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Effects of diagrams showing relationships between variables in solutions to problems concerning Speed.

1. Introduction

Some studies have claimed that “Speed” taught in the 6th is said to be the most difficult among the contents of mathematics taught in Japanese elementary schools, as with “Relative values” taught in the 5th grade. Speed problems also can be classified into three types. If y is the distance, a is the speed, and x is the time, then we have the relations as below.

The 1st usage: $x = y / a$, The 2nd usage: $y = a\,x$, The 3rd usage: $a = y / x$

The guidance of speed as usual in Japan is as follows.

- Order: 3rd usage(Definition of speed) → 2nd usage → 1st usage
- Diagram: Diagram of two number lines.
- How to describe: There is no common method, and it is left to each teacher.

Elementary school students in Japan are not good at solving verbal speed problems and describing how to solve them. This bad situation has not been improved for a long time. The problem seems to be in contents and method to teach.

2. Experiment

Teaching Objectives: The present study was undertaken in order to improve the ability of elementary school students in Japan to solve and explain the verbal speed problems.

Participants: Participants were 36 6th graders from one classes of an elementary school. They had learned “Box Diagram” to solve relative values problems in 5th grade.

Teaching steps and Teaching Process: Teaching steps and teaching process ware changed as follows.

- Order: 2nd usage → 3rd usage(Definition of speed) → 1st usage
- Diagram: “Box diagram”(It was learned in relative values and was effective.)
- How to describe: The content and order to describe are clearly indicated by the teacher using a box diagram. And the contents and order of the correct description are repeatedly written by the participant using the worksheet.

A. Frank, S. Krauss & K. Binder (Hrsg.),
Beiträge zum Mathematikunterricht 2019. Münster: WTM-Verlag.

The Box diagram on speed is as follows. “1” is always written in the upper left triangle. The box diagram can be read counterclockwise from “1” in the upper left, saying "If you go *‘a’* km in *‘1’* hour, go *‘y’* km in *‘x’* hours." By reading the box diagram, you can understand the relationship of numbers. Because the 2nd usage is easy for participants to understand.

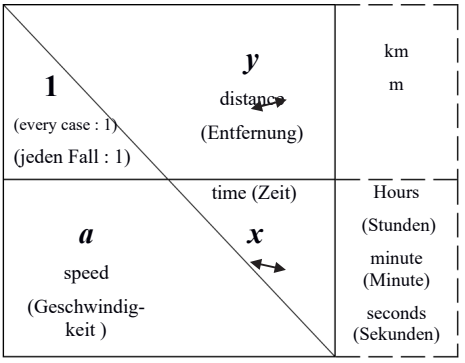


Fig. 1: Box diagram

Moreover, it is possible to correspond to the conversion of time and distance by adding a sub box to the right side. (Fig. 1)

The order of description was taught as follows.

Declare the numbers and units to find→

Understand and organize numerical values→

Make box diagram→

Read the box diagram and understand the relationship of numbers→

Create an expression with the second usage→

Transformation of expression→

calculation→

Conversion of answer.

The following worksheets were used in the lesson. The same worksheet was used for home learning.

The problem of post-test was created independently

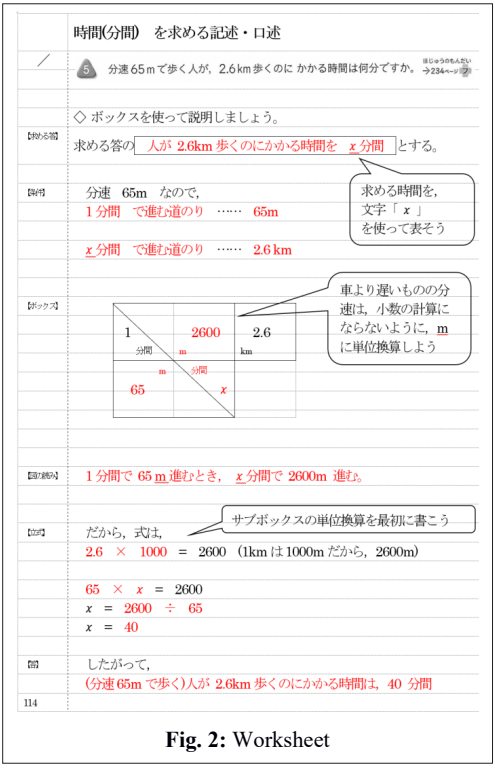
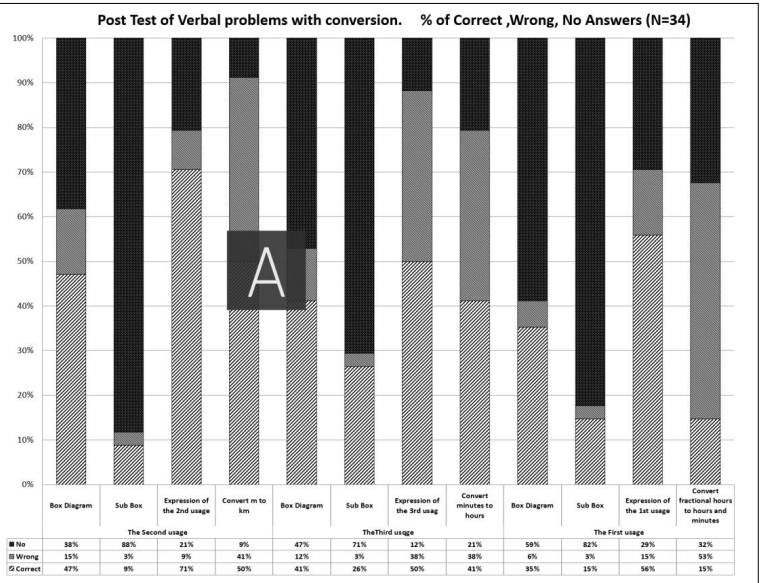


Fig. 2: Worksheet

by us. The post-test used are as follows. (Fig. 2)

Simple problem ① (The 2nd usage) A bicycle can run at 20 km in 1 hour. How long can this bicycle run in 3 hours? ②(The 3rd usage) A marathon player can run about 42 km in about two hours. Find the speed of the marathon player, and express by hour speed. ③ (The 1st usage) A car is moving at 60 km in 1 hour. Find the time it will take to go 300 km, and express by hour.
Verbal speed problem with conversion ① (The 2nd usage and Convert m to km) After the fireworks glow far in the distance, I heard the sound of fireworks in 12 seconds. Find out how many kilometers away from you where the fireworks glowed. However, the speed of the sound is about 333 m per second. ② (The 3rd usage and Convert minutes to hours) One athlete ran 1,000 m for 30 minutes. Find the speed of this athlete, and express by hour speed. ③ (The 1st usage and Convert fractional hours to hours and minutes) From F airport to G airport, there is 9750 km. How many hours and minutes does it take for an airplane at 1000 km / h to arrive at G airport from F airport?

The results of the post-test are shown below. (Tab. 1)



Tab. 1: The post test of verbal problems with conversion.

3. Discussion

About correct answer rate: The correct answer rate of the simple problem was 97% in the 2nd usage, 94% in the 3rd usage, 94% in the 1st usage, which was a very good achievement.

Table 1 shows the results of the post-test of verbal problems with conversion. The participants made many mistakes in unit conversion just before the answer. In particular, many participants could not convert hours expressed by decimal to hours and minutes. The correct answer rate until creating the formula was 71% for the 2nd usage, 50% for the 3rd usage, 56% for the 1st usage, which was a little better than what was expected by us.

On the effect of box diagram: When the box diagram was able to be correctly write, the solution process was able to be correctly described and the correct answer rate was high. Particularly, participants who drawn sub-boxes exactly had a high rate of making complete correct answers. Participants who wrote all the contents to be described had a high rate of correct answers. On the other hand, participants who did not write “declare the numbers and units to find” or read “box diagrams” were often incorrect.

It was suggested that organizing the numerical values in the box diagram and understanding the relationship of numbers is effective for improving the ability of verbal problem solving.

About the effect of worksheet: It was suggested that it is effective to clearly teach contents and order to be described by using worksheet. And it was also suggested that repeating the description using the worksheet enhances the ability of the description.

Based on the above results, it seems natural to conclude that the use of a box diagram and worksheets is effective for improving problem solving ability and descriptive ability. However, further studies should be conducted on the development of perfect descriptive abilities that do not skip the description.

4. References

Kato, T. & Moriya, S. & Shindo, T. (2017) Effects of educational practice on relative values problems using box diagram., *Proceedings of the Spring Annual Meeting of the Society of Mathematical Education 2017, Mathematics Education Society of Japan*, pp. 164-166.

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