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The possibility to use benchmarking strategies speeds up adults' response times in fraction comparison tasks

Introduction

The decision which of two fractions is the bigger one can be reached in a variety of ways. Literature distinguishes two kinds of strategies for this task (cf. e.g. Faulkenberry & Pierce, 2011; Reinhold, Reiss, Hoch, Werner, & Richter-Gebert, 2018): On the one hand, one can apply *component-based strategies.* These strategies make use of rules regarding the comparison of fractions with the same enumerator or denominator and, by bringing the fractions to the same enumerator or denominator, work for every pair of fractions to compare. On the other hand, *holistic strategies* are based on an understanding of the magnitude of both fractions. One particular kind of these strategies make use of *benchmarks*, i.e. certain numbers like 1 or 1/2 that may fit between the two fractions to compare. By comparing both fractions to a benchmark, the bigger fraction may be chosen transitively. For example, it is 3/7 < 2/3, as 3/7 < 1/2 and 1/2 < 2/3. It is not always possible to fit a benchmark between the two fractions to compare; the possibility to use a benchmarking strategy is therefore a feature of a pair of fractions.

There is empirical evidence that both students and adults use both kinds of strategies to compare fractions (Clarke & Roche, 2009; Faulkenberry & Pierce, 2011; Post & Cramer, 1987; Reinhold et al., 2018) and that for adults, holistic strategies lead to faster answers than component-based ones (Faulkenberry & Pierce, 2011).

Other influences on the response times include the numerical distance, expertise and *congruency*. Here, congruency refers to the faulty notion that the bigger the numbers, the bigger the fraction – an overgeneralization from the natural numbers, part of a so called *natural number bias* (Ni & Zhou, 2005): Congruent items (e.g. 2/5 vs. 7/8) in which the notion leads to a correct response are processed faster than incongruent items (e.g. 2/3 vs. 3/7). This effect is found even in expert mathematicians when comparing fractions with common components (Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013).

These studies indicate that strategy use and item congruency both influence fraction comparison for students as well as for adults. Yet, their interrelation has not been investigated systematically. Moreover, it is unclear to what extend mathematical knowledge can overrule or interact with these influences.

Research Aims

This study examines whether there is an effect of a natural number bias when providing adults with fraction comparison tasks with non-common components in which benchmarking is a feasible solution strategy. Furthermore, it was examined whether adults with a potentially higher prior knowledge benefit more from this possibility.

Method and Sample

24 adults participated in the study. All participants had a university degree, 13 had a background in mathematics educations (hereafter *experts*).

Participants were presented with 24 pairs of fractions and were tasked to quickly pick the bigger one. Tasks were adapted from DeWolf & Vosniadou (2011) and fit into 2 × 2 categories: in one dimension, whether they were congruent or incongruent with respect to a natural number bias, and in the other dimension whether a solution could be found using 1 or 1/2 as a benchmark, or if no benchmark seemed feasible.

The testing environment was implemented on the iPad. The user was presented with two "cards" with the fractions in symbolic representation and could pick the bigger one by tapping the corresponding card (cf. Figure 1). Participants could not change their decisions. By pressing the button below the cards, participants proceeded to the next item. Order of the items and which fraction was presented on the left was randomly decided by the iPad.

Fig. 1: Item display on the iPad, before (left) and after (right) choosing an answer.

For each item, both fractions, the time between display of the item and the selection of a fraction (response time, RT), and the correctness of the answer were recorded and saved on the iPad. After the testing session, data were send to a server and saved in a database for analysis.

For the analysis, only correct responses were included. Overall, a solution rate of 88.37 % was achieved. However, experts had a significant higher solution rate, $t(14.357)=-3.727$, $p=.002$, $d=1.563$. 845 observations were included in the analysis. Data were analyzed using a linear mixed effects model to account for the non-independence of the data. To ease interpretation, all variables were centered at the grand mean. The following model was approximated using the lme4 package (Bates, Mächler, Bolker, & Walker, 2015) for R (R Core Team, 2016):

$$RT = \beta_0 + \beta_1 \cdot \text{distance} + \beta_2 \cdot \text{congruent} + \beta_3 \cdot \text{benchmarking} \times \text{experts} + u_0 + v_0,$$

where u_0 and v_0 denote the random by-participant and by-item intercept, respectively. By including these random intercepts, this model respects that general response speed may vary across both participants and items.

Results

Table 1 gives an overview of the model estimations. In particular, there was no significant effect of a natural number bias or the distance between the two fractions on the response times. As expected, experts were significantly faster than non-experts. Both groups showed significant shorter response times when working on items where benchmarking strategies were feasible. However, there was no significant interaction between the possibility to use benchmarking strategies and expertise.

Table 1: Predictors of Solution Time for fraction comparison tasks with non-common components.

	Fixed		Random	
Variable	*B*	*SE*	*N*	σ^2
Fixed effects				
Constant	5.926^{***}	0.497		
Distance	−1.352	0.679		
Congruency	0.763	0.532		
Benchmarking possible	-2.054^{*}	0.774		
Experts	-1.315^{*}	0.581		
Benchmarking possible×Experts	−0.699	0.651		
Random effects				
Item			24	1.086
Participant			24	1.379

Note. R^2 = .34, 509 Observations; Fixed effects: *B* = Unstandardized Beta; *SE* = Standard Error for the Unstandardized Beta; Random parameters: *N* = Number of groups, σ^2 = Variance; Levels of significance: $^{*}p < .05$, $^{***}p < .001$

Discussion

No significant effect of a natural number bias was found, likely due to the combination of an adult sample and fraction pairs with non-common components, where previous studies also revealed no effect (Obersteiner et al., 2013). In contrast, a significant effect of the possibility to use benchmarking strategies was observed. Experts in our sample did not benefit more than non-experts from the possibility to use benchmarking strategies in a significant way. As the estimation is negative as expected, however, it remains to be seen whether a significant interaction may be found with a larger sample. Therefore, as a next step, we wish to expand the sample to starting university students. Next to answering the question how this sample performs, they study may be used to sensitize beginning education students to the different strategies. One open question is the unbiasing recording of the actual strategy used by adults.

The study also tested the iPad as a testing device. The implementation allowed efficient data processing and largely automated analyses; integrating new data involves little effort.

References

Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting Linear Mixed-Effects Models Using lme4. *Journal of Statistical Software*, *67*(1), 1–48. doi:10.18637/jss.v067.i01

Clarke, D., & Roche, A. (2009). Students' Fraction Comparison Strategies as a Window into Robust Understanding and Possible Pointers for Instruction. *Educational Studies in Mathematics*, *72*, 127–138. doi:10.1007/s10649-009-9198-9

DeWolf, M., & Vosniadou, S. (2011). The whole number bias in fraction magnitude comparisons with adults. In L. Carlson, C. Hoelscher, & T. F. Shipley (Eds.), *Proceedings of the 33rd Annual Conference of the Cognitive Science Society* (pp. 1751–1756). Cognitive Science Society.

Faulkenberry, T. J., & Pierce, B. H. (2011). Mental Representations in Fraction Comparison. *Experimental Psychology*, *58*(6), 480–489. doi:10.1027/1618-3169/a000116

Ni, Y., & Zhou, Y.-D. (2005). Teaching and Learning Fraction and Rational Numbers: The Origins and Implications of Whole Number Bias. *Educational Psychologist*, *40*(1), 27–52. doi:10.1207/s15326985ep4001_3

Obersteiner, A., Van Dooren, W., Van Hoof, J., & Verschaffel, L. (2013). The natural number bias and magnitude representation in fraction comparison by expert mathematicians. *Learning and Instruction*, *28*, 64–72. doi:10.1016/j.learninstruc.2013.05.003

Post, T. R., & Cramer, K. A. (1987). Children's strategies when ordering rational numbers. *Arithmetic Teacher*, *35*(2), 33–35.

R Core Team. (2016). *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. Retrieved from http://www.R-project.org

Reinhold, F., Reiss, K., Hoch, S., Werner, B., & Richter-Gebert, J. (2018). *Comparing Fractions: The Enactive Way. Supporting Students' Choice of Appropriate Strategies with iPad-Assisted Instruction*. Paper presented at the 2018 annual meeting of the American Educational Research Association, New York, AERA.