Iddo GAL, Haifa

Developing statistical literacy in mathematics education? Navigating between current gaps and new needs and contents

Abstract: This paper summarizes selected points raised during my plenary talk, which examined challenges facing the promotion of statistical literacy and statistical ideas within mathematics education, and pointed to some new developments related to statistical literacy. The paper focuses on one of these new developments, i.e., ideas regarding the knowledge and skills related to engaging with "civic statistics", based on work by ProCivicStat project, and on some implications.

Introduction: About statistical literacy

The place of statistics education within mathematics education has been problematic for many years, and continues to challenge mathematics educators and mathematicians, as well as school systems. This is despite the fact that "Data and Chance," "Statistics and Probability," or "Stochastics" are a major sub-area in the mathematics curriculum in all western countries.

Within the domain of statistics education, teaching for the development of statistical literacy has a special place. Statistical literacy is a construct related to adult numeracy (Gal, 1997) and mathematical literacy, but goes beyond them. It refers to comprehension of and engagement with statistical messages that may appear in the media, at work, and other contexts.

Statistical Literacy has been defined in several ways in the literature (Haack, 1979). My own definition (Gal, 2002) reads:

> *"The motivation and ability to access, understand, interpret, critically evaluate, and if relevant express opinions, regarding statistical messages, data-related arguments, or issues involving uncertainty and risk"*

The definition above implies that statistical literacy is part of a larger family of "literacies" or competencies that all adults are expected to possess, including not only numeracy but also scientific literacy, financial literacy, health literacy, and more.

It is important to emphasize that statistics by itself, and statistical literacy, extend beyond regular mathematical content. They involve more "big ideas" and knowledge and skills, given known differences between mathematics and statistics. Examples are the need to have knowledge about methods for data production and their relative advantages or limitations;

sampling; generalization to populations; informal inference vs. formal inference under uncertainty; the notion of error (e.g., in sampling, measurement, prediction or inference); risk and uncertainty (rather than probability per se); confounding variables, and much more.

To further illustrate the ideas regarding statistical literacy, let us focus on just one idea in the definition above, "Interpret". This term relates to the role of adults as "critical consumers" of messages that involve information of a statistical or probabilistic nature, which are often found in the print and digital media and other channels. Critical interpretation is a complex cognitive activity. Readers have to extract meaning from the text, consider the arguments presented and the quality or nature of the data or evidence supporting them, have habits of mind that enable them to ask "worry questions" about the statistical messages they engage with (e.g., what is the data source? how credible it is? do the data provide a sound base for a given generalization?), and when needed express their reactions in a reasoned and well-justified way.

Readers have to reason about statistical statements from a logical perspective – but to do so based on their understanding of the *issues* that the data refers to, i.e., they need to activate their world knowledge and connect quantitative statements they engage with the real-world context to which the data refer.

Statistical literacy and mathematics education in schools

From the perspective presented above, engaging students with "interpretation" (in the context of statistical literacy) fits very well with the goals of the mathematics curriculum in general. These goals aim, among other things, to develop students' mathematical reasoning and ability to think logically about or with numbers and mathematical constructs. Yet, in my interactions with mathematics teachers, most are not comfortable with anything that involves reading texts. Texts are often seen by mathematics teachers as a distraction, as external to the world of pure mathematics, and mostly shunned from the classroom, and minimized in examinations.

Statistical literacy is strongly hinted in most national curriculum statements, which usually state that they aspire to develop learners' understanding of quantitative statements in the world or in the media. However, in actuality, few teachers invest much time in teaching for statistical literacy. When high-school teachers do teach some statistics, the emphasis is on training students to be producers of data, to crunch numbers, and on teaching procedures and computations, not on big ideas in statistics, or understanding the role and uses of statistics in society.

There are many other gaps pertaining to the place of statistics in the mathematics curriculum, some related to curriculum issues, some to teachers and teaching. For example, on the curriculum side, we know that the time devoted to statistics is usually shorter (fewer teaching hours) compared to other major areas in the mathematics (national) curriculum. Also, topics in the mathematics curriculum develop from year to year, and more sophisticated ideas are presented over time. When teaching statistics, however, the same basic statistical topics are recycled, e.g., teachers at the high school still teach about measures of center (average, median) and dispersion, or basic graphing and graph-reading skills, etc. that were taught years earlier. Almost no new "big ideas" in statistics are introduced, except for in advanced courses in statistics that few students normally reach.

In short, when schools do invest in teaching statistical topics, they do so in an inconsistent way, and mainly prepare students to be producers of data, rather than (critical) consumers or decision makers with data or evidence. There are many reasons for that, such as the fact most mathematics teachers at the high-school level have never learned statistics, or have engaged in a project involving data collection, data analysis, and reporting of results and conclusions. This means that most mathematics teachers lack needed content knowledge and pedagogical content knowledge regarding statistics.

The factors and issues listed above, when taken in combination, lead to dire results. As I see it, what is taught in school mathematics, at least as far as content related to statistics, is too dissociated from the demands in the real world for which schools are supposed to prepare their graduates.

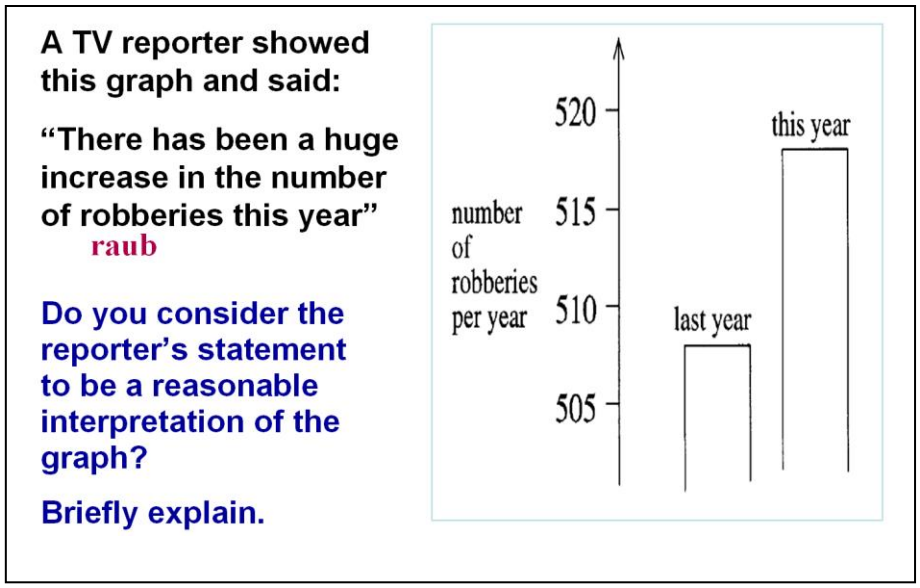

Figure 1: Robberies task from TIMSS 1998

Let's practice some reflection...

Please look at Figure 1. What do you think about the question presented?

Figure 1 illustrates several of the issues noted above, related to the need for critical interpretation skills on the one hand, and the gaps in teaching statistics in general and statistical literacy in particular. It presents the "Robberies" task, an item that was first used in the TIMMS 1998 survey (Mullis, Martin, Beaton, Gonzalez, Kelly, & Smith, 1998), which focused on performance of students in their *final* year of schooling (usually grade 12) (The same task was later used in PISA 2003). This task asked students to explain whether a reporter's statement about a "huge increase" was a reasonable interpretation of a bar graph showing the number of robberies in two years. The task involves quite minimal text and the bar graph is simple, normally taught in the primary school, just with a truncated scale.

On average, across all countries that took TIMSS 1998, *less than half of all graduating students* were able to cope (at least partially) with this task. In Germany itself, *20% gave full responses and 26% showed partial understanding*. Note that this is on a seemingly simple task with little text, which exemplifies one of the most basic skills educators usually use as an example for what they hope their pupils can do after learning mathematics at the high-school level: The ability to detect a discrepancy between displayed data (the "facts") and a given interpretation of these data. Keep in mind that in many countries a sizable proportion of students drop out or leave *before* the final year of high school, i.e., the overall percentage of the *age cohort* who can cope with such tasks is bound to be even lower. Comparable results were obtained in PISA 2003.

So, you may say: "Why bother? The TIMSS 1998 data are old and nobody should care much for findings from 20 years ago". My answer: No! These data are not old; they describe the skills of real people. These students, who were at age 17 or so years old when TIMSS took place, are now close to age 40, in various positions in the labor market; they have to function in the real world and interact with data about society. However, the school system has not equipped many of them well enough for their role as effective consumers of statistical data. Further, there is no reason to assume that in the last 20 years things have improved much, based on the earlier analysis of the place of statistics and statistical literacy in school mathematics.

New needs, contents & tools

The conceptualization of the statistical knowledge and skills that adults need is changing over time. In my talk I described several new developments in recent years, which are bound to change in radical ways

the required skill set related to statistics in general and statistical literacy in particular, and may impact on what we teach, how and when, regarding these topics. Among other things, I noted the *open data* revolution, which implies that citizens have direct access to more types of data and they can bypass traditional channels for communication of statistics. I also noted the shift to a *predictive society* which relies on new sources for data (e.g., "big data") and the resulting implications for new knowledge (e.g., of algorithms, data science concepts). These developments are interesting not only because they have implications for what we should teach, but also because they begin to blur the boundaries between a "producer" and a "consumer" of data. These and other developments ascribe new roles for citizens, who need to stay on guard since they are now part of both the data production and data consumption system.

A third type of new development relates to the area of *civic statistics*, one that is more directly related to the ideas on which statistical literacy was originally founded. The area of civic statistics was defined by ProCivicStat, a collaborative project of six universities in five countries (Germany, Hungary, Israel, Portugal, and the United Kingdom), funded by the European Commission's ERASMUS+ program. ProCivicStat aims to promote civic engagement via better understanding of statistics about key societal phenomena. As part of the ProCivicStat work, Engel, Gal, and Ridgway (2016) claim that to be fully engaged, citizens need to understand statistics regarding past trends, present situations, and possible future changes in diverse areas of importance to society such as demographics, employment, wages, migration, health, poverty, access to services, education, human rights, equity, pollution, and many other domains**.**

The ProCivicStat analysis argues that civic statistics are created and communicated to the public by multiple providers, both official agencies and public organizations as well as private actors (e.g., NGOs, newspapers). ProCivicStat argues that regardless of their source, civic statistics have five general characteristics, which relate to:

1. **Multivariate phenomena** that are often inter-correlated and involve non-linear relationships,
2. Data that are analysed and reported **through aggregation or disaggregation** on various levels,
3. **Dynamic data** that change or are updated over time,
4. Since data and findings about social phenomena are multivariate, aggregated at multiple levels, and dynamic, their description across time or comparison units requires the use of diverse types of

representations. Hence, they may often be conveyed to the public via **rich texts** (printed or spoken), such as via articles in the printed or digital media, TV news programs, social networks, and the like,

5. Data and findings about trends in society are also communicated via **rich visualizations** that are broader in scope and at times more sophisticated compared with the (limited) range of ideas and representations included in introductory statistics classes or in high-school curricula.

ProCivicStat argues that traditional instruction in statistics, at either the high-school or university level, seldom covers all aspects of civic statistics, hence does not prepare graduates for their life roles and undermines their ability to engage with social issues. Accordingly, on its website (www.procivicstat.org), ProCivicStat offers many teaching materials and suggestions (some of them in multiple languages) which offer tools and ideas for introducing and improving the teaching and learning of civic statistics, thereby also contributing to the development of statistical literacy and deeper understanding of statistics in general. The website also includes papers about new conceptual models that describe the knowledge bases and skills needed to understand civic statistics (Nicholson, Ridgway & Gal, 2017), elements of official statistics literacy (e.g., Gal & Ograjenšek, 2017), and other enabling processes and attitudes that underlie the understanding of statistics and data about societal changes.

Summary and Implications

The ideas presented in this brief paper are only the 'tip of the iceberg' with regard to the challenges related to improving statistical literacy. “Improving” means better and more teaching in schools, as well as spreading statistical literacy in the general adult population (MacCuirc, 2015). My actual plenary talk presented additional examples and ideas.

In summary, I argue that we have to think along two paths in parallel:

a. **Reflect on the teaching for statistical literacy and its place at the high-school level:** we should examine why we have failed in the past to promote statistical literacy, and what we can do about it, but do so from a systemic perspective that covers all key elements in the "competency system". From this lens, we should examine: curriculum design, teachers' preparation and professional development, teachers attitudes and beliefs about statistic, produce new teaching materials, embrace relevant technology geared for teaching statistical literacy (not just programs for data analysis per se), and so forth.

b. **Attend to new developments and their implications:** we should continue to update our views regarding what is the nature of statistical literacy by itself, as a target competence. After all, views about needed adult competencies do not sit still, but evolve over time in line with changes in society, results of new research, new technologies, and emerging insights about new demands facing citizens. All this requires that we update the conceptualization of statistical literacy, and accordingly fix our educational practices, at multiple levels of the system. The ProCivicStat project is a potent example for new conceptual models and new tools and suggestions that show a direction for needed changes, but there are additional projects and suggestions in the wider professional literature in statistics education.

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