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A Cross-sectional Examination of Children's Judgment of Expected Value

1. Problem and Methodology

In modern Japan, study of probability is positioned after the secondary education stage and probability education is rarely conducted in the elementary school stage. On the other hand, there have been discussions on probability as one of the important contents of primary education in other countries for a long time, and this has been reflected in the curriculum as well (Jones, Langrall & Mooney, 2007). In recent years, discussions on the necessity of properly introducing the concept of probability in elementary schools have begun in Japan as well. Given this, this study aims to obtain suggestions for probability education at the elementary school stage.

One of the issues in probability education that we consider here is the problem of the difference between the study of probability at school and probability judgment scenarios in everyday situations (Gal, 2005). There have been various discussions on the approach to solve this problem, but it has been pointed out that one of these is the importance of teaching by associating the concept of risk with probability (Borovcnik & Kapadia, 2018). The concept of risk is an important one in the context of safety in society, and is closely related to the living environment of children. Also, probability is a mathematical tool for dealing with risk. Therefore, how to teach probability as a means of dealing with risk is an important research theme in probability education (Borovcnik & Kapadia, 2018).

The concept of risk is handled in various research fields including insurance and risk management and finance, psychology and sociology, and its interpretations are diverse (Matsushita, 2018). There are also various definitions of the concept of risk, but according to a simple definition by the National Research Council (1989) it is a "product of the probability of the occurrence of damage and the severity of the damage". Severity of damage can be mathematically interpreted as a random variable value (hereafter, V). Regarding this V, one of the points at issue in defining risk is whether to term only adverse cases such as loss as risk or to also include favorable cases such as beneficial ones (Matsushita, 2018). In this study, it is assumed that risk includes both the meaning of disadvantage and advantage. Based on these discussions, the judgment that is required to deal with risk can be understood as judgment related to expected value taking into consideration the two variables of probability value (hereafter, P) and the probability variable value. In

Siller, H.-S., Weigel, W. & Wörler, J. F. (Hrsg.). *Beiträge zum Mathematikunterricht 2020.* Münster: WTM-Verlag, 2020.
doi: 10.37626/GA9783959871402.0

this way, in order to deal with risk properly, expected value judgment, which is one of the rational judgment criteria considering the two variables of P and V, is important.

Empirical studies on the children's judgment of expected value include research on expected utility theory in the field of psychology (Schlottmann & Anderson, 1994 et al.). These research findings show that children begin to acquire the concept of expected value in their daily lives before receiving any formal education about probability or expected value. On the other hand, these capture aspects of subject's subjective judgment of expected value and do not have direct implications for education. As this study aims to obtain suggestions for probability education in elementary school, it is necessary to understand the actual conditions of children's objective judgments of expected value. Therefore, we use Siegler's rule-assessment approach as a framework for understanding (Siegler, 1981). Siegler's method is effective as a framework for objective understanding of the judgment of the subject and has the potential to provide suggestions for education.

It cannot be said that an adequate number of studies that have tried to clarify the actual conditions of the children with respect to objective judgment of expected value from an empirical perspective have been conducted. Therefore, the aim is to understand the actual conditions of judgment of expected value first from the perspective of basic research by limiting to the Bernoulli trial. Basically, the purpose of this study is to clarify the following two points empirically.

Objective 1：To clarify the developmental process of children's judgment of expected value at the elementary school stage

The hypothesis for this is that the developmental process of the judgment of expected value is expressed by four rules obtained by the rule-assessment approach, and with progression in school year the order of the rule is also seen to be higher. (Hypothesis 1).

Objective 2：To clarify the period of acquisition of Rule IV.

The hypothesis for this is that the proportion of children who acquire Rule IV begins to increase from Grade 5 and Grade 6 (Hypothesis 2). It can be considered that the studies such as study of ratio in Grade 5, or of proportion and inverse proportion in Grade 6 has an impact.

2. Method

The subjects were 66 Grade 1 students, 64 Grade 2 students, 61 Grade 3 students, 64 Grade 4 students, 56 Grade 5 students, and 66 Grade 6 students

of Elementary School B in Prefecture A. The survey was conducted from February 12-22, 2019. The time required was about 45 minutes for each grade. The problems were limited to the Bernoulli trials, and problems consisted of selection of the one with a higher expected value from between two spinners. The question was, “Choose the spinner that will receive more points”.

3. Results

The analysis of the results was conducted in the following steps. The first step was to identify the rules using Siegler’s rule-assessment approach. The second step was examining the difference among the Grades. The test of difference in ratios was conducted by the direct probability calculation method and the analysis was conducted using the φ coefficient as the effect size. The following Table shows the number of persons that the rules applied to for each grade in the expected value judgment problem and the relevant percentages.

	NR	Ⅰ	Ⅱ	Ⅲ	Ⅳ
Grade 1	5 (7.6)	12 (18.2)	9 (13.6)	38 (57.6)	2 (3.0)
Grade 2	3 (4.7)	4 (6.3)	15 (23.4)	38 (59.3)	4 (6.3)
Grade 3	3 (4.9)	3 (4.9)	17 (27.9)	30 (49.2)	8 (13.1)
Grade 4	3 (4.7)	2 (3.1)	23 (35.9)	25 (39.1)	11 (17.2)
Grade 5	0	0	6 (10.7)	29 (51.8)	21 (37.5)
Grade 6	0	0	6 (9.1)	17 (25.8)	43 (65.1)

Based on the results in this Table, the results of the test of difference of the rule application rate for the grades through the direct probability calculation method were as follows. Compared to Grade 1, there were many Grade 2 children who encoded two variables in fixed tasks ($p<.05$, $\varphi=.19$, two-sided test). Compared to Grade 4, there was greater encoding of two variables in the respective fixed tasks and conflict tasks in the case of Grade 5($p<.10$, $\varphi=.20$, $p<.05$, $\varphi=.37$, two-sided test), and in addition, there were also more children who integrated the two variables ($p<.05$, $\varphi=.23$, two-sided test). compared to Grade 5, Grade 6 had more children who integrated the two variables ($p<.05$, $\varphi=.28$, two-sided test).

4. Considerations

Verification of Hypothesis 1

According to the results, a higher order of the rules was observed as the grade advanced. Further, each rule was represented as encoding of one variable,

encoding of two variables in a fixed task, encoding of two variables in a conflict task, and the integration of two variables. This was not just an ordering, but also the higher order rule being inclusive of the lower order rule. From this property as well, Hypothesis 1 about the developmental process was supported.

Verification of Hypothesis 2

Rule IV was seen from Grade 1. However, the rate gradually increased with advancement in grade and the rate of increase from Grade 4 to Grade 5 and Grade 5 to Grade 6 was high. This can be considered as due to the difference in learning experiences in related areas such as ratio and proportion, and it can be said that Hypothesis 2 is therefore supported.

As this paper considers children's judgment of expected value from the analysis of response patterns, future topics include an examination of qualitative aspects such as the quality of errors and relevant factors that have not been thoroughly examined. This aspect needs to be further examined from the point of view of conducting a strategic analysis. Also, it is necessary to further examine the factors that regulate the higher order of rules and teaching strategies that promote higher orders in the future.

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