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Stochastics content knowledge of pre-service Chilean mathematics teachers

Probability and Statistics (Stochastics) knowledge has been considered a necessary part of the cultural heritage in order to function effectively in the information society (Ben-Zvi & Makar 2016, Engel 2017). Therefore, the incorporation of stochastic learning into the Mathematics curriculum of many countries has boosted its teaching at different levels of compulsory education (for example, Chile: Ministry of Education [MINEDUC], 2009; Spain: Ministry of Education, Culture, and Sport [MECD], 2014, 2015; and USA: www.corestandards.org/Math; National Council of Teachers of Mathematics [NCTM], 2000). This modification establishes new demands and challenges for the teachers responsible for teaching these concepts. However, the literature suggests that many teachers are not sufficiently prepared, and many do not feel competent in their abilities to teach this material (Batanero, Burrill & Reading, 2011; Groth & Meletiou-Mavrotheris, 2018).

Therefore, we are interested in analysing the understanding of stochastics content of these professionals. Specifically, we seek to report some typical misconceptions about stochastics content of a sample of 269 pre-service Chilean mathematics teachers, that could guide possible reformulations of their initial training process.

Framework

Research has shown that Stochastic content can be difficult for novice students as well as perspective teachers. Several references now suggest organizational schemes for this content knowledge to enable teachers to effectively organize, implement, and evaluate the teaching and learning processes. For example, we highlight the books of Batanero et al. (2011), Batanero & Chernoff (2018), Ben-Zvi & Makar (2016), Ben-Zvi, Makar & Garfield (2018), and Garfield & Ben-Zvi (2008).

To explore teachers' content knowledge, we used the Onto-Semiotic Approach to knowledge and mathematics instruction [OSA] (Godino, Batanero & Font, 2007). With OSA tools, a model is established for the training of mathematics teachers, called Teacher's Didactic-Mathematical Knowledge and Competence [DMKC] (Godino, Giacomone, Batanero & Font, 2017). The foundation to this didactic knowledge, teacher content knowledge, is broken into two types: (1) The Common Content Knowledge, which is considered sufficient to solve the problems proposed in the Mathematics school

Siller, H.-S., Weigel, W. & Wörler, J. F. (Hrsg.). *Beiträge zum Mathematikunterricht 2020.* Münster: WTM-Verlag, 2020.
doi: 10.37626/GA9783959871402.0

curriculum; (2) The Extended Content Knowledge, which represents the mathematical basis for linking school stochastics with subsequent educational levels.

Method

This research is listed within the quantitative approach, and it is an exploratory study in the Chilean context (Hernández, Fernández & Baptista, 2014). We used the Scale of Common and Expanded Stochastic Content Knowledge (CESK), an instrument designed exclusively for this purpose, and were previously applied in a pilot sample of 126 prospective math teachers from Chile (21 men and 21 women) and Spain (49 men and 35 women). In the pilot results, the items range from easy to difficult (% of correct answers between 90 to 10%), adequate discrimination (biserial correlation coefficient higher than 0.2) and sufficient reliability (Cronbach' alpha = 0.564). As a result, we established the CESK Scale with 20 multiple choice items (10 for common content knowledge and 10 for extended content knowledge), each with three distractors and one correct option.

The full study consists of 269 Chilean prospective math teachers (150 men and 119 women, aged between 19 and 51, with an average of 23.8 years old), enrolled in 15 of the 30 universities in Chile that offer the pedagogy in mathematics career (http://www.cned.cl). Regarding prior training, all the participants had taken all the subjects about the Stochastic content considered in their curricula, but they not necessarily approved them all. In summary, for those who have at least one scheduled subject in their university (N=261), approximately 82% (N=214) have already passed all the planned subjects on stochastic content.

Results

Overall, the scores obtained in the questionnaire ranged from 0 to 13 points (out of 20 possible), with a mean of 5.29 points and a SD of 2.72 points. Cronbach's Alpha, 0.571, was slightly higher than the pilot study and still sufficient for this type of evaluation. The proportion of correct answers for each item (difficulty index) ranged from 0.05 for the item that evaluated an application of Bayes' theorem in a problem of conditional decision, to 0.59, for the item which evaluated the ability to compare the distribution of data between two groups. Based on these proportions, the CESK assessment was between moderate and difficult for this new sample. Regarding the ability to discriminate from each item, the point-biserial correlation coefficient ranged between 0.167, for the item that assessed the ability to distinguish between correlation and causality, and 0.452, for the item in which effects on the width of the confidence intervals are identified. Therefore, we conclude that

the strength of the items is adequate to discriminate between those who respond well because they know and those who do it by chance.

On the other hand, we define the variables Common Knowledge and Expanded Knowledge about the Stochastic Content (CKSC and EKSC respectively), as the number correct for the items that make up each category of knowledge. These variables will each range from 0 to 10 points. Regarding common knowledge, the mean of CKSC variable was 3.67 points, the median 4 points, and the standard deviation 1.98 points. While for expanded knowledge, the mean of EKSC variable was 1.63 points, the median of 1 point, and the standard deviation of 1.29 points. This exploration makes us suppose possible differences between the answers declared by participants according to both Content Knowledge categories.

To verify if there are differences between the two distributions, we calculate some summary measures on the differences between the two variables (D = CKSC-EKSC). Among the results, the minimum difference was -3 points, and the maximum was 8 points, while the 25th, 50th (median), and 75 th percentiles resulted in 1, 2 and 3 points respectively. Then, we used the Wilcoxon signed rank test for related samples to assess whether the median of D is equal to 0. Therefore, with a p-value of 0.000 (< 0.005), we reject the null hypothesis, that is, there are significant differences between Common and Expanded knowledge of the Stochastic contents for these participants. Moreover, we can see that subjects tended to perform better on the CSCK, because from the quartile 1 the difference D is greater than 0, reflecting less domain in the Expanded Content knowledge.

Discussion

Teachers need a solid understanding of the content they will teach (Godino et al., 2017), so that their weaknesses have no negative implications for classroom instruction and student learning. In our case, the conceptual knowledge of these pre-service teachers was generally poor, with significant differences between the scores obtained in both categories of content knowledge. A tendency for higher scores was obtained in Common Knowledge (CKSC), reflecting a greater deficiency in expanded knowledge (EKSC), which enables teachers to broaden their perspective on the content they are teaching and gives them enhanced ability to improve their teaching and better connect to topics their students will study in the future. The participants of this study represent a substantial number of professionals who will soon teach Stochastics in Chileans schools. Therefore, given the results, we highlight the need to align the initial training of teachers with local and international recommendations for this stage (Ruz, Díaz-Levicoy, Molina-Portillo & Ruiz-

Reyes, 2018), as well as to promote instances of professional development, where mathematics teachers can expand their content knowledge, and gain confidence in their teaching.

Acknowledgement

This work was supported by Chilean Government under a Ph.D. Scholarship number 72170025 and by the project B-SEJ-063-UGR18 (Spain).

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