

Takashi KATOU, Tohoku (Japan)

Solving ratio sentence problems classified according to arithmetic structure

Introduction

In Japan, the rate of correct answers with sentence problems related to ratio is low. A common solution to the problem of ratio is to substitute numerical values into the formula.

- 1st usage: Compared quantity \div Base quantity = Ratio
- 2nd usage: Base quantity \times Ratio = Compared quantity
- 3rd usage: Compared quantity \div Ratio = Base quantity

Complicated sentence problems are classified into varying types according to the structure of the operation. For single-line problems, only one main operation (hereinafter, it is described as "main operation") is conducted to obtain an answer. And multi-line problems involve performing two or more main operations and comparing the two values. In contrast, in the case of chain type problems, the value obtained by one main operation is substituted into the next main operation to obtain the answer. Numerical values or unit conversion may be required before or after multiplication or division. The classification according to arithmetic structure is organized as shown in Table 1. The row in the bottom row of the chain type column denotes that the corresponding question has not been asked in the past.

	Single type	Chain-type
Single-line	No processing or conversion	No processing or conversion
	Requires processing and conversion Before, after, before and after	Requires processing and conversion Before, after, before and after
Multi-line	No processing or conversion	—
	Requires processing and conversion Before, after, before and after	

Tab. 1: Classification by operation structure

As shown in Table 1, there are 12 basic problem types, including those that require various forms of processing and conversion. Because each of these 12 types of questions has three usages, there is actually a total of 36 types of sentence problems. The difficulty of solving different types of sentence problems was investigated by preparing and administering a preliminary survey test.

Experiment

The purpose of this study is to clarify the difficulty in solving problems with complex quantity sentences as classified by the arithmetic structure. A preliminary survey question was created and prepared for a full-scale survey. Because completing all 36 types of problems would take too much time, the preliminary survey was limited to 14 questions. The context of all text questions was to determine the length and proportion of ribbons. Processing and conversion were limited to numerical and percentage conversion problems. The multi-line type problems were created as a one sentence question, whereas the chain type problems were created as a divided question. The preliminary survey was conducted with 15 sixth graders in November 2019. Some examples of the single-line, multi-line, and chain type survey problems are presented in Tab. 2.

<p>• Single-line</p> <p>No. 1 (1st usage) The length of an 80 cm ribbon is expressed as 1 in ratio. How many times the length of a 0.2 m ribbon is the length of the 80 cm ribbon?</p> <p>No. 5 (2nd usage) The length of an 80 cm ribbon is expressed as 1 in ratio. The length of another ribbon is expressed as 20% of the first ribbon. How long is the second ribbon?</p> <p>No. 9 (3rd usage) The length of a ribbon is expressed as 1 in ratio. A 0.6 m ribbon can be expressed as 30% of the first ribbon. How long is the first ribbon?</p>
<p>• Multi-line</p> <p>No. 11 (3rd usage)</p> <p>When the length of Mr. A's ribbon is expressed as 1 in ratio, the length expressed as 0.6 in ratio to his ribbon is 0.9 m in length. When the length of Mr. B's ribbon is expressed as 1 ratio, the length expressed as 40% of Mr. B's ribbon is 80 cm. Whose ribbon is longer?</p>
<p>• Chain-type</p> <p>No. 12</p> <p>① (1st usage) The length of a 40 cm ribbon is expressed as 1 in ratio. How many times the length of a 24 cm ribbon is the length of the first ribbon?</p> <p>② (2nd usage) The ratio determined in ① was cut from the 120 cm ribbon. What is the length of the ribbon that was cut?</p> <p>③ (3rd usage)</p> <p>The length of a ribbon is expressed as 1 in ratio. A 30cm ribbon can be expressed as ① in ratio. How long is the first ribbon?</p>

Tab. 2: Survey problem examples

Table 3 shows the structure of calculations for the survey questions and the correct answer rates. Because of an error, the correct answer rate for the first part of question No. 10 is not shown.

	No.	Before conversion	Calculation 1	Calculation 2	After conversion	Correct (%)
Single-line	1	numerical	1st usage	—	—	47
	2	—	2nd usage	—	numerical	27
	3	numerical	3rd usage	—	—	7
	4	—	1st usage	—	percentage	60
	5	percentage	2nd usage	—	—	60
	6	percentage	3rd usage	—	—	27
	7	numerical	1st usage	—	percentage	47
	8	percentage	2nd usage	—	numerical	34
	9	numerical · percentage	3rd usage	—	—	21
Multi-line	10	—	2nd usage	—	numerical	/
		—	2nd usage	—	numerical	7
						Answer 7
	11	numerical	3rd usage	—	—	27
		percentage	3rd usage	—	—	20
					Answer 20	
Chain-type	12	—	1st usage	—	—	47
		—	1st usage	2nd usage	—	37
		—	1st usage	3rd usage	—	33
	13	—	2nd usage	—	—	34
		—	2nd usage	1st usage	—	20
		—	2nd usage	3rd usage	—	20
	14	—	3rd usage	—	—	40
		—	3rd usage	1st usage	—	20
		—	3rd usage	3rd usage	—	27

Tab. 3: Calculation structure and correct answer rates for survey questions

The survey results showed the following trends:

- None of the correct answer rates for single-line types that required unit conversion of numerical values (#1–3) exceeded 50%. The rate of correct answers for the first usage was highest, with a 20% decrease for the second usage and another 20% decrease for the third usage. Students were more adept at solving the single-line types that required the conversion of percentages (#4–6), however 40% did not correctly perform the first and second usages and over 70% gave incorrect answers for the third usage.

- In the case of single-line type problems that required conversion before, after, or before and after main operations, the correct answer rates were higher in the following order: (1) ratio conversion before main operation in the second usage (#5) or after the main operation in the first usage (#4); (2) numerical conversion before the main operation in the first usage (#1).
- In the case of multi-line type problems, the correct answer rate was almost the same as that of some of the more difficult single linear types.
- The correct answer rates for chain type problems were generally higher than those of most two-line problems but lower than the highest single-line correct answer rates. The product of the correct answer rates of the two usages is close to the correct answer rate of the answer.
- When solving problems involving the third usage, lower success rates occurred when expressions were formulated using the first usage, which resulted in difficulties in achieving the transformation of the expression.

Discussion

Although the sample was small, several conclusions could be inferred from the results of the survey and the children's calculation processes.

- The correct answer rate of single-line problems is higher in cases of ratio conversion than numerical value conversion.
- In the case of the third usage, children who formulate expressions using the first usage often do not successfully transform the expression.
- The full-scale survey requires the following improvements. The first survey items should comprise three pure single-line three usage problems that do not require processing or conversion, followed by single-line problems that require ratio conversion. Chain type problems should include both single and multiple problem sentences.

Acknowledgement This work was supported by JSPS KAKENHI Grant Number 18K02588.

References

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