technische universität
dortmund

Essays in Finance: Wrong-way Risk, Jumps and Stochastic Volatility

Dissertation zur Erlangung des akademischen Grades
Doctor rerum politicarum
der Fakultät für Wirtschaftswissenschaften
der Technischen Universität Dortmund

Janis Müller

March 11, 2019

Contents

List of Figures

List of Tables

Acknowledgment

First, I thank Professor Peter N. Posch for being my supervisor and arranging a creative environment at his chair. I also thank my research group, Johannes, Daniel, Philipp, Vladislav, Timotheos, Gerrit and Nils for valuable discussions. Furthermore, I thank Dirk Stemmer for giving me the opportunity to write this thesis. Finally, I thank Otilia Pasnicu for her unconditional support.

1 Introduction

The main focus of this thesis is about understanding the behavior of asset prices and asset returns regarding tail events, in the light of time-varying stochastic volatility and with respect to market efficiency. Thus, the contribution of this thesis is twofold: The first part deals with topics related to risk management whereas the second part deals with topics related to asset pricing.

With new regulations like the credit valuation adjustment (CVA) the assessment of wrong-way risk (WWR) is of utter importance. Wrong-way risk means a negative dependence of the exposure to a counterparty on the counterparty's credit quality. Thus, the **first paper** in chapter 2 studies the co-movement of counterparty credit risk and returns of different asset classes (equity, currency, commodity and interest rate). Whereas there exist several articles about WWR regarding the calculation of CVA (e.g. Glasserman and Yang, 2016; Hofer, 2016; Hull and White, 2012; Kenyon and Green, 2016; Rosen and Saunders, 2012), there is no explicit analysis of WWR for tail events which are of interest for calculation of, for example, margin requirements for non-centrally cleared derivatives. This framework was released by the Basel Committee on Banking Supervision (BCBS) and the International Organization of Securities Commissions (IOSCO) in 2013 with revision in 2015 (Basel Committe on Banking Supervision, 2015). The margin consists of a variation margin and an initial margin. The variation margin is used to settle daily profits and losses of derivatives. The initial margin is an insurance in case the counterparty defaults. It is used to cover any losses occurring from the default event date until the close-out of the derivatives position. The focus is on the initial margin which is exposed to WWR in extreme events, meaning major stress in the banking sector. Using extreme value theory to model the tail of the joint distribution of asset returns and counterparty credit risk, the impact on the risk measure expected shortfall is analyzed when conditioning on major stress in the banking sector for a period from December 2005 until January 2016. This is done for both the American and the European banking sector. There are two main results: Firstly, the correlation between assets belonging to the four mentioned asset classes and major stress in the European and American banking sector increases. Secondly, the expected shortfall conditioned on stress in the banking sector is significantly higher than the unconditioned expected shortfall. For instance, the weekly conditioned expected shortfall on a 97.5 percent level is about 2 to 440 percent higher depending on the asset.

Extreme movements in asset prices are often characterized by jumps and drying up liquidity. The **second paper** aims to improve the understanding of the (unconditioned) link between jumps and liquidity in chapter 3. Modelling price dynamics properly is of utter importance, especially in options pricing (e.g. Merton, 1976) and risk management (e.g. Duffie and Pan, 2001). Empirically, jump-diffusion processes lead to better results than pure diffusion processes (Andersen et al., 2002; Bollerslev et al., 2008; Eraker et al., 2003). Thus, this paper provides an improvement in the understanding of the occurrence of jumps of stocks by investigating stock liquidity as a possible driver. Liquidity is necessary to absorb temporary bulges of buy and sell orders without large price movements (Pagano, 1989). Three different measures for liquidity, the relative bid-ask spread, the turnover and the illiquidity measure of Amihud (2002a), and three different kinds of jump detection tests are considered, the tests of Barndorff-Nielsen and Shephard (2006), including modified tests by Andersen et al. (2012), as well as of Jiang and Oomen (2008) and of Lee and Mykland (2008). Sorting US stocks in three buckets according to their level of liquidity for the period 2007-2011, it is shown that the number of days with jumps increases by about 10% for the relative bid-ask spread and the turnover and about 34% for the illiquidity measure of Amihud (2002a) from the first bucket to the third bucket considering all three jump measures for the highest available frequency.

Apart from jumps, stochastic volatility is an important stylized fact of asset price processes. In the **third paper** the dynamics of asset prices are time-changed to study the influence of stochastic volatility on asset prices in a parameter-free approach (see chapter 4). Stochastic volatility seems necessary to understand features of financial markets (Bansal and Yaron, 2004; Barndorff-Nielsen and Veraart, 2013; Bollerslev et al., 2012; Carr and Wu, 2009). However, including stochastic volatility in asset pricing by assuming a specific process is critical. The choice of a specific process has significant consequences for results but empirically it is doubtful if one can choose the correct model out of all consistent alternatives (Carr and Lee, 2009). Applying the time-changing technique avoids to use a specific process for volatility to study the impact of stochastic volatility on asset prices. Firstly, formulas for the expected return of assets and the risk-free rate are derived. It is noteworthy that the risk-free rate becomes stochastic under the time-change. Based on the theoretical findings, stochastic consumption volatility is explored considering prevailing puzzles. Secondly, a factor is constructed mimicking the effect of stochastic volatility of the market portfolio on asset prices extending the five-factor model of Fama and French (2015). Considering anomalies targeted by existing factor models, the constructed factor especially helps to describe cross-sectional excess returns of portfolios formed on size and momentum. This finding indicates that the momentum effect is partly explicable by stochastic volatility.

The **fourth paper** deals with ambiguous volatility as an explanation for time-variation in the market's risk premium in chapter 5. Harvey (1989) and following Ferson and Harvey (1991) find equity premia exhibit cyclical variation. Popular asset pricing models allowing for variation in equity premia are habit formation (Campbell and Cochrane, 1999) and long-run risks (Bansal and Yaron, 2004). Whereas variation in equity premia is a consequence of variation in risk aversion in the habit model, the long-run risk model uses stochastic consumption volatility to let equity premia vary. In contrast to chapter 4 volatility is not only assumed to be stochastic but to be ambiguous. Recently, Jeong et al. (2015) find ambiguity to matter empirically. Following Epstein and Ji (2013) consumption volatility is assumed to lie in some interval and ambiguity is defined as the width of this interval. Considering the formulas for the expected market return and the risk-free rate, derived in a consumption-based asset pricing model (Cochrane, 2001), the implied consumption volatility is calculated from both formulas using a long data set for the U.S. market. Taking the difference of both resulting implied volatilities as a proxy for ambiguity, a regression shows that variation in this measure of ambiguity explains up to 69% of the post-war variation in the market's risk premium.

Finally, the **fifth paper** is about the currently discussed topic of market efficiency regarding cryptocurrencies. There exist many studies about the weak-form efficiency of Bitcoin (Urquhart, 2016; Nadarajah and Chu, 2017; Vidal-Tomás and Ibañez, 2018; Kristoufek, 2018; Jiang et al., 2018; Bariviera, 2017; Tiwari et al., 2018; Khuntia and Pattanayak, 2018; Alvarez-Ramirez et al., 2018) but little for the efficiency of cryptocurrencies in general (Brauneis and Mestel, 2018; Wei, 2018). Extending the existing literature about the efficiency of crytptocurrencies, this stduy investigates the average price delay of the cryptocurrency market to new information. According to the efficient market hypothesis of Fama (1970), prices should reflect new information instantaneously. Therefore, a delay in reflecting new information is a sign for inefficiency of a market. Using three delay measures as given in Hou and Moskowitz (2005) it is shown that news, affecting the cryptocurrency market, are much faster incorporated in prices during the last three years indicating that the cryptocurrency market becomes more efficient over time. Furthermore, the price delay is mainly driven by liquidity which is studied in the cross-section of 75 cryptocurrencies.

1.1 Publication details

Paper I (chapter 2):
Wrong-way risk in tails

Authors:
Janis Müller, Peter N. Posch

Abstract:
With new regulations like the credit valuation adjustment (CVA) the assessment of wrong-way risk (WWR) is of utter importance. We analyze the effect of a counterparty's credit risk and its influence on other asset classes (equity, currency, commodity and interest rate) in the event of extreme market movements like the counterparty's default. With an extreme value approach we model the tail of the joint distribution of different asset returns belonging to the above asset classes and counterparty credit risk indicated by changes in CDS spreads and calculate the effect on the expected shortfall when conditioning on counterparty credit risk. We find the conditional expected shortfall to be 2% to 440% higher than the unconditional expected shortfall depending on the asset class. Our results give insights both for risk management as well as for setting an initial margin for non-centrally cleared derivatives which becomes mandatory in the European Market Infrastructure Regulation (EMIR).

Publication details:
Journal of Asset Management, 19(4) (2018): 205-215.

Paper II (chapter 3):
Do illiquid stocks jump more frequently?

Authors:
Sebastian Kunsteller, Janis Müller, Peter N. Posch

Abstract:
We study the influence of stock liquidity on the stock price jump frequency using intraday data of 175 US stocks during 2007-11. Grouping these stocks according to their average liquidity we find less liquid stocks to jump more often than liquid stocks. Depending on the liquidity measure the least liquid stocks exhibit on average between 10% and 34% more jumps than the most liquid stocks. Our results are robust to different definitions of liquidity and jump measures as well prevail under different time frequencies.

Publication details:
Applied Economics, 51(25) (2019): 2764-2769.

Paper III (chapter 4):

How does stochastic volatility influence asset prices? – A parameter-free approach

Authors:

Janis Müller, Peter N. Posch

Abstract:

We disentangle the risk of time-varying volatility and return in a consumption-based asset pricing model by introducing stochastic volatility of consumption growth to asset prices moving in volatility units instead of moving in time. This time-change approach yields additional insights to risk premia's composition. We empirically explore stochastic volatility where it eases the risk-free rate puzzle and solves the equity premium puzzle if people are very impatient. As a factor it significantly improves the explanation of returns in the cross-section, solves the momentum effect among other anomalies and is not captured by other existing factors.

Publication details:

Submitted to The Review of Financial Studies

Paper IV (chapter 5):
Consumption volatility ambiguity and risk premium's time-variation

Authors:
Janis Müller, Peter N. Posch

Abstract:
In a consumption based asset pricing model one can calculate the volatility of (log-)consumption growth from the expected market return and from the risk-free rate. We propose to use the difference between these estimates to measure ambiguity about consumption volatility. Using a long dataset we show this measure explains up to 69% of post-war variation in the market risk premium.

Publication details:
Finance Research Letters, Forthcoming

Paper V (chapter 6):

Price delay and market frictions in cryptocurrency markets

Authors:

Gerrit Köchling, Janis Müller, Peter N. Posch

Abstract:

We study the efficiency of cryptocurrencies by measuring the price's reaction time to unexpected relevant information. We find the average price delay to significantly decrease during the last three years. For the cross-section of 75 cryptocurrencies we find delays to be highly correlated with liquidity.

Publication details:

Economics Letters, 174 (2019): 39-41

2 Wrong-way risk in tails

The following is based on Müller and Posch (2018b).
https://doi.org/10.1057/s41260-018-0076-9

3 Do illiquid stocks jump more frequently?

The following is based on Kunsteller et al. (2019).
https://doi.org/10.1080/00036846.2018.1558357

4 How does stochastic volatility influence asset prices? - A parameter-free approach

We explore the general influences of stochastic volatility on asset prices in a consumption-based asset pricing framework. We do not assume a specific process for the volatility but disentangle the risk of time-varying volatility and return risk by using time-changed asset prices.

Commonly asset prices are viewed as evolving over time. Under the assumption of consumption growth and asset returns to be iid. asset pricing models yield anomalies when tested empirically (cf. Mehra and Prescott, 1985). A variety of papers target these anomalies by extending the asset pricing model, i.e. generalizing Y_{t+1} in the general stochastic discount factor presentation $M_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} Y_{t+1}$ underlying most of these models (cf. Cochrane, 2017). Bansal and Yaron (2004) introduce stochastic volatility of the consumption growth as crucial for long-run risk models, empirically assessed by Beeler and Campbell (2012) and Bansal et al. (2012). Many authors followed making the case for this particular extension, eg. Carr and Wu (2009), Barndorff-Nielsen and Veraart (2013) or Bollerslev et al. (2012). However, including stochastic volatility in asset pricing by assuming a specific process is critical. The choice of a specific process has significant consequences for results but empirically it is doubtful if one can choose the correct model out of all consistent alternatives (Carr and Lee, 2009).

We propose a different approach to benefit from the extension of stochastic consumption growth volatility but not having to impose a specific driving process for consumption volatility as, for instance, in Boguth and Kuehn (2013). Instead of looking at asset prices evolving in time, we look at their evolution in realized variance of the (log-)consumption process. The volatility of this *time-changed* consumption process is deterministic by construction but time is now stochastic. When time is stochastic, we denote it by τ. It is a stopping time that measures how much time passes for some fixed amount of realized variance. In the time-changed setting we derive the corresponding risk-free rate and the expected return on risky assets. We find that risk premia depend on an additional correction term, the covariance between the random time τ and an assets return. Taking the time-changed version of the risk free-rate and the Hansen and Jagannathan (1991) bound, we explore the influence of stochastic volatility empirically. We find that it eases the risk-free rate puzzle and solves the equity premium puzzle if people are impatient. Regarding factor-models we construct a factor SVOL mimicking the effect of stochastic volatility represented by τ. It is the difference of high and low exposure of stocks to the random time τ. SVOL delivers an expected monthly return of 0.40% and is significant ($t - statistic : 3.57$) even by the standards of Harvey et al. (2016). Sorting stocks into 25 Size-Tau portfolios, the five-factor model of Fama and French (2015) is unable to describe the resulting monthly excess returns ranging from 0.23% to 1.22%. While the average expected return of SVOL cannot be explained by the five-factor model, the average expected return of the factor CVR of Boguth and Kuehn (2013) can be explained and CVR does not improve the performance of the five-factor model considering prevailing other anomalies. In contrast, adding the factor SVOL helps to explain cross-sectional risk premia by improving the performance of the five-factor model. Thus, our factor contains different information than CVR due to the differences in construction resulting from the assumed volatility process. Considering the momentum effect of Jegadeesh and Titman (1993), the loadings on SVOL suggest that the momentum effect is for the most part explicable by stochastic volatility. When volatility is just a constant the relation between time and realized variance is unique such that all formulas derived under the time-change collapse to already known expressions.

Among the first to use a time-change to analyze financial processes, Clark (1973) looks at future prices using a subordinated process to handle the non-normality of observed returns. Such a time-change is also called stochastic clock, transaction clock or business time. Time-changed Lévy processes enjoy increasing popularity in context of option pricing (Carr and Wu, 2004), variance risk premiums (Carr et al., 2012) and pricing non-Gaussian jump-like risks (Shaliastovich and Tauchen, 2011). Recently, Jeong et al. (2015) use time-changed Lévy processes to address ambiguity in empirical asset pricing. For a comprehensive overview of time-changes in asset price modeling, we refer to Geman (2005).

This paper is organized as follows: In the following section 4.1 we give an outline of the basic idea as well as of the time-changing technique. The theoretical implications in form of an additional risk correction for asset prices are subject of the following section 4.1.1. A test for the usefulness of the time-changing technique in empirical asset pricing is, firstly, given by considering the equity premium and risk-free rate puzzle (section 4.2) and, secondly, by constructing a mimicking factor SVOL extending existing factor-models (section 4.3). To understand the latter two sections 4.2 and 4.3 it is sufficient to skim through section 4.1.1 and skip the detailed studying of the risk corrections for the expected risk-free rate as well as the expected return of risky assets in section 4.1.2.

4.1 Stochastic volatility under time-change

Assuming a specific process for volatility in asset pricing models has significant consequences for results but empirically it is difficult to choose the correct one. Instead, we propose an approach without assuming a specific process for volatility. The idea is to change perspective by looking at asset price movements with the realized variance on the x-axis instead of time. For example, instead of observing prices daily we observe them after some amount of variance is realized. If we assume the price process to follow some geometric Brownian motion, the realized variance is the integrated variance of the log-price process which is empirically given by the sum of squared log-returns.[1] As an illustration we plot in figure 4.1 the S&P 500 Total Return Index from January 1998 until December 2016 in time (left) and realized variance (right).[2] The time-change stretches periods of high volatility and compresses periods of low volatility which often coincides with downward and upward trends in stock markets.

Throughout this paper we assume asset prices and consumption to be continuous. Volatility is assumed to be stochastic and is in general allowed to have jumps. In such a stochastic volatility model the dynamics of consumption C_t are given by the following

[1] Due to micro-structure in ultra high frequency data the estimator needs to be adjusted to be robust (c.f. At-Sahalia et al., 2011).

[2] We calculate the integrated variance using an EGARCH model based on daily log-returns, but one can also calculate a model free estimator using intraday log-returns (At-Sahalia et al., 2011).

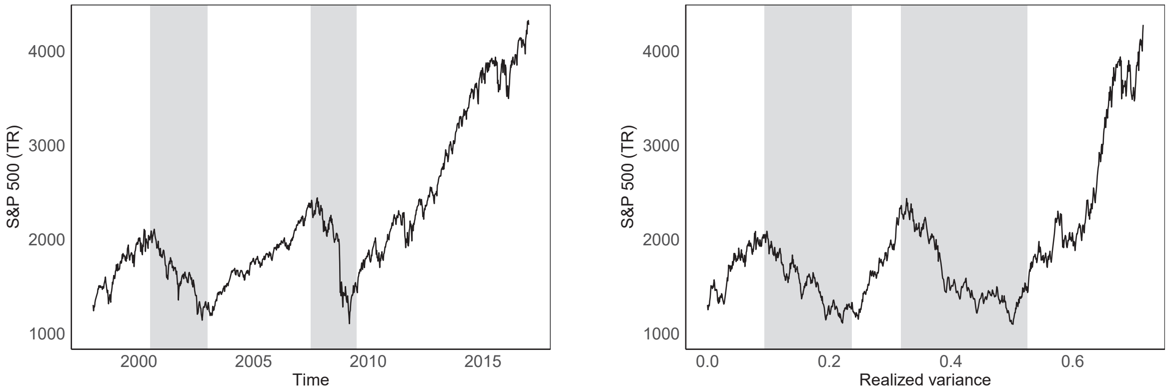

Figure 4.1: S&P 500 Total Return Index (TR) from January 1998 until December 2016 plotted in time (left) and realized variance (right)

stochastic differential equation (SDE):

$$\begin{aligned} \frac{dC_t}{C_t} &= \mu dt + \sqrt{V_t} dW_t^c \\ dV_t &= \alpha_t dt + \beta_t dW_t^v + \eta_t dJ_t \end{aligned} \tag{4.1}$$

where μ is a constant and α_t, β_t and η_t are stochastic processes such that the SDE is well defined. The processes W_t^v and W_t^c are standard Brownian motions with $dW_t^v dW_t^s = \rho dt$ where $\rho \in [-1, 1]$. The process J_t is some pure jump process. With $\eta_t = 0$ often used choices for α_t and β_t are $\alpha_t = \kappa(\theta - V_t)$ and $\beta_t = \xi\sqrt{V_t}$ as in Heston (1993) or with $\beta_t = \xi$ as in Hull and White (1987). However, empirically it is difficult to distinguish different choices of α_t and β_t but the selection is crucial for derivatives or optimization (cf. Epstein and Ji, 2013). Carr and Lee (2009) criticize the selection of a specific parametric model for the volatility process because the volatility is not directly observable and often noisy data leads to noisy estimates.

We do not assume a specific selection for α_t, β_t and η_t. To state the time-changed version of the consumption process in equation (4.1) we start by defining the following stopping time:

$$\tau := \tau_v = \inf\{t \geq 0; \int_0^t V_u du = v\}. \tag{4.2}$$

The idea is to observe the process C_t at the stopping time τ which is equivalent to waiting until the integrated variance exceeds some predefined level v:

$$C_\tau = C_0 \exp\left(\mu\tau - \frac{1}{2}\int_0^\tau V_u du + \int_0^\tau \sqrt{V_u} dW_u^c\right) \tag{4.3}$$

Observing C_τ instead of C_t is advantageous because of the time-change for martingales theorem according to Dambis (1965), Dubins and Schwarz (1965). The theorem as stated and proved in Karatzas and Shreve (1991) tells us that the integral $\int_0^\tau \sqrt{V_u} dW_u^c$ is a standard Brownian motion B_v^c. Using that $\int_0^\tau V_u du = v$ by construction of the stopping time τ, we can restate equation (4.3) as:

$$C_\tau = C_0 \exp\left(\mu\tau - \frac{v}{2} + B_v^c\right). \tag{4.4}$$

A more detailed derivation can be found in the appendix B.1. From equation (4.4) we see that a key advantage of the time-change is the separation of the risk arising from the Brownian motion and the stochasticity of the volatility which is absorbed by the stopping time τ. This disentanglement enables us to study the effect of return risk and the risk of time-variation in volatility separately without assuming a specific model for the volatility. When considering (4.1) in time, we have to deal with the integral $\int_0^t \sqrt{V_u} dW_u^c$. Separating the effect of the return risk represented by W_u^c and the risk of stochastic volatility V_u is in general impractical. For instance, if we condition on the volatility V_u to study the marginal impact of W_u^c, the process W_u^c is in general no Brownian motion and its law is unknown if the law of V_u is unknown. Under the time-change the separation appears as the sum of τ and B_v^c, where τ captures the effects of V_u.

4.1.1 Time-free rate and risk corrections under time-change

We derive the *time-free rate* as the risk-free asset under a time-change in a consumption-based asset pricing model. It can be interpreted as the return of a zero coupon bond which expires as soon as a predefined level of realized variance is exceeded. Thus, the maturity of this asset is random. Given the time-free rate we derive formulas for the risk-free rate, which is stochastic under the time-change, and for risky assets, such as stocks.

In classical asset pricing the asset which does not contain any uncertainty about its future return is called the *risk-free* asset and its return the risk-free rate. In the time-changed setting we look at intervals of integrated variance and the risk-free rate is not constant on such intervals. Instead, it is adjusted by an additional correction for the stochasticity of the volatility as we show. An analogous to the risk-free asset in time can be defined straightforwardly as the asset whose return in variance is know ex ante, the *time-free rate*:

$$R^{fv}_{v+1} = \frac{1}{\mathbb{E}(m_{v+1})} \tag{4.5}$$

As in Cochrane (2001) let the stochastic discount factor be similarly given in variance by

$$m_{v+1} = \beta^{\tau_{\Delta v}} \frac{u'(c_{v+1})}{u'(c_v)} \tag{4.6}$$

where Δv is the difference in integrated variance between v and $v+1$, u is the power utility function $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, β the subjective discount factor and τ_v the stopping time as defined in (4.2). Taking the time-changed consumption process $c_v := C_\tau$ as given in equation (4.4), the stochastic discount factor can be written as

$$m_{v+1} = e^{-(\gamma\mu+\delta)\tau_{\Delta v} + \gamma\frac{\Delta v}{2} - \gamma B^c_{\Delta v}} \tag{4.7}$$

where $\beta^{\tau_{\Delta v}} = e^{-\delta\tau_{\Delta v}}$. Starting with the expression for the time-free rate from equation (4.5), we get

$$\begin{aligned} \frac{1}{R^{fv}_{v+1}} &= \mathbb{E}\left(e^{-(\gamma\mu+\delta)\tau_{\Delta v}}\right)\mathbb{E}\left(e^{\gamma\frac{\Delta v}{2} - \gamma B^c_{\Delta v}}\right) + \mathbf{Cov}\left(e^{-(\gamma\mu+\delta)\tau_{\Delta v}}, e^{\gamma\frac{\Delta v}{2} - \gamma B^c_{\Delta v}}\right) \\ &= \mathbb{E}\left(e^{-(\gamma\mu+\delta)\tau_{\Delta v} + \gamma(\gamma+1)\frac{\Delta v}{2}}\right) + \mathbf{Cov}\left(e^{-(\gamma\mu+\delta)\tau_{\Delta v}}, e^{\gamma\frac{\Delta v}{2} - \gamma B^c_{\Delta v}}\right) \end{aligned} \tag{4.8}$$

since $B^c_{\Delta v}$ is a standard Brownian motion. Looking at equation (4.8), we see some important features of the time-free rate. If the variance is constant, i.e. there is no time-variation in volatility, the time-free rate is equal to the risk-free rate. Let $\Delta v = \sigma^2 \Delta t$, then $\tau_{\Delta v} = \Delta t$ by definition of τ and

$$R^{fv}_{v+1} = e^{(\gamma\mu+\delta)\Delta t - \gamma(\gamma+1)\frac{\sigma^2}{2}\Delta t} = R^{ft}_{t+1} \tag{4.9}$$

with Δt being the difference from time t to time $t+1$. If the covariance is zero, the time-free rate depends on the expectation over the stopping time τ only. To understand γ and δ let us assume $\tau_{\Delta v} \sim \mathcal{N}(\mu_\tau, \sigma_\tau^2)$, which is unrealistic since $\tau > 0\, a.s.$ but helps to study the effect of the stopping time τ. The time-free rate is in this case given by

$$R_{v+1}^{fv} = e^{(\gamma\mu+\delta)\mu_\tau - (\gamma\mu+\delta)^2 \frac{\sigma_\tau^2}{2} - \gamma(\gamma+1)\frac{\Delta v}{2}}. \tag{4.10}$$

The time-free rate is lowered by precautionary savings where the last term is equivalent to the precautionary savings term for the risk-free rate. The second term, however, captures the volatility about the stopping time and thus the volatility of the variance process of the stochastic volatility model in (4.1). The higher the volatility of the stopping time, the lower the time-free rate since people try to avoid additional uncertainty. Furthermore, the time-free rate depends on the expectation of τ which is always positive but its influence depends on risk aversion and impatiences. When people are impatient $\delta > 0$ and risk averse $\gamma > 0$ then $(\gamma\mu + \delta) > 0$ and a higher expected τ means a higher time-free rate. In contrast, if people prefer money tomorrow to money today and are risk loving, a higher expected τ means a lower time-free rate.

A positive covariance lowers the time-free rate and a negative covariance increases the time-free rate. For small values we can approximate the covariance by

$$\begin{aligned}\mathbf{Cov}\left(e^{-(\gamma\mu+\delta)\tau_{\Delta v}}, e^{\gamma\frac{\Delta v}{2} - \gamma B_{\Delta v}^c}\right) &\approx \mathbf{Cov}\left(-(\gamma\mu+\delta)\tau_{\Delta v}, \gamma\frac{\Delta v}{2} - \gamma B_{\Delta v}^c\right) \\ &= \gamma(\gamma\mu+\delta)\,\mathbf{Cov}\left(\tau_{\Delta v}, B_{\Delta v}^c\right).\end{aligned} \tag{4.11}$$

If people are risk averse and impatient, the sign of the covariance term depends on the covariance of the stopping time and the Brownian motion of the consumption process. A positive covariance means that consumption is negatively correlated with its variance since τ is larger when the variance is low and vice versa.

4.1.2 Risk corrections

In classical asset pricing expected asset returns equal the risk-free rate plus a risk correction which depends on the covariance of the stochastic discount factor and the assets return (e.g. see Cochrane, 2001). Assuming the stochastic discount factor and asset returns being modelled by geometric Brownian motions, the sign of this risk correction depends on the correlation of the Brownian motions. If volatility is stochastic, we find an additional term, the covariance between the assets return and the stopping time τ which is linked to consumptions volatility.

Starting with $1 = \mathbb{E}(m_{v+1} R^i_{v+1})$ and the expression for the time-free rate $R^{fv}_{v+1} = 1/\,\mathbb{E}(m_{v+1})$ we get

$$\mathbb{E}\left(R^i_{v+1}\right) = R^{fv}_{v+1} - R^{fv}_{v+1}\,\mathbf{Cov}\left(m_{v+1}, R^i_{v+1}\right). \tag{4.12}$$

With the expression for the stochastic discount factor in equation (4.7), we can approximate the covariance in equation (4.12) by:

$$\begin{aligned} &\mathbf{Cov}\left(e^{-(\gamma\mu+\delta)\tau_{\Delta v}+\gamma\frac{\Delta v}{2}-\gamma B^c_{\Delta v}}, R^i_{v+1}\right) \\ &\quad\approx \mathbf{Cov}\left(-(\gamma\mu+\delta)\tau_{\Delta v}+\gamma\frac{\Delta v}{2}-\gamma B^c_{\Delta v}, R^i_{v+1}\right) \\ &\quad= -(\gamma\mu+\delta)\,\mathbf{Cov}\left(\tau_{\Delta v}, R^i_{v+1}\right) - \gamma\,\mathbf{Cov}\left(B^c_{\Delta v}, R^i_{v+1}\right). \end{aligned} \tag{4.13}$$

Thus, any assets return is equal to the time-free rate adjusted by the covariance between the stopping time τ and the assets return as well as the driving Brownian motion of the consumption process $B^c_{\Delta v}$. Again, if the variance is constant, i.e. τ is constant, we arrive at the usual expression known from asset pricing in time. Let $\Delta v = \sigma^2 \Delta t$, then $\tau_{\Delta v} = \Delta t$ by definition of τ and according to equation (4.9) it holds $R^{fv}_{v+1} = R^{ft}_{t+1}$. Following the logic of equation (4.9) we have $m_{v+1} = m_{t+1}$ and $R^i_{v+1} = R^i_{t+1}$ and we get $\mathbb{E}\left(R^i_{t+1}\right) = R^{ft}_{t+1} - R^{ft}_{t+1}\,\mathbf{Cov}\left(m_{t+1}, R^i_{t+1}\right)$.

Thus equation (4.12) is a generalization of the risk correction equation known in classical asset pricing. Further, we study the covariance given in equation (4.13) for the expected risk-free rate as well as for the expected return of a risky asset.

Risk-free rate

First, we start with the risk-free rate. Let the risk-free rate be given by $R^{ft}_t = e^{r^{ft}t}$, where r^{ft} is the (constant) continuous risk-free rate. In variance it simply writes as $R^{ft}_{v+1} = e^{r^{ft}\tau_{\Delta v}}$. For small values we can do the approximation $R^{ft}_{v+1} \approx r^{ft}\tau_{\Delta v}$ and we get for the covariance in equation (4.13):

$$\begin{aligned}&\mathbf{Cov}\left(e^{-(\gamma\mu+\delta)\tau_{\Delta v}+\gamma\frac{\Delta v}{2}-\gamma B^c_{\Delta v}}, e^{r^{ft}\tau_{\Delta v}}\right)\\&\quad\approx -(\gamma\mu+\delta)r^{ft}\,\mathbf{Var}\left(\tau_{\Delta v}\right)-\gamma r^{ft}\,\mathbf{Cov}\left(B^c_{\Delta v},\tau_{\Delta v}\right).\end{aligned} \tag{4.14}$$

Considering equation (4.12), the expected risk free-rate is equal to the time-free rate plus some correction due to the variance of τ and the covariance of τ and $B^c_{\Delta v}$. If people are risk averse and impatient, the expected risk-free rate is higher than the time-free rate when the variance of the stopping time is high corresponding to a high volatility of volatility in time. Furthermore, if people are risk averse, the expected risk-free rate is higher than the time-free rate when the correlation of the stopping time τ and $B^c_{\Delta v}$ is positive, meaning consumption growth is high during times of low volatility and vice versa. This is reasonable because a zero coupon bond expires at a fixed time neglecting the level of realized variance during the lifetime of the bond. A zero coupon bond expiring as soon as some predefined level of realized variance is exceeded hedges against times of high volatility and low consumption growth. Thus in comparison, its price should be higher and its return, the time-free rate, lower.

Risky assets

Denote the risky asset by S_t given as the solution to a stochastic volatility model such as in equation (4.1). Since the time-change is always done according to the volatility of the consumption process, the time-changed version of the risky asset is

$$S_\tau = S_0 e^{\mu^s \tau_v - \frac{1}{2}\int_0^v a_u du + \int_0^v \sqrt{a_u} dB^s_u}. \tag{4.15}$$

A derivation is given in the appendix B.2. The stochastic process a_v is the ratio of the variance of the risky asset V^s_t and the variance of the consumption process V^c_t:

$$a_v = \frac{V^s_{\tau(v)}}{V^c_{\tau(v)}}. \tag{4.16}$$

Of course, since the risky assets variance process is different but we time change according to the variance process of the consumption process this ratio appears naturally. Let the stochastic discount factor again be given as in equation (4.7) and let $R^i_{v+1} = S_\tau / S_0$.

Let us further assume that the ratio of the volatility of the risky asset and the volatility of the consumption is a constant plus some independent random variable, i.e.

$$\sqrt{\frac{V_t^s}{V_t^c}} = \beta + \epsilon_t. \tag{4.17}$$

where $\epsilon_t > 0$ $a.s.$ and $\mathbb{E}\,\epsilon_t = 0$. Given the expression for the risky asset in equation (4.15) we can approximate the covariance in equation (4.13) by

$$\begin{aligned} &\mathbf{Cov}\left(m_{v+1}, R_{v+1}^i\right) \\ &\quad\approx -(\gamma\mu^c + \delta)\mu^s\,\mathbf{Var}\left(\tau_{\Delta v}\right) - \gamma\mu^s\,\mathbf{Cov}\left(B_{\Delta v}^c, \tau_{\Delta v}\right) - \\ &\quad(\gamma\mu^c + \delta)\,\mathbf{Cov}\left(\tau_{\Delta v}, \int_0^{\Delta v} \sqrt{a_u} dB_u^s\right) - \gamma\,\mathbf{Cov}\left(B_{\Delta v}^c, \int_0^{\Delta v} \sqrt{a_u} dB_u^s\right). \end{aligned} \tag{4.18}$$

Thus, a risky asset is also influenced by the variance of τ as well as the covariance of τ and $B_{\Delta v}^c$ but additionally by two further covariances.[3] Given the expression in equation (4.17) we can simplify the additional covariances in equation 4.18. First, we get

$$\mathbf{Cov}\left(\tau_{\Delta v}, \int_0^{\Delta v} \sqrt{a_u} dB_u^s\right) = \beta\,\mathbf{Cov}\left(\tau_{\Delta v}, B_{\Delta v}^s\right) + \mathbf{Cov}\left(\tau_{\Delta v}, \int_0^{\Delta v} \epsilon_{\tau(u)} dB_u^s\right). \tag{4.19}$$

Since ϵ is independent of τ, the latter term is zero. Secondly, we get

$$\begin{aligned} \mathbf{Cov}\left(B_{\Delta v}^c, \int_0^{\Delta v} \sqrt{a_u} dB_u^s\right) &= E\left(\int_0^{\Delta v} \beta + \epsilon_{\tau(u)} d\left[B^c B^s\right]_u\right) \\ &= \beta\rho_{c,s}\Delta v \end{aligned} \tag{4.20}$$

where we use that $d\left[B^c B^s\right]_u = \rho_{c,s} du$ with correlation coefficient $\rho_{c,s}$ and that $\mathbb{E}\,\epsilon_{\tau(u)} = 0$. This is the systematic risk part in the classical asset pricing theory. For instance, if $\beta = \sigma_s/\sigma_c$ and $\sqrt{\Delta v} = \sigma_c$, i.e. volatility is constant, the covariance expression is

[3] Note that

$$\begin{aligned} &\mathbf{Cov}\left(-(\gamma\mu^c + \delta)\tau_{\Delta v} - \gamma B_{\Delta v}^c, -\frac{1}{2}\int_0^v a_u du\right) \\ &= \mathbf{Cov}\left(-(\gamma\mu^c + \delta)\tau_{\Delta v} - \gamma B_{\Delta v}^c, -\frac{1}{2}\left(\beta v + \int_0^v \epsilon_{\tau(u)} du\right)\right) = 0 \end{aligned}$$

since ϵ is independent of τ and B^c.

$\sigma_s \sigma_c \rho_{c,s}$. Summarizing equations (4.19) and (4.20), the stochastic volatility of the asset does not influence asset prices as long as there is no persistent trend in the expectation of the ratio in equation (4.17).

4.2 Equity premium and risk-free rate puzzle

In the previous section we show time-varying volatility in consumption-growth to influence equity premia. Using inflation adjusted data for consumption-growth, U.S. market returns and the risk-free rate as described in Shiller (1992) and available in an updated version on Shiller's homepage, we quantify the effects of the time-varying volatility considering prevailing puzzles.

The equity premium puzzle of Mehra and Prescott (1985) and the risk-free rate puzzle emphasized by Weil (1989) arise when consumption-growth is assumed to be log-normally distributed. We restate the Hansen and Jagannathan (1991) bound in the time-changed setting to determine the effect of stochastic volatility on the equity premium puzzle. From equation (4.12) we derive

$$\frac{\mathbb{E}\left(R^i_{v+1}\right) - R^{fv}_{v+1}}{\mathbf{Var}\left(R^i_{v+1}\right)} = -\rho_{m_{v+1},R^i_{v+1}} \frac{\mathbf{Var}\left(m_{v+1}\right)}{\mathbb{E}(m_{v+1})}, \tag{4.21}$$

where $\rho_{m_{v+1},R^i_{v+1}}$ is the correlation between m_{v+1} and R^i_{v+1}. The expression $R^{fv}_{v+1} = 1/\,\mathbb{E}(m_{v+1})$ is the time-free rate as shown in equation (4.5) being the return of a zero-coupon bond expiring when the integrated variance exceeds a predefined level. With the expression for the time-changed risk-free rate according to equation (4.12) we can replace the time-free rate by

$$R^{fv}_{v+1} = \frac{\mathbb{E}\left(R^{ft}_{v+1}\right)}{1 - \mathbf{Cov}\left(m_{v+1}, R^{ft}_{v+1}\right)}, \tag{4.22}$$

to obtain

$$\left|\frac{\mathbb{E}\left(R^i_{v+1} - R^{ft}_{v+1}\right)}{\mathbf{Var}\left(R^i_{v+1}\right)}\right| \leq \frac{\sqrt{\mathbf{Var}\left(m_{v+1}\right)}}{\mathbb{E}(m_{v+1})} + \frac{\mathbf{Cov}\left(m_{v+1}, R^{ft}_{v+1}\right)}{\mathbb{E}(m_{v+1})\sqrt{\mathbf{Var}\left(R^i_{v+1}\right)}}. \tag{4.23}$$

with $m_{v+1} = e^{-(\gamma\mu+\delta)\tau_{\Delta v}+\gamma\frac{\Delta v}{2}-\gamma B^c_{\Delta v}}$. It is easy to check that the Hansen-Jagannathan bound in general depends on the subjective discount factor $\beta = e^{-\delta}$ when we consider stochastic volatility. If volatility is constant the above expression collapses to the well known results without β.

γ	β^{sv}_{rf}	β^{ln}_{rf}	β^{sv}_{HJ}	β^{ln}_{HJ}
1	1.0011	1.0001	$[0, 0.3177] \cup [1.2826, \infty)$	$\emptyset$
5	1.0684	1.0699	$[0, 0.3625] \cup [1.3451, \infty)$	$\emptyset$
10	1.1266	1.1321	$[0, 0.5281] \cup [1.3843, \infty)$	$\emptyset$
15	1.1486	1.1613	$\mathbb{R}_+$	$\mathbb{R}_+$
20	1.1276	1.1551	$\mathbb{R}_+$	$\mathbb{R}_+$
30	0.9490	1.0415	$\mathbb{R}_+$	$\mathbb{R}_+$
40	0.6443	0.8299	$\mathbb{R}_+$	$\mathbb{R}_+$
50	0.3352	0.5843	$\mathbb{R}_+$	$\mathbb{R}_+$

Table 4.1: Risk aversion parameters γ with corresponding intervals for the subjective discount factor β solving the equity premium and risk-free rate puzzle; We take consumption-growth to follow a geometric Brownian motion. The parameters β^{sv}_{rf} and β^{ln}_{rf} are the subjective discount factors for the risk-free rate with stochastic volatility and constant volatility, respectively. The parameters β^{sv}_{HJ} and β^{ln}_{HJ} are the subjective discount factors for the Hansen-Jagannathan bound with stochastic volatility andd constant volatility, respectively. We use inflation adjusted data for consumption-growth, U.S. market returns and the risk-free rate as described in Shiller (1992) and available in an updated version on Shiller's homepage. The timespan is from 1889 to 2008.

Table 4.1 shows the value of the subjective discount factor β necessary to solve the equity premium and risk-free rate puzzle, respectively. Assuming consumption-growth follows a geometric Brownian motion with constant volatility, i.e. consumption-growth is log-normally distributed, the subjective discount factor β^{ln}_{rf} is only smaller than one for risk aversion greater than 32. With stochastic volatility the subjective discount factor β^{sv}_{rf} is slightly smaller but still greater than one for risk aversion coefficients smaller than 28. Thus, allowing for stochastic volatility for consumption-growth eases the risk-free rate puzzle but does not solve it.

For the equity premium puzzle risk aversion must be as high as 12 in case of constant volatility such that the Hansen-Jagannathan bound is greater than the Sharpe ratio of the U.S. stock market which is about 0.38. In case of stochastic volatility the Hansen-Jagannathan bound additionally depends on the subjective discount factor allowing to solve the puzzle even for low risk aversion coefficients. However, people must be either very impatient or very patient such that the Hansen-Jagannathan bound is sufficiently high. Indeed, the subjective discount factor must be $\beta_{HJ}^{sv} \geq 1.3451$ or $\beta_{HJ}^{sv} \leq 0.3625$ having a risk aversion of five.

4.3 Stochastic volatility as a factor

Fama and French (1992) show the CAPM (Sharpe (1964), Lintner (1965) and Black (1972)) as an one-factor model to perform poorly in describing cross-sectional average returns. Extending the number of factors (Fama and French (1993, 2015)) improves the performance in describing cross-sectional average returns.

In section 4.1.2 we show stochastic volatility to have an explanatory power for cross-sectional average returns. Now we construct a factor $SVOL$ mimicking the effect of stochastic volatility on asset prices following the approach of Fama and French (1993): Sorting stocks independently into 25 Size-Tau portfolios we show the effect of time-varying volatility on asset prices to be neither captured by the three- or five-factor model of Fama and French (1993, 2015) nor by the momentum factor as in the factor-model of Carhart (1997). Adding the non-redundant factor SVOL to the five-factor model generally improves the performance in describing average returns. The resulting six-factor model especially helps to describe the average returns on the 25 Size-Prior 2–12 portfolios indicating that a significant part of the momentum effect results from stochastic volatility.

Following the results of section 4.1.2 we can state a beta pricing model having one factor equal to the well-known beta factor and one factor capturing the effect of stochastic volatility for describing an assets i excess return over the risk-free rate:

$$\mathbb{E}(R_{v+1}^{i} - R_{v+1}^{ft}) = \alpha_{v+1} + \beta_{i,\tau} \cdot \lambda_{\tau,m} + \beta_{i,B} \cdot \lambda_{B,m}, \tag{4.24}$$

where $\beta_{i,\tau}$ is the covariance between the stopping time τ and the assets i return R_{v+1}^{i} divided by the variance of τ and $\beta_{i,B}$ is the covariance between the driving Brownian motion B^{W} of the wealth portfolio and the assets i return R_{v+1}^{i} divided by the variance of B^{W}. The variables $\lambda_{\tau,m}$ and $\lambda_{B,m}$ as well as the intercept α_{v+1} are independent of the assets return and thus equal for all assets. The stopping time τ measures the time passing by until the integrated variance of the wealth portfolio passes a predefined threshold. The stock specific parameter $\beta_{i,\tau} =: Tau$ captures the effects of stochastic volatility of the wealth portfolio on the cross-section of expected stock returns. A more detailed derivation can be found in appendix B.3.

Data and construction of SVOL

We follow the methodology in Fama and French (1993, 2015) to construct our factor SVOL. At the end of each June we sort stocks to two Size (Small to Big), using the median of market capitalization, and independently to three Tau groups (Low to High), using medians of Tau for the 30th and 70th percentiles. The intersections of the two sorts produce 6 value-weight Size-Tau portfolios. To increase the robustness of our factor, we exclude the stocks falling in the 85% quantile of the absolute value of Tau.[4] We consider all NYSE, AMEX, and NASDAQ stocks on CRSP with share codes 10 or 11 having no missing return value for the last 12 months covering the span July 1963 – December 2016. We calculate the Tau groups based on the prior 12 months by calculating $\beta_{i,\tau}$ for each stock and sorting all stocks according to their value of $\beta_{i,\tau}$. For the calculation of $\beta_{i,\tau}$ we time-change all returns with respecto to the realized variance of the market portfolio. We compute the daily variance of the market portfolio using an EGARCH(1,1)[5] model on daily log-returns. The integrated variance is the sum of the daily variances. Now, we linearly approximate the return of the market and each stock i for a fixed unit of integrated variance. There is a trade off between having enough data points to estimate the covariance and variance for $\beta_{i,\tau}$ and having at least one observation per integrated variance unit. We find it optimal to divide the total integrated variance by 48 resulting in 48 time-changed returns R^i_{v+1} and realizations of τ. Dividing by a different number does not change the results significantly. Finally, we compute $\beta_{i,\tau}$ by computing $\mathbf{Cov}(\tau, R^i)$ and $\mathbf{Var}(\tau)$ using the 48 time-changed returns. For the five-factors $R_M - R_F$, SMB, HML, RMW and CMA as well as the momentum factor MOM, we take the data available on Kenneth French's homepage.

[4] The exclusion of 15% of the stocks does not change the mean of our factor (0.40 compared to 0.39 when considering all stocks) but increases its stability. We mainly exclude small and illiquid stocks for which the time-change can be inaccurate and subsequently the calculation of $\beta_{i,\tau}$. Although we exclude 15% of the stocks, they only represent 4.5% of the total market capitalization on average.

[5] We test different GARCH models and find the EGARCH model to perform best regarding AIC and BIC. However, the choice of GARCH model has only minor effects on our results.

In panel A of table 4.2 we report the average monthly return of the five-factors $R_M - R_F$, SMB, HML, RMW and CMA, the momentum factor MOM and the factor SVOL with standard deviation and t-statistic. The average monthly return on SVOL is significantly different from zero (t-statistic = 3.57), even by the standards of Harvey et al. (2016). With an average monthly return of 0.4% it is comparable to the average return of HML with 0.37%. Panel B shows the factors correlations. In Panel C we test SVOL for redundancy by regressing the five-factors and the five-factors and MOM on SVOL. The intercept is in both cases highly significant and leaves a monthly return of 0.54% and 0.39% unexplained by the five-factor model and the five-factor model plus MOM, respectively. Therefore, SVOL helps to explain average returns when added to the five-factor model. When regressing the five-factors and SVOL on MOM, the intercept is significantly reduced from 0.73 ($t-statistic = 4.73$) to 0.40 ($t-statistic = 2.48$) using the five-factors only. Considering the large loading of SVOL of 0.62 ($t-statistic = 9.52$) confirms it adds significantly to the five-factor model to explain the average return of the MOM factor. We investigate this finding further by testing the performance of SVOL for the 25 Size-Momentum sorted portfolios which is shown in tables 4.4 and 4.5. In panel D we show the monthly average returns in excess of the one-month U.S. Treasury bill rate for 25 value-weighted Size-Tau sorted portfolios resulting from the intersection of sorting stocks to five Size (Small to Big) and independently to five Tau groups (Low to High) where we exclude the stocks falling in the 85% quantile of the absolute value of Tau. The sample is all NYSE, AMEX, and NASDAQ stocks on CRSP with share codes 10 or 11 having no missing return value for the last 12 months. In each column returns tend to rise from Big to Small stocks showing the well-known size effect but the effect is weak for Low and High Tau portfolios. Average excess returns generally increase from Low to High Tau portfolios. For Big portfolios the returns increase monotonically from 0.23% for low Tau to 0.80% for high Tau. Considering the portfolios for small stocks the average excess return is 0.53% for Low Tau and 0.89% for High Tau but the highest monthly excess return is earned by the second highest Tau (4) portfolio with 1.22%.

Panel A: Averages, standard deviations and t-statistics for monthly returns

	$R_M - R_F$	SMB	HML	RMW	CMA	MOM	$SVOL$
Mean	0.51	0.26	0.37	0.25	0.31	0.67	0.40
Std dev.	4.42	3.04	2.82	2.23	2.01	4.22	2.81
t-Statistic	2.92	2.19	3.32	2.79	3.88	3.99	3.57

Panel B: Correlation between different factors

	$R_M - R_F$	SMB	HML	RMW	CMA	MOM	$SVOL$
$R_M - R_F$	1.00	0.28	-0.26	-0.23	-0.38	-0.13	0.32
SMB	0.28	1.00	-0.08	-0.35	-0.11	-0.03	0.28
HML	-0.26	-0.08	1.00	0.07	0.70	-0.18	-0.42
RMW	-0.23	-0.35	0.07	1.00	-0.03	0.11	-0.37
CMA	-0.38	-0.11	0.70	-0.03	1.00	-0.02	-0.30
MOM	-0.13	-0.03	-0.18	0.11	-0.02	1.00	0.29
$SVOL$	0.32	0.28	-0.42	-0.37	-0.30	0.29	1.00

Panel C: Using other factors in regressions to explain average returns on SVOL and MOM:

	Int	$R_M - R_F$	SMB	HML	RMW	CMA	MOM	SVOL
Coef	0.54	0.08	0.11	-0.36	-0.34	0.00		
t-Statistic	5.64	3.50	3.38	-7.82	-7.63	0.02		
R^2	0.32							
Coef	0.39	0.11	0.10	-0.25	-0.39	-0.08	0.20	
t-Statistic	4.31	4.89	3.18	-5.64	-9.23	-1.21	9.52	
R^2	0.41							
Using other factors in regressions to explain average returns on MOM:								
Coef	0.40	-0.18	-0.00	-0.31	0.45	0.39		0.62
t-Statistic	2.48	-4.65	-0.05	-3.98	5.86	3.52		9.52
R^2	0.20							

Panel D: Size-Tau sorted portfolios:

Tau →	Low	2	3	4	High
Small	0.53	0.78	0.95	1.22	0.89
2	0.49	0.81	0.96	0.97	0.91
3	0.48	0.64	0.88	0.90	1.00
4	0.47	0.68	0.73	0.80	0.83
Big	0.23	0.48	0.56	0.58	0.80

Table 4.2: Summary statistics for monthly factor percent returns; July 1963–December 2016, 642 months. $R_M - R_F$ is the value-weight return on the market portfolio of all sample stocks minus the one-month Treasury bill rate. SMB, HML, RMW, CMA and MOM are the size, book-to-market, profitability,investment and momentum factors as in Fama and French (2015, 2016). The factor SVOL is constructed by assigning stocks to two Size groups and also assigning stocks independently to three Tau groups, using medians of Tau for the 30th and 70th percentiles. To increase the robustness of our factor we exclude the stocks falling in the 85% quantile of the absolute value of Tau. In Panel A, SVOL uses the value-weighted portfolios formed from the intersection of the Size and Tau sorts ($2 \times 3 = 6$ portfolios). Panel A shows average monthly returns (Mean), the standard deviations of monthly returns (Std dev.) and the t-statistics for the average returns. Panel B shows the correlations for each set of factors. Panel C shows a test for redundancy of SVOL when using the five factors $R_M - R_F$, SMB, HML, RMW and CMA or the five factors plus MOM in regressions to explain SVOL. Panel D shows the intersections of the two sorts Size and Tau producing 25 value-weight Size-Tau portfolios. The table shows averages of monthly returns in excess of the one-month Treasury bill rate.

Size-Tau sorted portfolios

Although panel C of table 4.2 shows that SVOL is not a redundant factor, we test if the five-factor model can explain returns on the 5x5 Size-Tau sorted portfolios shown in panel D. The results are given in table 4.3 where we regress the five-factors ($R_M - R_F$, SMB, HML, RMW and CMA) with and without SVOL on the excess returns on the 25 Size-Tau portfolios. In panel A we report the intercepts for the five-factor model. The intercepts are significantly negative for the low Tau portfolios and significantly positive for the high Tau portfolios indicating the five-factor model is not able to capture the dispersion of excess returns on the 25 Size-Tau portfolios. This holds especially for the portfolios with the Big stocks. Adding SVOL improves the performance of the five-factor model as given in panel B. Except for the small and low Tau portfolio all intercepts are not significantly different from zero. The loadings of the factors HML, RMW and CMA show they cannot explain the average returns. HML is mostly decreasing with increasing excess returns, RMW shows no monotonic behavior and CMA is mostly insignificant. The loading on SVOL increase with increasing excess returns and is highly significant.

Testing other anomalies

Based on the previous results we test if SVOL can help to improve the five-factor model to explain anomalies targeted by the five factors. Additionally, we consider the anomalies net share issues (Ikenberry et al., 1995), accruals (Sloan, 1996) and momentum (Jegadeesh and Titman, 1993). We follow the procedure of Fama and French (2016) in evaluating the performance of the different factor models. The results are shown in table 4.4. For each of the 25 portfolio sets we test the five-factor model and the five-factor model plus SVOL. In case of the 25 Size-Tau and 25 Size-Prior 2–12 portfolios we additionally report the five-factor model plus MOM and plus MOM and SVOL. In the first column we show the test statistic GRS of Gibbons et al. (1989) with corresponding p-value in the second column. In the following three columns we evaluate the capability of the models to describe the dispersion of the returns resulting from the 25 portfolio sorts. First, we show the average absolute intercept $A|a_i|$ of the 25 portfolios. Setting the dispersion of the intercepts in relation to relative dispersion of the portfolio excess returns $\bar{r}_i$ is defined as the average excess return of portfolio i

minus the average excess return of the value-weighted market. Thus, high values of $\frac{A|a_i|}{A|\bar{r}_i|}$ and $\frac{Aa_i^2}{Ar_i^2}$ mean the dispersion of the intercepts is relatively low compared to the portfolios average returns. The ratio $\frac{As^2(a_i)}{Aa_i^2}$ is the average squared standard error of the intercepts divided by the average squared intercepts indicating the influence of estimation error on the dispersion of the intercepts.

For the 25 Size-Tau sorted portfolios the five-factor model fails the GRS test ($GRS =$ 3.08) and it is unable to explain the dispersion of average excess returns with an average absolute intercept of 0.203, $\frac{A|a_i|}{A|\bar{r}_i|} = 0.77$ and $\frac{Aa_i^2}{Ar_i^2} = 0.61$. Including SVOL to the five-factor model, the resulting six-factor model passes the GRS test on a 1% level ($GRS = 1.77; p - value = 0.012$). With 0.084 the average absolute intercept is less than half the value of the five-factor model. The model describes the dispersion of average excess well ($\frac{A|a_i|}{A|\bar{r}_i|} = 0.32; \frac{Aa_i^2}{Ar_i^2} = 0.14$) and the remaining dispersion is mostly due to estimation error ($\frac{As^2(a_i)}{Aa_i^2} = 0.60$). Adding MOM to the five-factor model improves the performance but is still significantly worse than the six-factor model with SVOL. Adding both, MOM and SVOL, to the five-factor model is only a small improvement compared to the five-factor model plus SVOL.

Although adding SVOL to the five-factor model is unable to fully describe average excess returns on the 25 Size-Prior 2–12 portfolios it significantly improves its performance. For instance, the average absolute intercept decreases from 0.278 to 0.168 and the dispersion statistic $\frac{Aa_i^2}{Ar_i^2}$ is more than cut by half from 0.79 to 0.34. These findings are confirmed by looking at the regression results shown in table 4.5. The loading on SVOL is mostly significant and increases from low Prior 2–12 to high Prior 2–12. This works best for big stocks leaving no intercept significantly different from zero but loses traction for smaller stocks. Based on these results we conclude that the momentum effect is for bigger stocks for the most part and for smaller stocks for some part a result of stochastic volatility.

For the value effect (Size-B/M), operating profitability (Size-OP), investment (Size-Inv), accruals (Size-Accruals) and net share issues (Size-Net Shares Issued) adding SVOL to the five-factors improves the performance regarding the GRS test as well as the dispersion measures.

The closest factor to SVOL is CVR of Boguth and Kuehn (2013). It also aims to capture the impact of stochastic consumption volatility but its construction and underlying assumptions are different and thus its empirical performance. Boguth and Kuehn (2013) assume a consumption based asset pricing model where the first and second moment of consumption growth follow a Markov chain. The mean and the volatility have two states which they switch independently and the volatility process is additionally assumed to be independent of the Brownian motion (cf. (4.1)). In contrast, we do not make such restrictive assumptions about the stochastic process for volatility nor its dependence on the Brownian motion for the consumption process. Using the time-change preserves both effects.

To test whether CVR adds additional information about expected returns, we regress CVR on the five-factors of Fama and French (2015):

$$\begin{aligned} CVR = &-0.24 - 0.05(R_M - R_F) - 0.27SMB - 0.11HML - 0.53RMW - 0.12CMA + e_t \\ &(-1.80)\,(-1.67) \qquad\qquad (-6.27) \qquad (-1.89) \qquad (-8.93) \qquad (-1.28)\; R^2 = 0.16 \end{aligned} \tag{4.25}$$

The five-factor model explains about 58% of the average return of CVR. The remaining intercept (-0.24) is insignificant ($t - statistic = -1.80$) such that CVR adds little to the explanation of average returns when added to the five-factor model.[6] This is confirmed by testing the performance of CVR considering the anomalies as for SVOL in table 4.4. The results given in the appendix B.4 show that CVR does not improve the performance of the five-factor model regarding other anomalies. In contrast SVOL improves the performance of the five-factor model and its average return cannot be explained by the five-factors nor momentum. Thus, SVOL and CVR contain different information due to the differences in the construction.

[6]Adding the momentum factor further decreases the intercept to -0.16 ($t - statistic = -1.23$).

$$R(t) - R_F(t) = a + b\,[R_M(t) - R_F(t)] + sSMB(t) + hHML(t) + rRMW(t) + cCMA(t) + \tau SVOL(t) + e(t)$$

$Tau \rightarrow$	Low	2	3	4	High	Low	2	3	4	High
Panel A: Five-factor intercepts: $R_M - R_F$, SMB, HML, RMW and CMA										
			a					$t(a)$		
Small	-0.18	0.08	0.30	0.55	0.28	-1.19	0.61	2.20	3.67	1.77
2	-0.30	-0.01	0.16	0.20	0.20	-3.04	-0.18	1.82	2.11	1.87
3	-0.43	-0.18	0.05	0.09	0.27	-4.92	-2.73	0.97	1.42	3.55
4	-0.38	-0.12	-0.09	0.01	0.14	-4.20	-2.04	-1.88	0.26	2.17
Big	-0.41	-0.12	0.01	0.12	0.41	-3.63	-1.89	0.15	1.91	4.73
Panel B: Six-factor coefficients: $R_M - R_F$, SMB, HML, RMW, CMA and SVOL										
			a					$t(a)$		
Small	-0.10	0.14	0.21	0.42	0.08	-0.61	0.96	1.51	2.77	0.51
2	-0.12	0.09	0.14	0.06	-0.04	-1.23	1.07	1.51	0.63	-0.39
3	-0.13	0.00	0.08	0.00	0.05	-1.86	-0.08	1.42	0.00	0.81
4	-0.04	0.05	-0.03	-0.01	0.02	-0.53	0.95	-0.53	-0.23	0.35
Big	0.05	0.07	0.02	-0.09	0.07	0.55	1.34	0.39	-1.71	1.08
			h					$t(h)$		
Small	0.09	0.28	0.19	0.15	0.15	1.23	4.04	2.77	2.01	1.93
2	0.17	0.16	0.22	0.22	0.01	3.63	3.84	5.00	4.76	0.12
3	0.11	0.13	0.18	0.15	0.08	3.10	4.51	6.98	5.16	2.57
4	0.04	0.07	0.15	0.10	0.01	1.24	2.70	6.32	3.77	0.22
Big	0.00	0.05	-0.02	0.11	0.10	0.04	1.85	-0.64	4.21	3.39
			r					$t(r)$		
Small	-0.27	-0.04	-0.12	-0.19	-0.32	-3.61	-0.55	-1.84	-2.65	-4.21
2	-0.27	0.05	-0.05	-0.11	-0.27	-5.83	1.18	-1.25	-2.51	-5.72
3	-0.12	-0.03	0.03	0.04	-0.18	-3.59	-1.22	1.36	1.23	-5.45
4	-0.09	0.05	0.18	0.05	-0.07	-2.69	1.91	7.53	1.87	-2.52
Big	-0.13	0.11	0.21	0.12	-0.09	-3.35	4.36	8.98	4.70	-2.93
			c					$t(c)$		
Small	0.05	-0.17	0.02	0.02	-0.04	0.49	-1.76	0.25	0.18	-0.41
2	-0.05	0.13	0.08	0.06	0.21	-0.72	2.20	1.37	0.96	3.11
3	0.08	0.10	0.08	0.11	0.04	1.58	2.36	2.25	2.61	0.96
4	0.12	0.19	0.14	0.14	0.00	2.53	5.50	3.98	3.54	-0.02
Big	0.07	0.14	0.14	-0.04	-0.13	1.27	3.82	4.30	-1.15	-2.94
			τ					$t(\tau)$		
Small	-0.16	-0.10	0.17	0.24	0.37	-2.53	-1.70	2.97	3.91	5.75
2	-0.33	-0.20	0.04	0.26	0.44	-8.72	-5.71	1.22	6.89	10.89
3	-0.54	-0.32	-0.05	0.16	0.40	-19.08	-13.46	-2.16	6.60	14.64
4	-0.63	-0.31	-0.13	0.05	0.22	-23.04	-14.92	-6.21	2.21	8.67
Big	-0.85	-0.34	-0.02	0.39	0.63	-25.77	-16.20	-1.13	18.42	24.64

Table 4.3: Regressions for 25 value-weight Size-Tau portfolios; July 1963 to December 2016, 642 months. At the end of June each year, stocks are allocated to five Size groups (Small to Big) and independently to five Tau groups (Low Tau to High Tau). The intersections of the two sorts produce 25 Size-Tau portfolios. The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 Size-Tau portfolios. The RHS variables are the excess market return, $R_M - R_F$, the Size factor, SMB, the value factor, HML, the profitability factor, RMW, and the investment factor, CMA, constructed using independent 2x3 sorts on Size and each of B/M, OP, Inv. and Tau. Panel A of the table shows five-factor intercepts produced by the Mkt, SMB, HML, RMW and CMA. Panel B shows six-factor intercepts, slopes for HML, RMW, CMA and SVOL, and t-statistics for these coefficients.

	GRS	$p(GRS)$	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|r_i\|}$	$\frac{Aa_i^2}{Ar_i^2}$	$\frac{As^2(a_i)}{Aa_i^2}$	$A(R^2)$
25 Size-Tau portfolio							
	3.08	0.000	0.203	0.77	0.61	0.15	0.853
SVOL	1.77	0.012	0.084	0.32	0.14	0.60	0.876
MOM	2.52	0.000	0.166	0.63	0.43	0.21	0.857
MOM SVOL	1.76	0.013	0.087	0.33	0.15	0.55	0.877
Panel A: 25 Size-Momentum portfolio							
	4.42	0.000	0.278	0.86	0.79	0.08	0.855
SVOL	3.59	0.000	0.168	0.52	0.34	0.18	0.865
MOM	3.63	0.000	0.116	0.36	0.14	0.23	0.921
MOM SVOL	3.37	0.000	0.118	0.37	0.14	0.24	0.922
Panel A: 25 Size-B/M portfolio							
	3.25	0.000	0.094	0.37	0.16	0.28	0.918
SVOL	2.88	0.000	0.084	0.33	0.13	0.35	0.920
Panel A: 25 Size-OP portfolio							
	2.28	0.000	0.067	0.31	0.09	0.62	0.929
SVOL	2.04	0.002	0.060	0.28	0.09	0.65	0.930
Panel A: 25 Size-Inv portfolio							
	3.44	0.000	0.083	0.34	0.14	0.28	0.930
SVOL	2.90	0.000	0.075	0.30	0.12	0.34	0.931
Panel A: 25 Size-Accruals portfolio							
	3.77	0.000	0.121	0.55	0.32	0.23	0.914
SVOL	3.11	0.000	0.111	0.50	0.27	0.28	0.916
Panel A: 25 Size-Net Shares Issued portfolio							
	3.52	0.000	0.100	0.32	0.15	0.36	0.893
SVOL	3.25	0.000	0.099	0.31	0.15	0.37	0.895

Table 4.4: Summary statistics for tests of five-factor and five-factor plus SVOL models; July 1963–December 2016, 642 months. The table tests the ability of five-factor and five-factor plus SVOL models to explain monthly excess returns on 25 Size-Tau portfolios, 25 Size-Momentum portfolios, 25 Size-B/M portfolios, 25 Size-OP portfolios, 25 Size-Inv portfolios, 25 Size-Accruals portfolios and 25 Size-Net Shares Issued portfolios. For the 25 Size-Tau and the 25 Size-Momentum portfolios, we additionally report the five-factor model plus momentum (MOM) and SVOL and MOM. For each set of 25 regressions, the table shows the GRS statistic testing whether the expected values of all 25 intercept estimates are zero, the average absolute value of the intercepts, $A|ai|$, $\frac{A|a_i|}{A|\bar{r}_i|}$, the average absolute value of the intercept a_i over the average absolute value of $\bar{r}_i$, which is the average return on portfolio i minus the average value-weighted market portfolio excess return, $\frac{Aa_i^2}{A\bar{r}_i^2}$, the average squared intercept over the average squared value of $\bar{r}^2$, $\frac{As^2(a_i)}{Aa_i^2}$, the average of the estimates of the variances of the sampling errors of the estimated intercepts over Aa_i^2 , and $A(R^2)$, the average value of the regression R^2 corrected for degrees of freedom.

$$R(t) - R_F(t) = a + b\,[R_M(t) - R_F(t)] + sSMB(t) + hHML(t) + rRMW(t) + cCMA(t) + \tau SVOL(t) + e(t)$$

Panel A: Five-factor intercepts: $R_M - R_F$, SMB, HML, RMW and CMA

	a					$t(a)$				
Small	-0.73	-0.21	0.06	0.25	0.59	-4.94	-2.7	1.02	3.90	6.30
2	-0.63	-0.20	-0.02	0.17	0.49	-4.56	-2.49	-0.43	3.19	5.64
3	-0.43	-0.19	-0.11	-0.06	0.51	-2.82	-2.35	-1.78	-1.01	5.38
4	-0.46	-0.23	-0.13	0.03	0.42	-2.78	-2.43	-1.95	0.51	4.05
Big	-0.40	-0.12	-0.17	-0.04	0.32	-2.48	-1.23	-2.78	-0.60	3.06

Panel B: Six-factor coefficients: $R_M - R_F$, SMB, HML, RMW, CMA and SVOL

	a					$t(a)$				
Small	-0.62	-0.10	0.10	0.20	0.40	-4.09	-1.33	1.59	3.01	4.48
2	-0.42	-0.05	0.04	0.18	0.34	-3.05	-0.70	0.80	3.20	4.04
3	-0.20	-0.07	0.01	-0.04	0.36	-1.32	-0.82	0.16	-0.69	3.88
4	-0.21	-0.05	-0.02	0.05	0.25	-1.29	-0.57	-0.34	0.81	2.46
Big	-0.17	0.04	-0.09	-0.07	0.13	-1.05	0.44	-1.46	-1.10	1.33
	h					$t(h)$				
Small	0.40	0.35	0.32	0.22	0.03	5.42	9.46	10.5	6.89	0.61
2	0.28	0.27	0.23	0.18	-0.09	4.27	7.09	8.52	6.70	-2.10
3	0.27	0.27	0.24	0.21	-0.13	3.69	6.98	8.19	6.98	-2.98
4	0.32	0.18	0.21	0.09	-0.13	4.05	4.10	6.84	3.08	-2.57
Big	0.27	0.17	0.11	0.00	-0.10	3.42	3.72	3.74	-0.10	-1.95
	r					$t(r)$				
Small	-0.56	-0.05	0.12	0.13	0.01	-7.75	-1.31	4.15	4.12	0.21
2	-0.48	0.05	0.17	0.17	0.03	-7.32	1.30	6.26	6.53	0.63
3	-0.50	0.06	0.17	0.30	0.11	-6.92	1.44	5.85	10.05	2.58
4	-0.53	0.06	0.21	0.32	0.14	-6.83	1.48	6.66	10.50	2.90
Big	-0.30	0.02	0.17	0.29	0.23	-3.97	0.33	5.62	9.42	4.70
	c					$t(c)$				
Small	-0.47	-0.07	0.01	0.11	0.08	-4.57	-1.35	0.28	2.44	1.31
2	-0.49	-0.10	0.02	0.08	0.01	-5.23	-1.95	0.61	2.24	0.21
3	-0.47	-0.10	0.01	0.09	0.04	-4.52	-1.85	0.32	2.12	0.59
4	-0.43	0.05	0.05	0.15	0.10	-3.86	0.77	1.13	3.46	1.46
Big	-0.43	-0.04	0.00	0.21	-0.01	-3.87	-0.61	0.05	4.74	-0.13
	τ					$t(\tau)$				
Small	-0.21	-0.19	-0.07	0.10	0.35	-3.52	-6.35	-2.68	3.86	9.66
2	-0.40	-0.27	-0.13	-0.01	0.27	-7.20	-8.55	-5.64	-0.37	7.93
3	-0.43	-0.23	-0.22	-0.03	0.27	-7.04	-7.13	-9.20	-1.36	7.27
4	-0.46	-0.33	-0.20	-0.04	0.31	-6.95	-8.95	-7.50	-1.44	7.67
Big	-0.43	-0.30	-0.15	0.06	0.34	-6.65	-7.76	-6.08	2.34	8.32

Table 4.5: Regressions for 25 value-weight Size-Momentum portfolios; July 1963 to December 2016, 642 months. At the end of June each year, stocks are allocated to five Size groups (Small to Big) using NYSE market cap breakpoints. Stocks are allocated independently to five Momentum groups (Low Tau to High Tau), again using NYSE breakpoints. The intersections of the two sorts produce 25 Size-Momentum portfolios. The LHS variables in each set of 25 regressions are the monthly excess returns on the 25 Size-Momentum portfolios. The RHS variables are the excess market return, $R_M - R_F$, the Size factor, SMB, the value factor, HML, the profitability factor, RMW, and the investment factor, CMA, constructed using independent 2x3 sorts on Size and each of B/M, OP, Inv. and Tau. Panel A of the table shows five-factor intercepts produced by the Mkt, SMB, HML, RMW and CMA. Panel B shows six-factor intercepts, slopes for HML, RMW, CMA and SVOL, and t-statistics for these coefficients.

4.4 Conclusion

We use a time-changing technique to show the fundamental influence of stochastic volatility for a consumption-based asset pricing model. Letting the stochastic discount factor being the subjectively discounted marginal isoelastic utility, we show that if the consumption process has a time-varying volatility, there is an additional risk correction not only for risky assets but also for the risk-free rate. Indeed, an asset that performs relatively well in times of highly volatile consumption growth has lower expected returns than an asset that performs relatively poor in times of high consumption growth volatility. Additionally, investors demand a higher expected return if consumption growth is low in times of high consumption growth volatility. These results hold for both, risky assets and the risk-free rate.

Furthermore, we test the influence of stochastic volatility on the risk-free rate and the equity premium puzzle. The risk-free rate is lower than compared to assuming consumption growth being log-normally distributed. However, we still need a high risk aversion to get a subjective discount factor below one. Therefore, stochastic volatility for consumption growth eases the risk-free rate puzzle but does not solve it. The results for the equity premium puzzle, however, are promising: If volatility is assumed to be time-varying the Hansen-Jagannathan bound also depends on the subjective discount factor. Thus, the equity premium puzzle can be solved even for low one digit risk aversions if investors are just impatient enough.

Finally, we test if the effects of stochastic volatility are included in the Fama and French (2015) five-factor model or in the four-factor model of Carhart (1997). Constructing a mimicking factor for stochastic volatility SVOL, we find that this effect is neither included in the five-factor model nor in the five factor-model plus the momentum factor. A sorting of 25 portfolios according to size and the the exposure to τ shows a wide range of monthly excess returns of 0.23% up to 1.22%. The five-factor model with and without the momentum factor can neither jointly explain the excess returns according to the GRS test nor can explain the dispersion of returns. Adding the mimicking factor SVOL to the five-factor model, the resulting six-factor model describes the excess returns well. To show the usefulness of the additional factor SVOL we also check other anomalies. For the 25 Size-Prior 2-12 portfolios (momentum) the six-factor model performs significantly better than the five-factor model according to the GRS-test as well as to the measures for dispersion. Although the intercepts are insignificant for bigger stocks the six-factor model has still difficulties to explain the excess returns on the small portfolios. However, based on the loadings of SVOL we conclude that the momentum effect is mainly a result of the effects of stochastic volatility. For the value effect (Size-B/M), operating profitability (Size-OP), investment (Size-Inv) and accruals (Size-Accruals) adding SVOL to the five factors slightly improves the performance regarding the GRS test as well as the measures for dispersion. Summarizing, our time-changing technique is a helpful tool for theoretical as well as empirical asset pricing allowing to disentangle the risk of time-varying volatility and return risk without assuming difficult and complex models being intractable for empirical research and too restrictive for theoretical work.

5 Consumption volatility ambiguity and risk premium's time-variation

The following is based on Müller and Posch (2018a).
https://doi.org/10.1016/j.frl.2018.08.016 https://doi.org/10.1016/j.frl.2018.08.016

6 Price delay and market frictions in cryptocurrency markets

The following is based on Köchling et al. (2019).
https://doi.org/10.1016/j.econlet.2018.10.025

Appendices

A Supplementary material for chapter 3

Additionally to the analysis of the whole sample, we also do robustness checks to our results when using periods of high and low volatility since the jump detection tests are sensitive to volatility. We split the sample from 2007-2011 in the period 2007-2008 and 2009-2011, where the first period is a period with high volatility and 2009-2011 a period of low volatility. Furthermore, we use additional jump detection tests: The quadpower (QPV) and tripower (TPV) variation of Barndorff-Nielsen and Shephard (2006), the MinRV of Andersen et al. (2012) as well as another configuration of the JO test with $\mathrm{MPV}_i(4,6)$. As shown in tables A.3, A.4, A.5 and A.6 these robustness checks lead to the same results as the analysis provided in section 3.2.1.

A.1 Further jump days information

In section 3.2.2 we argue that a higher turnover ratio is associated with larger jumps for all jump tests. Indeed, comparing the average jump size for the different liquidity buckets reveals that the sorting regarding the turnover leads to a higher average jump size for liquid stocks than for illiquid stocks shown in tables A.1 and A.2. When sorting regarding the bid-ask spread and $ILLIQ$ yields exactly the opposite. The more liquid a stock, the lower its average jump size. This behavior of the turnover ratio could give further insights in the discussion whether it is a measure of liquidity or uncertainty as studied in Barinov (2014).

	QPV			TPV			medRV			minRV		
rel. Bid-Ask	1min	5 min	15 min	1 min	5 min	15 min	1 min	5 min	15 min	1 min	5 min	15 min
ΔJumps in %												
Bucket (1)	0.51	0.90	1.13	0.52	0.90	1.14	0.66	1.02	1.23	0.67	1.03	1.31
Bucket (2)	0.63	1.06	1.34	0.65	1.07	1.33	0.84	1.21	1.52	0.84	1.18	1.58
Bucket (3)	0.80	1.32	1.67	0.81	1.34	1.67	1.06	1.58	1.92	1.08	1.64	2.25
Turnover												
ΔJumps in %												
Bucket (1)	0.78	1.34	1.63	0.80	1.36	1.62	1.01	1.54	1.87	1.02	1.55	2.02
Bucket (2)	0.61	1.04	1.35	0.62	1.05	1.34	0.81	1.22	1.51	0.82	1.20	1.60
Bucket (3)	0.54	0.89	1.16	0.55	0.91	1.18	0.71	1.03	1.29	0.74	1.09	1.45
ILLIQ												
ΔJumps in %												
Bucket (1)	0.52	0.93	1.19	0.54	0.93	1.19	0.66	1.03	1.28	0.67	1.02	1.32
Bucket (2)	0.61	1.05	1.32	0.62	1.06	1.30	0.80	1.20	1.48	0.81	1.20	1.59
Bucket (3)	0.80	1.29	1.62	0.81	1.31	1.64	1.10	1.58	1.93	1.13	1.64	2.23

Table A.1: BNS tests jump days information; This table displays information about jumps detected by the four BNS tests from 2007 until 2011 at the one-, five- and fifteen-minute frequency. The jump information is partitioned into three buckets based on their liquidity level during the sample period. Bucket (1) contains the most liquid and bucket (3) the most illiquid stocks. Jump sizes are given by the mean of all logarithmic returns in % within a bucket and are denoted by "ΔJumps".

	JO test						LM test		
	p6			p4					
rel. Bid-Ask	1min	5 min	15 min	1 min	5 min	15 min	1 min	5 min	15 min
ΔJumps in %									
Bucket (1)	0.68	0.87	0.97	0.74	0.95	1.04	0.67	1.27	2.17
Bucket (2)	0.83	1.05	1.17	0.91	1.14	1.25	0.83	1.51	2.58
Bucket (3)	1.10	1.39	1.46	1.19	1.52	1.59	1.19	2.11	3.40
Turnover									
ΔJumps in %									
Bucket (1)	1.02	1.33	1.42	1.11	1.46	1.52	1.04	1.90	3.15
Bucket (2)	0.83	1.06	1.16	0.92	1.14	1.25	0.85	1.52	2.56
Bucket (3)	0.75	0.92	1.02	0.78	1.00	1.10	0.79	1.44	2.47
ILLIQ									
ΔJumps in %									
Bucket (1)	0.68	0.91	1.01	0.73	0.98	1.07	0.68	1.31	2.34
Bucket (2)	0.81	1.02	1.12	0.88	1.11	1.20	0.81	1.46	2.43
Bucket (3)	1.13	1.39	1.50	1.25	1.54	1.63	1.21	2.14	3.41

Table A.2: JO and LM tests jump days information; This table displays information about jumps detected by the two JO tests and the LM test from 2007 until 2011 at the one-, five- and fifteen-minute frequency. The jump information is partitioned into three buckets based on their liquidity level during the sample period. Bucket (1) contains the most liquid and bucket (3) the most illiquid stocks. Jump sizes are given by the mean of all logarithmic returns in % within a bucket and are denoted by "ΔJumps".

A.2 Tables for bucket analysis for split sample

Table A.3: BNS jump test analysis (2007 - 2008)

This table summarizes the mean number of a stock's jump days during the time period from 2007 until 2008 (504 trading days) according to their liquidity bucket for the different BNS tests. The jump days are calculated on observations based on a one-, five- and fifteen-minute time frame, respectively. Each stock is categorized by its liquidity level which is given by the mean of relative bid-ask spread, turnover and *ILLIQ* during this period. Bucket (1) offers the highest degree of liquidity and bucket (3) the lowest. Additionally, the p-values of an ANOVA analysis are reported. The p-values of Tukey's HSD test to highlight the differences between the buckets are only reported if there is a significant p-value of the ANOVA test. N is the number of stocks with an available liquidity measure and jump days for each year in the period. ***, **, * denote statistical significance at the 1%, 5% and 10% level, respectively.

	QPV			TPV			medRV			minRV		
rel. Bid-Ask	1 min	5 min	15 min	1 min	5 min	15 min	1 min	5 min	15 min	1 min	5 min	15 min
Bucket (1)	142.2	38.0	19.7	123.0	34.1	17.9	69.9	27.8	15.4	38.8	14.9	4.3
Bucket (2)	150.2	38.0	19.0	130.8	33.9	17.7	69.6	27.6	15.2	41.7	14.6	5.2
Bucket (3)	172.5	43.5	22.0	152.9	37.5	19.9	82.7	28.4	16.7	50.4	15.2	6.1
ANOVA	0.1803	0.1738	0.007***	0.142	0.288	0.014**	0.1513	0.615	0.0025***	0.0563*	0.6384	0.0001***
(1)-(2)			0.9803			0.9951			0.9706	0.8255		0.1336
(1)-(3)			0.02			0.0242			0.0047	0.0543		0.00004
(2)-(3)			0.0124			0.0319			0.0098	0.1894		0.0248
N	130	130	130	130	130	130	130	130	130	130	130	130
Turnover												
Bucket (1)	147.5	40.8	21.4	130.5	35.6	19.4	71.7	27.4	16.1	43.4	14.7	5.7
Bucket (2)	150.3	37.5	18.6	130.4	33.6	16.9	71.4	27.6	15.2	41.2	14.9	4.9
Bucket (3)	186.4	44.5	22.3	164.0	38.8	20.8	83.9	29.9	16.5	48.6	15.8	5.2
ANOVA	0.0369**	0.2108	0.0954*	0.0487**	0.267	0.0521 *	0.1473	0.3063	0.3947	0.2856	0.5967	0.1775
(1)-(2)	0.985		0.1091	0.9999		0.0944						
(1)-(3)	0.0541		0.9441	0.0828		0.9987						
(2)-(3)	0.0794		0.203	0.0809		0.0828						
N	142	141	137	142	141	137	142	141	137	142	141	137
ILLIQ												
Bucket (1)	116.0	32.9	18.1	101.0	29.8	16.7	60.2	25.7	14.6	33.9	13.4	4.5
Bucket (2)	149.3	38.3	20.0	128.7	33.9	18.6	72.7	27.5	16.7	42.5	15.2	4.8
Bucket (3)	220.4	51.9	24.1	196.6	44.6	21.8	94.5	31.7	16.6	57.1	16.8	6.5
ANOVA	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0000***	0.0001***	0.0000***	0.0005***	0.0000***
(1)-(2)	0.0556	0.2931	0.3996	0.0989	0.3183	0.3257	0.1456	0.4393	0.0544	0.1154	0.1322	0.8086
(1)-(3)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0003	0.0000
(2)-(3)	0.0000	0.0002	0.0000	0.0000	0.0003	0.0001	0.0043	0.0038	0.0894	0.0031	0.0903	0.0000
N	142	141	137	142	141	137	142	141	137	142	141	137

Table A.4: BNS jump test analysis (2009 - 2011)

This table summarizes the mean number of a stock's jump days during the time period from 2009 until 2011 (756 trading days) according to their liquidity bucket for the different BNS tests. The jump days are calculated on observations based on a one-, five- and fifteen-minute time frame, respectively. Each stock is categorized by its liquidity level which is given by the mean of relative bid-ask spread, turnover and *ILLIQ* during this period. Bucket (1) offers the highest degree of liquidity and bucket (3) the lowest. Additionally, the p-values of an ANOVA analysis are reported. The p-values of Tukey's HSD test to highlight the differences between the buckets are only reported if there is a significant p-value of the ANOVA test. N is the number of stocks with an available liquidity measure and jump days for each year in the period. ***, **, * denote statistical significance at the 1%, 5% and 10% level, respectively.

	QPV			TPV			medRV			minRV		
rel. Bid-Ask	1 min	5 min	15 min	1 min	5 min	15 min	1 min	5 min	15 min	1 min	5 min	15 min
Bucket (1)	141.9	48.5	27.7	123.4	43.5	25.0	78.3	39.5	23.1	41.9	20.7	8.0
Bucket (2)	186.7	52.2	30.4	163.8	45.9	28.4	82.9	40.8	25.2	46.4	22.3	7.6
Bucket (3)	342.6	79.0	37.4	312.1	68.3	33.3	102.0	42.3	26.9	59.0	21.3	8.5
ANOVA	0.0000***	0.0000***	0.0009***	0.0000***	0.0000***	0.0018***	0.0356**	0.5	0.0463**	0.0222**	0.5056	0.3835
(1)-(2)	0.1927	0.7758	0.728	0.2172	0.8576	0.4526	0.8767		0.5554	0.7513		
(1)-(3)	0.0000	0.0000	0.0011	0.0000	0.0000	0.0014	0.0384		0.0364	0.0208		
(2)-(3)	0.0000	0.0000	0.0119	0.0000	0.0000	0.0452	0.1179		0.3001	0.1173		
N	133	133	133	133	133	133	133	133	133	133	133	133

Turnover												
Bucket (1)	210.4	60.0	31.0	188.8	53.5	28.1	80.6	40.5	25.1	46.8	21.5	7.1
Bucket (2)	214.4	58.7	32.0	191.4	51.3	29.1	86.9	40.9	24.3	49.0	21.6	8.2
Bucket (3)	287.6	70.3	36.7	258.3	59.9	32.9	108.5	41.3	27.6	59.8	22.0	9.3
ANOVA	0.0257**	0.2012	0.2495	0.0344**	0.3118	0.3078	0.033**	0.939	0.111	0.1898	0.9459	0.0669*
(1)-(2)	0.9912			0.9955			0.8313					0.6314
(1)-(3)	0.0416			0.0554			0.0334					0.0551
(2)-(3)	0.0566			0.0684			0.1274					0.3247
N	145	145	143	145	145	143	145	145	143	145	145	143

ILLIQ												
Bucket (1)	151.9	49.1	29.2	133.0	43.7	26.5	75.2	38.1	23.6	41.8	20.8	7.6
Bucket (2)	201.5	53.3	30.3	177.5	47.2	28.2	80.1	40.6	25.0	45.1	21.1	8.1
Bucket (3)	362.1	87.3	40.2	331.0	74.4	35.6	121.2	44.2	28.4	69.1	23.3	8.9
ANOVA	0.0000***	0.0000***	0.0002***	0.0000***	0.0000***	0.0007***	0.0000***	0.0263**	0.0016**	0.0003***	0.138	0.2074
(1)-(2)	0.1501	0.7705	0.8152	0.179	0.7624	0.65	0.8854	0.5045	0.38	0.8896		
(1)-(3)	0.0000	0,0000	0.0003	0.0000	0.0000	0.0007	0.0001	0.02	0.0011	0.0007		
(2)-(3)	0.0000	0.0000	0.0028	0.0000	0.0000	0.0134	0.0004	0.2435	0.0582	0.0032		
N	145	145	143	145	145	143	145	145	143	145	145	143

Table A.5: JO and LM jump test analysis (2007-2008)

This table summarizes the mean number of a stock's jump days during the time period from 2007 until 2008 (504 trading days) according to their liquidity bucket for the two JO tests and the LM test. The jump days are calculated on observations based on a one-, five- and fifteen-minute time frame, respectively. Each stock is categorized by its liquidity level which is given by the mean of relative bid-ask spread, turnover and *ILLIQ* during this period. Bucket (1) offers the highest degree of liquidity and bucket (3) the lowest. Additionally, the p-values of an ANOVA analysis are reported. The p-values of Tukey's HSD test to highlight the differences between the buckets are only reported if there is a significant p-value of the ANOVA test. N is the number of stocks with an available liquidity measure and jump days for each year in the period. ***, **, * denote statistical significance at the 1%, 5% and 10% level, respectively.

	JO test						LM test		
	p6			p4					
rel. Bid-Ask	1min	5 min	15 min	1 min	5 min	15 min	1 min	5 min	15 min
Bucket (1)	95.4	85.8	110.0	67.5	60.0	75.5	216.1	50.5	11.9
Bucket (2)	98.8	86.6	103.8	69.0	61.6	71.8	214.9	45.3	9.8
Bucket (3)	102.2	89.4	102.2	69.5	61.4	70.3	221.9	49.0	11.4
ANOVA	0.461	0.2513	0.1716	0.8982	0.6003	0.286	0.7693	0.0361**	0.0267**
(1)-(2)								0.9788	0.9748
(1)-(3)								0.072	0.0523
(2)-(3)								0.0519	0.0398
N	130	130	130	130	130	130	130	130	125
Turnover									
Bucket (1)	89.0	84.9	102.7	59.9	57.8	72.0	204.0	48.5	12.0
Bucket (2)	101.1	87.7	103.8	71.3	62.4	70.1	220.4	47.9	10.3
Bucket (3)	108.8	93.0	114.7	74.7	63.8	78.2	230.6	51.2	11.3
ANOVA	0.0004***	0.1846	0.139	0.0008***	0.166	0.1266	0.00205**	0.7745	0.1108
(1)-(2)	0.0379			0.0125			0.1919		
(1)-(3)	0.0003			0.0009			0.0161		
(2)-(3)	0.2704			0.6805			0.5347		
N	142	141	137	142	141	137	142	132	125
ILLIQ									
Bucket (1)	87.4	78.5	105.3	62.1	56.2	72.9	195.0	45.3	11.5
Bucket (2)	101.3	89.4	110.8	72.4	63.8	75.3	225.2	51.1	10.3
Bucket (3)	110.4	97.9	104.7	71.2	64.2	71.7	235.1	51.1	11.9
ANOVA	0.0000***	0.0000***	0.0023***	0.0248**	0.0004***	0.0092***	0.0001***	0.0000***	0.0001***
(1)-(2)	0.0121	0.0021	0.2153	0.033	0.0063	0.5762	0.0029	0.0053	0.9142
(1)-(3)	0.0000	0.0000	0.0014	0.0752	0.0007	0.0073	0.0001	0.0000	0.0026
(2)-(3)	0.1481	0.0032	0.1281	0.9567	0.7367	0.0879	0.5239	0.0057	0.0082
N	142	141	137	142	141	137	142	132	125

Table A.6: JO and LM jump test analysis (2009 - 2011)

This table summarizes the mean number of a stock's jump days during the time period from 2009 until 2011 (756 trading days) according to their liquidity bucket for the two JO tests and the LM test. The jump days are calculated on observations based on a one-, five- and fifteen-minute time frame, respectively. Each stock is categorized by its liquidity level which is given by the mean of relative bid-ask spread, turnover and *ILLIQ* during this period. Bucket (1) offers the highest degree of liquidity and bucket (3) the lowest. Additionally, the p-values of an ANOVA analysis are reported. The p-values of Tukey's HSD test to highlight the differences between the buckets are only reported if there is a significant p-value of the ANOVA test. N is the number of stocks with an available liquidity measure and jump days for each year in the period. ***, **, * denote statistical significance at the 1%, 5% and 10% level, respectively.

	JO test						LM test		
	p6			p4					
rel. Bid-Ask	1 min	5 min	15 min	1 min	5 min	15 min	1 min	5 min	15 min
Bucket (1)	120.4	125.7	157.5	88.0	92.0	110.1	307.3	70.1	12.5
Bucket (2)	129.4	131.6	168.3	90.1	92.8	116.3	323.7	76.7	13.5
Bucket (3)	125.9	135.7	165.1	75.7	87.1	114.9	297.2	75.2	15.0
ANOVA	0.3095	0.029**	0.1542	0.0035***	0.1297	0.3562	0.0892*	0.0498**	0.0017***
(1)-(2)		0.252		0.8775			0.3631	0.4906	0.9142
(1)-(3)		0.0225		0.0204			0.6822	0.0385	0.0026
(2)-(3)		0.513		0.0049			0.0753	0.3595	0.0082
N	133	133	133	133	133	133	133	133	133
Turnover									
Bucket (1)	119.3	135.8	163.6	79.7	93.1	114.0	305.5	78.6	14.2
Bucket (2)	124.8	126.3	161.5	84.6	89.5	113.3	309.7	71.1	12.9
Bucket (3)	134.3	134.1	166.1	88.0	88.6	115.8	306.5	71.7	14.4
ANOVA	0.0246**	0.0296**	0.1011	0.1785	0.264	0.2604	0.9323	0.025**	0.3949
(1)-(2)	0.5683	0.0334						0.0644	
(1)-(3)	0.0193	0.8895						0.0361	
(2)-(3)	0.1988	0.1062						0.9727	
N	145	145	143	145	145	143	144	138	134
ILLIQ									
Bucket (1)	113.1	123.7	162.9	80.3	87.9	112.6	286.6	67.1	12.6
Bucket (2)	128.0	133.0	163.6	87.8	92.9	114.7	320.9	77.6	13.3
Bucket (3)	137.4	139.7	164.7	84.0	90.4	115.9	314.2	76.8	15.6
ANOVA	0.0000***	0.0001***	0.4202	0.241	0.2273	0.187	0.0073***	0.0011***	0.0019***
(1)-(2)	0.0137	0.0282					0.0086	0.026	0.8326
(1)-(3)	0.0000	0.0000					0.0437	0.001	0.0023
(2)-(3)	0.1775	0.16					0.8268	0.5226	0.0137
N	145	145	143	145	145	143	144	138	134

B Appendix for chapter 4

B.1 Time-change of the stochastic volatility model

Here we derive the time-changed version of C_t to show the principle mechanism. We start with

$$d\ln(C_t) = \mu dt + \sqrt{V_t} dW_t^c - \frac{1}{2} V_t dt \tag{B.1}$$

which is equivalent to writing

$$\ln(C_t) = \ln(C_0) + \mu t + \int_0^t \sqrt{V_u} dW_u^c - \frac{1}{2} \int_0^t V_u du. \tag{B.2}$$

Now we replace t by the stopping time $\tau_v = \inf\{t \geq 0; [M]_t \geq v\}$ with $M_t = \int_0^t \sqrt{V_u} dW_u^c$. Thus the stopping time is defined as $\tau_v = \inf\{t \geq 0; \int_0^t V_u du \geq v\}$ and the stochastic process $C_{\tau(v)}$ writes as

$$\ln(C_{\tau(v)}) = \ln(C_0) + \mu\tau(v) + \int_0^{\tau(v)} \sqrt{V_u} dW_u^c - \frac{1}{2} \int_0^{\tau(v)} V_u du. \tag{B.3}$$

According to the Dambis, Dubins & Schwarz Theorem as stated in Karatzas and Shreve (1991) we have

$$M_{\tau(v)} = \int_0^{\tau(v)} \sqrt{V_u} dW_u^c =: B_v^c \tag{B.4}$$

where B_v^c is a standard Brownian motion. With $\int_0^{\tau(v)} V_u du = v$ by construction of the stopping time $\tau(v)$, the differentiate expression of (B.3) writes

$$\ln(C_{\tau(v)}) = \mu\tau_v + B_v^c - \frac{v}{2}. \tag{B.5}$$

This is the time-changed version C_τ stated in (4.4).

B.2 Time change of the risky asset

To derive the time-changed version of the risky asset, we start with

$$\begin{aligned} d\ln(S_t^s) &= \mu^s dt + \sqrt{V_t^s} dW_t^s - \frac{1}{2} V_t^s dt \\ \Leftrightarrow \ln(S_t^s) &= \ln(S_0^s) + \mu^s t + \int_0^t \sqrt{V_u^s} dW_u^s - \frac{1}{2} \int_0^t V_u^s du. \end{aligned} \tag{B.6}$$

We define $M_t^s = \int_0^t \sqrt{V_u^c} dW_u^s$ where V^c is the variance process of the consumption process. Then the stopping time for the time-change is given by $\tau_v = \inf\{t \geq 0; \int_0^t V_u^c du \geq v\}$ and it holds

$$\begin{aligned} \int_0^{\tau(v)} \sqrt{V_u^s} dW_u^s &= \int_0^{\tau(v)} \sqrt{\frac{V_u^s}{V_u^c}} d \int_0^u \sqrt{V_y^c} dW_y^s \\ &= \int_0^{\tau(v)} \sqrt{\frac{V_u^s}{V_u^c}} dM_u^s. \end{aligned} \tag{B.7}$$

Taking the differentiate representation we get

$$\sqrt{\frac{V_{\tau(v)}^s}{V_{\tau(v)}^c}} dM_{\tau(v)}^s = \sqrt{\frac{V_{\tau(v)}^s}{V_{\tau(v)}^c}} dB_v^s \tag{B.8}$$

where B_v^s is a standard Brownian motion according to the Dambis, Dubins & Schwarz Theorem. With equation (B.3) and $a_v = \frac{V_{\tau(v)}^s}{V_{\tau(v)}^c}$ it holds

$$\ln(S_{\tau(v)}^s) = \mu^s \tau_v - \frac{1}{2} \int_0^v a_u du + \int_0^v \sqrt{a_u} dB_u^s. \tag{B.9}$$

This is the stated time-changed version of the risky asset.

B.3 Beta pricing model with wealth portfolio

Recall, that any assets expected return can be written as

$$\mathbb{E}(R^i_{v+1}) = R^{fv}_{v+1} - R^{fv}_{v+1}\,\mathbf{Cov}(m_{v+1}, R^i_{v+1}). \tag{B.10}$$

We linearize the stochastic discount factor doing a Taylor approximation $m_{v+1} = a_{v+1} + b_{v+1}R^W_{v+1}$ where R^W_{v+1} is the return on the wealth portfolio. Using the above equation to the expected return on the wealth portfolio gives

$$\begin{aligned}\mathbb{E}(R^W_{v+1}) &= R^{fv}_{v+1} - R^{fv}_{v+1}\,\mathbf{Cov}(a_{v+1} + b_{v+1}R^W_{v+1}, R^W_{v+1})\\ &= R^{fv}_{v+1} - R^{fv}_{v+1}b_{v+1}\,\mathbf{Var}(R^W_{v+1})\end{aligned} \tag{B.11}$$

and solving for the constant b_{v+1} we get

$$b_{v+1} = -\frac{\mathbb{E}(R^W_{v+1} - R^{fv}_{v+1})}{R^{fv}_{v+1}\,\mathbf{Var}(R^W_{v+1})}. \tag{B.12}$$

With a positive expected excess return on the wealth portfolio and a positive time-free rate the constant b_{v+1} is always negative. Assuming that the wealth portfolio follows a geometric Brownian motion with stochastic volatility, i.e. $R^W_t = R^W_0 e^{\mu^W t - \frac{1}{2}\int_0^t V^W_u du + \int_0^t \sqrt{V^W_u} dW^W_u}$, the time-changed process is given by $R^W_v = R^W_0 e^{\mu^W \tau_v - \frac{v}{2} + B^W_v}$ with $\tau_v = \inf\{t \geq 0; \int_0^t V^W_u du \geq v\}$. If the exponent is sufficiently small we can approximate the covariance by

$$\mathbf{Cov}(R^W_{v+1}, R^i_{v+1}) \approx \frac{\mathbf{Cov}(\tau_{v+1}, R^i_{v+1})}{\mathbf{Var}(\tau_{v+1})} \cdot \frac{\mu^W\,\mathbf{Var}(\tau_{v+1})}{\mathbb{E}(m_{v+1})} + \frac{\mathbf{Cov}(B^W_{v+1}, R^i_{v+1})}{\mathbf{Var}(B^W_{v+1})} \cdot \frac{\mathbf{Var}(B^W_{v+1})}{\mathbb{E}(m_{v+1})}. \tag{B.13}$$

With the expression for the time-free rate

$$R^{fv}_{v+1} = \frac{\mathbb{E}\left(R^{ft}_{v+1}\right)}{1 - \mathbf{Cov}\left(m_{v+1}, R^{ft}_{v+1}\right)}, \tag{B.14}$$

we get the beta pricing model

$$\mathbb{E}(R^i_{v+1} - R^{ft}_{v+1}) = \alpha_{v+1} + \beta_{i,\tau} \cdot \lambda_{\tau,m} + \beta_{i,B^W} \cdot \lambda_{B^W,m}, \tag{B.15}$$

where $\alpha_{v+1} = R^{ft}_{v+1} \frac{\mathbf{Cov}(m_{v+1}, R^{ft}_{v+1})}{1-\mathbf{Cov}(m_{v+1}, R^{ft}_{v+1})}$, $\lambda_{\tau,m}$ and $\lambda_{B^W,m}$ are equal for all assets. The factor $\beta_{i,\tau}$ is the covariance between the stopping time τ and the assets i return R^i_{v+1} divided by the variance of τ and β_{i,B^W} is the covariance between the driving Brownian motion B^W_v of the wealth portfolio and the assets i return R^i_{v+1} divided by the variance of B^W_v which is just v since B^W_v is a standard Brownian motion.

B.4 Testing anomalies and CVR

	GRS	$p(GRS)$	$A\|a_i\|$	$\frac{A\|a_i\|}{A\|r_i\|}$	$\frac{Aa_i^2}{Ar_i^2}$	$\frac{As^2(a_i)}{Aa_i^2}$	$A(R^2)$
Panel A: 25 Size-Tau portfolio							
	2.69	0.000	0.213	0.71	0.50	0.16	0.857
SVOL	1.44	0.079	0.075	0.25	0.09	0.86	0.881
CVR	2.73	0.000	0.218	0.73	0.51	0.16	0.859
Panel A: 25 Size-Momentum portfolio							
	4.22	0.000	0.284	0.82	0.70	0.09	0.854
SVOL	3.43	0.000	0.161	0.46	0.29	0.23	0.865
CVR	4.12	0.000	0.273	0.78	0.63	0.10	0.857
Panel A: 25 Size-B/M portfolio							
	2.97	0.000	0.099	0.35	0.14	0.30	0.919
SVOL	2.54	0.000	0.089	0.32	0.11	0.39	0.921
CVR	2.92	0.000	0.100	0.36	0.14	0.29	0.919
Panel A: 25 Size-OP portfolio							
	1.92	0.005	0.075	0.30	0.08	0.60	0.930
SVOL	1.67	0.023	0.064	0.26	0.08	0.66	0.931
CVR	1.94	0.004	0.075	0.30	0.09	0.58	0.931
Panel A: 25 Size-Inv portfolio							
	3.32	0.000	0.087	0.31	0.12	0.30	0.931
SVOL	2.77	0.000	0.077	0.28	0.10	0.38	0.932
CVR	3.27	0.000	0.088	0.32	0.12	0.30	0.932
Panel A: 25 Size-Accruals portfolio							
	4.01	0.000	0.135	0.51	0.26	0.23	0.916
SVOL	3.32	0.000	0.122	0.46	0.22	0.30	0.917
CVR	4.06	0.000	0.133	0.51	0.26	0.23	0.916
Panel A: 25 Size-Net Shares Issued portfolio							
	3.27	0.000	0.103	0.30	0.13	0.37	0.900
Tau	3.11	0.000	0.106	0.30	0.14	0.37	0.902
CVR	3.21	0.000	0.105	0.30	0.13	0.37	0.901

Table B.1: Summary statistics for tests of five-factor, five-factor plus SVOL and five-factor plus CVR models; July 1964–January 2010-December, 564 months. The table tests the ability of five-factor, five-factor plus SVOL and five-factor plus CVR models to explain monthly excess returns on 25 Size-Tau portfolios, 25 Size-Momentum portfolios, 25 Size-B/M portfolios, 25 Size-OP portfolios, 25 Size-Inv portfolios, 25 Size-Accruals portfolios and 25 Size-Net Shares Issued portfolios. The factor CVR is taken from Oliver Boguth's homepage as used in Boguth and Kuehn (2013). For each set of 25 regressions, the table shows the GRS statistic testing whether the expected values of all 25 intercept estimates are zero, the average absolute value of the intercepts, $A|ai|$, $\frac{A|a_i|}{A|\bar{r}_i|}$, the average absolute value of the intercept a_i over the average absolute value of $\bar{r}_i$, which is the average return on portfolio i minus the average value-weighted market portfolio excess return, $\frac{Aa_i^2}{A\bar{r}_i^2}$, the average squared intercept over the average squared value of $\bar{r}^2$, $\frac{As^2(a_i)}{Aa_i^2}$, the average of the estimates of the variances of the sampling errors of the estimated intercepts over Aa_i^2 , and $A(R^2)$, the average value of the regression R^2 corrected for degrees of freedom.

C Derivation of expected market return

As given in (Cochrane, 2001) the expected market return can be written as

$$\begin{aligned}
\mathbb{E}\, R_t^m &= \frac{1}{\mathbb{E}\, m_t} - \frac{1}{\mathbb{E}\, m_t} \mathbf{Cov}(m_t, R_t^m) \\
&= \frac{1}{\mathbb{E}\, m_t} - \frac{1}{\mathbb{E}\, m_t} \sqrt{\mathbf{Var}(m_t)\, \mathbf{Var}(R_t^m)} \rho_{m,R^m} \qquad \text{(C.1)}
\end{aligned}$$

where ρ_{m,R^m} is the correlation between the market return and the stochastic discount factor $m_t = \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}$ with subjective discount factor $\beta = e^{-\delta}$, risk aversion γ and consumption c_t. Assuming consumption growth to be lognormal we get for the mean and the variance of m_t:

$$\mathbb{E}\, m_t = e^{-\delta-\gamma\mu+\frac{\gamma^2}{2}\sigma_t^2} \qquad \text{(C.2)}$$

$$\mathbf{Var}(m_t) = e^{-2\delta-2\gamma\mu+\gamma^2\sigma_t^2} \left(e^{\gamma^2\sigma_t^2} - 1\right). \qquad \text{(C.3)}$$

Inserting equations (C.2) and (C.3) in (C.1) leads to the formula given in equation (5.2).

Bibliography

Alter, A. and Schüler, Y. S. (2012). Credit spread interdependencies of European states and banks during the financial crisis. *Journal of Banking and Finance*, 36(12):3444–3468.

Alvarez-Ramirez, J., Rodriguez, E., and Ibarra-Valdez, C. (2018). Long-range correlations and asymmetry in the Bitcoin market. *Physica A: Statistical Mechanics and its Applications*, 492:948–955.

Amihud, Y. (2002a). Illiquidity and stock returns: Cross-section and time-series effects. *Journal of Financial Markets*, 5(1):31–56.

Amihud, Y. (2002b). Illiquidity and stock returns: cross-section and time-series effects. *Journal of Financial Markets*, 5(1):31–56.

Andersen, T. G., Benzoni, L., and Lund, J. (2002). An empirical investigation of continuous-time equity return models. *The Journal of Finance*, 57(3):1239–1284.

Andersen, T. G., Dobrev, D., and Schaumburg, E. (2012). Jump-robust volatility estimation using nearest neighbor truncation. *Journal of Econometrics*, 169(1):75–93.

At-Sahalia, Y., Mykland, P. A., and Zhang, L. (2011). Ultra high frequency volatility estimation with dependent microstructure noise. *Journal of Econometrics*, 160(1):160–175.

Aït-Sahalia, Y. and Yu, J. (2009). High frequency market microstructure noise estimates and liquidity measures. *The Annals of Applied Statistics*, 3(1):422–457.

Bansal, R., Kiku, D., and Yaron, A. (2012). An Empirical Evaluation of the Long-Run Risks Model for Asset Prices. *Critical Finance Review*, 1(1):183–221.

Bansal, R. and Yaron, A. (2004). Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles. *Journal of Finance*, 59(4):1481–1509.

Barinov, A. (2014). Turnover: liquidity or uncertainty? *Management Science*, 60(10):2478–2495.

Bariviera, A. F. (2017). The inefficiency of Bitcoin revisited: a dynamic approach. *Economics Letters*, 161:1–4.

Barndorff-Nielsen, O. and Shephard, N. (2006). Econometrics of testing for jumps in financial economics using bipower variation. *Journal of Financial Econometrics*, 4(1):1–30.

Barndorff-Nielsen, O. E. and Veraart, A. E. D. (2013). Stochastic Volatility of Volatility and Variance Risk Premia. *Journal of Financial Econometrics*, 11(1):1–46.

Basel Committe on Banking Supervision (2015). Margin requirements for non- centrally cleared derivatives. Technical report.

Baur, D. G. and Mcdermott, T. K. (2010). Is gold a safe haven ? International evidence. *Journal of Banking and Finance*, 34(8):1886–1898.

Beeler, J. and Campbell, J. Y. (2012). The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment. *Critical Finance Review*, 1(1):141–182.

Black, F. (1972). Capital Market Equilibrium with Restricted Borrowing. *Journal of Business*, 45(3):444.

Boguth, O. and Kuehn, L. A. (2013). Consumption Volatility Risk. *Journal of Finance*.

Bollerslev, T., Law, T. H., and Tauchen, G. (2008). Risk, jumps, and diversification. *Journal of Econometrics*, 144(1):234–256.

Bollerslev, T., Sizova, N., and Tauchen, G. (2012). Volatility in equilibrium: Asymmetries and dynamic dependencies. *Review of Finance*, 16(1):31–80.

Boudt, K. and Petitjean, M. (2014). Intraday liquidity dynamics and news releases around price jumps: Evidence from the djia stocks. *Journal of Financial Markets*, 17:121–149.

Brauneis, A. and Mestel, R. (2018). Price discovery of cryptocurrencies: Bitcoin and beyond. *Economics Letters*, 165:58–61.

Campbell, J. Y. and Cochrane, J. H. (1999). By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. *Journal of Political Economy*, 107(2):205–251.

Carhart, M. M. (1997). On Persistence in Mutual Fund Performance. *Journal of Finance*, 52(1):57.

Carr, P. and Lee, R. (2009). Volatility Derivatives. *Annual Review of Financial Economics*, 1(1):319–339.

Carr, P., Lee, R., and Wu, L. (2012). Variance swaps on time-changed Lévy processes. *Finance and Stochastics*, 16(2):335–355.

Carr, P. and Wu, L. (2004). Time-changed Lévy processes and option pricing. *Journal of Financial Economics*, 71(1):113–141.

Carr, P. and Wu, L. (2009). Variance Risk Premiums. *Review of Financial Studies*, 22(3):1311–1341.

Chordia, T., Roll, R., and Subrahmanyam, A. (2002). Order imbalance, liquidity, and market returns. *Journal of Financial Economics*, 65(1):111–130.

Chordia, T., Roll, R., and Subrahmanyam, A. (2008). Liquidity and market efficiency. *Journal of Financial Economics*, 87(2):249 – 268.

Clark, P. K. (1973). A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices. *Econometrica*, 41(1):135.

Cochrane, J. H. (2001). *Asset Pricing*. Princeton University Press, Princeton, NJ.

Cochrane, J. H. (2011). Presidential Address: Discount Rates. *Journal of Finance*, 66(4):1047–1108.

Cochrane, J. H. (2017). Macro-Finance*. *Review of Finance*, 21(3):945–985.

Corwin, S. A. and Schultz, P. (2012). A simple way to estimate bid-ask spreads from daily high and low prices. *Journal of Finance*, 67(2):719–760.

Dambis, K. E. (1965). On decompositions of continuous submartingales. *Theory of Probability & Its Applications*, 10(3):401–410.

Dubins, L. E. and Schwarz, G. (1965). On Continuous Martingales. *Proceedings of the National Academy of Sciences*, 53(5):913–916.

Duffie, D. and Pan, J. (2001). Analytical value-at-risk with jumps and credit risk. *Finance and Stochastics*, 5(2):155–180.

Duffie, D., Pan, J., and Singleton, K. (2000). Transform analysis and asset pricing for affine jump-diffusions. *Econometrica*, 68(6):1343–1376.

Epstein, L. G. and Ji, S. (2013). Ambiguous Volatility and Asset Pricing in Continuous Time. *Review of Financial Studies*, 26(7):1740–1786.

Eraker, B., Johannes, M., and Polson, N. (2003). The impact of jumps in volatility and returns. *Journal of Finance*, 58(3):1269–1300.

Fama, E. F. (1970). Efficient Capital Markets : A Review of Theory and Empirical. *The Journal of Finance*, 25(2):383–417.

Fama, E. F. and French, K. R. (1992). The Cross-Section of Expected Stock Returns. *Journal of Finance*, 47(2):427–465.

Fama, E. F. and French, K. R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1):3–56.

Fama, E. F. and French, K. R. (2015). A five-factor asset pricing model. *Journal of Financial Economics*, 116(1):1–22.

Fama, E. F. and French, K. R. (2016). Dissecting Anomalies with a Five-Factor Model. *Review of Financial Studies*, 29(1):69–103.

Ferson, W. E. and Harvey, C. R. (1991). The Variation of Economic Risk Premiums. *Journal of Political Economy*, 99(2):385–415.

Geman, H. (2005). From measure changes to time changes in asset pricing. *Journal of Banking & Finance*, 29(11):2701–2722.

Gibbons, M. R., Ross, S. A., and Shanken, J. (1989). A Test of the Efficiency of a Given Portfolio. *Econometrica*, 57(5):1121.

Glasserman, P. and Yang, L. (2016). Bounding wrong-way risk in cva calculation. *Mathematical Finance*, 28(1):268–305.

Goyenko, R., Holden, C. W., and Trzcinka, C. A. (2009a). Do liquidity measures measure liquidity? *Journal of Financial Economics*, 92(2):153–181.

Goyenko, R. Y., Holden, C. W., and Trzcinka, C. A. (2009b). Do liquidity measures measure liquidity? *Journal of Financial Economics*, 92(2):153–181.

Gumbel, E. J. (1961). Multivariate extremal distributions. *Bulletin de l'Institut International de Statistiques*.

Hansen, L. P. and Jagannathan, R. (1991). Implications of Security Market Data for Models of Dynamic Economies. *Journal of Political Economy*, 99(2):225–262.

Harvey, C. R. (1989). Time-varying conditional covariances in tests of asset pricing models. *Journal of Financial Economics*, 24(2):289–317.

Harvey, C. R., Liu, Y., and Zhu, H. (2016). … and the Cross-Section of Expected Returns. *Review of Financial Studies*, 29(1):5–68.

Heston, S. L. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *Review of Financial Studies*, 6(2):327–343.

Hofer, M. (2016). Path-consistent wrong-way risk: a structural model approach. *The Journal of Risk*, 19(1):25–42.

Hou, K. and Moskowitz, T. J. (2005). Market Frictions, Price Delay, and the Cross-Section of Expected Returns. *Review of Financial Studies*, 18(3):981–1020.

Huang, X. and Tauchen, G. (2005). The relative contribution of jumps to total price variance. *Journal of Financial Econometrics*, 3(4):456–499.

Hull, J. and White, A. (1987). The Pricing of Options on Assets with Stochastic Volatilities. *Journal of Finance*, 42(2):281–300.

Hull, J. and White, A. (2012). CVA and wrong way risk. *Financial Analyst Journal*, 68(5):58–69.

Ikenberry, D., Lakonishok, J., and Vermaelen, T. (1995). Market underreaction to open market share repurchases. *Journal of Financial Economics*, 39(2-3):181–208.

Jegadeesh, N. and Titman, S. (1993). Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance*, 48(1):65.

Jeong, D., Kim, H., and Park, J. Y. (2015). Does ambiguity matter? Estimating asset pricing models with a multiple-priors recursive utility. *Journal of Financial Economics*, 115(2):361–382.

Jiang, G. J. and Oomen, R. C. A. (2008). Testing for jumps when asset prices are observed with noise - a swap variance approach. *Journal of Financial Econometrics*, 144(2):352–370.

Jiang, Y., Nie, H., and Ruan, W. (2018). Time-varying long-term memory in Bitcoin market. *Finance Research Letters*, 25:280–284.

Karatzas, I. and Shreve, S. E. (1991). *Brownian Motion and Stochastic Calculus*. Graduate Texts in Mathematics. Springer New York, New York, NY.

Kenyon, C. and Green, A. (2016). Option-Based Pricing of Wrong Way Risk for CVA.

Khuntia, S. and Pattanayak, J. K. (2018). Adaptive market hypothesis and evolving predictability of Bitcoin. *Economics Letters*, 167:26–28.

Kou, S. G. (2002). A jump-diffusion model for option pricing. *Management Science*, 48(8):1086–1101.

Kristoufek, L. (2018). On Bitcoin markets (in)efficiency and its evolution. *Physica A*, 503(November 2017):257–262.

Kunsteller, S., Müller, J., and Posch, P. N. (2019). Do illiquid stocks jump more frequently? *Applied Economics*, 51(25):2764–2769.

Köchling, G., Müller, J., and Posch, P. N. (2019). Price delay and market frictions in cryptocurrency markets. *Economics Letters*, 174:39 – 41.

Lee, S. S. and Mykland, P. A. (2008). Jumps in financial markets: A new nonparametric test and jump dynamics. *Review of Financial studies*, 21(6):2535–2563.

Lewis, J. B. and Linzer, D. A. (2005). Estimating regression models in which the dependent variable is based on estimates. *Political Analysis*, 13(4):345–364.

Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics and Statistics*, 47(1):13.

Longin, F. and Solnik, B. (2001). Extreme Correlation of International Equity Markets. *The Journal of Finance*, 56(2):649.

McNeil, A. J. and Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance*, 7(3-4):271–300.

Mehra, R. and Prescott, E. C. (1985). The equity premium: A puzzle. *Journal of Monetary Economics*, 15(2):145–161.

Merton, R. (1976). Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics*, 3(1-2):125–144.

Müller, J. and Posch, P. N. (2018a). Consumption volatility ambiguity and risk premium's time-variation. *Finance Research Letters*.

Müller, J. and Posch, P. N. (2018b). Wrong-way-risk in tails. *Journal of Asset Management*, 19(4):205–215.

Nadarajah, S. and Chu, J. (2017). On the inefficiency of Bitcoin. *Economics Letters*, 150:6–9.

Pagano, M. (1989). Endogenous market thinness and stock price volatility. *Review of Economic Studies*, 56(2):269–287.

Rosen, D. and Saunders, D. (2012). CVA the wrong way. *Journal of Risk Management in Financial Institutions*, 5(3):252–272.

Sarr, A. and Lybek, T. (2002). Measuring liquidity in financial markets. *IMF Working paper*.

Shaliastovich, I. and Tauchen, G. (2011). Pricing of the time-change risks. *Journal of Economic Dynamics and Control*, 35(6):843–858.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, 19(3):425–442.

Shiller, R. J. (1992). *Market Volatility*. MIT Press, Cambridge.

Sloan, R. G. (1996). Do Stock Prices Fully Reflect Information in Accruals and Cash Flows about Future Earnings? *Accounting Review*, 71(3):289–315.

Tiago de Oliveira, J. (1973). Statistical Extremes-a survey. *Center of Applied Mathematics*.

Tiwari, A. K., Jana, R. K., Das, D., and Roubaud, D. (2018). Informational efficiency of Bitcoin—An extension. *Economics Letters*, 163:106–109.

Urquhart, A. (2016). The inefficiency of Bitcoin. *Economics Letters*, 148:80–82.

Vidal-Tomás, D. and Ibañez, A. (2018). Semi-strong efficiency of Bitcoin. *Finance Research Letters*.

Wei, W. C. (2018). Liquidity and market efficiency in cryptocurrencies. *Economics Letters*, 168:21–24.

Weil, P. (1989). The equity premium puzzle and the risk-free rate puzzle. *Journal of Monetary Economics*, 24(3):401–421.

Welch, I. and Goyal, A. (2008). A Comprehensive Look at The Empirical Performance of Equity Premium Prediction. *Review of Financial Studies*, 21(4):1455–1508.

Yamai, Y. and Yoshiba, T. (2005). Value-at-risk versus expected shortfall: A practical perspective. *Journal of Banking & Finance*, 29(4):997–1015.