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## **What meanings do German and Croatian students assign to geometry? A comparative study**

Geometry, one of the oldest branches of mathematics, is well-known for its diversity and coherence (Jones, 2000), but the geometry education seems to have lost these features (Mammana & Villani, 1998). These issues initiated reassessing the role of contemporary geometry education (Mammana & Villani, 1998). One of the approaches focuses on the idea of a coherent geometry curriculum by framing it in terms of fundamental ideas (Wittmann, 1999).

The insight into Croatian and German curricula for mathematics (Ministarstvo znanosti, obrazovanja i sporta [MZOS], 2006; Senatsverwaltung für Bildung, Jugend und Wissenschaft Berlin [RLP], 2015) in primary grades shows that these curricula reflect the multi-dimensional view of geometry, although some topics, namely geometric forms and measurement are much more emphasized than others (Glasnović Gracin & Kuzle, 2018; Kuzle, Glasnović Gracin, & Klunter, 2018). The work presented in this paper focuses on providing insight into what meanings German and Croatian primary students assign to geometry through the lenses of fundamental ideas.

### **Theoretical perspective**

Starting in the late 1970s, researchers have advocated structuring mathematics curriculum around fundamental ideas, sometimes called “overarching ideas”. In 1999, Wittmann proposed organizing geometry curriculum around seven fundamental ideas, which reflected the multi-dimensional view of geometry. These included: (1) geometric forms and their construction, (2) operations with forms, (3) coordinates, (4) measurement, (5) patterns, (6) geometric forms in the environment, and (7) geometrization. As such, these ideas provide a basis for a contemporary geometry education, reflecting both richness and coherence of geometry phenomena, which are in line with the recommendations of the ICME-7 study for new geometry curricula (Mammana & Villani, 1998).

### **Using drawings as a research method**

To gain insight into young students’ understandings of geometry, adequate and age-appropriate methods are important. In the past decade, drawings have been recognized as a new method for children’s expressions (e.g., Halverscheid & Rolka, 2006), which, combined with usual methods, such as interviews and surveys, may help to better understand different phenomena of interest (e.g., beliefs, emotions, classroom interactions).

Glasnović Gracin and Kuzle (2018) conducted a case study in Croatia with the focus on four students' fundamental ideas of geometry revealed through drawings. Similarly, Kuzle et al. (2018) conducted an exploratory study in Germany with a larger sample, which included Grade 3-6 students. The findings from both studies showed a rather narrow understanding of geometry with respect to exhibited diversity of fundamental ideas, which encouraged us to conduct a comparative study between the two countries with the following research questions: What fundamental ideas of geometry can be seen in Croatian and German primary grade students' drawings? What similarities and differences exist among Croatian and German participants' drawings?

## Method

The study participants were students aged 8 to 10 years. In total, 86 students from Germany (Berlin and Brandenburg) and 249 students from Croatia (mainly the Zagreb area) participated in the study. Student work was based on an adaptation of the instrument originally designed by Halverscheid and Rolka (2006). In the study, the students were asked to draw an image of what geometry is for them, which was followed by a semi-structured interview. All data were analyzed by using an adaptation of Wittmann's (1999) model of fundamental ideas. Detailed information about data collection, analysis, and the analytical tool are described in Kuzle et al. (2018) and Kuzle (2019).

## Results

The results from both countries showed that „Geometric forms and their construction“ (F1) was the most frequently coded fundamental idea of geometry (74% in Germany, and 87% in Croatia), as presented in Table 1.

Country	F1	F2	F3	F4	F5	F6	F7	Total
Germany (DE)	74%	7%	3%	4%	1%	12%	1%	642
Croatia (HR)	87%	0%	1%	3%	1%	8%	0%	1835

Table 1. Relative Frequencies of Students' Fundamental Ideas of Geometry. Abbreviations: F1 - Geometric forms and their construction; F2 - Operations with forms; F3 - Coordinates, spatial relationships, and reasoning; F4 - Measurement; F5 - Geometric patterns; F6 - Geometric forms in the environment; F7 - Geometrization.

The fundamental idea „Operations with forms“ (F2) refers to translation, rotation, mirroring, and other operations in geometry. Interestingly, 7% of German coded items were assigned to this idea, while the Croatian participants did not illustrate this fundamental idea at all. Fundamental idea „Geometric

forms in the environment“ (F6) focuses on the real-world objects. German participants presented this idea in 12% and Croatian participants in 8% of coded items. Other ideas were minimally present in students’ drawings independent of the country, with less than 5% of all coded items (see Table 1).

Since „Geometric forms and their construction“ (F1) was the most often presented fundamental idea in both countries, we examined the proportion of its sub-categories, which included 0-, 1-, 2-, 3-dimensional objects, geometric properties, drawing tools, and angles. The results are presented in Table 2.

<b>F1</b>	<b>0-dim objects</b>	<b>1-dim objects</b>	<b>2-dim objects</b>	<b>3-dim objects</b>	<b>Geom. properties</b>	<b>Draw. tools</b>	<b>An-gles</b>
DE	0.2%	7%	51%	22%	5%	12%	1%
HR	2%	19%	38%	17%	5%	4%	2%

Table 2. Proportions of components of the fundamental idea F1

Findings indicate the domination of 2-dimensional objects (e.g., triangle, square, quadrilateral, circle) in both countries. However, Croatian students presented 0-dimensional (i.e., points) and 1-dimensional objects (i.e., lines, curves) more often than the German participants, while the German students more often illustrated plane shapes, solids, and drawing tools.

## **Discussion and conclusions**

The results showed that the participants’ drawings and interviews from both countries revealed a rather narrow understanding of geometry through the lens of fundamental ideas. In both countries, „Geometric forms and their construction“ dominated in the drawings. This result is not surprising given that both mathematics curricula (MZOS, 2006; RLP, 2015) place considerable emphasis on this fundamental idea. Nonetheless, the data revealed that the students associate this fundamental idea mainly with geometric shapes, disregarding its other aspects. Additionally, 1-dimensional objects were more often illustrated by the Croatian participants. Lastly, German participants illustrated „Operations with forms“, whereas this was not illustrated at all by Croatian participants. Since these differences correspond to different emphases in national curricula, it seems that the existing mathematics curricula are crucial in developing learners’ understanding of geometry and geometrical concepts.

Interestingly, the fundamental idea „Measurement“ (F4) was not significantly present in the students' data, though both curricula place a great emphasis on it. This may be due to difficulties in illustrating this fundamental idea, which leads to some shortcomings of the method. Therefore, this approach needs additional data sources to increase the validity of the results.

The study findings suggest the necessity of re-questioning the curricular requirements regarding the multi-dimensional nature of geometry, and the coherence of its topics, as proposed by Mammana and Villani (1998). Moreover, the implemented curriculum needs to be researched too, in order to attain a deeper insight into current practices in geometry education.

## References

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