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Junior High School Students' Understanding of Mathematical Functions

1. Introduction

Previous studies on teaching functions are many in Japan (Japan Society of Mathematical Education, 2010). However, the National Institute for Educational Policy Research (2014, 2017) indicates that Japanese students often have problems comprehending mathematical functions. To remedy this problem, it is necessary to clarify the stage of instructional content of mathematical functions that students are unable to comprehend.

2. Background

To identify the stage of non-comprehension, it is necessary to grasp the process by which students understand math. To understand certain mathematical concepts, multiple appropriate examples are considered necessary. Furthermore, to understand abstract mathematical concepts, it is necessary to gradually proceed to abstraction from specific events. van Hiele (1986) divides students' understanding into five levels, and the relationship of each level with its next is such that a certain level of "thought results" becomes the "subject of thinking" of the next level. Therefore, it is necessary to form concepts for understanding abstract concepts through several examples.

Prior research in Japan on the teaching of functions, based on the awareness of students and educational practice, examined function guidance and made certain evaluations. Yokochi (1962) presented the function of the instructional system for students (elementary school fifth and sixth graders to junior high school second graders or so). The discrete finite set, relationships, and functions were first taught, and next, after teaching the function concept of a continuous amount, the formulaic expression of functions and the graph of functions were taught. Okamori (1983) first teaches 'Extract two variables from the actual event and make them correspond' to first-grade junior high school students. He then demonstrates an instructional system leading to the expression of functions and instruction regarding graphs. This part is in common with the research of Yokochi. From these two studies, we understand that it is necessary to form concepts to understand the concept of abstract functions.

Van Hiele's theory covers the entire process of students' understanding over the long term. However, it does not sufficiently cover short-term instruction content, for example, ‘function definition’ i.e. ‘meaning of functions’, etc. Hence, this research aims to construct a theoretical framework that can target such guidance contents.

3. Method

To analyse factors hindering students’ understanding of functions, I establish a theoretical model based on the previous research described above.

In junior high school, students often learn mathematics through real events and advanced mathematics through mathematical events. Therefore, knowledge and concepts of mathematics taught to students can be classified into those that belong to the ‘real world’ and those of the ‘mathematical world’, as seen in mathematical modelling.

From the research of Yokochi and Okamori, the mathematics world can be classified into an ‘abstracted stage’ and an ‘abstracting stage’.

An ‘abstracting stage’ is one where students form abstract mathematical knowledge and concepts. For example, [I] Extract a variate from a phenomenon, [II] Relate the 2 changed variates etc. to be taught.

With the guidance of the ‘abstracting stage’, from the study of van Hiele, concepts for understanding abstract mathematical knowledge and concepts are formed. In this research, we call this the ‘developing concept’. While teaching the ‘abstracted stage’, mathematical concepts are formed.

4. Result

From the above, guidance of functions can be arranged as follows.

First, while teaching in the ‘real world’ stage, the student recognizes the change in the indicated event.

Next, during the guidance in the ‘stage of abstraction’, they form a developing concept concerning [I] and [II] for the indicated event, for example.

Finally, with the guidance in the ‘abstracted stage’ they form concepts of functions based on the developing concepts, that is, the meanings of functions.

Fig.1 shows the above results.

Based on this theoretical framework, students summarize the changes in ‘real world’ events, present them in graphs or, under guidance, express them in formulas to form function concepts.

5. Discussion (Based on Nisawa (2017))

We examine the effectiveness of the theoretical framework mentioned above. Learning a function begins with understanding its definition. This is called understanding the ‘meaning of function relation’, and it is pointed out that this is where the problem lies (National Institute for Educational Policy Research, 2014, 2017). Therefore, unless a developing concept concerning (I) and (II) is formed, it is difficult to understand the definition of the function from the research of Yokochi and Okamori.

In February 2015, Nisawa et al. (2017) conducted a series of surveys to investigate the degree to which students were taught the concepts underlying mathematical functions. They estimated the current level of aptitude in middle school students with respect to (I) and (II). They studied 39 first-grade students (12-13-years old) at a public junior high school in Kobe by asking them to answer questions regarding the relationship between variables while pouring water into a cup (Figure 2: Parts (1), (i), and (ii) are especially relevant to our study because the students have already learned about the relationship between two changing quantities in elementary school).

It shows the result of (1). They found that 66.7% of the students gave two or more appropriate answers regarding the extraction of the ‘changing quantity’. However, 33.3% of the students could give only one appropriate answer and another 10.3% could not answer at all. Regarding the correspondence between variables (i) and (ii), an overall 38.5% of students, could give 1 correct answer, and 25.6% of students could give 2 or more correct answers. However, 35.9% of students were completely unable to perform correspondence between two variables.

These findings indicate that a certain number of students had insufficient ability to extract variables from observed phenomena and form a relationship between the two variables. Therefore, from a theoretical framework, they have difficulty in understanding the definition of functions. Even if they learn the contents of the next level in this state, they may not fully understand them.

The students who had a problem, as pointed out in the Nisawa survey results described above, clearly show that intentional guidance on (I) and (II) is necessary. This would help in making the teaching content clear by placing the knowledge and concept of mathematics that the student should understand in an 'abstracted stage' and setting a developing concept to understand it. In addition, conducting surveys on developing concepts revealed that students could determine the guidance contents of developing concepts and grasp the actual situation. Therefore, the theoretical framework of Fig.1 is considered effective for students to grasp teaching content and understand it.

In the future, it is important to organize knowledge and concepts for function understanding, apply the theoretical framework to it, and examine the guidance contents to form developing concepts.

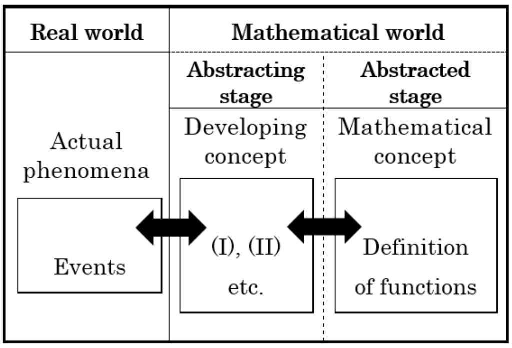

Figure 1: Theoretical framework of teaching method

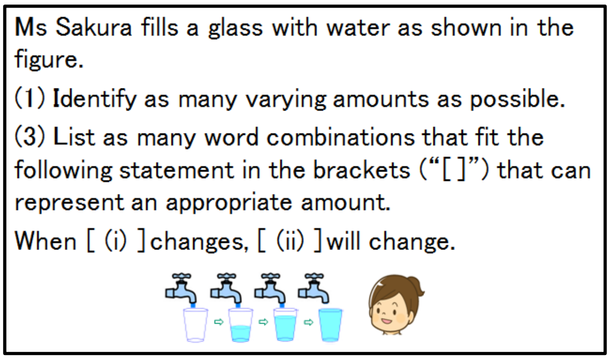

Figure 2: Survey questions (Nisawa et al., 2017)

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