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Mathematical proficiency of quadratic function - a structure of doctoral dissertation

Introduction

Since Felix Klein in "Merano program" (1905) emphasized functions as a central concept of mathematics education, they became an important subject for many mathematics education researchers. According to Yerushalmy and Shternberg functions are one of the most important mathematical tools for helping students make sense of the world around them, as well as for preparing them for further study in mathematics (as paraphrased in National Council of Teachers of Mathematics, 2009, p. 41). The concept of function provides us with interconnection of formulas, models and graphs, as well as functional thinking. Many different aspects of teaching and learning function concept have been researched, especially in the last 20 years of 20th century, from when most of the related literature originate (e.g. Dubinsky & Harell, 1992, DeMarois & Tall, 1996, Vinner, 1983, and Dubinsky & Wilson, 2013). Educationalists agree that students in general have difficulties with forming meanings for function concept. Conceptual difficulties observed in last 50 years, mostly remained the same (Dubinsky & Wilson, 2013) and we are confronted with similar or even the same problems as Felix Klein was 100 years ago (Weigand et al., 2017). Furthermore, Denbel (2015) state that the best way for teaching and presenting functions to students, has not been identified so far. Stated conclusions create opportunity to consider some other, more comprehensive approach to teaching and learning functions.

Mathematical proficiency framework

In the modern mathematics education being able to merely use computational procedures and reproduce large quantities of knowledge isn't enough, reasoning, problem solving, connecting and communicating mathematical ideas play important role in persons mathematical proficiency (Schoenfeld, 2007). Research has shown that rule-based teaching doesn't provide students with opportunities to create meaning for the rules, just when to use them, which leads to forgetting, errors and poor strategic decisions (Kilpatrick et al., 2001). On the contrary, more broad-based curricula develops skills but also conceptual understanding and problem solving (Groves, 2012). Kilpatrick et al. (2001) define a concept of five, equally important and mutually interdependent strands, necessary for successful mathematics learning, named mathematical proficiency. The strands are: conceptual understanding,

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procedural fluency, strategic competence, adaptive reasoning and productive disposition. Conceptual understanding is a comprehension of mathematical concepts, operations and relations; procedural fluency is skill in carrying out procedures flexibly, accurately, efficiently and appropriately; strategic competence is the ability to formulate, represent, and solve mathematical problems; adaptive reasoning is the capacity for logical thought, reflection, explanation, and justification; and productive dis-position is defined as habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

The structure of doctoral dissertation

The title of doctoral dissertation is „Students' competencies relating to functions from the perspective of mathematical proficiency". The aim of the research is to offer a more comprehensive educational approach to teaching and learning concept of function. An approach that will set achievable outcomes for the development of the sense of function concept while considering vertical coherence in the function concept formation. The aim will be achieved by observing the concept through the lenses of mathematical proficiency framework. Denbel (2015) states that the common expectation from students finishing secondary education is to know function concept in general, and to be familiar with specific types of functions, such as linear, quadratic, exponential etc. This thesis will explore possibility of applying theory of mathematical proficiency on teaching and learning functions on the example of the concept of quadratic function.

To achieve the aim of the dissertation three research questions have been set:

[RQ1] What is mathematical proficiency in field of quadratic function?

[RQ 2] What level of competencies relating to quadratic function, with regard to the mathematical proficiency characteristics, do students demonstrate in the administered test?

[RQ3] What is the nature of understandings, and nature and cause of cognitive conflicts and deficits, regarding mathematical proficiency characteristics, that appear in the process of solving tasks relating to quadratic function?

The existing framework of mathematical proficiency is a theoretical framework that needs to be further elaborated in order to be implemented. By considering existing literature, each mathematical proficiency strand needs to be elaborated to define indicators of each strand, and then, by considering subject matter, interpreted in the terms of quadratic function.

To answer RQ2 and RQ3, that is, to gain the insight into how students understand quadratic function, a mixed method study, including testing and interviews, will be conducted. The first subgoal of answering RQ2 is to work out the method for evaluating these hidden aspects of mathematical competencies. This will be achieved by developing a test based on the normative point of view (RQ1) which aims on detecting students' understandings as well as their cognitive conflicts and learning deficits (descriptive aspect). A sample of 200 Croatian second grade students attending general gymnasium program will be tested in a form of a written test of quadratic function competencies test and self-assessment of associated beliefs. The second subgoal of answering RQ2 is to identify some interesting characteristics of mathematical proficiency regarding quadratic functions, which can be furtherly investigated in the form of interviews. The interviewer will examine the students' cognitive strategies that appear in the process of solving tasks relating to functions. Special attention will be given to those characteristics of mathematical proficiency that could not be properly researched through the written test (e.g. productive disposition). This in-depth analysis will provide the answer to Q3.

Conclusions

Outcomes of answering RQ2 and RQ3 will potentially allow revision of the answer of the RQ1, and thus produce some didactical consequences for teaching and learning the concept of quadratic function. It is expected that didactical guidelines, with the aim of improving mathematical proficiency relating to quadratic functions, achieving sense making and assuring vertical concept development, will be given. Finally, as a road to further research, it could be possible to consider teaching methods and designing tasks which can support development of mathematical proficiency of quadratic function.

At the present the RQ1 has been answered, that is, a normative aspect of mathematical proficiency for the quadratic function has been defined. For example, for defining conceptual understanding strand, the theory of Grundvorstellungen (vom Hofe, 1995) was considered to define four basic understanding regarding quadratic function. The process of the elaboration of the strands on the example of the adaptive reasoning and the strategic competence strand can be observed at the paper Gusic (2019). Next, the test necessary for answering the RQ2 has been developed and pilot tested on the sample of 44 Croatian students. The preparations for the real testing are in the progress.

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