Residual-Based Tests for Fractional Cointegration: A Monte Carlo Study[#]

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Abstract:

This paper reports on an extensive Monte Carlo study of seven residual-based tests of the hypothesis of no cointegration. Critical values and the power of the tests under the alternative of fractional cointegration are simulated and compared.

It turns out that the Phillips-Perron t-test when applied to regression residuals is more powerful than Geweke-Porter-Hudak tests and the Augmented Dickey-Fuller test. Only the Modified Rescaled Range test is more powerful than the Phillips-Perron test in a few situations. Moreover in large samples, the power of the Phillips-Perron test increases if a time trend is included in the cointegrating regression.

Keywords: Fractional cointegration; Monte Carlo experiment; Geweke-Porter-Hudak test; Modified rescaled range test; Phillips-Perron test; Augmented Dickey-Fuller test.

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1. Introduction

Fractional cointegration has become an important and relevant topic in time series analysis in recent years. Cheung & Lai (1993) examine a model of purchasing power parity, Baillie & Bollerslev (1994) investigate exchange rates, Booth & Tse (1995) interest rate futures and Dittmann (1998) stock prices. All of them find evidence for fractional cointegration in their data.

Two integrated time series are called fractionally cointegrated, if there is a linear combination (possibly including an intercept or a time trend) that is fractionally integrated. An important issue in this class of models is testing the hypothesis of no cointegration with a test that is powerful against fractionally cointegrated alternatives.

As in cointegration analysis in general, such tests can be constructed in two different ways. One possibility is to specify and estimate a full parametric model, followed by an appropriate test for fractional cointegration. This approach is pursued by Baillie & Bollerslev (1994) and Dueker & Startz (1997). The other way is to estimate the potential cointegration equation by an OLS regression and to test the residuals for a unit root with a semiparametric test, as in Cheung & Lai (1993) or Booth & Tse (1995). When conducting these semiparametric residual-based tests, one only needs to estimate those parameters that determine the long run behavior of the system. This property is especially appealing when financial time series are considered, since these often have complicated short run characteristics. Therefore, only residual-based tests are considered in this paper.

Classical residual based tests are the Phillips-Perron test and the Augmented Dickey-Fuller test when applied to regression residuals (see Phillips & Ouliaris (1990)). In a Monte Carlo study, Diebold & Rudebusch (1991) showed that the power of Dickey-Fuller type unit root tests against fractionally integrated alternatives is quite low. Presumably, this is the reason why these tests were not used as tests for fractional cointegration in the literature.

Another residual-based test for fractional cointegration is the modified rescaled range test as described by Lo (1991) when applied to the first differences of regression residuals. To the best of the author's knowledge, this test has not been used in fractional cointegration analysis either, though it seems to be a promising candidate.

Cheung & Lai (1993) as well as Booth & Tse (1995) employ t-tests based on Geweke & Porter-Hudak (1983) estimates of the long memory parameter of the regression residuals. However, there are several possibilities of constructing such a test. Cheung &

Lai (1993) estimate the long memory parameter of the first differences of the regression residuals and decide by a t-test whether it is zero. On the other hand, Booth and Tse (1995) apparently estimate the long memory parameter from the residuals themselves and test whether this is equal to one. Hurvich & Ray's (1995) and Velasco's (1997) research suggests that the last test may be improved by tapering the residuals' periodogram before estimating the long memory parameter.

The aim of this study is to determine that residual-based test of the hypothesis of no cointegration which is most powerful against fractionally cointegrated alternatives. This is done by Monte Carlo experiments, since there is no theory at hand that can give an answer to this problem.[1]

Since financial time series often contain a deterministic linear trend, we consider cointegrating regressions both with and without an additional time trend (which we call unrestricted and restricted regression, respectively). We also investigate the case that no time trend is included in the cointegrating regression while the individual series have such a trend.

The paper is organized as follows. Section 2 introduces the concept of fractional cointegration and seven residual based tests which are to be compared. Section 3 reports Monte Carlo results for restricted estimation in the absence of deterministic trends, and Section 4 gives an account of the corresponding results for unrestricted estimation. Section 5 discusses the question whether the (known but possibly wrong) asymptotic variance of the GPH estimator should be used when computing the corresponding tests. In Section 6, the consequences of performing a restricted estimation when the individual series contain a time trend are considered. Section 8 summarizes the most important results. The Appendix contains a description of the conducted Monte Carlo experiments and their results.

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1 Krämer & Marmol (1998) derive divergence rates of the Phillips-Perron test and the Augmented Dickey-Fuller test under the alternative of fractional cointegration. Unfortunately, similar results are not available for Geweke-Porter-Hudak tests or the modified rescaled range test. Besides, divergence rates are only of limited use if finite samples are considered.

2. Fractional Cointegration and Seven Residual Based Tests

Let x_t and y_t be two I(1) time series, i.e. Δx_t and Δy_t are stationary and have a finite positive spectral density at frequency zero. We do not assume that the mean of Δx_t or Δy_t is zero, so both processes x_t and y_t may exhibit a linear time trend. We call $\{x_t, y_t\}$ fractionally cointegrated, if and only if there exists a cointegrating equation

$$x_t = \alpha + \beta \cdot y_t + \gamma \cdot t + u_t \tag{1}$$

where u_t is I(d) with $0 < d < 1$, so that the spectral density of u_t is unbounded at frequency zero and behaves like λ^{-2d} as $\lambda \to 0$. If u_t is I(0), the system is called classically cointegrated.

In the remainder of this section, seven residual-based tests for the hypothesis H_0: "x_t and y_t are not cointegrated, i.e. u_t is I(1)" versus H_1: "x_t and y_t are cointegrated, i.e. u_t is I(d) with $d < 1$" are presented. These tests build upon the OLS residuals of the regression

$$x_t = \alpha + \beta \cdot y_t + \gamma \cdot t. \tag{2}$$

(2) is called *unrestricted* regression. If the term $\gamma \cdot t$ is omitted, it is called *restricted* regression. Let $\hat{u}_t$ denote the residuals of either regression.

Geweke Porter-Hudak Tests (GPH)

One possibility to construct a test for fractional cointegration is to estimate the long memory parameter d* of $\hat{u}_t$ and to test for "d* = 1". It is important to distinguish between d and d*. d is the long memory parameter of the true residuals u_t while d* is the long memory parameter of the OLS regression residuals $\hat{u}_t$. Since the OLS regression method tends to reduce too much of the residual's variance, the regression residuals are likely to be biased towards stationarity. So, one might expect that d* < d.

Geweke & Porter-Hudak (1983) proposed to estimate d, the long memory parameter of u_t, by regressing

$$\ln(I(\lambda_k)) = \hat{c} - 2\hat{d}\ln(2\sin(\lambda_k/2)), \tag{3}$$

where $\lambda_k = 2\pi k/T$ and $I(\lambda_k)$ is the periodogram of u_t at frequency λ_k. T is the sample size and k runs from 1 to n, where $n = T^{\mu}$ and μ is chosen – usually from [0.5, 0.6]. Robinson (1995) and Velasco (1997) show that the t-test-statistic $(\hat{d} - d) \div \widehat{\sigma}_{\hat{d}}$ is asymptotically normal if $d < ¾$, u_t is Gaussian and under some further asymptotic restrictions on the range $\{\ell, \ell + 1, ..., n\}$ over which regression (3) is carried out. Moreover, $\hat{d}$ is shown to be consistent if $d < 1$.

Velasco (1997) also presents an estimator for d that is asymptotically normal for $d \in [0.5, 1.5)$ under similar assumptions. This estimator $\widehat{d_T}$ can be obtained by using the cosine bell tapered periodogram $I^T(\lambda_k)$ instead of $I(\lambda_k)$ in the periodogram regression (3) and regressing over only every third frequency, i.e. $k = 1, 4, 7, ..., n$. In order to obtain $I^T(\lambda_k)$ (up to a factor that is irrelevant for our purpose), one simply uses the tapered series $h_t \cdot u_t$, where $h_t = ½(1 - \cos(2\pi t/T))$, instead of u_t when calculating the periodogram.

These theoretical results hold, if u_t can be observed, what we assume not to be the case. Instead, we are concerned with the estimated $\hat{u}_t$ from regression (2) and it is not clear whether any of these statements is still true. Nevertheless, these results may serve as valuable guidelines for the construction of possibly powerful tests. In this paper, we will examine the following four GPH tests:

- **s-GPH Test:** Under the null hypothesis of no cointegration u_t is I(1), so the obvious way is to estimate $\widehat{d^*}$ from regression (3) with the periodogram of the residuals $\hat{u}_t$ and to use the t-Test $(\widehat{d^*} - 1) \div \hat{\sigma}(\widehat{d^*})$, which we call *standard Geweke Porter-Hudak test*. Unfortunately, we have no theoretical justification to believe that this test statistic converges.
- **d-GPH Test:** Since the t-statistic for $\hat{d}$ is asymptotically normal if $d < ¾$ and since Δu_t is I(0) under the null hypothesis, it might be fruitful to estimate $\widehat{d_\Delta^*}$, the long memory parameter of $\Delta\hat{u}_t$, from regression (3) with the periodogram of the differenced residuals $\Delta\hat{u}_t$. The appropriate t-test $\widehat{d_\Delta^*} \div \hat{\sigma}(\widehat{d_\Delta^*})$ shall be called *differenced Geweke Porter-Hudak test*. This test was first discussed by Cheung & Lai (1993). Hurvich & Ray (1995) demonstrate that the GPH estimator $\hat{d}$ can differ considerably from $\widehat{d_\Delta} + 1$, so we can expect d-GPH and s-GPH to have different properties.
- **t-GPH Test:** Another way to obtain an asymptotically normal estimate for d is to use the cosine bell tapered periodogram. We calculate $\widehat{d_T^*}$ from the regression residuals $\hat{u}_t$ as described above and employ the t-Test $(\widehat{d_T^*} - 1) \div \hat{\sigma}(\widehat{d_T^*})$, which we call *tapered Geweke Porter-Hudak test*.

- **t*-GPH Test:** As the periodogram regression (3) for the t-GPH test runs over only one third of the frequencies $\lambda_1, \lambda_2, \ldots, \lambda_n$, the test probably gains some power, if the regression is carried out over all of these frequencies. We call the thus resulting test *full tapered Geweke Porter-Hudak test*.

Modified Rescaled Range Test (MRR)

Lo (1991) developed a test of the hypothesis of no long range dependence (i.e. d = 0) that is robust to short-range dependence. Let z_t be the time series that is to be tested for H_0: "z_t is I(0)". Then the modified rescaled range test statistic is given by:

$$\frac{1}{\left(\sqrt{T}\,\hat{\sigma}_{ps}(q)\right)}\left[\max_{1 \le k \le T}\sum_{j=1}^{k}(z_j - \bar{z}_T) - \min_{1 \le k \le T}\sum_{j=1}^{k}(z_j - \bar{z}_T)\right] \tag{4}$$

where $\hat{\sigma}_{ps}^{\,2}(q)$ is a consistent estimator of the partial sum's variance:

$$\hat{\sigma}_{ps}^{\,2}(q) = \frac{1}{T}\sum_{j=1}^{T}(z_j - \bar{z}_T)^2 + \frac{2}{T}\sum_{j=1}^{q-1}\left(1 - \frac{j}{q}\right)\sum_{i=j+1}^{T}(z_i - \bar{z}_T)(z_{i-j} - \bar{z}_T) \tag{5}$$

We choose the lag truncation parameter q by Andrews' (1991) data-dependent formula, where q is the greatest integer less than or equal to k_T with

$$k_T = \left(\frac{3T}{2}\right)^{\frac{1}{3}}\left(\frac{2\,\hat{r}}{1 - \hat{r}^2}\right)^{\frac{2}{3}}. \tag{6}$$

$\hat{r}$ is the estimated first-order autocorrelation of z_t.

Cheung (1991) found in a related Monte Carlo study that the modified rescaled range test has more power against fractional alternatives than the Geweke Porter-Hudak test when d < 0.25. Since Δu_t is I(0) under the null hypothesis of no cointegration, this test, if calculated for $z_t = \Delta\hat{u}_t$, might therefore be more powerful than the d-GPH Test.

We call the test given by (4), (5) and (6) for $z_t = \Delta\hat{u}_t$ the Modified Rescaled Range test (MRR).

Phillips-Perron-t-Test (PP)

A classical test of the hypothesis of no cointegration is the Phillips-Perron t-test (see e.g. Hamilton (1994)). This test is a modified version of the OLS t-test of the null hypothesis "$\rho = 1$" in the regression

$$\hat{u}_t = \rho\,\hat{u}_{t-1} + e_t\,. \tag{7}$$

$$Z_t = \sqrt{\frac{\hat{c}}{\hat{\sigma}_{ps}^{\,2}(q)}}\frac{\hat{\rho}-1}{\hat{\sigma}(\hat{\rho})} - \frac{1}{2}\left(\hat{\sigma}_{ps}^{\,2}(q) - \hat{c}\right)\frac{1}{\hat{\sigma}_{ps}(q)}\left[\frac{(T-1)\,\hat{\sigma}(\hat{\rho})}{s}\right] \quad (8)$$

where $\hat{c} = \frac{1}{T-1}\sum_{t=2}^{T}\hat{e}_t^{\,2}$, $\quad \hat{\sigma}(\hat{\rho}) = s^2 \div \sum_{t=2}^{T}\hat{u}_{t-1}^{2}$, and $s^2 = \frac{1}{T-2}\sum_{t=2}^{T}\hat{e}_t^{\,2}$

$\hat{\sigma}_{ps}^{\,2}(q)$ is given by (5) and (6) with $z_t = \hat{e}_t$. Again, we choose the lag truncation parameter by Andrews' (1991) formula, as proposed by Cheung & Lai (1997).

Augmented Dickey-Fuller Test (ADF):

The augmented Dickey-Fuller t-test statistic is the OLS t-test of the null hypothesis "$\rho = 1$" in the regression

$$\hat{u}_t = \rho\,\hat{u}_{t-1} + \zeta_1\,\Delta\hat{u}_{t-1} + \zeta_2\,\Delta\hat{u}_{t-2} + \ldots + \zeta_{p-1}\,\Delta\hat{u}_{t-p+1} + e_t\,. \quad (9)$$

In view of his simulation results, Hall (1994) recommends not to fix the dimension p of model (9) but to estimate p from the data. In this paper, we use the MPE (Mean square Prediction Error) criterion as described by Fuller (1996), i.e. we choose the p that minimizes[2]

$$MPE(p) = \frac{T}{T-p}\frac{1}{T-2p}\sum_{t=2}^{T}\hat{e}_t^{\,2}. \quad (8)$$

The MPE criterion is closely related to the Akaike Information Criterion (AIC). The advantage of MPE is that no likelihood specification is needed and that its calculation is simple.

3. Restricted Estimation in the Absence of Deterministic Trends

In this section, we assume that $E(\Delta x_t) = 0 = E(\Delta y_t)$, so that x_t and y_t have no deterministic trend and γ in (1) is zero. Further, all tests under consideration are calculated from the residuals $\hat{u}_t$ of the restricted regression

$$x_t = \alpha + \beta \cdot y_t. \quad (2')$$

Table 3 to Table 6 in Appendix B contain simulated critical values for s-GPH, d-GPH, t-GPH und t*-GPH under the null hypothesis of no cointegration for sample sizes 100, 250, 500 and 1000, respectively. We consider two ranges of the periodogram regression {1, 2, ..., n}: $n = T^{0.5}$ and $n = T^{0.6}$.

2 More specifically, we choose the smallest p for which MPE (p) < MPE (p+1).

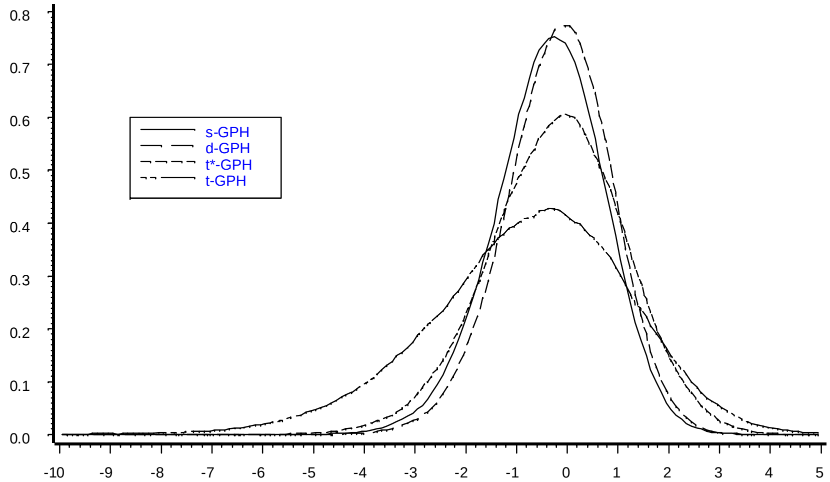

Figure 1: Estimated density of s-GPH, d-GPH, t-GPH and t*-GPH under the null hypothesis of no cointegration with T=1000 and n=T**0.6

Figure 1 shows the four empirical distributions for T = 1000 and n = $T^{0.6}$ computed from 100,000 simulations. Compared to the standard normal distribution, which is the limiting distribution of d-GPH and t-GPH when calculated from u_t instead of $\hat{u}_t$, these distributions are biased and skewed to the left. This finding complies with the idea of the "bias towards stationarity" of the GPH estimator due to the preceding OLS regression.

Table 1 in Appendix B contains the corresponding critical values for the MRR test. Figure 2 displays the empirical distribution of MRR with T = 1000 together with the asymptotic distribution of MRR when calculated from u_t (which is the range of a Brownian Bridge) as given by Lo (1991). It illustrates that MRR's distribution is biased to the left, too. It is remarkable that the empirical variance of MRR *increases* slightly with the sample size.

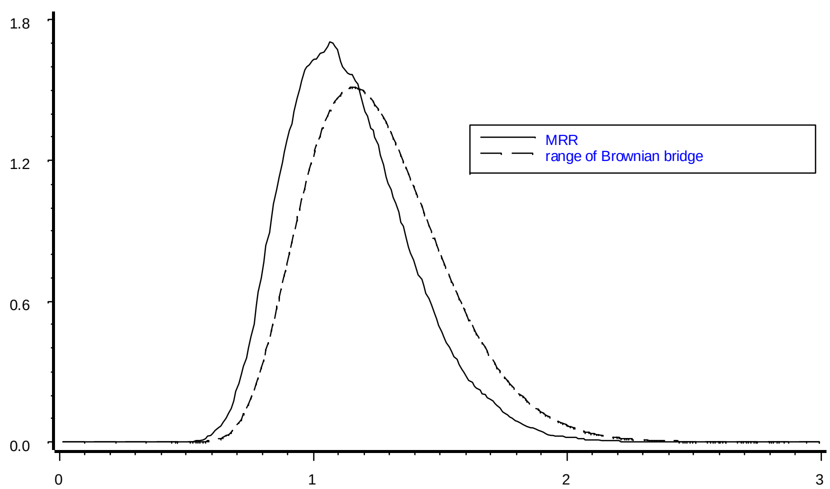

Figure 2: Estimated density of MRR under the null hypothesis of no cointegration with T=1000 and densitiy function of the range of a Brownian bridge

Table 2 contains the corresponding critical values for PP and ADF. These empirical distributions must be simulated anew, because the tables in Phillips & Ouliaris (1990) are only valid for fixed lag truncation parameters (or model dimensions, respectively). However, PP and ADF as defined in Section 2 contain data-dependent parameter selection methods, so that the critical values might be different from the usual ones. Nevertheless, our critical values do not differ significantly from those given by Phillips & Ouliaris (1990).

Tables 7 to 10 in Appendix B display the simulated power of the eleven tests under the alternative of fractional cointegration. We consider ten different long memory parameters d = 0, 0.1, 0.2, ..., 0.9 and four sample sizes 100, 250, 500 and 1000. The following findings can be reported:

1. For a given n, t*-GPH dominates t-GPH considerably, i.e. the power of the full tapered GPH test is always larger than the power of the tapered GPH test. This is not surprising, as the periodogram regression of t*-GPH runs across thrice as many points as the regression of t-GPH.
2. For all GPH tests, the test with n = $T^{0.6}$ dominates the one with n = $T^{0.5}$. This is not surprising either, since residuals of the data generating process (henceforth DGP) are ARFIMA (0, d, 0). Consequently, the spectral density of the DGP is undisturbed by short range influences, so that it would be best to run the periodogram regression over all frequencies (i.e. n = T/2).
3. Except for small samples (T ≤ 500) and large d (d ≥ 0.8), s-GPH dominates t*-GPH. This can be explained by the fact that the influence of high order autocorrelations is reduced by tapering in t*-GPH. On the one hand, this is important to ensure convergence, because these high order autocorrelations are calculated from only few

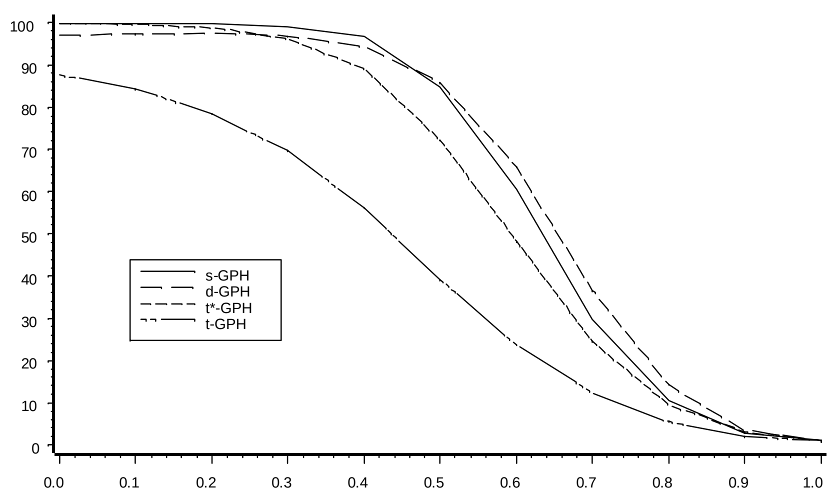

Figure 3: Power of s-GPH, d-GPH, t*-GPH and t-GPH against fractional cointegrated alternatives with T = 1000, n = T and significance level 1%

observations. On the other hand, they contain much information about the long memory of the series. So this information is not used completely by t*-GPH.t*-GPH is slightly better than s-GPH for T = 100 and d = 0.9.

4. For all sample sizes, the best GPH test is d-GPH if d > 0.5. For d < 0.5 (0.4), s-GPH is better than d-GPH for sample sizes T > 100 (T = 100).

   Figure 3 shows the power of the four GPH tests with significance level 1%, T = 1000 and $n = T^{0.6}$. It illustrates results 1, 3 and 4.

5. PP dominates ADF and all GPH tests. This result is quite surprising, since PP has originally not been designed as a test against fractionally cointegrated alternatives. Reasons for this might be the relative simplicity of PP and the fact that the periodogram regression is not very robust.

6. MRR is more powerful than any GPH test if $d \geq 0.7$ and more powerful than PP in large samples ($T \geq 500$) if d = 0.9.

7. The power of MRR increases as d decreases only on [0.5, 1]. For $d \leq 0.4$ and for small sample sizes (T < 500), MRR's power declines considerably with decreasing d. MRR performs poorly if T = 100 and d < 0.5.

   Figure 4 displays the power of d-GPH, MRR, PP and ADF with significance level 1% for T = 1000 dependent on d. It illustrates findings 5 and 7.

In contrast to our results, Cheung & Lai (1993) found that d-GPH is more powerful than ADF – especially if d lies between 0.35 and 0.65. We suspect that the reason is that Cheung & Lai (1993) fixed the dimension p of the ADF model (to p = 4) whereas p is determined by a data dependent model selection criterium in the present study. Hall (1994) showed that the power of ADF can be considerably increased if data-based model selection criteria are employed. Another reason for the different results of the

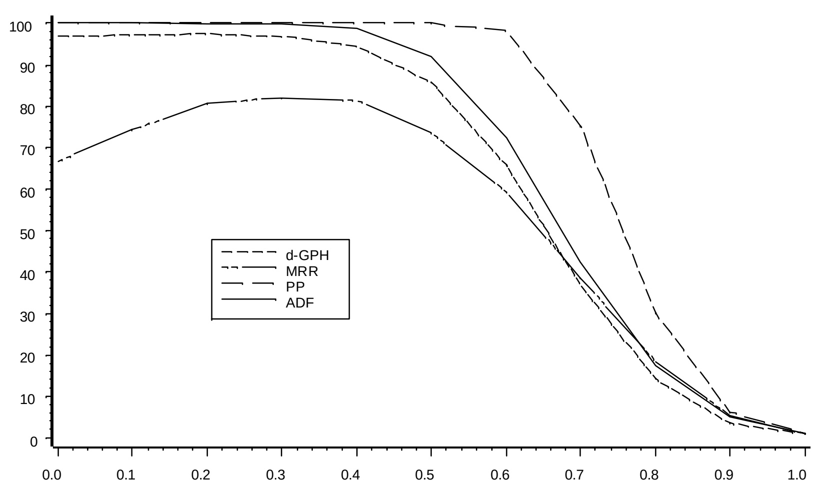

Figure 4: Power of d-GPH, MRR, PP and ADF against fractional cointegrated alternatives with T = 1000, n = T and significance level 1%

two studies might be that the cointegrating regression in Cheung & Lai (1993) does not contain a constant. Further, Cheung & Lai (1993) use the asymptotic variance of the periodogram regression residuals when calculating d-GPH. This alteration of d-GPH is discussed in Section 5 and can result in further efficiency gains.

4. Unrestricted Estimation

In this section, we consider the unrestricted estimation (2) so that any deterministic linear trend in x_t or y_t is removed automatically. Simulation results for critical values and the power of the eleven tests, when applied to residuals of an unrestricted regression, can be found in Appendix C, which is organized according to Appendix B. In what follows, only those results are reported that are different from the findings of Section 3.

ad 3. s-GPH dominates t*-GPH (without exceptions for small samples and large d).

ad 4. d-GPH is more powerful than s-GPH, if $d > 0$ and $T = 100$. Moreover, d-GPH is more powerful than s-GPH, if $d \geq 0.5$ and $T \geq 250$, as before.

ad 6. PP dominates MRR for all sample sizes and all d.

8. For large sample sizes ($T \geq 500$), the power of PP and ADF when applied to residuals from the unrestricted regression is *higher* than the power of the two tests in the restricted estimation case. So even if we know that the individual series have no linear time trend and that $\gamma = 0$ in (1) consequently, it is better not to assume that γ is zero in the cointegrating regression (2) – provided that $T \geq 500$. It is surprising that one can increase the power of PP or ADF by not using all available information. Therefore, Hansen's (1992) conjecture that "excess detrending will reduce the test's power" (p.103) seems to be wrong.
9. The empirical variance of the null distribution for the GPH tests is larger if estimation is unrestricted, whereas things are the other way round for MRR, PP and ADF.

5. GPH Test with Asymptotic Residual Variance

Robinson (1995) shows that the residuals of the periodogram regression (3) have the asymptotical variance $\pi^2/6$ and that the GPH estimator's asymptotical variance is $\pi^2/24n$. This result is obtained under the assumption that the process u_t is Gaussian, though Robinson (1995) conjectures "that a limit distribution theory can be obtained under more general distributional assumptions" (p. 1052). Cheung & Lai (1993) as well as Booth & Tse (1995) use the residuals' asymptotic variance when calculating the GPH tests. We have to keep in mind however that we are concerned with regression residuals $\hat{u}_t$ in finite samples and not with the true residuals u_t. It is not clear whether using the asymptotic variance really is an improvement in this situation. Moreover, the assumption of Gaussianity seems especially questionable if financial time series are considered, which often exhibit complicated non-Gaussian patterns. For this reason, Robinson's (1995) asymptotics have not been used when calculating the GPH tests in previous sections. This section now investigates whether the d-GPH test can be improved thereby.

Table 1 in Appendix D displays the critical values for d-GPH for T=500 and $n = T^{0.6}$. Since the residuals' or the estimator's asymptotic variance can be used and the test can be applied to the restricted or the unrestricted regression, there are four cases to distinguish. Tables 2 and 3 contain the power of these four tests. Additionally, Tables 4 to 6 contain the corresponding simulation results for the small sample size T = 100.

We first observe that it makes virtually no difference which asymptotic variance is used (even though the critical values are quite different): The difference in power never exceeds 0.03%.

Compared to the corresponding tests with estimated variance, using the asymptotic variance improves the power of the test, if $d < 0.7$. In large samples (T = 500), the maximum difference is obtained for d = 0.6 when 5% critical values are considered (1.62% for restricted and 1.85% for unrestricted estimation). In small samples (T = 100), the maximum difference is more substantial (6.09% for restricted estimation for d = 0.4, 7.72% for unrestricted estimation for d = 0). For $d \geq 0.7$ however, there does not seem to be any advantage of the test with asymptotic variance over that with estimated variance. On the contrary, the test with estimated variance seems to be slightly better if d = 0.9.

One must keep in mind that the simulated processes in this study are "well behaved" in the sence that there are no short term disturbances and that errors are normally distributed. Since this is surely not true for financial time series, the d-GPH test with asymptotic variance might well perform considerably worse in practice than in this study. Further, in the region where power is low (i.e. $d \geq 0.7$), the test with asymptotic variance does not improve the power significantly. Altogether, the use of the asymptotic variance can only be recommended in small samples.

6. Restricted Estimation in the Presence of Deterministic Trends

Consider the case in which both series x_t and y_t have a linear time trend and γ in (1) is zero. Further assume that the alternative holds, so that x_t and y_t are indeed cointegrated and cointegrating regression (2) is not spurious. In this case $(1, -\beta)'$ is the cointegrating vector for the stochastic trends and simultaneously for the deterministic trends in $(x_t, y_t)'$. If the time trend $\gamma \cdot t$ is now included in the cointegrating regression (2) – which effectively detrends the two series x_t and y_t – $\hat{\beta}$ converges to β at the rate of $O_p\left(T^{1-d}\right)$, as derived by Cheung & Lai (1993). On the other hand, if the time trend $\gamma \cdot t$ is not included, i.e. if the cointegrating regression is restricted, Hassler & Marmol (1998) show that $\hat{\beta}$ coverges faster, namely at the rate of $O_p\left(T^{1.5-d}\right)$. An intuition for this is that the time trend stretches the regression points along the true regression line, so that estimation becomes easier. This suggests that the power of the tests for fractional cointegration increases if the cointegrating regression is restricted. In finite samples, this effect can be expected to be the stronger the larger the drift in x_t and y_t compared to the increments' variance is.

Therefore, I also examined three tests (PP, d-GPH and MRR) when applied to T = 500 residuals from a restricted regression if both series have a time trend. Four different drifts δ were considered: 1, 0.1, 0.01 and 0.001 (The increments' variance is 1). Hansen (1992) shows that the asymptotic distribution of PP under the null hypothesis of no cointegration (i.e. $\gamma = 0$ and $u_t \sim I(1)$) depends on whether the individual series contain a deterministic trend. Therefore, the simulated critical values given in Appendix B are no longer applicable (at least for PP). Tables 1, 2 and 3 in Appendix E display the adequate critical values for each of the four levels of drift for PP, d-GPH and MRR, respectively. These critical values vary significantly with the size of the drift for all three considered tests and are significantly different from the critical values of Appendix B for large trends ($\delta \geq 0.1$). The critical values for PP given by Hansen (1992) are significantly different from ours for two of the four considered trends. This is due to the fact that Hansen (1992) simulates critical values of the asymptotic distribution and thus does not consider the effect of different sizes of drift in finite samples. As a consequence, critical values should be simulated suitably for each data set anew.

Table 4 contains the corresponding power of the three tests under the alternative of fractional cointegration (i.e. $\gamma = 0$, $u_t \sim I(d)$, $d < 1$) for four values of d. Here the critical values from Tables 1 – 3 were employed. In comparison to the unrestricted estimation approach, d-GPH and MRR show substantial power gains. PP exhibits power gains only for large trends ($\delta = 0.1, 1$). For small trends, PP's power is larger if the cointegrating regression is unrestricted. Still PP dominates d-GPH and MRR for $d < 0.9$. For $d = 0.9$ and $\delta < 1$, MRR's power slightly exceeds the power of PP.

On the other hand, if the key assumption "$\gamma = 0$" does not hold, the power of the tests is very low. This is illustrated by Table 5 in Appendix E which shows the power of the three tests if $\gamma \neq 0$ and $u_t \sim I(0.9)$.

To sum up it can be said that restricted estimation in the presence of deterministic trends leads to power gains compared to unrestricted estimation if the trends are large and γ in (1) is known to be zero. If $\gamma \neq 0$ however, restricted estimation results in serious power losses.

7. Conclusions

This paper shows that the Phillips-Perron t-test when applied to regression residuals is clearly the best test when testing the null hypothesis of no cointegration against fractionally cointegrated alternatives. In particular it is more powerful than any of the four GPH tests in the study, including the tests used by Cheung & Lai (1993) and Booth & Tse (1995). Merely, the modified rescaled range test is more powerful than the Phillips-Perron test if sample size is large, the cointegration regression is restricted and the true long memory parameter is close to 1. This study also shows that the power of the Phillips-Perron test *increases* in large samples ($T \geq 500$) if a time trend is included in the cointegrating regression – even if there is no time trend in reality.

The d-GPH test proposed by Cheung & Lai (1993), which tests whether the long memory parameter of the regression residuals' first differences is zero, turned out to be the best test among the GPH tests. In fact, the s-GPH test, which tests whether the long memory parameter of the regression residuals is one, is slightly more powerful than d-GPH if the true long memory parameter lies in [0, 0.5), but in that region also d-GPH's power is quite high. On the other hand, d-GPH is more powerful than s-GPH if the true long memory parameter comes from (0.5, 1) where power is low in general. Velasco's

(1997) proposal to taper the residuals' periodogram before estimating the long memory parameter turned out to be useless for our purposes.

Using the asymptotic variance of periodogram regression residuals as given by Robinson (1995), increases the power of d-GPH for $d < 0.7$. However, there is some evidence that doing so might decrease d-GPH's power if d lies in the crucial region close to 1. All in all, the use of the asymptotic variance can only be recommended in small samples.

We also pointed at some pitfalls that arise if the individual series contain a time trend but no time trend is included in the cointegrating regression. We therefore recommend to include such a time trend whenever there is evidence of a drift in one of the individual series.

Appendix A: Description of the Monte-Carlo experiment

For the simulation of the critical values, 100,000 replications are conducted. For each replication two random walks of appropriate length are generated by calculating the partial sums of two streams of uncorrelated standard normal variates. The 95% confidence intervals of the 5% critical value are computed as described in Rohatgi (1984), pp. 496-500.

10,000 replications are used for the power simulations. Each time a random walk u_{1t} and a fractionally integrated series u_{2t} of length T+50 are calculated, where u_{2t} is generated as described in Hosking (1984). Then the fractionally cointegrated system is modeled by $x_t = 2u_{1t} - u_{2t}$ and $y_t = u_{2t} - u_{1t}$ and the first 50 observations are discarded. Under the null hypothesis of no cointegration the 95% confidence interval of the rejection percentage is given by 1% ± 0.2%, 5% ± 0.43% or 10% ± 0.59%, depending on the desired significance level.

For the simulations reported in Appendix E, the trend $\delta \cdot t$ is added to the random walk u_{1t} if $\gamma = 0$. If $\gamma \neq 0$, the trends $\delta_1 \cdot t$ and $\delta_2 \cdot t$ are added to x_t and y_t, respectively. All calculations were performed in SAS/IML.

Appendix B: Simulation Results for Restricted Estimation in the Absence of Deterministic Trends

Table 1: Critical values for MRR

Percentile	T=100	T=250	T=500	T=1000
1.0%	0.6584	0.6730	0.6846	0.6953
2.5%	0.7037	0.7214	0.7324	0.7435
5.0%	0.7469	0.7668	0.7788	0.7916
7.5%	0.7783	0.7989	0.8113	0.8235
10.0%	0.8028	0.8240	0.8382	0.8494
12.5%	0.8243	0.8467	0.8602	0.8720
15.0%	0.8438	0.8669	0.8802	0.8922
Mean	1.080	1.111	1.128	1.142
Variance	0.05294	0.05771	0.05964	0.06067
Skewness	0.5787	0.6356	0.6547	0.6504
Kurtosis	0.3204	0.4667	0.5278	0.5156
95% CI for 5% critical value	[0.7452, 0.7491]	[0.7645, 0.7691]	[0.7765, 0.7807]	[0.7897, 0.7937]

Table 2: Critical values for PP and ADF

	Phillips-Perron-Test				ADF-Test			
Percentile	T=100	T=250	T=500	T=1000	T=100	T=250	T=500	T=1000
1.0%	-4.027	-3.944	-3.927	-3.910	-4.108	-3.973	-3.939	-3.920
2.5%	-3.688	-3.632	-3.624	-3.604	-3.756	-3.656	-3.634	-3.608
5.0%	-3.407	-3.373	-3.362	-3.350	-3.463	-3.392	-3.372	-3.351
7.5%	-3.234	-3.206	-3.192	-3.182	-3.278	-3.221	-3.199	-3.184
10.0%	-3.099	-3.077	-3.064	-3.055	-3.140	-3.088	-3.070	-3.059
12.5%	-2.993	-2.969	-2.960	-2.946	-3.028	-2.980	-2.968	-2.951
15.0%	-2.899	-2.876	-2.869	-2.858	-2.931	-2.885	-2.879	-2.860
Mean	-2.062	-2.048	-2.044	-2.037	-2.076	-2.052	-2.047	-2.038
Variance	0.7002	0.6824	0.6785	0.6718	0.7302	0.6932	0.6844	0.6718
Skewness	0.1541	0.1866	0.2020	0.2107	0.1041	0.1637	0.1911	0.2063
Kurtosis	0.4728	0.5276	0.4931	0.4848	0.4694	0.5320	0.4959	0.4831
95% CI for 5% critical value	[-3.420, -3.395]	[-3.383, -3.361]	[-3.373, -3.351]	[-3.361, -3.338]	[-3.477, -3.449]	[-3.404, -3.381]	[-3.383, -3.360]	[-3.362, -3.339]

Table 3: Critical values for the eight GPH tests with T = 100 observations

Percentile	s-GPH ($n=T^{0.5}$)	s-GPH ($n=T^{0.6}$)	d-GPH ($n=T^{0.5}$)	d-GPH ($n=T^{0.6}$)	t*-GPH ($n=T^{0.5}$)	t*-GPH ($n=T^{0.6}$)	t-GPH ($n=T^{0.5}$)	t-GPH ($n=T^{0.6}$)
1.0%	-4.037	-3.658	-3.538	-3.238	-5.296	-4.530	-22.916	-14.544
2.5%	-3.346	-3.062	-2.864	-2.655	-4.217	-3.690	-14.306	-10.252
5.0%	-2.747	-2.568	-2.318	-2.177	-3.411	-3.020	-9.809	-7.794
7.5%	-2.399	-2.246	-1.995	-1.887	-2.936	-2.620	-7.796	-6.520
10.0%	-2.145	-2.022	-1.762	-1.673	-2.591	-2.337	-6.567	-5.633
12.5%	-1.945	-1.835	-1.563	-1.506	-2.323	-2.104	-5.694	-5.005
15.0%	-1.770	-1.678	-1.415	-1.360	-2.098	-1.907	-5.027	-4.487
Mean	-0.539	-0.495	-0.264	-0.246	-0.531	-0.412	-2.094	-1.608
Variance	1.582	1.386	1.407	1.238	2.701	2.260	63.028	15.574
Skewness	-0.591	-0.436	-0.525	-0.403	-0.733	-0.467	-34.227	-4.004
Kurtosis	1.436	0.677	1.684	0.735	2.049	0.976	3337.565	81.720
95% CI for 5% critical value	[-2.775, -2.727]	[-2.586, -2.546]	[-2.339, -2.298]	[-2.197, -2.160]	[-3.447, -3.382]	[-3.046, -2.996]	[-9.962, -9.673]	[-7.897, -7.708]

Table 4: Critical values for the eight GPH tests with T = 250 observations

Percentile	s-GPH ($n=T^{0.5}$)	s-GPH ($n=T^{0.6}$)	d-GPH ($n=T^{0.5}$)	d-GPH ($n=T^{0.6}$)	t*-GPH ($n=T^{0.5}$)	t*-GPH ($n=T^{0.6}$)	t-GPH ($n=T^{0.5}$)	t-GPH ($n=T^{0.6}$)
1.0%	-3.687	-3.326	-3.227	-2.958	-4.471	-3.965	-14.359	-8.160
2.5%	-3.047	-2.799	-2.630	-2.447	-3.650	-3.278	-10.326	-6.512
5.0%	-2.557	-2.353	-2.163	-2.050	-3.016	-2.715	-7.815	-5.299
7.5%	-2.240	-2.076	-1.872	-1.786	-2.625	-2.369	-6.492	-4.591
10.0%	-2.007	-1.875	-1.662	-1.592	-2.329	-2.120	-5.612	-4.091
12.5%	-1.824	-1.712	-1.492	-1.431	-2.095	-1.911	-4.972	-3.695
15.0%	-1.667	-1.569	-1.348	-1.296	-1.896	-1.735	-4.456	-3.349
Mean	-0.489	-0.428	-0.240	-0.213	-0.407	-0.303	-1.602	-1.055
Variance	1.391	1.243	1.239	1.135	2.253	1.984	15.003	5.858
Skewness	-0.461	-0.341	-0.396	-0.291	-0.454	-0.310	-3.023	-0.953
Kurtosis	0.804	0.384	0.809	0.402	0.998	0.488	35.220	4.912
95% CI for 5% critical value	[-2.577, -2.534]	[-2.371, -2.335]	[-2.183, -2.144]	[-2.065, -2.034]	[-3.041, -2.991]	[-2.736, -2.690]	[-7.912, -7.717]	[-5.355, -5.253]

Table 5: Critical values for the eight GPH tests with T = 500 observations

Percentile	s-GPH ($n=T^{0.5}$)	s-GPH ($n=T^{0.6}$)	d-GPH ($n=T^{0.5}$)	d-GPH ($n=T^{0.6}$)	t*-GPH ($n=T^{0.5}$)	t*-GPH ($n=T^{0.6}$)	t-GPH ($n=T^{0.5}$)	t-GPH ($n=T^{0.6}$)
1.0%	-3.435	-3.187	-3.055	-2.839	-4.089	-3.682	-8.223	-6.686
2.5%	-2.877	-2.661	-2.529	-2.383	-3.380	-3.082	-6.547	-5.500
5.0%	-2.430	-2.261	-2.098	-1.984	-2.807	-2.580	-5.307	-4.582
7.5%	-2.143	-1.998	-1.821	-1.733	-2.440	-2.263	-4.579	-3.995
10.0%	-1.930	-1.802	-1.625	-1.548	-2.176	-2.025	-4.066	-3.575
12.5%	-1.757	-1.645	-1.460	-1.395	-1.959	-1.830	-3.656	-3.243
15.0%	-1.612	-1.508	-1.323	-1.266	-1.782	-1.657	-3.320	-2.948
Mean	-0.452	-0.390	-0.228	-0.193	-0.333	-0.248	-1.060	-0.860
Variance	1.292	1.185	1.166	1.091	2.045	1.871	5.750	4.404
Skewness	-0.374	-0.292	-0.334	-0.257	-0.353	-0.233	-0.885	-0.555
Kurtosis	0.467	0.281	0.509	0.231	0.569	0.263	2.908	1.018
95% CI for 5% critical value	[-2.447, -2.412]	[-2.279, -2.246]	[-2.114, -2.082]	[-2.003, -1.967]	[-2.827, -2.781]	[-2.599, -2.562]	[-5.358, -5.256]	[-4.616, -4.550]

Table 6: Critical values for the eight GPH tests with T = 1000 observations

Percentile	s-GPH ($n=T^{0.5}$)	s-GPH ($n=T^{0.6}$)	d-GPH ($n=T^{0.5}$)	d-GPH ($n=T^{0.6}$)	t*-GPH ($n=T^{0.5}$)	t*-GPH ($n=T^{0.6}$)	t-GPH ($n=T^{0.5}$)	t-GPH ($n=T^{0.6}$)
1.0%	-3.281	-3.065	-2.918	-2.787	-3.879	-3.591	-7.092	-6.103
2.5%	-2.766	-2.566	-2.406	-2.314	-3.218	-2.973	-5.807	-5.082
5.0%	-2.334	-2.175	-2.013	-1.939	-2.658	-2.488	-4.760	-4.250
7.5%	-2.061	-1.927	-1.742	-1.701	-2.322	-2.171	-4.144	-3.734
10.0%	-1.859	-1.739	-1.563	-1.515	-2.068	-1.941	-3.684	-3.353
12.5%	-1.690	-1.581	-1.411	-1.364	-1.863	-1.750	-3.331	-3.043
15.0%	-1.549	-1.451	-1.276	-1.237	-1.694	-1.586	-3.041	-2.781
Mean	-0.418	-0.353	-0.205	-0.172	-0.287	-0.212	-0.932	-0.761
Variance	1.221	1.141	1.109	1.074	1.928	1.803	4.689	3.972
Skewness	-0.341	-0.244	-0.287	-0.234	-0.288	-0.202	-0.682	-0.437
Kurtosis	0.366	0.214	0.347	0.225	0.412	0.206	1.676	0.611
95% CI for 5% critical value	[-2.350, -2.318]	[-2.190, -2.161]	[-2.029, -1.997]	[-1.955, -1.924]	[-2.682, -2.635]	[-2.508, -2.469]	[-4.801, -4.721]	[-4.285, -4.218]

Table 7: Power of the nine tests for fractional cointegration with T = 100 observations

(all entries in percent)

		d									
Size	Test	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1%	s-GPH (.5)	20.20	19.99	18.72	16.19	13.91	8.69	5.73	3.19	2.14	1.35
	s-GPH (.6)	54.79	50.92	44.33	36.93	29.83	17.77	10.80	5.49	2.68	1.40
	d-GPH (.5)	21.27	20.07	18.87	17.46	14.63	9.87	6.52	4.33	2.42	1.52
	d-GPH (.6)	50.04	46.87	43.59	37.28	30.43	20.14	13.03	6.92	3.34	1.68
	t*GPH (.5)	17.81	16.43	15.88	12.65	11.19	6.88	5.08	3.13	1.77	1.19
	t*GPH (.6)	44.02	40.12	34.66	28.31	22.28	14.56	8.97	5.53	2.66	1.65
	t-GPH (.5)	6.50	5.76	5.29	4.98	4.43	3.27	2.83	2.23	1.57	1.21
	t-GPH (.6)	11.70	11.12	10.15	9.46	7.51	5.59	4.22	2.67	1.82	1.37
	MRR	1.81	3.34	5.11	7.51	10.20	11.15	10.37	8.24	4.52	2.15
	PP	100.00	100.00	100.00	99.96	98.74	85.13	56.64	25.30	8.50	2.33
	ADF	97.27	95.55	92.31	84.19	73.35	53.93	35.41	17.14	6.01	1.96
5%	s-GPH (.5)	51.86	50.35	47.29	43.64	37.48	26.87	19.37	13.50	8.69	6.19
	s-GPH (.6)	85.24	82.36	77.26	69.38	60.86	43.79	30.39	19.15	11.01	6.89
	d-GPH (.5)	48.53	47.53	46.11	42.91	37.86	29.70	22.78	15.95	10.83	6.86
	d-GPH (.6)	79.10	77.05	73.92	68.37	61.53	48.01	34.51	23.53	13.52	7.25
	t*GPH (.5)	47.14	44.56	42.32	37.37	33.41	24.62	18.71	13.42	8.59	6.68
	t*GPH (.6)	77.63	74.01	68.91	61.34	52.95	39.29	28.06	18.86	11.31	7.20
	t-GPH (.5)	26.22	25.08	23.90	22.28	20.35	15.55	13.28	9.44	7.68	6.52
	t-GPH (.6)	40.08	38.45	36.49	32.92	29.31	22.16	17.36	12.37	9.05	6.87
	MRR	10.58	15.75	20.46	24.56	29.52	30.37	28.82	24.38	16.14	8.88
	PP	100.00	100.00	100.00	100.00	99.88	95.77	79.02	49.64	23.86	9.72
	ADF	99.17	98.35	96.80	93.13	86.91	71.93	55.88	36.78	19.27	8.65
10%	s-GPH (.5)	70.81	69.24	66.13	60.58	55.24	42.52	32.53	24.05	16.31	12.32
	s-GPH (.6)	93.84	92.32	89.12	83.90	76.76	61.53	45.91	32.28	20.49	13.36
	d-GPH (.5)	64.51	64.86	62.94	59.47	54.57	45.39	36.76	27.35	19.08	13.15
	d-GPH (.6)	88.93	87.32	85.67	81.82	76.29	64.77	50.36	37.39	23.58	14.09
	t*GPH (.5)	65.67	63.83	60.96	55.11	50.56	39.35	31.85	23.90	16.79	13.24
	t*GPH (.6)	89.41	86.79	83.35	77.01	70.08	55.71	43.09	31.13	20.60	13.77
	t-GPH (.5)	44.22	43.29	41.68	39.22	35.86	28.55	24.98	18.95	15.15	12.55
	t-GPH (.6)	59.85	58.24	55.25	50.99	46.97	37.87	30.83	23.21	16.56	12.97
	MRR	22.61	29.42	35.69	40.80	45.52	46.11	43.89	38.06	27.91	16.46
	PP	100.00	100.00	100.00	100.00	99.97	98.08	87.12	62.53	35.98	17.27
	ADF	99.55	99.15	98.27	95.90	92.09	80.49	66.75	48.88	30.42	15.64

Table 8: Power of the nine tests for fractional cointegration with T = 250 observations

(all entries in percent)

		d									
Size	Test	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1%	s-GPH (.5)	55.52	53.01	48.15	40.48	31.55	19.46	11.36	5.92	3.23	1.71
	s-GPH (.6)	97.70	96.33	93.56	87.21	75.18	54.26	32.34	14.60	6.17	2.03
	d-GPH (.5)	40.95	41.05	40.26	36.60	30.21	21.24	13.70	7.20	4.02	2.01
	d-GPH (.6)	88.10	87.42	86.29	80.74	72.44	56.62	36.36	18.81	8.07	2.39
	t*GPH (.5)	46.53	44.52	38.85	32.51	24.46	16.28	10.17	5.72	3.25	1.59
	t*GPH (.6)	92.26	89.43	83.28	73.99	60.44	40.80	24.90	12.55	5.47	2.12
	t-GPH (.5)	11.86	12.43	11.34	9.82	8.16	5.89	4.31	2.73	1.89	1.38
	t-GPH (.6)	49.71	47.58	41.81	35.06	27.60	17.73	11.77	6.01	3.21	1.78
	MRR	16.23	22.45	29.35	33.84	36.17	33.96	27.50	20.55	11.05	3.53
	PP	100.00	100.00	100.00	100.00	100.00	99.74	91.11	56.95	20.78	4.13
	ADF	99.98	99.94	99.61	98.20	92.89	77.55	54.38	30.24	13.77	3.67
5%	s-GPH (.5)	85.78	84.27	80.03	73.84	63.54	47.89	33.37	20.37	12.26	7.76
	s-GPH (.6)	99.78	99.64	99.20	97.83	94.24	82.92	62.69	38.73	20.75	9.43
	d-GPH (.5)	68.09	69.59	69.27	66.86	60.61	49.75	36.98	24.22	15.14	8.25
	d-GPH (.6)	96.56	96.41	96.65	94.48	91.95	83.09	67.20	44.86	24.22	10.45
	t*GPH (.5)	78.19	76.91	71.74	64.82	55.05	42.42	30.39	20.04	12.28	7.79
	t*GPH (.6)	99.10	98.44	96.99	93.58	87.20	73.32	53.72	34.74	18.94	9.64
	t-GPH (.5)	39.66	39.09	37.79	35.19	29.45	22.93	18.27	12.62	9.74	6.86
	t-GPH (.6)	83.63	81.40	76.56	70.28	61.27	48.22	35.31	22.88	13.66	8.15
	MRR	55.02	62.13	69.29	73.01	74.13	69.02	59.93	47.40	30.20	13.68
	PP	100.00	100.00	100.00	100.00	100.00	99.95	96.72	76.30	40.22	13.99
	ADF	100.00	100.00	99.94	99.45	97.73	90.36	74.13	50.92	30.02	12.31
10%	s-GPH (.5)	94.15	93.46	90.68	86.73	79.16	66.21	50.24	33.67	22.54	14.28
	s-GPH (.6)	99.98	99.95	99.84	99.32	97.91	92.14	77.05	54.86	33.28	17.28
	d-GPH (.5)	79.67	81.55	81.76	79.95	75.45	66.40	53.78	38.20	26.28	15.39
	d-GPH (.6)	98.55	98.52	98.76	97.56	96.59	91.64	80.99	61.24	38.42	19.12
	t*GPH (.5)	90.18	89.00	85.17	80.73	72.28	59.15	46.05	33.33	21.91	14.44
	t*GPH (.6)	99.72	99.55	99.24	97.71	94.48	85.66	69.81	51.15	31.58	17.15
	t-GPH (.5)	59.49	59.82	57.36	53.94	47.66	39.36	31.99	24.21	18.04	13.32
	t-GPH (.6)	93.19	91.80	89.20	85.20	78.20	65.72	51.61	36.75	24.11	15.31
	MRR	75.87	81.02	85.45	87.13	87.51	83.45	76.23	63.82	45.12	23.16
	PP	100.00	100.00	100.00	100.00	100.00	99.98	98.31	84.29	52.57	23.05
	ADF	100.00	100.00	99.96	99.80	98.93	94.76	83.20	62.91	41.49	21.19

Table 9: Power of the nine tests for fractional cointegration with T = 500 observations

(all entries in percent)

		d									
Size	Test	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1%	s-GPH (.5)	90.34	89.12	84.06	75.19	61.74	40.52	23.10	11.11	4.38	1.75
	s-GPH (.6)	99.99	99.96	99.93	99.09	96.80	84.80	60.60	29.75	10.54	2.75
	d-GPH (.5)	61.17	63.92	65.85	62.78	55.21	42.06	25.52	13.78	5.99	2.11
	d-GPH (.6)	97.00	97.29	97.51	96.91	94.43	85.83	65.84	36.61	14.23	3.49
	t*GPH (.5)	80.99	77.72	70.92	60.95	47.78	31.40	18.45	10.08	4.98	1.81
	t*GPH (.6)	99.89	99.65	98.87	96.21	89.17	72.20	48.35	24.48	9.62	3.19
	t-GPH (.5)	42.05	39.58	36.51	30.92	23.75	15.72	10.22	5.89	3.32	1.67
	t-GPH (.6)	87.62	84.40	78.45	69.61	56.21	39.19	23.72	12.36	5.52	2.12
	MRR	66.73	74.35	80.70	82.10	81.46	73.55	59.33	38.61	18.50	5.34
	PP	100.00	100.00	100.00	100.00	100.00	100.00	98.36	75.36	30.11	6.21
	ADF	100.00	100.00	99.99	99.91	98.83	92.00	72.37	42.48	17.35	5.12
5%	s-GPH (.5)	98.69	98.38	96.90	94.04	87.36	71.87	52.27	32.21	16.41	8.55
	s-GPH (.6)	100.00	100.00	100.00	99.94	99.64	97.13	86.17	59.72	29.25	11.24
	d-GPH (.5)	81.33	84.74	86.11	85.32	82.24	71.92	55.42	37.24	20.05	9.12
	d-GPH (.6)	99.34	99.45	99.48	99.38	99.14	96.86	88.48	67.03	35.93	13.06
	t*GPH (.5)	96.61	95.03	92.30	86.94	78.44	62.61	44.52	29.35	16.86	8.20
	t*GPH (.6)	100.00	99.99	99.94	99.68	98.27	91.82	76.99	53.27	26.98	11.42
	t-GPH (.5)	76.42	74.43	71.41	65.10	56.49	43.02	30.98	21.41	13.26	7.59
	t-GPH (.6)	98.26	97.36	95.56	92.47	84.94	70.87	53.87	35.03	18.15	9.16
	MRR	93.22	95.18	97.01	97.24	97.03	94.25	86.26	69.03	43.46	17.55
	PP	100.00	100.00	100.00	100.00	100.00	100.00	99.59	88.13	50.64	17.34
	ADF	100.00	100.00	100.00	100.00	99.82	97.22	87.47	64.40	35.83	15.26
10%	s-GPH (.5)	99.73	99.75	99.06	97.78	94.35	84.91	68.54	47.69	27.88	15.89
	s-GPH (.6)	100.00	100.00	100.00	100.00	99.91	99.11	93.92	74.19	43.51	20.34
	d-GPH (.5)	88.70	91.32	92.71	92.52	91.00	84.72	71.43	52.38	33.33	17.21
	d-GPH (.6)	99.82	99.81	99.85	99.75	99.70	98.93	94.74	80.51	51.32	22.88
	t*GPH (.5)	99.08	98.54	97.23	94.76	89.56	77.59	61.65	44.93	27.54	15.73
	t*GPH (.6)	100.00	100.00	100.00	99.93	99.52	96.76	87.77	68.33	40.76	20.19
	t-GPH (.5)	89.33	88.05	85.79	80.85	73.66	61.08	47.51	35.41	23.04	14.75
	t-GPH (.6)	99.58	99.22	98.69	97.28	94.00	85.30	71.10	51.56	30.62	17.21
	MRR	97.99	98.61	99.33	99.36	99.41	98.32	94.41	82.50	59.39	28.56
	PP	100.00	100.00	100.00	100.00	100.00	100.00	99.81	93.00	62.46	27.50
	ADF	100.00	100.00	100.00	100.00	99.98	98.78	93.15	75.23	48.26	24.63

Table 10: Power of the nine tests for fractional cointegration with T = 1000 observations

(all entries in percent)

		d									
Size	Test	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1%	s-GPH (.5)	99.44	99.28	98.11	94.46	86.65	67.02	42.76	20.71	7.33	2.39
	s-GPH (.6)	100.00	100.00	100.00	100.00	99.94	98.79	88.76	56.63	20.24	4.00
	d-GPH (.5)	75.27	80.66	83.41	83.29	79.37	66.35	47.46	24.87	9.6	2.88
	d-GPH (.6)	99.56	99.74	99.86	99.82	99.61	98.19	91.29	63.58	25.83	4.92
	t*GPH (.5)	97.13	96.48	93.01	84.62	72.30	53.22	32.82	16.46	6.47	2.43
	t*GPH (.6)	100.00	100.00	99.99	99.93	99.17	93.84	75.07	45.06	16.8	3.76
	t-GPH (.5)	70.41	68.52	61.93	52.78	41.19	28.86	17.84	9.44	4.34	1.73
	t-GPH (.6)	99.10	98.60	96.83	92.92	82.59	67.00	43.01	22.60	8.38	2.68
	MRR	97.58	98.37	98.96	99.09	99.05	96.84	88.52	64.66	32.40	8.29
	PP	100.00	100.00	100.00	100.00	100.00	100.00	99.92	88.93	42.38	8.24
	ADF	100.00	100.00	100.00	100.00	99.92	98.36	87.66	59.27	25.23	6.05
5%	s-GPH (.5)	99.97	99.94	99.83	99.37	97.58	89.89	72.87	47.70	23.48	9.46
	s-GPH (.6)	100.00	100.00	100.00	100.00	99.99	99.92	98.10	82.63	46.68	14.95
	d-GPH (.5)	88.79	92.60	94.02	95.03	94.05	88.83	76.94	54.06	28.25	11.91
	d-GPH (.6)	99.98	99.98	99.99	99.98	99.96	99.81	98.54	87.50	54.26	18.33
	t*GPH (.5)	99.82	99.72	99.09	97.39	92.81	82.45	64.68	42.69	22.21	10.46
	t*GPH (.6)	100.00	100.00	100.00	99.99	99.93	99.47	93.86	73.79	41.70	14.11
	t-GPH (.5)	93.42	92.59	89.09	83.83	74.96	61.61	45.37	29.51	16.58	8.15
	t-GPH (.6)	99.98	99.94	99.73	99.16	96.83	90.62	75.04	51.60	26.89	10.50
	MRR	99.87	99.95	99.94	99.95	99.95	99.83	98.42	88.65	59.96	24.38
	PP	100.00	100.00	100.00	100.00	100.00	100.00	99.97	95.52	63.51	20.84
	ADF	100.00	100.00	100.00	100.00	100.00	99.60	95.88	78.93	46.54	17.43
10%	s-GPH (.5)	99.99	100.00	99.94	99.85	99.32	96.11	85.40	64.04	36.84	17.50
	s-GPH (.6)	100.00	100.00	100.00	100.00	100.00	99.99	99.38	91.53	61.71	25.25
	d-GPH (.5)	93.34	95.85	97.17	97.67	97.54	95.08	87.52	69.93	43.34	21.22
	d-GPH (.6)	99.99	99.99	100.00	100.00	100.00	99.96	99.49	94.32	69.37	29.77
	t*GPH (.5)	99.97	99.96	99.86	99.34	97.51	91.80	79.18	59.33	35.81	18.87
	t*GPH (.6)	100.00	100.00	100.00	100.00	99.99	99.85	97.75	85.40	57.26	24.36
	t-GPH (.5)	97.84	97.82	96.05	92.91	87.93	77.92	63.13	46.34	28.64	16.14
	t-GPH (.6)	100.00	100.00	99.94	99.85	99.14	96.49	86.97	68.76	42.05	18.82
	MRR	99.98	100.00	100.00	100.00	99.98	99.96	99.49	94.81	73.68	36.17
	PP	100.00	100.00	100.00	100.00	100.00	100.00	100.00	97.60	73.85	31.70
	ADF	100.00	100.00	100.00	100.00	100.00	99.82	97.91	86.97%	59.44	27.23

Appendix C: Simulation Results for Unrestricted Estimation

Table 1: Critical values for MRR

Percentile	T=100	T=250	T=500	T=1000
1.0%	0.6336	0.6480	0.6567	0.6668
2.5%	0.6779	0.6922	0.7015	0.7118
5.0%	0.7190	0.7361	0.7458	0.7551
7.5%	0.7481	0.7653	0.7760	0.7870
10.0%	0.7714	0.7894	0.8012	0.8118
12.5%	0.7927	0.8113	0.8225	0.8333
15.0%	0.8114	0.8306	0.8417	0.8526
Mean	1.039	1.066	1.082	1.095
Variance	0.05084	0.05478	0.05695	0.05804
Skewness	0.6568	0.6965	0.7075	0.7230
Kurtosis	0.4577	0.6088	0.6550	0.6993
95% CI for 5% critical value	[0.7170, 0.7206]	[0.7341, 0.7378]	[0.7438, 0.7476]	[0.753, 0.7573]

Table 2: Critical values for PP and ADF

	Phillips-Perron-Test				ADF-Test			
Percentile	T=100	T=250	T=500	T=1000	T=100	T=250	T=500	T=1000
1.0%	-4.524	-4.411	-4.370	-4.333	-4.637	-4.448	-4.383	-4.350
2.5%	-4.189	-4.095	-4.066	-4.039	-4.271	-4.125	-4.078	-4.050
5.0%	-3.913	-3.830	-3.809	-3.791	-3.979	-3.851	-3.819	-3.797
7.5%	-3.733	-3.663	-3.643	-3.631	-3.790	-3.682	-3.654	-3.638
10.0%	-3.598	-3.536	-3.519	-3.509	-3.654	-3.552	-3.526	-3.515
12.5%	-3.488	-3.431	-3.415	-3.407	-3.538	-3.446	-3.421	-3.411
15.0%	-3.395	-3.339	-3.327	-3.322	-3.439	-3.354	-3.332	-3.324
Mean	-2.604	-2.563	-2.554	-2.549	-2.623	-2.569	-2.556	-2.550
Variance	0.6074	0.5873	0.5771	0.5710	0.6433	0.6003	0.5824	0.5741
Skewness	-0.09598	0.007453	0.01722	0.03129	-0.1496	-0.0177	0.006837	0.02468
Kurtosis	0.3332	0.3199	0.3611	0.3074	0.3481	0.3322	0.3641	0.3026
95% CI for 5% critical value	[-3.924, -3.902]	[-3.839, -3.820]	[-3.821, -3.799]	[-3.801, -3.780]	[-3.994, -3.968]	[-3.863, -3.840]	[-3.830, -3.807]	[-3.808, -3.788]

Table 3: Critical values for the eight GPH tests with T = 100 observations

Percentile	s-GPH ($n=T^{0.5}$)	s-GPH ($n=T^{0.6}$)	d-GPH ($n=T^{0.5}$)	d-GPH ($n=T^{0.6}$)	t*-GPH ($n=T^{0.5}$)	t*-GPH ($n=T^{0.6}$)	t-GPH ($n=T^{0.5}$)	t-GPH ($n=T^{0.6}$)
1.0%	-4.999	-4.400	-3.998	-3.562	-6.015	-5.133	-28.002	-17.088
2.5%	-4.156	-3.723	-3.212	-2.939	-4.883	-4.264	-17.262	-12.326
5.0%	-3.487	-3.187	-2.635	-2.446	-4.062	-3.558	-12.079	-9.461
7.5%	-3.105	-2.870	-2.280	-2.137	-3.566	-3.141	-9.710	-8.042
10.0%	-2.837	-2.638	-2.033	-1.917	-3.207	-2.843	-8.290	-7.038
12.5%	-2.621	-2.449	-1.834	-1.743	-2.916	-2.601	-7.269	-6.306
15.0%	-2.438	-2.291	-1.669	-1.589	-2.676	-2.399	-6.488	-5.742
Mean	-1.119	-1.035	-0.458	-0.427	-1.046	-0.880	-3.451	-2.781
Variance	1.807	1.537	1.550	1.348	2.865	2.339	75.022	18.907
Skewness	-0.743	-0.509	-0.602	-0.463	-0.933	-0.562	-27.675	-7.393
Kurtosis	1.721	0.874	1.610	1.029	2.740	1.036	1716.015	219.571
95% CI for 5% critical value	[-3.512, -3.461]	[-3.205, -3.167]	[-2.655, -2.609]	[-2.466, -2.425]	[-4.100, -4.032]	[-3.588, -3.536]	[-12.268, -11.904]	[-9.567, -9.361]

Table 4: Critical values for the eight GPH tests with T = 250 observations

Percentile	s-GPH ($n=T^{0.5}$)	s-GPH ($n=T^{0.6}$)	d-GPH ($n=T^{0.5}$)	d-GPH ($n=T^{0.6}$)	t*-GPH ($n=T^{0.5}$)	t*-GPH ($n=T^{0.6}$)	t-GPH ($n=T^{0.5}$)	t-GPH ($n=T^{0.6}$)
1.0%	-4.353	-3.964	-3.499	-3.229	-5.115	-4.492	-16.317	-9.161
2.5%	-3.701	-3.411	-2.916	-2.702	-4.260	-3.784	-12.082	-7.417
5.0%	-3.180	-2.942	-2.426	-2.282	-3.565	-3.197	-9.274	-6.214
7.5%	-2.862	-2.654	-2.121	-2.014	-3.142	-2.831	-7.873	-5.503
10.0%	-2.632	-2.440	-1.901	-1.805	-2.848	-2.568	-6.938	-4.989
12.5%	-2.435	-2.261	-1.726	-1.642	-2.604	-2.355	-6.226	-4.583
15.0%	-2.271	-2.110	-1.571	-1.496	-2.398	-2.173	-5.677	-4.242
Mean	-1.022	-0.914	-0.415	-0.379	-0.873	-0.717	-2.728	-1.903
Variance	1.517	1.365	1.334	1.209	2.357	2.051	17.792	5.877
Skewness	-0.499	-0.360	-0.432	-0.330	-0.568	-0.356	-10.395	-0.961
Kurtosis	0.772	0.336	0.818	0.395	1.090	0.422	598.675	2.937
95% CI for 5% critical value	[-3.199, -3.159]	[-2.960, -2.923]	[-2.449, -2.407]	[-2.300, -2.262]	[-3.596, -3.533]	[-3.220, -3.171]	[-9.364, -9.173]	[-6.264, -6.168]

Table 5: Critical values for the eight GPH tests with T = 500 observations

Percentile	s-GPH ($n=T^{0.5}$)	s-GPH ($n=T^{0.6}$)	d-GPH ($n=T^{0.5}$)	d-GPH ($n=T^{0.6}$)	t*-GPH ($n=T^{0.5}$)	t*-GPH ($n=T^{0.6}$)	t-GPH ($n=T^{0.5}$)	t-GPH ($n=T^{0.6}$)
1.0%	-4.056	-3.708	-3.295	-3.097	-4.683	-4.195	-9.450	-7.415
2.5%	-3.498	-3.200	-2.755	-2.613	-3.901	-3.548	-7.633	-6.231
5.0%	-3.033	-2.780	-2.326	-2.197	-3.301	-3.007	-6.282	-5.287
7.5%	-2.735	-2.510	-2.046	-1.940	-2.920	-2.677	-5.542	-4.719
10.0%	-2.514	-2.309	-1.837	-1.742	-2.644	-2.424	-4.991	-4.306
12.5%	-2.334	-2.144	-1.667	-1.588	-2.423	-2.222	-4.568	-3.972
15.0%	-2.171	-1.997	-1.525	-1.453	-2.234	-2.050	-4.215	-3.684
Mean	-0.952	-0.835	-0.390	-0.349	-0.762	-0.622	-1.914	-1.592
Variance	1.420	1.277	1.246	1.162	2.145	1.944	6.044	4.384
Skewness	-0.383	-0.281	-0.340	-0.258	-0.419	-0.282	-1.165	-0.613
Kurtosis	0.426	0.226	0.427	0.251	0.639	0.316	4.101	1.149
95% CI for 5% critical value	[-3.053, -3.011]	[-2.798, -2.764]	[-2.345, -2.310]	[-2.213, -2.182]	[-3.325, -3.277]	[-3.031, -2.987]	[-6.341, -6.231]	[-5.326, -5.247]

Table 6: Critical values for the eight GPH tests with T = 1000 observations

Percentile	s-GPH ($n=T^{0.5}$)	s-GPH ($n=T^{0.6}$)	d-GPH ($n=T^{0.5}$)	d-GPH ($n=T^{0.6}$)	t*-GPH ($n=T^{0.5}$)	t*-GPH ($n=T^{0.6}$)	t-GPH ($n=T^{0.5}$)	t-GPH ($n=T^{0.6}$)
1.0%	-3.852	-3.557	-3.193	-2.987	-4.393	-3.960	-8.046	-6.774
2.5%	-3.308	-3.077	-2.653	-2.512	-3.692	3.374	-6.631	-5.768
5.0%	-2.864	-2.661	-2.241	-2.121	-3.140	-2.871	-5.627	-4.950
7.5%	-2.590	-2.401	-1.982	-1.881	-2.785	-2.550	-5.003	-4.443
10.0%	-2.382	-2.201	-1.784	-1.689	-2.521	-2.314	-4.551	-4.049
12.5%	-2.219	-2.045	-1.623	-1.537	-2.304	-2.116	-4.177	-3.721
15.0%	-2.074	-1.909	-1.483	-1.407	-2.125	-1.949	-3.865	-3.457
Mean	-0.883	-0.762	-0.367	-0.317	-0.682	-0.549	-1.704	-1.432
Variance	1.330	1.230	1.199	1.121	2.025	1.854	4.822	3.967
Skewness	-0.322	-0.249	-0.293	-0.234	-0.325	-0.222	-0.778	-0.461
Kurtosis	0.326	0.144	0.375	0.163	0.398	0.180	1.818	0.523
95% CI for 5% critical value	[-2.883, -2.847]	[-2.679, -2.643]	[-2.257, -2.226]	[-2.138, -2.108]	[-3.166, -3.119]	[-2.894, -2.848]	[-5.667, -5.586]	[-4.987, -4.915]

Table 7: Power of the nine tests for fractional cointegration with T = 100 observations

(all entries in percent)

		d									
Size	Test	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1%	s-GPH (.5)	7.33	7.87	7.98	7.77	8.21	5.12	3.57	2.54	1.58	1.38
	s-GPH (.6)	27.77	27.26	25.00	22.12	19.52	11.71	7.33	4.30	2.76	1.54
	d-GPH (.5)	8.16	9.03	8.13	9.13	8.83	5.69	4.26	2.90	1.81	1.29
	d-GPH (.6)	29.46	29.33	26.50	24.10	21.81	13.73	8.27	4.73	2.92	1.74
	t*GPH (.5)	6.68	6.78	7.54	7.26	7.72	5.15	3.66	2.58	1.86	1.48
	t*GPH (.6)	20.51	20.28	18.98	17.33	16.20	9.90	6.55	3.98	2.56	1.52
	t-GPH (.5)	3.27	3.45	3.20	2.78	3.17	2.47	2.22	1.61	1.17	1.12
	t-GPH (.6)	5.78	5.97	5.77	5.59	5.62	4.11	3.01	2.09	1.61	1.22
	MRR	0.73	1.70	2.61	3.79	6.91	6.75	7.18	5.62	3.07	1.93
	PP	100.00	100.00	99.99	99.70	97.72	80.17	49.67	21.35	7.51	2.44
	ADF	92.05	89.21	83.35	76.73	70.05	50.53	32.12	14.46	5.35	1.86
5%	s-GPH (.5)	27.91	28.50	28.34	27.97	27.52	19.34	14.88	10.70	8.04	6.31
	s-GPH (.6)	62.60	61.74	58.02	53.08	48.74	33.84	24.15	16.02	10.83	7.48
	d-GPH (.5)	28.19	30.31	29.50	30.23	29.39	21.67	16.44	11.93	8.53	6.63
	d-GPH (.6)	61.83	61.99	58.85	55.74	51.61	37.51	27.56	17.91	11.47	7.60
	t*GPH (.5)	24.24	24.82	25.08	24.85	24.59	18.13	14.10	9.98	7.95	6.26
	t*GPH (.6)	53.20	50.68	48.99	46.31	41.75	29.57	21.82	14.54	10.06	7.29
	t-GPH (.5)	14.83	15.34	15.23	14.72	15.60	12.20	10.45	8.25	6.78	5.70
	t-GPH (.6)	22.78	23.38	23.43	22.46	21.69	16.10	13.23	9.60	7.85	6.08
	MRR	5.70	8.36	12.02	16.71	22.35	23.11	23.07	19.31	13.57	8.11
	PP	100.00	100.00	100.00	99.97	99.75	94.01	74.96	44.95	21.90	9.95
	ADF	95.86	95.08	92.61	93.17	83.86	68.09	53.51	34.38	17.93	8.77
10%	s-GPH (.5)	44.25	44.64	45.57	44.08	43.19	32.81	25.46	19.42	15.13	12.29
	s-GPH (.6)	78.81	77.32	74.42	69.95	65.38	49.95	37.91	26.45	19.33	13.62
	d-GPH (.5)	46.00	47.36	46.89	46.61	46.10	35.75	28.81	21.69	16.53	12.81
	d-GPH (.6)	77.27	77.60	75.20	71.99	68.80	55.30	42.55	29.92	20.53	14.23
	t*GPH (.5)	40.03	40.80	41.04	40.03	39.54	30.56	25.02	18.34	15.11	12.05
	t*GPH (.6)	71.46	69.46	67.61	63.71	58.81	44.79	35.51	25.38	18.37	13.58
	t-GPH (.5)	27.21	28.33	28.80	27.74	28.03	22.78	19.46	15.65	12.97	10.97
	t-GPH (.6)	38.09	39.20	39.00	37.76	37.05	28.94	24.03	18.23	14.96	11.78
	MRR	13.65	18.05	23.61	30.43	36.64	36.88	36.38	31.62	23.48	15.20
	PP	100.00	100.00	100.00	99.99	99.92	97.29	84.86	58.42	34.06	17.77
	ADF	97.61	97.08	95.38	93.17	90.11	76.86	64.78	46.33	28.69	15.85

Table 8: Power of the nine tests for fractional cointegration with T = 250 observations

(all entries in percent)

Size	Test	d 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1%	s-GPH (.5)	29.64	30.86	31.48	27.86	23.68	13.68	8.94	5.16	2.64	1.69
	s-GPH (.6)	88.56	87.10	82.86	75.22	64.42	40.77	24.73	11.59	4.95	2.10
	d-GPH (.5)	25.03	28.48	29.39	26.44	24.13	15.58	10.56	5.86	3.39	1.70
	d-GPH (.6)	79.90	79.48	78.03	72.32	65.09	45.24	29.21	14.60	6.24	2.47
	t*GPH (.5)	22.00	22.45	22.81	20.61	17.72	11.14	7.44	3.96	2.58	1.62
	t*GPH (.6)	77.16	73.79	68.19	59.45	49.02	30.71	18.05	9.64	4.33	1.99
	t-GPH (.5)	6.68	7.11	7.74	7.44	6.27	4.68	3.60	2.86	2.08	1.40
	t-GPH (.6)	30.63	30.89	29.33	24.75	21.38	13.35	8.96	5.44	2.76	1.59
	MRR	7.27	11.40	16.62	22.17	27.30	24.45	21.10	15.72	8.62	3.27
	PP	100.00	100.00	100.00	100.00	100.00	99.72	92.21	58.55	19.86	4.16
	ADF	99.84	99.67	99.05	96.88	91.69	74.68	52.35	30.42	13.47	3.70
5%	s-GPH (.5)	63.23	64.59	64.24	58.98	52.89	37.71	27.55	17.76	10.93	7.32
	s-GPH (.6)	98.43	97.92	97.02	93.84	88.91	72.20	52.72	33.18	17.80	8.79
	d-GPH (.5)	54.23	56.50	58.68	55.78	52.48	40.64	30.66	20.44	11.99	7.43
	d-GPH (.6)	93.87	93.77	93.62	91.49	88.00	75.43	59.23	38.15	19.61	9.95
	t*GPH (.5)	53.98	54.88	54.84	50.39	43.92	32.07	23.99	16.24	10.85	7.02
	t*GPH (.6)	95.31	94.22	91.98	87.11	79.75	62.16	45.38	28.64	15.88	8.58
	t-GPH (.5)	23.83	25.63	26.25	25.07	23.61	17.90	15.33	11.08	8.66	6.32
	t-GPH (.6)	66.15	64.88	63.43	57.62	52.90	39.11	28.29	19.29	11.40	7.83
	MRR	37.56	46.48	53.52	59.72	63.33	58.17	51.26	40.03	25.10	12.55
	PP	100.00	100.00	100.00	100.00	100.00	99.96	97.76	79.62	51.06	14.71
	ADF	99.99	99.89	99.77	99.15	97.22	88.78	73.80	52.80	30.81	13.14
10%	s-GPH (.5)	78.66	79.70	78.77	74.75	69.33	54.22	41.90	30.17	19.61	13.63
	s-GPH (.6)	99.52	99.42	99.10	97.64	95.39	85.56	68.66	48.60	28.58	16.12
	d-GPH (.5)	69.59	71.49	74.14	71.56	68.40	57.32	46.51	33.57	22.03	14.33
	d-GPH (.6)	97.50	97.40	97.46	96.55	94.72	87.18	74.45	54.41	32.19	17.83
	t*GPH (.5)	71.55	72.57	72.31	67.65	61.61	48.39	37.77	27.91	18.94	13.55
	t*GPH (.6)	98.60	98.21	97.19	94.78	89.84	77.79	62.01	43.88	26.97	16.28
	t-GPH (.5)	39.12	41.56	42.81	40.65	38.82	31.40	26.14	20.06	15.61	11.82
	t-GPH (.6)	81.18	81.27	78.87	74.04	69.61	55.99	44.07	31.91	20.47	14.36
	MRR	60.13	68.21	74.17	78.36	80.60	74.88	68.21	56.67	38.06	21.43
	PP	100.00	100.00	100.00	100.00	100.00	99.99	99.01	87.30	54.32	24.04
	ADF	100.00	99.96	99.91	99.49	98.71	93.52	82.48	64.22	42.13	21.76

Table 9: Power of the nine tests for fractional cointegration with T = 500 observations

(all entries in percent)		d									
Size	Test	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1%	s-GPH (.5)	71.61	72.11	69.29	61.23	50.56	32.25	19.44	10.15	4.39	1.74
	s-GPH (.6)	99.85	99.67	99.32	98.00	93.65	78.51	53.11	26.59	9.32	2.87
	d-GPH (.5)	51.84	55.39	57.23	54.57	48.02	35.27	22.02	11.64	4.90	2.08
	d-GPH (.6)	95.29	95.95	95.87	94.68	90.68	80.00	58.12	30.78	11.20	3.25
	t*GPH (.5)	54.85	55.56	52.95	46.13	37.20	22.98	14.30	7.77	3.54	1.47
	t*GPH (.6)	98.71	98.28	96.07	91.85	82.34	62.40	39.14	19.71	7.56	2.44
	t-GPH (.5)	22.40	24.17	24.31	21.05	17.78	11.67	7.96	4.76	2.92	1.46
	t-GPH (.6)	73.63	72.12	67.36	59.28	50.06	33.47	20.16	10.73	4.81	1.88
	MRR	50.58	59.66	66.64	70.71	71.60	62.78	48.48	30.83	15.18	4.73
	PP	100.00	100.00	100.00	100.00	100.00	100.00	99.18	81.64	34.16	6.72
	ADF	100.00	100.00	99.99	99.87	98.54	92.08	74.83	44.81	18.67	5.57
5%	s-GPH (.5)	92.88	93.03	91.57	86.58	79.32	62.28	45.09	28.20	15.10	8.38
	s-GPH (.6)	99.99	99.98	99.97	99.84	99.23	94.76	80.74	54.72	26.77	11.02
	d-GPH (.5)	75.05	78.79	80.61	79.77	76.03	64.48	50.18	32.31	17.56	9.30
	d-GPH (.6)	98.93	99.15	99.38	98.97	98.06	94.98	84.42	61.16	30.71	12.55
	t*GPH (.5)	86.19	86.40	83.51	78.05	69.86	53.65	38.17	25.31	15.17	7.78
	t*GPH (.6)	99.95	99.91	99.64	99.07	96.55	88.15	70.60	46.30	24.26	9.92
	t-GPH (.5)	55.47	58.04	57.09	52.61	47.19	35.02	26.94	18.16	12.21	6.81
	t-GPH (.6)	93.91	93.46	90.84	86.62	79.32	64.64	48.21	31.16	17.88	8.37
	MRR	86.71	90.71	92.75	94.26	93.80	89.71	80.75	61.74	38.01	16.37
	PP	100.00	100.00	100.00	100.00	100.00	100.00	99.87	92.36	57.09	19.00
	ADF	100.00	100.00	99.99	99.97	99.61	97.42	88.74	67.15	38.45	16.23
10%	s-GPH (.5)	97.61	97.49	96.59	93.91	89.95	77.21	61.41	42.93	26.07	15.59
	s-GPH (.6)	100.00	100.00	100.00	99.98	99.80	98.31	91.02	70.76	40.73	19.95
	d-GPH (.5)	84.89	87.37	89.20	89.30	87.01	78.69	66.76	48.23	28.99	16.84
	d-GPH (.6)	99.64	99.83	99.80	99.72	99.41	98.15	92.71	75.44	45.73	22.24
	t*GPH (.5)	94.54	94.50	93.12	89.45	83.34	69.78	55.27	39.07	25.68	15.08
	t*GPH (.6)	100.00	100.00	99.95	99.77	98.92	94.46	83.45	62.81	38.13	18.60
	t-GPH (.5)	73.23	74.74	74.48	70.39	64.34	51.42	41.85	30.28	21.81	13.59
	t-GPH (.6)	97.99	97.89	96.53	94.26	89.95	79.00	65.02	46.75	29.00	15.87
	MRR	95.36	97.20	97.82	98.13	97.99	96.04	90.87	77.29	53.18	26.49
	PP	100.00	100.00	100.00	100.00	100.00	100.00	99.96	95.91	68.50	29.63
	ADF	100.00	100.00	99.99	99.99	99.80	98.88	93.62	77.72	51.22	26.10

Table 10: Power of the nine tests for fractional cointegration with T = 1000 observations

(all entries in percent)

		d									
Size	Test	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1%	s-GPH (.5)	96.44	96.18	94.30	89.15	79.54	59.03	36.92	18.78	6.69	2.34
	s-GPH (.6)	100.00	100.00	100.00	99.97	99.82	97.44	85.07	52.67	18.44	3.65
	d-GPH (.5)	67.09	74.41	77.92	77.72	73.08	59.00	40.15	20.73	8.15	2.72
	d-GPH (.6)	99.23	99.68	99.75	99.68	99.39	97.50	87.93	59.04	22.84	4.77
	t*GPH (.5)	88.03	88.24	84.01	76.08	62.95	43.80	26.28	13.76	5.46	2.23
	t*GPH (.6)	100.00	99.99	99.98	99.80	98.37	90.59	70.35	39.54	14.31	3.48
	t-GPH (.5)	48.60	50.38	48.86	41.08	34.15	21.83	13.92	7.45	3.69	1.89
	t-GPH (.6)	96.87	95.66	93.53	88.22	78.69	59.50	39.15	19.59	7.46	2.66
	MRR	94.45	96.65	97.59	97.85	97.60	94.32	83.05	58.07	27.24	7.42
	PP	100.00	100.00	100.00	100.00	100.00	100.00	99.99	94.18	49.91	10.01
	ADF	100.00	100.00	100.00	100.00	99.95	98.66	90.08	63.31	27.63	7.38
5%	s-GPH (.5)	99.71	99.55	99.39	98.36	95.11	85.13	67.23	43.51	22.25	10.16
	s-GPH (.6)	100.00	100.00	100.00	100.00	99.97	99.76	96.90	80.19	43.72	14.08
	d-GPH (.5)	83.80	89.24	92.04	92.49	91.39	85.21	71.18	48.59	25.96	11.11
	d-GPH (.6)	99.85	99.96	99.97	99.99	99.96	99.73	97.53	84.86	50.32	17.27
	t*GPH (.5)	98.39	98.26	97.46	94.29	88.69	74.63	56.53	37.12	19.58	9.41
	t*GPH (.6)	100.00	100.00	100.00	99.99	99.88	98.43	91.36	69.14	37.04	13.07
	t-GPH (.5)	81.18	82.33	79.63	74.33	66.78	52.10	38.20	25.00	15.09	8.06
	t-GPH (.6)	99.68	99.53	99.25	98.07	95.02	85.78	70.06	46.38	23.73	10.53
	MRR	99.50	99.73	99.83	99.91	99.77	99.48	96.74	83.85	53.80	21.80
	PP	100.00	100.00	100.00	100.00	100.00	100.00	100.00	97.98	70.44	24.77
	ADF	100.00	100.00	100.00	100.00	100.00	99.84	96.76	82.49	50.94	20.41
10%	s-GPH (.5)	99.93	99.89	99.85	99.43	98.37	93.24	81.12	60.24	35.47	17.81
	s-GPH (.6)	100.00	100.00	100.00	100.00	100.00	99.98	98.89	89.81	59.94	24.31
	d-GPH (.5)	89.85	94.03	95.98	96.44	96.08	92.64	83.61	64.37	40.03	19.80
	d-GPH (.6)	99.93	99.98	100.00	100.00	100.00	99.95	99.25	92.44	65.98	28.33
	t*GPH (.5)	99.55	99.58	99.36	98.07	95.56	86.49	72.39	52.62	31.84	17.30
	t*GPH (.6)	100.00	100.00	100.00	100.00	99.99	99.60	96.56	82.44	52.43	23.00
	t-GPH (.5)	91.32	92.19	90.83	86.75	81.41	69.77	55.13	39.70	25.52	14.90
	t-GPH (.6)	99.92	99.90	99.81	99.42	98.29	93.93	83.42	63.03	37.83	18.66
	MRR	99.92	99.93	99.98	99.98	99.98	99.91	98.91	92.57	68.97	34.39
	PP	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.04	79.69	36.39
	ADF	100.00	100.00	100.00	100.00	100.00	99.95	98.45	89.23	63.20	30.51

Appendix D: Simulation Results for d-GPH with Asymptotic Variance

Table 1: Critical values for d-GPH when the asymptotic variance of (A) regression residuals or (B) OLS-estimator is used with $T = 500$ and $n = T^{0.6}$

Percentile	restricted estimation A	restricted estimation B	unrestricted estimation A	unrestricted estimation B
1.0%	-2.760	-3.202	-3.013	-3.496
2.5%	-2.306	-2.676	-2.539	-2.947
5.0%	-1.933	-2.243	-2.127	-2.468
7.5%	-1.686	-1.957	-1.884	-2.186
10.0%	-1.507	-1.748	-1.697	-1.969
12.5%	-1.362	-1.581	-1.547	-1.795
15.0%	-1.234	-1.432	-1.415	-1.641
Mean	-0.189	-0.220	-0.336	-0.390
Variance	1.042	1.403	1.105	1.488
Skewness	-0.234	-0.234	-0.274	-0.274
Kurtosis	0.243	0.243	0.248	0.248
95% CI of 5% critical value	[-1.949, -1.917]	[-2.261, -2.225]	[-2.144, -2.112]	[-2.488, -2.451]

Table 2: Power of d-GPH when the asymptotic variance of (A) regression residuals or (B) OLS-estimator is used with *restricted regression*, $T = 500$ and $n = T^{0.6}$. (C) gives the corresponding power of d-GPH when the variance is estimated (as in previous tables).

(all entries in percent)

Size	Test	d = 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1%	A	97.60	98.12	98.37	98.11	96.43	88.97	67.31	36.37	13.51	3.35
	B	97.61	98.13	98.37	98.11	96.44	88.97	67.33	36.40	13.53	3.35
	C	97.00	97.29	97.51	96.91	94.43	85.83	65.84	36.61	14.23	3.49
5%	A	99.50	99.63	99.77	99.74	99.36	97.87	90.10	67.91	36.01	12.49
	B	99.50	99.63	99.77	99.74	99.36	97.87	90.10	67.91	36.02	12.49
	C	99.34	99.45	99.48	99.38	99.14	96.86	88.48	67.03	35.93	13.06
10%	A	99.82	99.92	99.92	99.94	99.78	99.45	96.07	81.56	52.63	22.86
	B	99.82	99.92	99.92	99.94	99.78	99.45	96.10	81.58	52.65	22.87
	C	99.82	99.81	99.85	99.75	99.70	98.93	94.74	80.51	51.32	22.88

Table 3: Power of d-GPH when the asymptoti3c variance of (A) regression residuals or (B) OLS-estimator is used with *unrestricted regression*, $T = 500$ and $n = T^{0.6}$. (C) gives the corresponding power of d-GPH when the variance is estimated (as in previous tables).

(all entries in percent)

Size	Test	d = 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1%	A	95.91	96.78	97.04	96.32	93.93	83.17	58.39	29.20	10.49	3.03
	B	95.91	96.78	97.04	96.32	93.93	83.18	58.42	29.20	10.50	3.03
	C	95.29	95.95	95.87	94.68	90.68	80.00	58.12	30.78	11.20	3.25
5%	A	99.18	99.43	99.46	99.34	99.10	96.54	86.25	61.26	31.51	12.98
	B	99.18	99.43	99.46	99.34	99.10	96.54	86.27	61.26	31.52	12.99
	C	98.93	99.15	99.38	98.97	98.06	94.98	84.42	61.16	30.71	12.55
10%	A	99.69	99.83	99.79	99.69	99.66	98.92	93.59	76.58	47.16	21.85
	B	99.69	99.83	99.79	99.69	99.66	98.92	93.59	76.59	47.17	21.85
	C	99.64	99.83	99.80	99.72	99.41	98.15	92.71	75.44	45.73	22.24

Table 4: Critical values for d-GPH when the asymptotic variance of (A) regression residuals or (B) OLS-estimator is used with T = 100 and $n = T^{0.6}$

Percentile	restricted estimation A	restricted estimation B	unrestricted estimation A	unrestricted estimation B
1.0%	-2.864	-3.842	-3.157	-4.236
2.5%	-2.367	-3.175	-2.633	-3.532
5.0%	-1.958	-2.627	-2.193	-2.942
7.5%	-1.713	-2.298	-1.940	-2.603
10.0%	-1.525	-2.046	-1.745	-2.342
12.5%	-1.374	-1.843	-1.590	-2.133
15.0%	-1.246	-1.671	-1.454	-1.951
Mean	-0.207	-0.277	-0.381	-0.511
Variance	1.049	1.888	1.125	2.026
Skewness	-0.304	-0.304	-0.308	-0.308
Kurtosis	0.410	0.410	0.437	0.437
95% CI of 5% critical value	[-1.976, -1.940]	[-2.651, -2.602]	[-2.212, -2.177]	[-2.968, -2.920]

Table 5: Power of d-GPH when the asymptotic variance of (A) regression residuals or (B) OLS-estimator is used with *restricted regression*, T = 100 and $n = T^{0.6}$. (C) gives the corresponding power of d-GPH when the variance is estimated (as in previous tables).

(all entries in percent)

Size	Test	d = 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1%	A	56.49	53.80	48.06	40.33	31.81	20.04	11.86	6.29	3.02	1.71
	B	56.50	53.81	48.08	40.34	31.82	20.04	11.86	6.29	3.02	1.71
	C	50.04	46.87	43.59	37.28	30.43	20.14	13.03	6.92	3.34	1.68
5%	A	83.37	82.06	79.83	74.46	66.25	51.03	36.33	22.95	12.91	8.01
	B	83.37	82.06	79.83	74.46	66.25	51.03	36.33	22.95	12.90	8.00
	C	79.10	77.05	73.92	68.37	61.53	48.01	34.51	23.53	13.52	7.25
10%	A	91.43	90.63	89.92	85.72	80.76	68.44	53.81	37.29	23.86	15.35
	B	91.43	90.63	89.92	85.71	80.76	68.44	53.81	37.29	23.86	15.35
	C	88.93	87.32	85.67	81.82	76.29	64.77	50.36	37.39	23.58	14.09

Table 6: Power of d-GPH when the asymptotic variance of (A) regression residuals or (B) OLS-estimator is used with *unrestricted regression*, T = 100 and $n = T^{0.6}$. (C) gives the corresponding power of d-GPH when the variance is estimated (as in previous tables).

(all entries in percent)

Size	Test	d = 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1%	A	31.08	29.57	26.83	24.30	20.82	11.04	6.67	3.68	2.50	1.40
	B	31.05	29.56	26.83	24.30	20.80	11.01	6.67	3.68	2.50	1.40
	C	29.46	29.33	26.50	24.10	21.81	13.73	8.27	4.73	2.92	1.74
5%	A	69.55	67.52	65.38	60.39	54.82	38.56	27.86	18.27	11.32	7.08
	B	69.55	67.54	65.38	60.39	54.84	38.56	27.86	18.27	11.32	7.08
	C	61.83	61.99	58.85	55.74	51.61	37.51	27.56	17.91	11.47	7.60
10%	A	83.21	82.62	79.95	76.63	71.83	57.01	43.57	31.07	20.55	13.58
	B	83.18	82.59	79.92	76.61	71.81	57.00	43.55	31.05	20.52	13.55
	C	77.27	77.60	75.20	71.99	68.80	55.30	42.55	29.92	20.53	14.23

Appendix E: Simulation Results for Restricted Estimation in the Presence of Deterministic Trends

Table 1: Critical values for PP if $\gamma = 0$, $u_t \sim I(1)$ and T = 500 (10.000 replications)

Percentile	Size of deterministic trend			
	$\delta = 1$	$\delta = 0.1$	$\delta = 0.01$	$\delta = 0.001$
1.0%	-4.034	-3.975	-3.921	-3.955
2.5%	-3.716	-3.642	-3.600	-3.632
5.0%	-3.454	-3.377	-3.344	-3.376
7.5%	-3.287	-3.217	-3.174	-3.180
10.0%	-3.155	-3.095	-3.052	-3.052
12.5%	-3.060	-2.993	-2.955	-2.952
15.0%	-2.971	-2.902	-2.867	-2.868
Mean	-2.196	-2.073	-2.034	-2.038
Variance	0.575	0.710	0.687	0.681
Skewness	0.035	0.355	0.241	0.196
Kurtosis	0.399	0.799	0.583	0.511
95% CI for 5% critical value	[-3.490, -3.424]	[-3.412, -3.355]	[-3.378, -3.315]	[-3.422, -3.331]

Table 2: Critical values for d-GPH if $\gamma = 0$, $u_t \sim I(1)$, T = 500 and $n = [T^{0.6}]$ (10.000 replications)

Percentile	Size of deterministic trend			
	$\delta = 1$	$\delta = 0.1$	$\delta = 0.01$	$\delta = 0.001$
1.0%	-2.684	-2.631	-2.890	-2.863
2.5%	-2.167	-2.148	-2.383	-2.344
5.0%	-1.731	-1.806	-1.961	-1.978
7.5%	-1.511	-1.582	-1.696	-1.736
10.0%	-1.321	-1.394	-1.532	-1.553
12.5%	-1.169	-1.240	-1.390	-1.394
15.0%	-1.061	-1.102	-1.268	-1.269
Mean	-0.004	-0.057	-0.193	-0.205
Variance	1.065	1.065	1.084	1.087
Skewness	-0.267	-0.196	-0.309	-0.249
Kurtosis	0.390	0.208	0.335	0.264
95% CI for 5% critical value	[-1.790, -1.686]	[-1.872, -1.752]	[-2.028, -1.916]	[-2.030, -1.930]

Table 3: Critical values for MRR if $\gamma = 0$, $u_t \sim I(1)$ and T = 500 (10.000 replications)

Percentile	Size of deterministic trend			
	$\delta = 1$	$\delta = 0.1$	$\delta = 0.01$	$\delta = 0.001$
1.0%	0.7139	0.7130	0.6826	0.6755
2.5%	0.7639	0.7588	0.7377	0.7261
5.0%	0.8142	0.8105	0.7842	0.7770
7.5%	0.8489	0.8477	0.8142	0.8096
10.0%	0.8809	0.8743	0.8382	0.8373
12.5%	0.9057	0.8997	0.8606	0.8611
15.0%	0.9317	0.9212	0.8810	0.8818
Mean	1.202	1.185	1.128	1.129
Variance	0.0721	0.0672	0.0596	0.0599
Skewness	0.599	0.574	0.643	0.600
Kurtosis	0.313	0.260	0.469	0.350
95% CI for 5% critical value	[0.8057, 0.8225]	[0.8040, 0.8174]	[0.7781, 0.7902]	[0.7699, 0.7850]

Table 4: Power of PP, d-GPH and MRR if $\gamma = 0$, $u_t \sim I(d)$, T = 500 and critical values from Tables 1 - 3 are used

		PP				d-GPH				MRR			
Size	δ	d=0.5	d=0.7	d=0.8	d=0.9	d=0.5	d=0.7	d=0.8	d=0.9	d=0.5	d=0.7	d=0.8	d=0.9
1%	1	100.00	88.38	45.18	10.10	89.49	45.00	18.80	4.58	82.60	49.10	27.06	8.48
	0.1	100.00	84.50	39.47	8.43	89.51	45.80	19.76	5.33	82.93	48.21	26.25	8.69
	0.01	99.99	75.71	31.15	5.99	84.22	34.81	12.37	3.38	73.50	37.82	17.76	5.96
	0.001	100.00	74.99	29.02	5.88	85.52	35.45	13.04	3.46	70.50	34.49	16.70	4.97
5%	1	100.00	96.42	69.02	26.48	98.43	77.89	49.39	18.97	97.33	78.40	54.50	24.40
	0.1	100.00	94.65	63.14	23.41	97.83	74.15	45.08	17.23	97.05	77.17	53.06	24.54
	0.01	100.00	88.29	52.71	18.02	97.12	67.80	36.40	14.53	94.79	70.29	44.71	19.37
	0.001	100.00	88.14	50.67	17.38	97.03	66.24	35.73	13.96	94.29	68.18	42.84	18.10
10%	1	100.00	98.37	80.17	39.56	99.51	88.41	64.09	31.06	99.34	89.98	70.81	38.53
	0.1	100.00	96.97	73.63	34.13	99.28	85.71	60.36	29.02	99.20	88.39	68.81	38.63
	0.01	100.00	92.84	63.97	27.91	98.97	80.80	52.16	24.00	98.40	82.50	58.80	29.55
	0.001	100.00	92.91	63.07	28.34	98.81	79.84	50.99	24.17	98.27	81.64	58.92	29.78

Table 5: Power of PP, d-GPH and MRR if $\gamma \neq 0$, $u_t \sim I(0.9)$, T = 500 and critical values from Appendix B are used

		δ_1 and δ_2									
all entries in percent		1				0.1			0.01		0.001
Size	Test	1	0.1	0.01	0.001	0.1	0.01	0.001	0.01	0.001	0.001
1%	PP	2.41	2.15	0.74	0.90	1.24	1.09	1.22	4.09	5.43	5.77
	d-GPH	0.89	1.33	1.27	1.20	0.84	1.69	1.97	3.19	3.17	3.64
	MRR	1.09	1.52	1.62	1.78	1.15	2.37	2.58	4.30	5.19	5.45
5%	PP	9.42	8.09	3.73	3.52	6.21	4.44	4.79	12.16	16.17	16.83
	d-GPH	4.82	5.58	5.87	5.94	4.55	7.87	8.13	12.22	12.57	13.20
	MRR	5.26	6.79	7.44	7.45	5.22	9.66	10.31	15.69	17.02	17.44
10%	PP	17.55	14.03	7.23	7.03	11.80	8.33	8.70	20.39	25.49	27.16
	d-GPH	9.50	11.39	12.42	12.24	9.44	15.28	15.51	21.21	23.08	23.75
	MRR	9.91	12.65	13.60	13.66	10.29	16.62	17.48	26.20	28.53	29.04

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