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Implementation Research on the Formation of Students' Concept of Mathematical Functions

1. Introduction

It is a well-known fact that establishing the relationship between two variables is the basis of mathematical functions and the following points constitute its foundation: (I) quantifying changes in a phenomenon; that is, deriving variables from the phenomenon; and (II) establishing the relationship between the two variables derived.

However, previous studies indicate that there are issues with students' understanding of (I) and (II) regarding mathematical functions.

It is said that infants have a proportional relational concept about time and distance in accordance with the "large is large" schema, though it is implicit. Lateron, they acquire "large is small" schema and make the connection between two variables. Therefore, even before learning functions at school, students have already made the connection between the relationship among two variables, even though it is not accurate or based on numerical values, and they possess the groundwork to understand mathematical functions.

From this, we can infer that if there is a problem in students' understanding of mathematical functions, there is a possibility that the current learning methodology of mathematical functions is not taking advantage of this concept of relationship between two variables inherently possessed by students.

2. Method

Dixon, Moore et al. (1991, 1996) have shown that performing intuitive problem solving first, without the use of numerical values, and intuitive understanding, is necessary to generate mathematical solution strategies. Therefore, transitioning from "learning without the use of numerical values" to "learning with the use of numerical values" is deemed to be one of the approaches that promote the understanding of mathematical functions.

One of the factors indicated in previous studies that contributes to students' problem of learning mathematical functions is skipping the step of "learning without the use of numerical values"; it can be concluded that the students are probably not able to draw on the foundation of understanding mathematical functions that they already have.

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Therefore, in this study, an educational framework comprising a logical structure for the formation of qualitative relational concepts has been set to facilitate the understanding of functions in students. The relationship between the two variables is set as "one variable increases when the other variable increases," "one variable decreases when the other variable increases," or a combination of these. The educational framework comprises two components: "Learning without the use of numerical values" and "Learning with the use of numerical values" (Fig. 1).

The aim of "Learning without the use of numerical values" is to enable children to form developing concepts about (I) and (II) that have a logical structure.

Concept formation is determined by deriving variables after observing changes in a phenomenon and establishing the relationship between the two variables as "one variable increases when the other variable increases" and "one variable decreases when the other variable increases."

This educational exercise targets children between 9-10 years of age, the age at which children in Japan learn about the relationship between two variables.

The children recorded changes in the shadow of a solid cone on a drawing sheet (Fig. 2). These sheets were used as teaching material to study the relationship between two variables with time as the main axis.

The procedure of the educational exercise was as follows:

- (i) The shape of the shadow recorded by the child on the drawing sheet was copied and cut out. Variables such as length of the sides, width of the shadow, and size of the angles were derived by direct comparison.
- (ii) Relationship between two variables was established. Specifically, the relationship between time and other variables (length of sides, width of shadow, size of angles, etc.) was established first.
- (iii) Relationship between two variables that involve time was established (length of sides, width of shadow, size of angles, etc.).

The target of this educational exercise were children between the ages of 9 and 10 (two children). The exercise was conducted on Sunday, June 30, 2019, from 9:15 to 10:00 and 10:30 to 11:10 at the university laboratory.

3. Result

The results of the exercise are described as follows:

(i) Deriving the variables: First, the cut-outs were placed on top of each other to directly compare the shapes of the shadows. By doing this, both the children were able to identify the left and right sides, base, width, and size of angles as variables. The three sides and angles of the triangles were named A, B, and C etc., to make it easier for the children to explain the variation in variables.

(ii) Establishing the relationship between the passage of time and other variables: It was confirmed that both children understood that the change in the variables derived in (i) corresponds to the passage of time. It was also confirmed that they could explain the correspondence between the two variables as "Side B becomes shorter as time passes." The children verbally discussed the correspondence between other variables with each other and could talk about it without any difficulty.

(iii) Establishing the relationship between two variables that involve time: I explained to the children that if "Side A becomes shorter as time passes," we could say that "Side A becomes shorter because time passes," and if "Side B becomes shorter as time passes," we could say that "When Side A becomes shorter, side B also becomes shorter." Both the children immediately understood the relationship between the change in the lengths of sides A and B. I asked them to write down the correspondence between other variables in their notebooks. Both the children were able to write down the relationships between different sides, angle and side, side and area, etc. In the end, they were able to present the relationship between the two variables that they had written in their notebooks.

4. Discussion

In this paper, to facilitate the learning of mathematical functions, a learning stage without the use of numerical values was provided before learning functions with the use of numerical values in order to take advantage of the foundation of functions inherently possessed by children. The aim was to form a developing concept for (I) and (II) which have a logical structure.

An educational exercise involving two children was conducted where they learned the following without any difficulty: (i) Deriving variables (ii) Establishing the relationship between the passage of time and other variables (iii) Establishing the relationship between two variables that involve time. They were able to explain the relationship between the two variables to them. Therefore, it can be said that the developing concept was formed in both children. Some points that could not be described in this paper were that there was a questionnaire survey conducted before and after the educational exer-

cise, and that (iii) had been difficult to explain before the educational exercise but was explained after it. The results also indicate that the formation of the developing concept has been achieved. It can be said that the explanation of (i), (ii), and (iii) was made possible through the educational exercise that took advantage of the foundation of mathematical functions possessed by children.

It can be concluded that the content of the educational exercise conducted in this study is appropriate as it drew upon the foundation of relationship between two variables, which children already possess.

In the future, we need to study how we can use these results of learning without the use of numerical values toward learning with the use of numerical values, conduct educational exercises as a part of elementary school activities, and study their effectiveness.

Learning without the use of numerical values		Learning with the use of numerical values
Attempts to develop the concepts of (I) and (II) by observing changes in a cone's shadow.	⇒	On the foundation of the developing concepts of (I) and (II), expressing such changes in the form of numbers, charts, and graphs, and thereby considering the relationship between the two variables.

Fig. 1: Educational Framework for the Formation of Concepts Regarding (I) and (II)

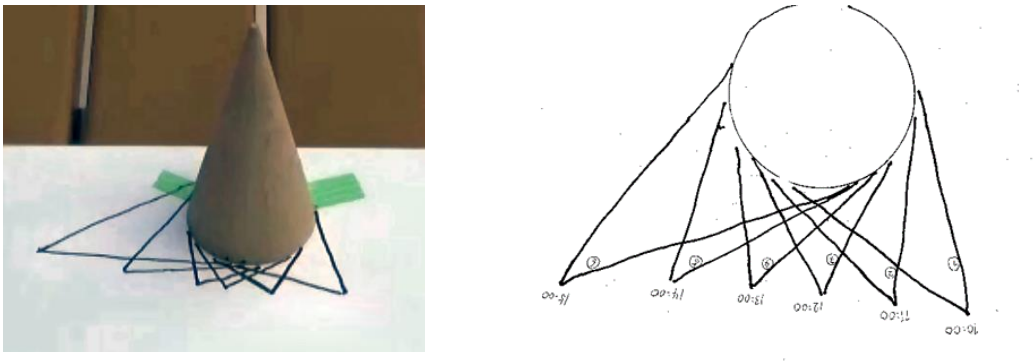

Fig. 2: Recording Changes in the Shadow of a Solid Cone (left) and Attaching Numerical Values to the Recording (right)

Literatur

Dixon, J., A. & Moore, C., F. (1996). The developmental role of intuitive principles in choosing mathematical strategies, *Developmental Psychology*, 32, 241–253.

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