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An empirical study on mathematical thinking of first-year university students in Chile

This paper presents a brief contextualization of the educational system in Chile, identifying certain problems of education and specifically with mathematics in students who are in the process of transition from high school to university. Then, the main ideas that constitute the concept of Grundvorstellungen as a didactic discipline are presented. This study aims to develop a didactic analysis oriented to the mathematical thinking that students have about contents seen at school. For this purpose, a research design is presented that will investigate a group of first-year students from two private Chilean universities belonging to engineering careers.

Education system in Chile

The education in Chile underwent a resounding change in the 80s with the new constitution created during the military dictatorship. The role of the state would now become subsidiary and the development of privatization was encouraged. As mentioned by Villalobos & Quaresma (2015), the way in which resources were allocated was changed and the labor status of teachers was dismantled and diminished, responding to a market logic that favored the deterioration of public education. Nowadays there are three types of school administration: public, private subsidized and private paid, where public education, in general terms, is the one that presents the greatest social, economic and learning problems.

For example, the latest results of the PDT (Transition test that allows the selection of students for university), show that only two public schools managed to enter the top 100 of the best average scores. Out of a total of almost 11,300 educational centres in the country. According to the last official report of the Department of Educational Evaluation, Measurement and Registration (DEMRE, 2022), in the mathematics PDT, students from private paid schools obtained an average score of 601 points (scale 150 - 850), private subsidized schools 496 points and public schools just 471 points. We can easily recognize the huge social gap that has occurred due to an educational model that responds to accountability systems and reinforces inequality, segregation and segmentation (Villalobos & Quaresma, 2015).

Mathematics in first-year University

There is a belief in Chile that accessing university is the best way to achieve social mobility (López et al., 2018), which has resulted in an increase in enrolment in higher education compared to years ago. Therefore, today we

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have an enormous number of students who finish school with deficits in mathematical content and also wish to study in the university as confirmed by the PDT scores or the research of Peters et al. (2021). Here more than a thousand first-year students from a private university were diagnosed in school math concepts such as: fractions, percentages, variables, equations and data analysis, obtaining alarming results. It should be noted that most of the diagnosed students came from public or subsidized private schools.

All first-year students are at the beginning of a transition process known as Klein's first discontinuity, which has been studied from multiple didactic perspectives (Gueudet et al., 2017). Some studies point out that cognitive prerequisites are one of the most important determinants of study success in a first-year of university (Kosiol et al., 2019). Luk (2005), from his part confirms that school mathematics and college mathematics require different mathematical knowledge and skills. However, as Gueudet et al. (2017), point out, there is a large number of studies that analyse the initial and final states of first-year university students, but very few that go further into the processes of change in mathematical thinking. For all of the above reasons, it was decided to design an investigation to understand and explain the difficulties that these students have with the mathematical contents of the school.

It has been decided to design a research that allows us understand and explain the difficulties that this student have with mathematical contents of the school, but necessary to understand the contents of a first-year of university. The following section describes the didactic framework chosen to support the research, called Grundvorstellungen (GV or GVs).

Grundvorstellungen

A GV in the words of vom Hofe (1995), would be the relationship between mathematical content and the individual phenomenon of concept formation. Which allows characterizing mathematical concepts or procedures with their possible interpretations in real life or mathematical situations. Vom Hofe & Blum (2016), point out that we can consider three aspects that allow us to distinguish the different treatments that GVs have had: the constitution of meaning of a mathematical concept by linking it back to a familiar knowledge or experiences. The generation of a corresponding mental representation of that concept and the ability to apply a concept to real-life and mathematical situations.

They also describe three aspects of GVs that allow us to position and represent different but related methods of work. The first aspect is the normative, which operates through educational guidelines. The second aspect is the descriptive, which allows characterizing students' mental representations,

through information about individual images and explanatory models for each student. Finally, we have the constructive aspect, which refers to the construction of GVs through the treatment of mathematical concepts in a teaching situation.

We can identify two types of GVs primary and secondary. As vom Hofe & Blum (2016), point out, primary GVs focus on concrete actions with real objects and have a representational character. Secondary GVs on the other hand, focus on operations with symbolic objects and have a symbolic character. This theoretical framework is a fundamental part for the development of the research and therefore, the design explained below is based on the didactic guides provided by it.

Research design and methodology

This study is inserted in a qualitative paradigm with quantitative support, focusing on the complex cognitive reality of students in their first-year of university. The study group corresponds to students from engineering schools belonging to two private universities in Chile, the second one integrated by students with better results in the mathematics PDT than the first one and a higher economic-social stratum. The engineering school in both universities was chosen because of its proximity to mathematics and the problems they usually present in mathematics subjects (Thomas et al., 2015).

A first step, which is already underway, is the realization of a normative analysis using GVs that will allow a refinement of the test used in Peters et al. (2021). In addition, a new questionnaire will be created that considers equations and functions as main content.

Stage 1: Quantitative test application with the contents of fractions, percentages, variables, equations and functions. Results analysis and select contents with greater difficulty. ***Objective 1***: To know the level of mathematical knowledge of students at their entry to university. ***Stage 2***: Selection of students through inclusion and exclusion criteria. Application of in-depth interviews. ***Objective 2***: To know the meanings that students give to mathematical contents. ***Stage 3***: Descriptive analysis through GVs and discourse analysis through the categorization of free nodes with NVivo software. ***Objective 3***: To characterize students' mathematical thinking in the selected contents and to understand the problems that students have in the mathematical contents.

Final comments

The aim of this research is to learn about a reality that has been rarely explored by the scientific community, allowing the development of new

knowledge about the mathematics thinking and problems of first-year university students and to provide guidelines on how they can be addressed to smooth their transition process. Therefore, this study could be a contribution for the remedial programs that are currently being carried out in different universities that integrate mathematics as one of the subjects where "leveling" is desired. So that these institutions can rethink their curricular and guidelines for the first-year of university. It is also a great opportunity to expand and use GVs as a theoretical model to a Spanish-speaking country.

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