Received: 10 July 2020 | Accepted: 10 November 2020

DOI: 10.1002/pamm.202000157

Modeling gradient-enhanced anisotropic ductile damage: Application to low cycle fatigue

Kai Langenfeld[1,*], **Kerstin Möhring**[2], **Frank Walther**[2], and **Jörn Mosler**[1,**]

[1] Institute of Mechanics, TU Dortmund University, Leonhard-Euler-Str. 5, 44227 Dortmund, Germany
[2] Department of Materials Test Engineering, TU Dortmund University, Baroper Str. 303, 44227 Dortmund, Germany

Low cycle fatigue is in general associated with large plastic strains accompanied by the formation and coalescence of pores. Considering continuum damage mechanics, many local constitutive models for ductile damage can be found in the literature. These local models however, that do not involve any length scale, lead to mesh dependent results. In order to regularize these local models, the micromorphic approach will be applied. Within this talk, it will be shown that the standard micromorphic approach is not suitable to regularize the underlying ductile damage model. A modified version will be therefore proposed and compared to the standard approach. This modified model will be extended to low cycle fatigue and its capabilities will be shown.

1 Kinematics

Let $\boldsymbol{u}(\boldsymbol{X})$ be the displacement field of all points $\boldsymbol{X}$ of body $\mathcal{B}$. Then, the Hencky strain tensor follows as $\boldsymbol{\varepsilon} = \frac{1}{2}\ln(\boldsymbol{C})$ with $\boldsymbol{C}$ being the right Cauchy-Green tensor. The additive split of the Hencky strain tensor in an elastic part and a plastic part $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{\mathrm{e}} + \boldsymbol{\varepsilon}^{\mathrm{p}}$ allows to extend linearized kinematics to finite strains while preserving the structure of the local constitutive model, cf. [1].

2 Constitutive model

The ductile damage model is adopted from [2,3]. Its micromorphic enhanced Helmholtz energy reads

$$\psi = \psi^{\mathrm{e}}(\boldsymbol{\varepsilon},\, \boldsymbol{\varepsilon}^{\mathrm{p}},\, \boldsymbol{b}) + \psi^{\mathrm{p}}(\boldsymbol{a},\, \boldsymbol{k};\, \boldsymbol{b}) + \psi^{\mathrm{pen}}(\boldsymbol{b},\, \boldsymbol{\varphi_b}) + \psi^{\mathrm{grad}}(\nabla\boldsymbol{\varphi_b}) \tag{1}$$

with $\boldsymbol{a}$ and $\boldsymbol{k}$ being two second-order tensors associated with kinematic and isotropic hardening, cf. [4]. The tensor $\boldsymbol{b}$ describes the integrity of material points, i.e., damage is defined as $\boldsymbol{d} = \boldsymbol{I} - \boldsymbol{b}$. By applying the micromorphic regularization, additional tensor $\boldsymbol{\varphi_b}$ is coupled to integrity tensor $\boldsymbol{b}$ and its gradient is minimized [5,6]. The dual variables then follow as

$$\boldsymbol{\sigma} = \frac{\partial\psi}{\partial\boldsymbol{\varepsilon}}, \quad \boldsymbol{\omega} = \frac{\partial\psi}{\partial\boldsymbol{\varphi_b}}, \quad \boldsymbol{\alpha} = -\frac{\partial\psi}{\partial\boldsymbol{a}}, \quad \boldsymbol{\kappa} = -\frac{\partial\psi}{\partial\boldsymbol{k}}, \quad \boldsymbol{\beta} = -\frac{\partial\psi}{\partial\boldsymbol{b}}\,. \tag{2}$$

The model is completed by the plastic potential $g = \Phi + \Gamma$ from which the evolution equations of internal variables follow as

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{p}} = \lambda\,\frac{\partial g}{\partial\boldsymbol{\sigma}}, \quad \dot{\boldsymbol{a}} = \lambda\,\frac{\partial g}{\partial\boldsymbol{\alpha}}, \quad \dot{\boldsymbol{k}} = \lambda\,\frac{\partial g}{\partial\boldsymbol{\kappa}}, \quad \dot{\boldsymbol{b}} = \lambda\,\frac{\partial g}{\partial\boldsymbol{\beta}}\,. \tag{3}$$

Since this is in line with the principle of generalized standard materials, cf. [7], the dissipation inequality is automatically fulfilled. The aforementioned modification of the micromorphic approach, cf. [4], is the modification of the yield function of the type

$$\Phi \leftarrow \Phi + f(\boldsymbol{\omega})\,. \tag{4}$$

Since $\boldsymbol{\omega}$ can be rewritten as a gradient contribution, the modification indeed corresponds to a gradient-enhancement.

Fig. 1 compares the modified and the standard micromorphic approach by means of a one-dimensional truss with an imperfection. The left figure shows the typical localization effect when considering models without internal length scale. Contrary, both the standard micromorphic approach and the modified version yield mesh independent results. Yet, only the modified version shows a softening response. By considering two-dimensional boundary value problems, it is shown in [4] that the standard approach leads, similar to the local model, to mesh-dependent results.

3 Extension to low cycle fatigue

In order to adapt the model to low cycle fatigue, certain changes to the constitutive equations are introduced. The major change is the introduction of an additional internal variable, which triggers the damage evolution by means of the dissipated plastic work during deformation, cf. [8].

---

[*] Corresponding author: e-mail kai.langenfeld@tu-dortmund.de, phone +49 231 755-3765
[**] Corresponding author: e-mail joern.mosler@tu-dortmund.de, phone +49 231 755 5744, fax +49 231 755 2688

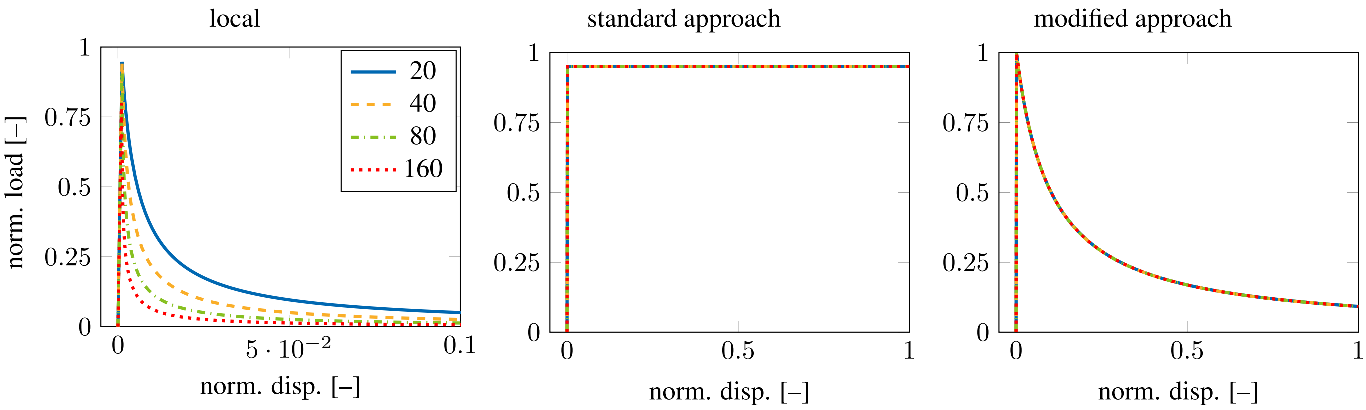

Fig. 1: One-dimensional truss with an imperfection: Load-displacement diagrams predicted by: (left) the local model, (middle) the standard micromporphic model and (right) the modified micromorpic model for different finite element meshes (20, 40, 80 and 160 finite elements)

A first impression of the final model is given in Fig. 2, in which the model is calibrated with respect to a cyclic tensile test. The left plot shows the load-displacement diagram up to 77 cycles and the right plot the load-displacement diagram until failure after 93 cycles. It can be seen that the extended model is able to simulate cyclic hardening as well as material degradation in a mesh objective manner.

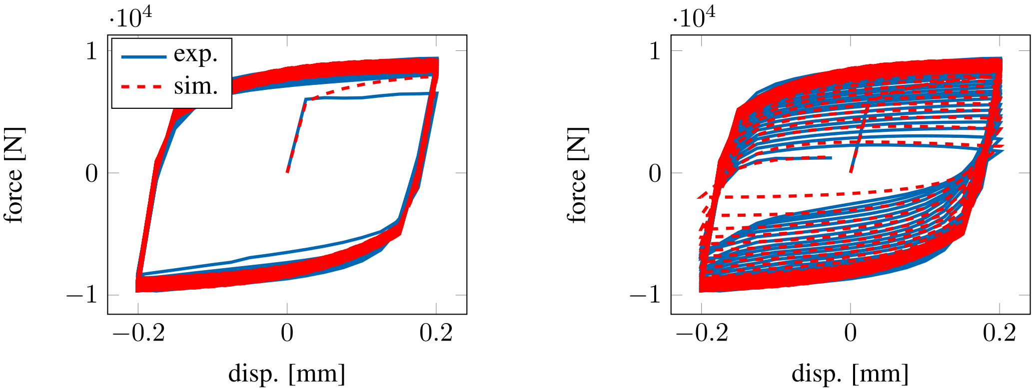

Fig. 2: Cyclic tensile test – left: load-displacement diagram corresponding to 77 cycles; right: load-displacement diagram corresponding to 93 cycles (failure)

4 Conclusion

A constitutive model suitable for to low cycle fatigue was presented. It is based on the anisotropic ductile damage model proposed in [2, 3] and has been gradient-enhanced by a modified version of the micromorphic approach [5, 6]. Based on an illustrative example, the capabilities of this modification were shown. The resulting gradient-enhanced model was adapted to low cycle fatigue. It was shown that the final model indeed captures the low cycle fatigue behaviour observed in experiments – qualitatively as well as quantitatively.

Acknowledgements Financial support from the German Research Foundation (DFG) via SFB/TR TRR 188 (projectnumber 278868966), project C01, is gratefully acknowledged.
Open access funding enabled and organized by Projekt DEAL.
Correction added on 19 February 2021, after first online publication: Kai Langenfeld was designated as corresponding author.

References

[1] P. Papadopoulos and J. Lu, Comput. Methods. Appl. Mech. Eng. **159**, 1–18 (1998)
[2] A. Menzel, M. Ekh, P. Steinamann and K. Runesson, Int. J Numer. Meth. Eng. **54**, 1409–1430 (2002)
[3] M. Ekh, A. Menzel, K. Runesson and P. Steinmann, Int. J. Eng. Sci. **41**, 1535–1551 (2003)
[4] K. Langenfeld, J. Mosler, Comput. Methods. Appl. Mech. Eng. **360**, (2020)
[5] B. J. Dimitrijevic, K. Hackl Tech. Mech. **28**, 43–52, (2008)
[6] S. Forest, J. Eng. Mech. **135**, 117–131 (2009)
[7] B. Halphen , Q. S. Nguyen JMec. **14**, 39–63 (1975)
[8] J. Lemaitre, R. Desmorat, Engineering Damage Mechanics. (Springer, Berlin, Heidelberg, 2005)