	0	0	0.1000	0.1000	0.3200
	n=100	0.05	0.15	3.0	0	0	0	0.0600	0.0600	0.2200
	n=200	0.05	0.15	3.0	0	0	0	0.0450	0.0450	0.1400
	n=400	0.05	0.15	3.0	0	0	0	0.0375	0.0375	0.1025
	n=800	0.05	0.15	3.0	0	0	0	0.0269	0.0269	0.0688
	min	0.01	0.01	0.2	0	0	0	0.0200	0.0200	0.0450
	med	0.03	0.05	2.0	0	0	0	0.0259	0.0259	0.0694
	mean	0.03	0.07	4.2	0	0	0	0.0343	0.0343	0.1008
	max	0.05	0.15	15.0	0	0	0	0.1000	0.1000	0.3200

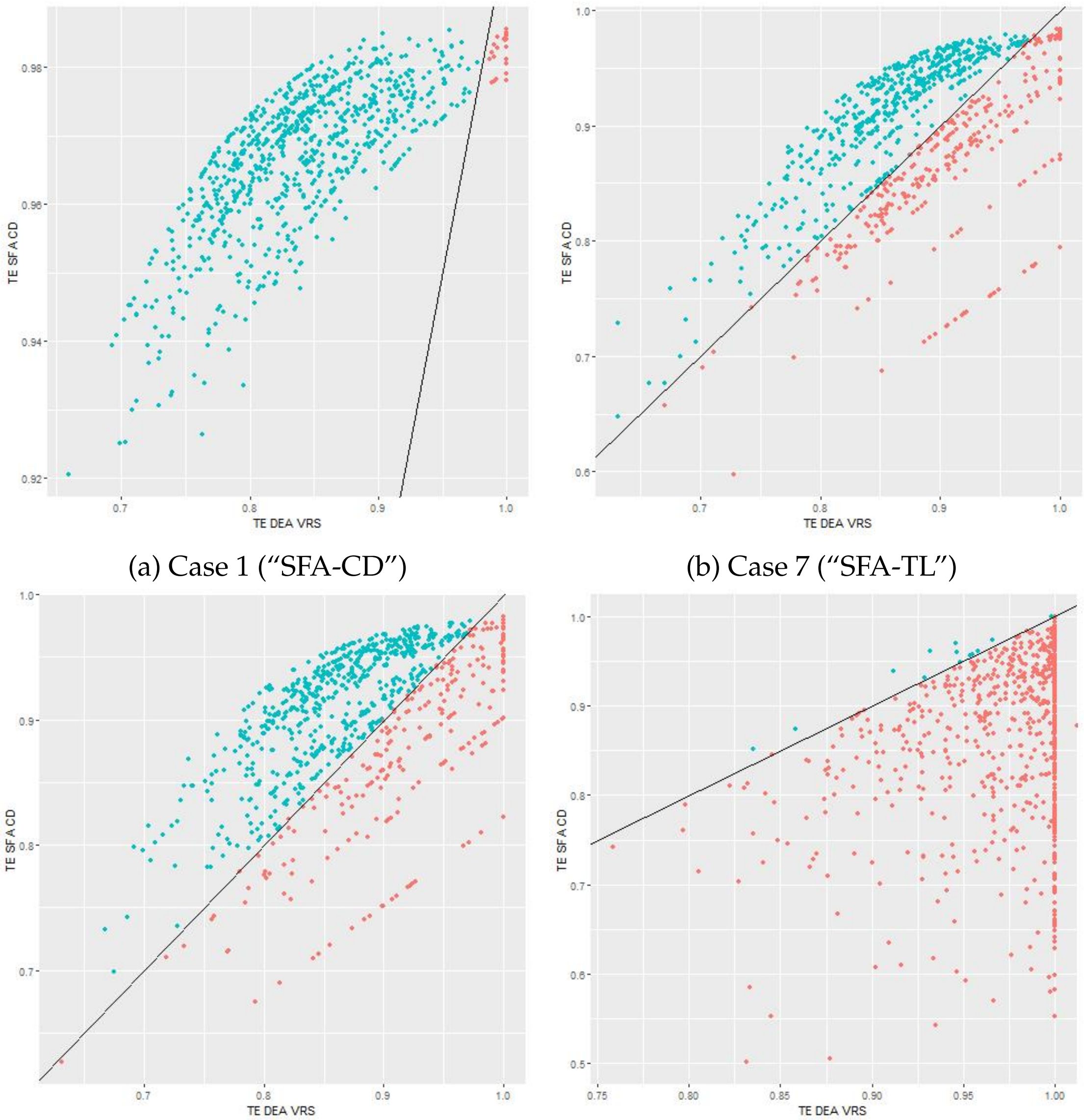
Note: The data generating process is based on DEA on the fitted values of a Translog cost function.

Table A52: Percentage of Violations of the Monotonicity Assumption for Case 16 ("DEA")

	N	σ_v	σ_u	λ	Y1-CD	Y2-CD	Y3-CD	Y1-TL	Y2-TL	Y3-TL
I	n=50	0	0.01	Inf	0.0000	0.0000	0.0000	0.0800	0.0800	0.1000
	n=100	0	0.01	Inf	0.0000	0.0000	0.0000	0.0700	0.0700	0.0700
	n=200	0	0.01	Inf	0.0000	0.0000	0.0000	0.0600	0.0600	0.0650
	n=400	0	0.01	Inf	0.0000	0.0000	0.0000	0.0625	0.0625	0.0650
	n=800	0	0.01	Inf	0.0000	0.0000	0.0000	0.0625	0.0625	0.0650
II	n=50	0	0.05	Inf	0.0000	0.0000	0.0000	0.1000	0.1000	0.1200
	n=100	0	0.05	Inf	0.0000	0.0000	0.0000	0.0700	0.0700	0.0800
	n=200	0	0.05	Inf	0.0000	0.0000	0.0000	0.0650	0.0650	0.0650
	n=400	0	0.05	Inf	0.0000	0.0000	0.0000	0.0625	0.0625	0.0650
	n=800	0	0.05	Inf	0.0000	0.0000	0.0000	0.0619	0.0619	0.0650
III	n=50	0	0.15	Inf	0.0000	0.0000	0.0000	0.1800	0.1800	0.2000
	n=100	0	0.15	Inf	0.0000	0.0000	0.0000	0.1000	0.1000	0.1100
	n=200	0	0.15	Inf	0.0000	0.0000	0.0000	0.0750	0.0750	0.0750
	n=400	0	0.15	Inf	0.0000	0.0000	0.0000	0.0700	0.0700	0.0650
	n=800	0	0.15	Inf	0.0000	0.0000	0.0000	0.0650	0.0650	0.0638
	min	–	–	–	0.0000	0.0000	0.0000	0.0600	0.0600	0.0638
	med	–	–	–	0.0000	0.0000	0.0000	0.0700	0.0700	0.0650
	mean	–	–	–	0.0000	0.0000	0.0000	0.0790	0.0790	0.0849
	max	–	–	–	0.0000	0.0000	0.0000	0.1800	0.1800	0.2000

Note: The data generating process is based on DEA.

B Figures



(a) Case 1 (“SFA-CD”)

(b) Case 7 (“SFA-TL”)

(c) Case 11 (“TL-DEA”)

(d) Case 16 (“DEA”)

Figure A1: Comparison of DEA-VRS and SFD-CD estimates

In Cases 1, 7, and 11, we assume that $N = 800$ and $\sigma_u = \sigma_v = 0.05$, while in Case 16, we assume that $\sigma_v = 0$.

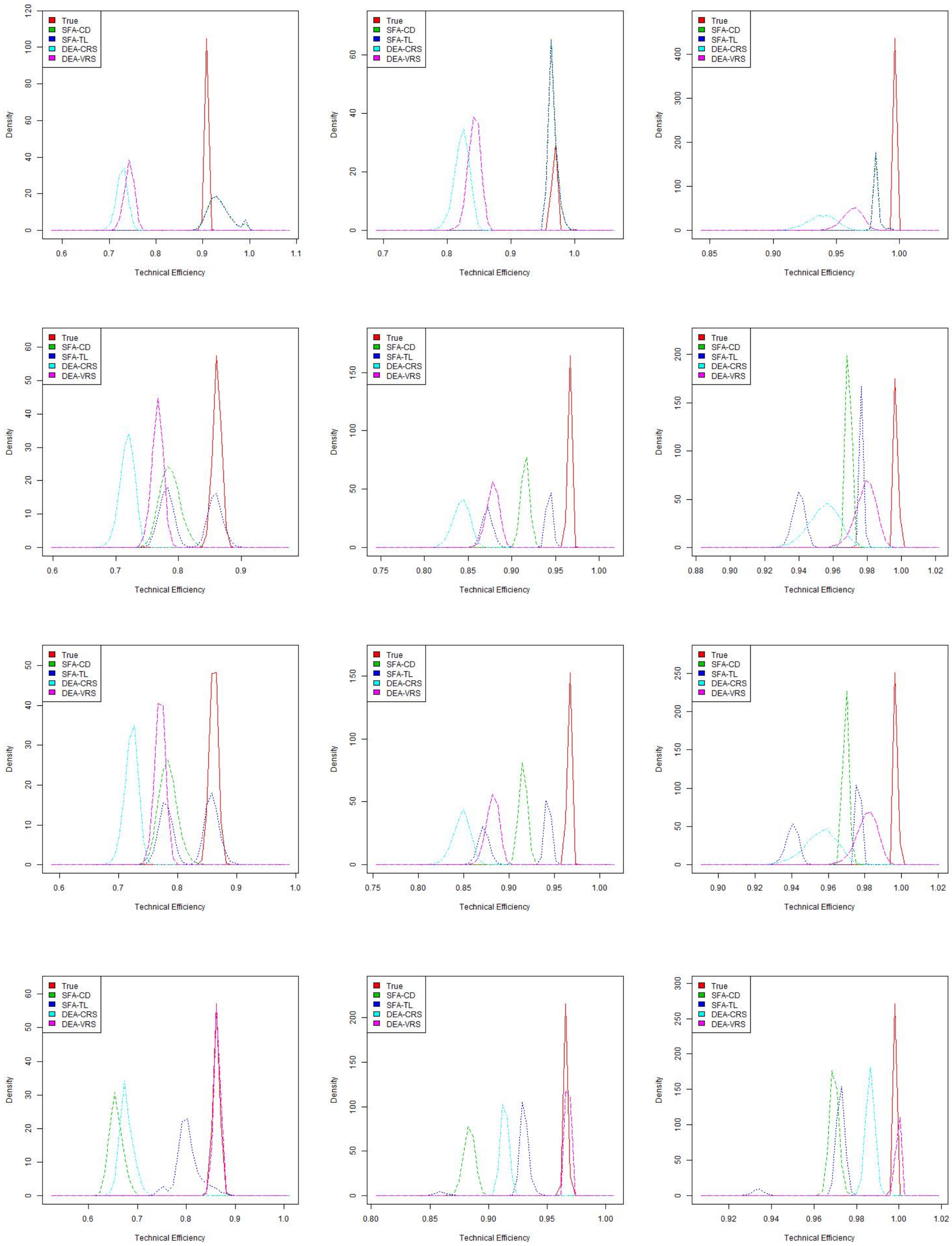


Figure A2: Distributions of 5th, 50th, and 95th Percentiles of Technical Efficiency Estimates in Case 1 ("SFA-CD"), Case 7 ("SFA-TL") Case 11 ("TL-DEA"), and Case 16("DEA") for the True Efficiency Scores, SFA-CD, SFA-TL, DEA-CRS, and DEA-VRS

In Cases 1, 7, and 15, we assume that $N = 800$ and $\sigma_u = \sigma_v = 0.05$, while in Case 20, we assume that $\sigma_v = 0$.