Drawing Topology

Ariadne

Moritz L. Sümmermann
✉ moritz.suemmermann@uni-koeln.de
📞 +49 1577 6490488

Motivation

- Topology is an important field in modern mathematics
- Topology and in particular homotopies are inherently visual, but are taught with formulas
- There is no real-world material able to emulate homotopies
- Ariadne makes it possible to learn about topology without formalism

Sample questions

- Which points can be connected by paths?
- Are these paths homotopic?
- How many different paths can be drawn on a surface from a point to itself?
- Are all paths with the same winding number homotopic?
- Why is the winding number of a closed path an integer?

Mathematics

- Fundamental group $\pi_1(X, x_0)$
- Winding number of a path
- Homotopies from a path f to a path g

$$H : [0,1] \times [0,1] \to Y$$
$$H(x,0) = f(x)$$
$$H(x,1) = g(x)$$

- Cutting and gluing along paths
- Realization of all orientable surfaces with boundary curves and arbitrary genus

Try it

Virtual reality

- Manipulation of paths and surfaces using Windows Mixed Reality controllers
- Overcomes the limitations of a 2D interface
- Experience self-made surfaces with the sense of place

Implementation

- Programmed in C# using Unity3D
- Multitouch and virtual reality roomscale interface
- Built-in computation of homotopy classes in 2D