Combining German Macro Economic Forecasts using Rank-Based Techniques

Matthias Klapper

Department of Statistics, University of Dortmund,

44221 Dortmund, Germany

klapper@amadeus.statistik.uni-dortmund.de

Abstract: Macro economic forecast data for Germany is gathered and combined using various combination techniques. Rank-based combination methods are presented that outperform the Simple Average and Least Squares based methods.

Keywords: Macro economic forecasts, combination of forecasts.

Acknowledgement: Supported by Deutsche Forschungsgemeinschaft, Graduiertenkolleg „Angewandte Statistik".

AMS 1991 Subject Classification: 62G30

1. Introduction

The history of research in the field of combination of forecasts shows that combining forecasts has an advantage over using individual forecasts, see Gulledge and Ringguest (1986), Jungmittag (1993) and others. Usually the Simple Average turns out to be the most successful technique to combine forecasts in literature. There are many attempts made to improve over the Simple Average for example by using Least Squares techniques with fixed and changing weights. The fundamental paper for this approach is that of Granger and Ramathan (1984). An overview of the development of combining forecasts can be found in Appendix A.
Less explored are combining techniques based on rankings of past performance. Most literature on this subject is being written in English and most research is being done in the

United States, therefore literature of combinig forecast mostly considers US data. In this paper we will try to use German macro economic forecast data. The difference to most previous studies is that the forecasts are done by using econometric models instead of time series models.
The description of the data can be found in Chapter 2. Chapter 3 applies traditional combination techniques to the data and Chapter 4 presents some nonparametric rank-based techniques. Chapter 5 closes with a conclusion. Appendix A gives a brief history of the combination of forecasts, Appendix B gives a sample VGR table, Appendix C lists the data and Appendix D states the most important formulas.

2. The Data

The main forecasters of macro economic variables in Germany are the economic research institutes that make up the „Arbeitsgemeinschaft Deutscher Wissenschaftlicher Forschungsinstitute", the DIW (Berlin), HWWA-Institut (Hamburg), Ifo-Institut (München), Institut für Weltwirtschaft (Kiel), RWI (Essen) and since 1993 the IfW (Halle). These institutes also release a combined consensus forecast twice a year. Many other national and international organizations do their own forecasts, for example the Sachverständigenrat, a council of independent experts, the Deutsche Gewerkschaftsbund (the labor union), and the OECD. An overview of the most important macro economic forecasters in Germany and their publications can be found in Table 1. The anomalities of the most crucial variables of these forecasters are listed in Table 2.
In the Mid-1960s German economic research institutes started publishing numeric macro economic forecasts. They predicted certain variables of the „Volkswirtschaftliche Gesamtrechnung" in form of a table. By the Mid-1970s most institutes adopted that table for their macro economic forecast presentations. A sample VGR table can be found in Appendix B. Sometimes the number of variables doubles, when actual numbers and percentage change are forecasted. Most institutes increased the number of variables forecasted over time. More recent forecasts for example also include currency exchange rates and more detailed employment/ unemployment variables. Unemployment forecasts generally begin in the early 1980s when the rates started to rise to become a major economic factor. Sections 1-2, e.g. Composition and Use of the GDP are the same for most for all forecasters. The detail of Sections 3-5, e.g. Income, Household Income and Government Revenue and Expenditures varies between the forecasters, we displayed only the most common variables.
All forecasters are doing a year-end forecast (Oct-Jan) for the following year. These forecasts have the most historic data available which makes them the most promising for a combining analysis. Looking at Table 1 there are 8 forecasters that do year-end forecasts for at least 20 years. There are only five that do mid-year forecasts and only five that do ½-year forecast horizons. We therefore concentrate in this analysis on the year-end forecasts with 1 year forecasting horizons.
Many of the variables published are related to each other e.g. a variable not price adjusted, the same variable price adjusted, and the corresponding price development. Since the not price adjusted variable can be calculated from the price adjusted and the price development it is enough to look only at the price adjusted variables and the prices. Of the prices, most previous analyses only look at the consumer prices since price developments are highly correlated.
To check whether all forecasters are having the same variable definitions, we compare the realizations with each other. The realization in our case is the value released by the Statistische

Bundesamt at the end of the following year. Most variables seem to have the same definitions for all forecasters with some exceptions: Most forecasters seem to have different definitions of productivity, we therefore drop that variable.
Except the WSI, all unemployment forecasts begin in the mid 1980s. There are also different definitions by using either the number of employees or the number of dependent employees as the denominator. Since there is not enough historic data available we will not consider that variable. International forecasts for example for the „industrial countries“ are also problematic because there are different definitions which countries belong to that group. Since the IfW starts publishing employment forecasts as late as 1987 and the OECD does not publish the investment variables regularily and has a definition problem with the disposable income, we drop these variables as well.
The RWI Essen started forecasting regularily at the end of the year in 1987 which does not give us enough historic data for our analysis. We could have used the RWI forecast closest to the end of the year but that could have been very early, e.g. July in 1983 and we therefore do not include the RWI forecasts. The HWWA forecasts were excluded from this study since there are no HWWA forecasts for 1980.
We are still left with six variables across seven forecasters for 21 years of history. These variables are the real changes of Gross National Product, private consumption, public consumption, exports, imports, and the change of consumer prices. The table with these forecasts and the corresponding realizations as of the end of the following year can be found in Appendix C.

Table 1 - Overview of the Most Important Macro Economic Forecasts in Germany

Source	Publication	ISSN	Variables	Approx. Number of Vars.	Detail	Release Dates	Horizon	Start Date
DIW	Wochenbericht, DIW Berlin	0012-1304	VGR Set	130	1 year	January	1 year	1960
Deutsches Institut für Wirtschaftsforschung	"Grundlinien der Wirtschaftsentwicklung"		Basic International		1/2 year	July	current + 1 year	1982
HWWA	Konjunktur von Morgen,	0023-3439	VGR Set	45	1 year	August	current + 1 year	1978
HWWA-Institut für Wirtschaftsforschung	HWWA Hamburg		Basic International			December	1 year	1970
Ifo	Wirtschaftskonjunktur,	0043-6283	VGR Set	144	1 year	July	current + 1 year	1962
Ifo Institut für Wirtschaftsforschung	Ifo München		Basic International		1/2 year	December	1 year	1962
IfW	Die Weltwirtschaft, IfW Kiel	0043-2652	VGR Set	40	1 year	March	current + 1 year	1992
Institut für Weltwirtschaft			Basic International	8 (Mar,Sep)		June	current + 1 year	1976
						September	current + 1 year	1992
						December	1 year	1971
RWI	RWI Konjunkturberichte,	0931-8801	VGR Set	37	1 year	Jan/Feb	1 year	1987
Rheinisch-Westfälisches Insti- tut für Wirtschaftsforschung	RWI Essen		Basic International			June/July	current + 1 year	1987
AWF	Wochenbericht, DIW Berlin	0012-1304	VGR Set	134	1 year	April	current year	1965
Arbeitsgemeinschaft Deutscher wissenschaftlicher Forschungsinstitute	"Die Lage der Weltwirtschaft und der Deutschen Wirt- schaft im Frühjahr/Herbst"		Basic International		1/2 year	October	current + 1 year	1965
SVR	Jahresgutachten, Stuttgart	-	VGR Set	32	1 year	November	1 year	1965
Sachverständigenrat zur Begutachtung der Gesamt- wirtschaftlichen Entwicklung	"Die Voraussichtliche Entwicklung"		Basic International		1/2 year			
WSI	WSI Mitteilungen,	0342-300x	VGR Set	61	1 year	May	current year	1991
Wirtschafts- und Sozialwiss. Institut des Deutschen Gewerkschaftsbundes	WSI Düsseldorf "Zur Wirtschaftsentwicklung in der Bundesr. Deutschland"					November	1 year	1969
OECD	OECD Economic Outlook	0029-7011	Key Econ.	29	1 year	June	current + 1 year	1967
Organization for Economic			Indicators		1/2 year	December	1 year	1967

Cooperation and Development			(varying)			

Table 2 - Data Anomalities
(year = forecasted year)

All	Until 1994 West Germany, thereafter Germany.
DIW	GNP industrial countries (1988): 1.8-2.5 (range).
HWWA	1972 two scenarios: with and without actions. 1974,1980 no forecast. 1975, 1987-1993 January, 1978 February. From 1997: ½ year forecasts. Range GNP: 1989: 2-2.5
Ifo	1971 no forecast. Until 1980 and 1983: Beginning of the year (January/February). 1995 also forecasts for 1996 and 1999. Ranges GNP: 1984: 2-2.5, 1985: 2-2.5, 1987: 2-2.5, 1989: 2-2.5, 1991: 3-3.5 Consumer Prices: 1987: 0.5-1
IfW	1973, 1974 no forecast. From 1995: Quarterly forecasts (some variables). Employment, unemployment: No rates, or percent changes, only absolute numbers.
RWI	1969-1970: Forecasts in February, September 1969, March 1970. 1975-1986: Forecasts spring and fall each year except 1982-1983. From December 1992: ½ year forecasts. From 1997 two year horizon.
AWF	From July 1962 numeric. Until July 1964 in July/ December. Until Fall 1969: Including „Institut für Wirtschaftsforschung Braunschweig-Völkenrode". From Fall 1993: Including „Institut für Wirtschaftsforschung Halle".
SVR	1964 only ½ year horizon. Until 1973 only nominal forecasts. Ranges: GNP: 1980: 2.5-3 Consumer Prices: 1973: 5.5-6, 1975: 5.5-6
WSI	1970 no forecast. 1971 two scenarios A and B. 1972 first unemployment rate forecasted, well before other forecasters. 1974 productivity only per employee hour. 1995 only East and West Germany, weighted with 0.20 and 0.80.
OECD	1972 only first half 1971 to first half 1972. From 1989 two year horizon.
Realizations	Source: SVR from November of following year except:

1994: Import, Export, Household disposable income: DIW 30/95.

3. Traditional Combining Techniques

3.1 Simple Combination Techniques

Before testing different combining techniques to find out the best technique for these data, we have to think about what criterion to use to define "best". Döpke and Langfeldt (1995) use the mean error (ME), the mean absolute difference (MAD) the root mean squared error (RMSE) the Mean Absolute Percentage Error (MAPE), and the Theil's U (TU), that takes the squareroot of the MSE of the forecast and divides it by the MSE of the "naive" forecast. There are three versions of the Theil's U, made up by different definitions of "naive": Taking the forecast value zero, taking last years forecast value and taking the average of the last five years forecast values. The exact definitions are given in Appendix C.
Table 3 displays the criteria mentioned above for forecasts of the seven forecasters compared to the actual numbers published by the Statistische Bundesamt at the end of the following year for the performance period 1987-1996. The ME seems to be less than zero which is an indication of a possible bias. A closer look shows that this is the same for all forecasters across most of the variables and part of the nature of these data where a great source of error is the under- and respectively overestimation due to sudden turns in economic trends like booms and recessions. This means that the performance window 1987-1996 is responsible for the ME and for 1976-1996 the ME is closer to zero and sometimes above zero. For our analysis we therefore consider the forecasts to be unbiased.
For the Gross Domestic Product (GDP) the Ifw has the lowest RMSE followed by Ifo, AWF, DIW, SVR, OECD, and WSI. All three TU's rank the forecasters the same, which also happens for the Private Consumption and all the other variables not displayed here. The MAD and the MAPE rank the forecasters slightly different.

Table 3 - Comparison of Forecast Quality Criteria

	GDP							**Private Consumption**						
	ME	MAD	RMSE	MAPE	TU1	TU2	TU3	ME	MAD	RMSE	MAPE	TU1	TU2	TU3
DIW	-0,84	1,00	1,38	0,30	0,49	0,69	0,72	-0,38	0,70	0,93	0,67	0,36	0,65	0,61
Ifo	-0,44	1,04	1,28	0,25	0,45	0,64	0,67	-0,23	0,69	0,86	0,63	0,33	0,60	0,56
IfW	-0,07	1,01	1,17	0,23	0,41	0,59	0,62	-0,12	0,70	0,83	0,59	0,32	0,58	0,55
AWF	-0,09	1,27	1,36	0,29	0,48	0,69	0,72	-0,13	0,91	1,05	0,99	0,41	0,74	0,69
SVR	-0,29	1,17	1,39	0,29	0,49	0,70	0,73	-0,28	0,74	0,94	0,69	0,37	0,66	0,62
WSI	-0,20	1,50	1,65	0,33	0,58	0,83	0,87	-0,51	1,01	1,15	1,23	0,45	0,81	0,75
OECD	-0,18	1,26	1,42	0,28	0,50	0,71	0,75	-0,08	0,72	0,89	1,00	0,35	0,63	0,59

The Simple Average seems to be a very popular method to combine forecasts. Many researchers in the past were not able to find significantly better combination techniques for many different data sets like Clemen and Winkler (1986) and Makridakis et al. (1982) with their famous data sets.
The underlying assumption for using the Simple Average, which assigns each forecaster the same weight, is that all forecasters are predicting equally well. Varying RMSEs and MADs in Table 3 indicate that this is not the case. The performance of individual forecasters also changes over time. This makes us think that there must be an advantage to use past performance information of the forecasters to assign different weights to them.

There are very simple methods to use past performance information. Clemen and Winkler (1986) propose to use the best forecaster in the previous forecast since he showed to be able to be the most accurate. Using a different argument they also propose to use the worst forecaster in the previous forecast occasion since he might be the one that improved the most due to motivation through previous bad performance.
The combined forecasts are compared to the actual numbers published by the Statistische Bundesamt at the end of the following year. Using the data in Appendix C we can use all 21 forecasts for the Simple Average. Since we need past performance for the „best" and „worst" strategies we can start with the 1978 forecast that is done at the end of 1977 when we have the actual numbers and therefore the performance for 1976. Since other combining strategies presented in the following chapters require more history, we will only show the performance for the last 10 years 1987-1996 to be able to give the OLS and other methods at least 10 historic timepoints of data to build their models on.
The resulting MAD and RMSE are displayed in Table 4 in relation to the corresponding values of the Simple Average. For comparison, this table also contains the performance of the individual forecasts and techniques explained in later chapters. The WSI forecasts have the highest RMSE for all six variables but excluding the WSI does not improve the combining results. „Best Last", IfW, three times Ifo, and „Worst Last" have the lowest RMSE and even excluding the WSI forecasts the combining methods do not result in a lower RMSE.

3.2 Ordinary Least Squares

In the Ordinary Least Squares (OLS) approach we do one-step (two-year) ahead forecasts by fitting a model at each step. First we take a certain time point to start. It makes sense to take about the midpoint of the data to be able to have a big enough performance window (10 years) and enough history (10 years). Then we fit a model using all the available data at that point for example at the end of 1986 we take all available data (1976-1985) to fit a model. Then we forecast 1987 and compare the result to the actual number reported at the end of 1988. The second step we are at the end of 1987 where we have 11 years of historic data that we can use to fit a new model to predict 1989 and compare the result with the actual number for 1989. We keep on doing this step by step until 1996. We therefore get 10 combined forecasts and compare them to 10 actual numbers. Using these residuals we calculate the MAD and the RMSE.
There are several variations of this procedure that make sense to explore: We can take all available history at each step in time to fit the model or only take the last 10 years. We can also limit the number of forecasters included in the model by doing stepwise regression. We can either do stepwise regression once and keep the order for doing all 10 years of fitting and forecasting or repeat the stepwise regression at each step.
Looking at the RMSEs in Table 4 we can see that for the GDP the OLS stepwise using all history for three and four variable models (OLSM3-4) and the OLS stepwise once using all available history (OLSO3-5) outperform the Simple Average. For the Private Consumption the OLSF1, for the Public Consumption the OLSM3, for the Export the OLSM2, for the Import the OLSF1-3 and for the Consumer Prices no OLS

Table 4 - MAD and MSE for different Combining Techniques 1987-1996 performance window relative to Simple Average

	GDP		Private Cons.		Public Cons.		Expor t		Import		Consumer P	
	MAD	RMSE	MAD	RMSE	MAD	RMSE	MAD	RMSE	MAD	RMSE	MAD	RMSE
DIW	0,88	1,05	1,05	1,07	1,20	1,31	1,08	1,09	1,13	1,10	1,32	1,28
Ifo	0,91	0,97	1,04	0,99	0,72	0,78	0,95	0,98	0,92	0,91	1,00	1,08
IfW	0,89	0,89	1,05	0,96	1,00	0,98	1,03	1,03	0,87	0,87	0,93	1,01
AWF	1,12	1,04	1,37	1,21	0,87	0,93	1,02	1,01	1,01	1,02	1,18	1,27
SVR	1,03	1,06	1,11	1,08	0,94	0,97	0,97	1,02	1,09	1,16	1,14	1,22
WSI	1,32	1,26	1,52	1,32	1,41	1,49	1,18	1,14	1,07	1,10	1,07	1,20
OECD	1,11	1,08	1,08	1,03	1,09	1,12	0,97	0,93	0,96	0,98	0,72	0,92
Simple Avg.	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**
Best Last	0,88	0,89	1,19	1,05	1,13	1,11	0,99	1,02	1,01	1,04	0,99	1,04
Worst Last	1,24	1,23	1,57	1,33	1,26	1,36	1,09	1,02	0,99	0,96	1,17	1,18
OLSM 1	1,06	1,13	1,19	1,21	1,09	1,08	1,09	1,07	1,22	1,17	1,17	1,34
OLSM 2	0,91	1,09	1,41	1,40	1,16	1,15	0,81	0,93	1,52	1,39	1,24	1,38
OLSM 3	0,75	0,78	1,64	1,53	0,98	0,96	0,85	1,08	1,27	1,18	1,60	1,68
OLSM 4	0,85	0,92	1,54	1,40	1,54	2,00	0,95	1,23	1,42	1,38	1,59	1,75
OLSM 5	1,06	1,26	1,39	1,30	1,99	2,76	1,10	1,38	1,51	1,47	1,69	1,70
OLSM 6	1,21	1,43	1,47	1,41	2,12	2,74	1,18	1,46	1,57	1,56	1,66	1,76
OLSM 7	1,30	1,50	1,46	1,40	2,10	2,74	1,15	1,46	1,61	1,59	1,67	1,77
OLSF 1	0,97	1,01	0,95	0,96	1,07	1,10	1,21	1,16	0,91	0,92	1,33	1,49
OLSF 2	1,04	1,10	1,23	1,06	1,60	1,77	1,78	2,13	1,03	0,92	1,88	1,87
OLSF 3	0,93	1,07	1,63	1,37	2,22	2,71	2,21	2,60	1,06	0,97	2,17	2,11
OLSF 4	0,94	1,16	1,62	1,38	2,26	2,76	2,01	2,24	1,28	1,19	2,73	3,41
OLSF 5	0,89	1,41	1,72	1,59	3,04	3,67	2,12	2,29	1,44	1,48	3,24	4,08
OLSF 6	1,38	1,72	1,79	1,64	3,20	3,91	2,60	2,90	1,54	1,54	3,40	3,86
OLSF 7	1,99	2,84	2,23	2,23	3,64	4,41	2,89	3,40	1,84	2,09	3,82	4,29
OLSO 1	1,14	1,17	1,09	1,02	1,01	1,11	1,08	1,06	1,09	1,09	1,17	1,34
OLSO 2	1,06	1,19	1,02	1,01	1,30	1,33	1,05	1,14	1,33	1,29	1,44	1,53
OLSO 3	0,75	0,81	1,31	1,27	1,34	1,63	1,16	1,34	1,21	1,20	1,65	1,72
OLSO 4	0,66	0,76	1,48	1,38	1,57	1,96	1,22	1,42	1,35	1,32	1,47	1,56
OLSO 5	0,79	0,86	1,41	1,31	2,10	2,73	1,20	1,43	1,46	1,41	1,61	1,63
OLSO 6	1,21	1,43	1,50	1,40	2,10	2,74	1,06	1,32	1,49	1,52	1,69	1,73
OLSO 7	1,30	1,50	1,46	1,40	2,10	2,74	1,15	1,46	1,61	1,59	1,67	1,77
ERLS 1	0,85	0,94	1,58	1,38	1,12	1,17	1,10	1,16	1,19	1,12	1,33	1,37
ERLS 2	0,92	1,04	1,39	1,27	1,21	1,25	1,03	1,09	1,01	0,96	1,56	1,52
ERLS 3	0,72	0,83	1,37	1,29	1,72	2,35	1,18	1,22	1,17	1,07	1,78	1,75
ERLS 4	0,74	0,92	1,28	1,24	1,91	2,68	1,19	1,22	1,33	1,25	1,68	1,66
ERLS 5	1,01	1,39	1,24	1,23	1,76	2,55	1,16	1,16	1,40	1,32	1,69	1,70
ERLS 6	1,03	1,40	1,27	1,21	1,72	2,40	1,14	1,08	1,40	1,36	1,76	1,74
NN IfW/DIW	0,89	0,90	0,98	1,00	0,99	0,97	1,13	1,13	1,12	1,07	1,30	1,26
NN IfW/Ifo	0,94	0,95	1,03	0,98	0,84	0,87	0,99	1,01	0,90	0,88	0,99	1,03
NN IfW/AWF	0,95	0,95	1,00	0,96	0,96	0,96	1,10	1,11	0,92	0,92	0,94	1,00
NN IfW/SVR	0,99	0,99	1,00	0,96	0,94	0,94	0,99	1,06	1,01	1,07	1,03	1,04
NN IfW/WSI	0,93	0,93	1,00	0,96	0,99	0,97	1,20	1,20	0,91	0,91	0,94	1,00
NN IfW/OECD	1,00	0,98	1,02	0,97	1,02	1,01	1,06	0,99	0,96	0,97	0,86	0,96
NN Ifo/WSI	0,93	1,00	1,12	1,01	0,71	0,78	0,99	1,01	0,90	0,89	0,99	1,03
CMSE	0,95	0,97	1,03	1,00	0,93	0,95	0,93	0,96	1,03	1,02	0,97	0,96
CMAD	0,99	0,99	0,99	0,99	0,97	0,96	0,98	0,99	0,99	0,99	1,00	0,99
CMAPE	0,98	0,98	0,99	0,99	0,97	0,96	0,98	0,99	0,99	0,99	1,00	0,99
CCIV	0,99	0,99	0,99	0,99	0,97	0,97	0,99	0,99	0,99	0,99	1,00	0,99
CCIV3	0,98	0,99	1,02	0,99	0,93	0,94	0,96	0,97	0,99	0,98	0,99	0,97
CAVE	0,99	0,99	0,99	0,99	0,98	0,97	0,99	0,99	0,99	0,99	1,00	0,99

	GDP		Private Cons.		Public Cons.		Expor t		Import		Consumer P	
	MAD	RMSE	MAD	RMSE	MAD	RMSE	MAD	RMSE	MAD	RMSE	MAD	RMSE
Simple Avg.	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**	**1,00**
RANK	0,99	0,99	0,99	0,99	0,97	0,97	0,99	0,99	0,99	0,99	1,00	0,99
RANK2	0,99	0,99	0,99	0,99	0,96	0,95	0,98	0,99	1,00	0,99	1,00	0,99
RANK3	0,99	0,98	0,99	0,98	0,94	0,94	0,97	0,98	1,00	0,99	0,99	0,98
RANK4	0,99	0,98	0,99	0,98	0,93	0,93	0,97	0,98	1,00	0,99	0,99	0,98
RANK 6	0,99	0,99	0,99	0,97	0,91	0,91	0,96	0,98	1,00	1,00	0,98	0,97
RANK H	0,99	0,99	0,98	0,98	0,97	0,98	0,98	0,99	1,00	1,00	0,99	0,98
RANK W0.5	0,99	0,99	1,00	0,98	0,93	0,93	0,97	0,98	1,00	1,00	0,99	0,98
RANK GEO	0,97	0,98	1,00	1,00	0,96	0,96	0,99	0,99	0,99	0,99	1,01	0,99

Explanations to Table 4:
OLSM = Stepwise each step, all available history.
OLSF = Stepwise each step, 10 years history.
OLSO = Stepwise once, all available history.
ERLS = Equality Restricted Least Squares.
CMSE, CMAD, CMAPE, CCIV, CCIV3, CAVE = Russell and Adam (1987).
1-7 = Number of Variables (e.g. forecasters).
NN x/y = Non Negativity restricted Least Squares combining Forecasters x and y.
RANK = Rank methods.
2 = sqared, 3 = tripled, etc., H = all History, W0.x = Last years number weighted x% and current year (100-x)%, GEO = using geometric mean.

model outperforms the Simple Average. All other models perform worse for our six variables, sometimes much worse so that there is no OLS method that outperforms the Simple Average for all six economic variables.
Looking closer at the data we can see, that the OLS methods perform well during economic up- and downturns and perform extremely poor when some forecasters are not able to forsee a turn in the economy. Since the main cause of trouble are the estimated model parameters, we will put some restrictions on them.

3.3 Restriction: LS Coefficients Add to 1

A possible restriction introduced in Aksu and Gunter (1992) is that the LS coefficients have to add to one. The results are also displayed in Table 4. The results are similar to the OLS approaches beating the Simple Average only for GDP and Imports but mosty being much worse. A cause of this problem are very high positive and negative coefficients like for the OLS model.

3.4 Restriction: LS Coefficients Greater than 0

If we restrict the parameters to be greater than zero we usually get high coefficients for two forecasters and zero or close to zero for al others even when using all forecasters to fit the model. We therefore only look at models containing two forecasters. For the GDP there are seven two-forecaster models that beat the Simple Average and are shown in Table 4. But these

are not consistently better since for the Exports and Consumer Prices only the IfW/OECD model outperforms the SA while this model does not beat the SA for Public Consumption and Imports.
The coefficients are usually between zero and one but not adding up to one exactly. This indicates that a method might help that combines both constraints from Section 3.3 and 3.4.

4. Rank Techniques

4.1 Ranks using 10 years of history

A possible solution to the problems in Chapter 3 is not to use regression but nonparametric methods based on the ranks of the previous performance of the forecasters. There are several possible approaches. One presented by Russell and Adam (1987) ranks the forecasters in each time period separately. They pay only attention to the best or the best three forecasters respectively each time period. The results of their techniques are displayed in Table 4. The CMAD, CMAPE, CCIV, and CAVE methods outperform the Simple Average for the RMSE for all variables among them two of the methods Russell and Adam (1987) proposed best, the CMAD and CAVE.
In our approach we use the ranks for all forecasters over all time periods. With historic data from 1976-1985 we have 10 years multiplied by the number of forecasters (7) which makes 70 forecast errors. They are now ranked by absolute size with Table 5 showing an example for the years 1976-1985 for the GNP. This table shows us the ranks of the negative absolute forecast error that means 1 for the highest and 70 for the lowest absolute forecast error. We now add up the ranks for each forecaster and divide it by the total sum of ranks. The result is displayed in the last column of Table 5 and used as coefficients to generate a combined forecast.
The advantage of this method is that we are getting coefficients that are always greater than zero and that add up to 1. We repeat this procedure for the next year, the following year, etc., the coefficients are displayed in graphical format in Figure 1. For every year we calculate a combined forecast that is compared to the actual number to calculate a combined forecast error. The corresponding MAD and RMSE for these errors can be found in Table 4 under „RANK“. As we can see, this method improves the MAD as well as the RMSE compared to the Simple Average or performs at least equally well.
Instead of taking the ranks, we can transform the ranks by taking the squared, tripled, etc. ranks. To visualize how the transformed ranks look like, we put the squared and quadrupled ranks for 1987 into Table 5. These transformations change the coefficients since it creates more extreme coefficients and increases their variance. We can see this by looking at Figure 1 that displays the change of the shares each forecaster has of the combined forecast and the boxplots in Figure 2. Looking at the series of boxplots for simple ranks (Reg), squared (Squ), tripled (Tri), quadrupled (Qua), and sixtupled ranks (Six), we can see an increase in size of the box and notches with the median staying constant first but decreasing at the end getting significantly lower for the sixtuple transformation. This can be explained by the lower boundary of 0 for the coefficients. Looking at the MAD and RMSE in Table 4 we can see that the higher the transformation power the lower the MAD and RMSE for all variables except the Imports. This variable has its minimum for both criteria using the simple ranks, but the RMSE for the Imports using the transformed ranks is still lower than the RMSE for the Simple

Average for the simple, squared, tripled and quadrupled ranks. This indicates that quadrupling the ranks could be a good transformation to use.
Looking at the change of shares over time in Figure 1 we can also see that the stability of the share for a certain forecaster decreases with increasing the power of the transformation since the influence of a newly added high rank has more influence on the coefficient.

4.2 Other Rank Techniques

We will explore some variations of the rank combining technique. We can use all available history instead of the last 10 years at each step. The corresponding coefficients are displayed in Figure 1 and also in the boxplot in Figure 2. We used the quadrupled ranks since they performed well among the previously mentioned transformations and therefore have to compare the results to the line labeled „RANK4" in Table 4, Figure 1 and the boxplot in Figure 2. As we can see in Table 4 under „RANK H" the MAD and the RMSE are higher than if we use only the last 10 years of history. The boxplot in Figure 2 shows less variation than if using only 10 years. An explanation would be that forecast errors are decreasing over time and later absolute errors are generally smaller and being outweighted by some past very big ones that stay influential over the years.
Another technique is to take the average of the new determined coefficient and the previous' year's coefficient. The corresponding boxplot labeled W0.5 in Figure 2 again shows less variation of the coefficients since change takes more time. This is also visible in Figure 1 changes of coefficients are much smoother over time. MAD and RMSE are again higher than for the regular quadrupled ranks.
We can also take the mean of the inverse ranks instead of the mean of the negative absolute ranks. This technique is labeled „Geo" in Table 4 and Figure 2. Even though the box and notches are less wide than for the regular quadrupled ranks we can see some outliers. It is giving different forecasters the highest weight as we can see comparing the first and the third table from Figure 1 and the corresponding graphs. This method performs better for the GDP but much worse for Exports and Government Consumption.

5. Conclusion

We applied various techniques to combine the forecasts of German macroeconomic variables. Methods described in Chapter 4 in detail based on the ranks of former performance were able to outperform the Simple Average. These techniques were also better than Least Squares techniques favored in most of the literature. There are many different ways to utilize the ranks of previous forecasts as shown in Chapter 4 and most methods were better than the Simple Average. Using the quadrupled ranks performed best for most variables.

Table 5 - Ranks for 1987 GNP for the seven forecasters

Ranks, 1987

	1976	*1977*	*1978*	*1979*	*1980*	*1981*	*1982*	*1983*	*1984*	*1985*	*Sum*	
DIW	41,5	2,5	59	41,5	63,5	23,5	18,5	18,5	35	35	338,5	0,136
Ifo	15	11,5	59	41,5	63,5	23,5	8,5	18,5	53	53	347	0,140
IfW	15	6	56	41,5	50,5	13	69	27,5	63,5	35	377	0,152
AWF	15	2,5	59	41,5	35	66	8,5	27,5	35	35	325	0,131
SVR	25,5	11,5	56	25,5	30	48,5	18,5	69	63,5	50,5	398,5	0,160
WSI	7	1	56	29	46	69	10	46	21,5	46	331,5	0,133
OECD	4	31	67	41,5	53	48,5	5	21,5	35	61	367,5	0,148
											2485	

Squared Ranks, 1987

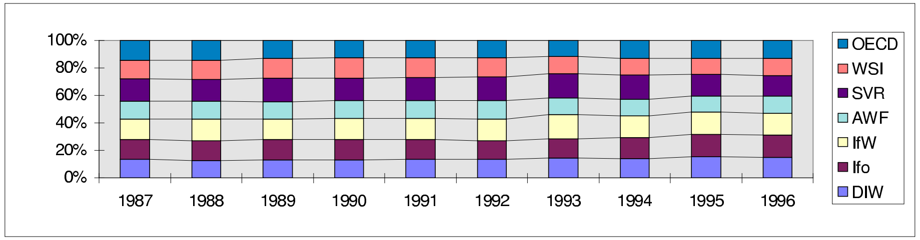
	1976	*1977*	*1978*	*1979*	*1980*	*1981*	*1982*	*1983*	*1984*	*1985*	*Sum*	
DIW	1722	6	3481	1722	4032	552	342	342	1225	1225	14651	0,126
Ifo	225	132	3481	1722	4032	552	72	342	2809	2809	16178	0,139
IfW	225	36	3136	1722	2550	169	4761	756	4032	1225	18613	0,159
AWF	225	6	3481	1722	1225	4356	72	756	1225	1225	14294	0,122
SVR	650	132	3136	650	900	2352	342	4761	4032	2550	19507	0,167
WSI	49	1	3136	841	2116	4761	100	2116	462	2116	15698	0,134
OECD	16	961	4489	1722	2809	2352	25	462	1225	3721	17783	0,152
											116723	

Quadrupled Ranks, 1987

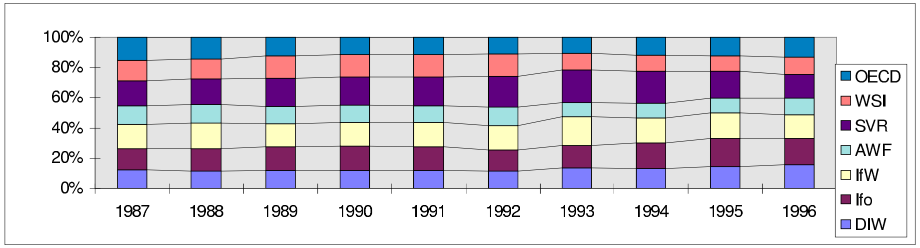
	1976	*1977*	*1978*	*1979*	*1980*	*1981*	*1982*	*1983*	*1984*	*1985*	*Sum*	
DIW	2966145	39	12117361	2966145	16259040	304980	117135	117135	1500625	1500625	37849231	0,109
Ifo	50625	17490	12117361	2966145	16259040	304980	5220	117135	7890481	7890481	47618958	0,137
IfW	50625	1296	9834496	2966145	6503775	28561	22667121	571914	16259040	1500625	60383598	0,174
AWF	50625	39	12117361	2966145	1500625	18974736	5220	571914	1500625	1500625	39187915	0,113
SVR	422825	17490	9834496	422825	810000	5533080	117135	22667121	16259040	6503775	62587788	0,180
WSI	2401	1	9834496	707281	4477456	22667121	10000	4477456	213675	4477456	46867343	0,135
OECD	256	923521	20151121	2966145	7890481	5533080	625	213675	1500625	13845841	53025370	0,153
											347520203	

Figure 1 - Coefficients for the Variable GDP

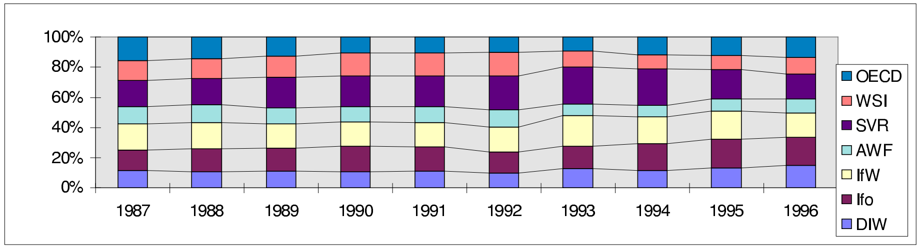
Ranks



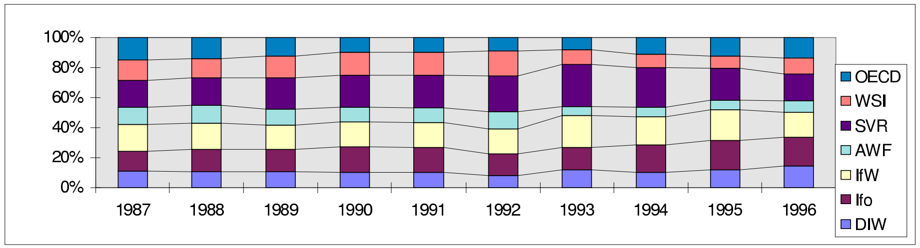
Squared ranks



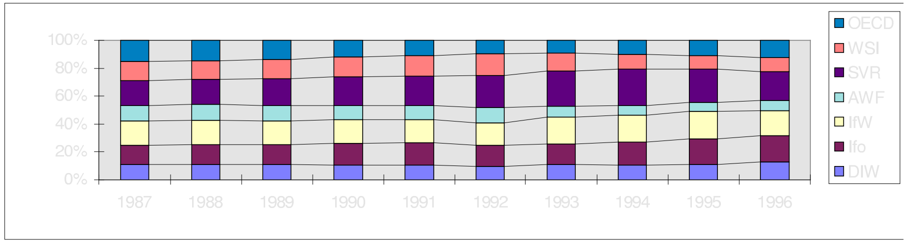
Tripled ranks



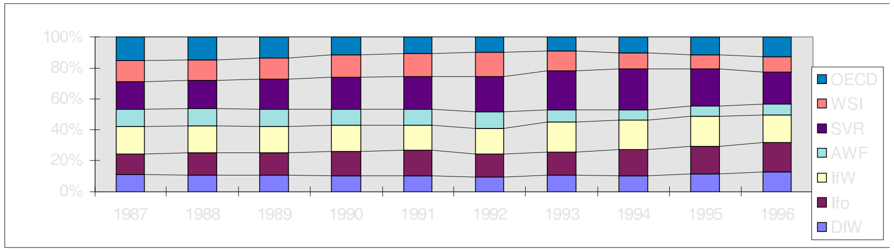
Quadrupled Ranks



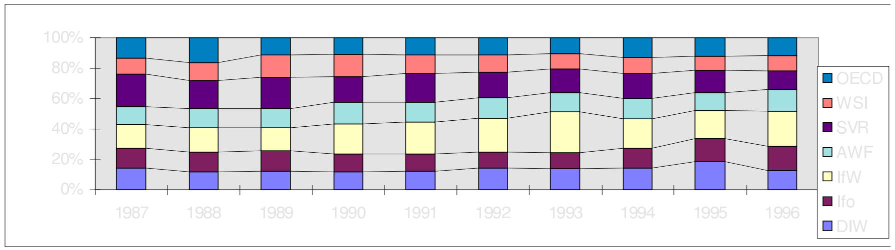
Quadrupled ranks using all history



Quadrupled ranks 0.5/0.5



Quadrupled ranks, geometric mean



Sixtupled ranks

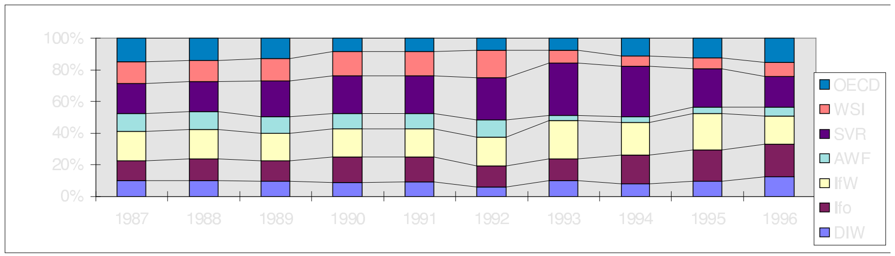
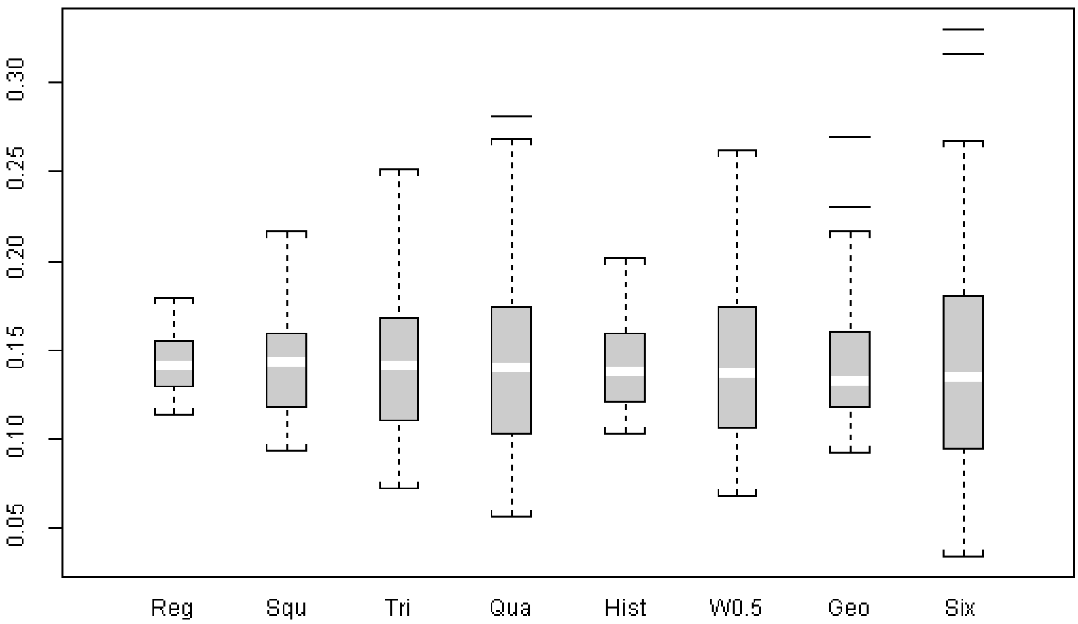


Figure 2 - Boxplot of the Rank Method Coefficients



Reg = ranks, Squ = squared ranks, Tri = tripled ranks, Qua = quadrupled ranks, Hist = using all available history, W0.5 = Avg. of current and previous coefficient, Geo = geometric mean, Six = sixtupled ranks.

Bibliography

- Aksu, Celal and Sevket I. Gunter (1992): An Empirical Analysis of the Accuracy of SA, OLS, ERLS, NRLS Combination Forecasts, International Journal of Forecasting Vol. 8, 27-43
- Anandalingam, G. and Lian Chen (1989): Bayesian Forecast Combination and Kalman Filtering, International Journal of Systems Science, 1499-1507
- Bates, J.M. and C.W.J. Granger (1969): The Combination of Forecasts, Operational Research Quarterly Vol. 20 No. 4, 451-468
- Clemen, Robert T. and Robert L. Winkler (1986): Combining Economic Forecasts, Journal of Business & Economic Statistics Vol. 4, 39-46
- Deutsch, Melinda, Clive W.J. Granger and TimoTeräsvirta (1994): The Combination of Forecasts Using Changing Weights, International Journal of Forecasting Vol. 10, 47-57
- Diebold, Francis X. (1988): Serial Correlation and the Combination of Forecasts, Journal of Business and Economic Statistics Vol. 6, 105-111
- Diebold, Francis X. and Peter Pauly (1987): Structural Change and the Combination of Forecasts, Journal of Forecasting Vol. 6, 21-40
- Döpke, Jörg und Enno Langfeldt (1995): Zur Qualität von Konjunkturprognosen für Westdeutschland 1976-1994, Kiel Discussion Papers No. 247, Kieler Institut für Weltwirtschaft
- Fan, Dennis K., Kin-Nam Lau and Pui-Lam Leung (1996): Combining Ordinal Forecasts with an Application in a Financial Market, Journal of Forecasting Vol. 15, 37-48
- Faria, A.E. and R.C. Souza (1995): A Re-evaluation of the Quasi-Bayes Approach to the Linear Combination of Forecasts, Journal of Forecasting Vol. 14, 533-542
- Fomby, Thomas A. and Subarna K. Samanta (1991): Application of Stein Rules to Combination Forecasting, Journal of Business and Economic Statistics Vol. 9, 391-407
- Granger, Clive W.J. and Ramu Ramanathan (1984): Improved Methods of Combinig Forecasts, Journal of Forecasting Vol. 3, 197-204
- Guerard, John B. Jr. and Robert T. Clemen (1989): Collinearity and the Use of Latent Root Regression for Combining GNP Forecasts, Journal of Forecasting Vol. 8, 231-238
- Gulledge, Thomas R. Jr., Jeffrey L Ringuest and James A. Richardson (1986): Subjective Evaluation of Composite Economic Policy Inputs, Socio-Econ. Plan. Sci. Vol. 20, 51-55
- Gunter, Sevket I. and Celal Aksu (1989): N-Step Combinations of Forecasts, Journal of Forecasting Vol. 8, 253-267
- Gupta, Sunil and Peter C. Wilton (1987): Combination of Forecasts: An Extension, Management Science Vol. 33, 356-372
- Jungmittag, Andre (1993): Die Kombination von Prognosen: Ein Überblick mit Anwendungen, Jahrbuch für Nationalök. u. Stat., 58-72
- LeSage, James P. and Michael Magura (1992): A Mixture-Model Approach to Combining Forecasts, Journal of Business and Economic Statistics Vol 10, 445-452
- Makridakis, S., et al. (1982): The Accuracy of Extrapolation (time-series) Methods: Results of a Forecasting Combination, Journal of Forecasting Vol. 1, 111-153.
- Nanayakkara, N. and Noel Cressie (1991): Combining Two Unbiased Estimators of a Common Mean od Two Normal Populations, Austral. J. Statist. Vol. 33, 43-56
- Reid, David J. (1968): Combining Three Estimates of Gross Domestic Product, Economica Vol. 35, 431-444
- Russell, Thomas D. and Everett E. Adam, Jr. (1987): An Empirical Evaluation of Alternative Forecast Combinations, Management Science Vol. 33, 1267-1276

- Sessions, D.H. and S. Chatterjee (1989): The Combining of Forecasts Using Recursive Techniques with Non-stationary Weights, Journal of Forecasting Vol. 8, 239-251
- Tibiletti, Luisa (1994): A Non-Linear Combination of Experts' Forecasts: A Bayesian Approach, Journl of Forecasting Vol 13, 21-27
- Trenkler, G. and G. Ihorst (1995): Improved Estimation by Weak Covariance Adjustment Technique
- Trenkler, G. and E.P. Liski (1986): Linear Constraints and the Efficiency of Combined Forecasts, Journal of Forecasting Vol. 5, 197-202
- Wall, K. D. and Correia, C. (1989): A Preference-Based Method for Forecast Combination, Journal of Forecasting Vol. 8, 269-292
- West, Carol Taylor (1996): System-Based Weights Versus Series-Specific Weights in the Combination of Forecasts, Journal of Forecasting Vol. 15, 369-383
- Wiper, Michael P. and Simon French (1995): Combining Experts' Opinions using a Normal-Wishart Model, Journal of Forecasting Vol. 14, 25-34

Appendix A - History of the Combination of Forecasts

As an historical overview, I will present a list of the main papers that contributed to the advance of research in the area of combining forecasts. The papers will be in the order published and include only papers that contained a new aspect to this subject matter. While the first ideas were published by Reid (1968) thirty years ago the main development did not start before the mid 1980s led by the Makridakis et al. (1982) extensive numerical study and the paper of Granger of Ramanathan (1984). Clemen (1989) created a large annotated bibliography containing more than 200 sources.
Although these 24 papers were published in nine diffenent scientific journals, half of them were printed in the Journal of Forecasting. The detailed sources of these papers will be listed in the Bibliography.

Reid 1968	The core paper, the first ideas.
Bates & Granger 1969	Most others refer to as the most fundamental.
Granger & Ramanathan 1984	Regression methods.
Gulledge, Ringuest & Richardson 1986	Linear optimization.
Trenkler & Liski 1986	Additions to GR (1984).
Gupta & Wilton 1987	Odds matrix method.
Diebold & Pauly 1987	Extension to GR (1984) with structural change.
Russell & Adam 1987	Weights based on realative MSE and ranks.
Diebold 1988	Serial correlation.
Anangalingam & Chen 1989	Kalman filtering to avoid multicollinearty.
Guerard & Clemen 1989	Latent root regression.
Sessions & Chatterjee 1989	Recursive and ad hoc methods compared.
Gunter & Aksu 1989	N-step combinations.
Wall & Correia 1989	Turning points, ordinal data.
Nanayakkara & Cressie 1991	Efficency of estimates.
Fomby & Samanta 1991	Stein rules.
LeSage & Magura 1992	Multiprocess mixture models.
Tibiletti 1994	Bordley model extension.
Deutsch, Granger & Teräsvirta 1994	Changing weights over time.
Trenkler & Ihorst 1995	Weak covariance technique.
Wiper & French 1995	Wishart model, expert opinions.
Faria & Sousa 1995	Quasi Bayes.
Fan, Lau & Leung 1996	Ordinal forecasts.
West 1996	System based weights.

GR = Granger & Ramathan

Appendix B - Volkswirtschaftliche Gesamtrechnung (VGR)

	Actual	Forecast	
	Last (1996)	Current (1997)	Next (1998)

1A. Composition of the GDP

Employment			
Productivity			
Unemployment			

2A. Use of the GDP, not price adjusted

Private consumption			
Public consumption			
Investment in equipment and machinery			
Investment in construction			
Export			
Import			
Gross Domestic Product			

2B. Use of the GDP, fixed (1991) prices

Private consumption			
Public consumption			
Investment in equipment and machinery			
Investment in construction			
Export			
Import			
Gross Domestic Product			

2C. Price development

Private consumption			
Public consumption			
Investment in equipment and machinery			
Investment in construction			
Export			
Import			
Gross Domestic Product			

	Actual	Forecast	
	Last (1996)	Current (1997)	Next (1998)

3A. Income, not price adjusted

Distribution of disposable income from dependent labor			
Distribution of disposable income from companies and savings			
National Income			
Gross National Product			

3B. Income, fixed (1991) prices

Distribution of disposable income from dependent labor			
Distribution of disposable income from companies and savings			
National Income			
Gross National Product			

4. Household Income

Household disposable income			
Household savings rate / ratio			

5. Government Revenue and Expenditures

Taxes			
Social security			
Money transfers			
Debt payments			
Investments			

Sample years in parentheses.

Appendix C

		1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
DIW	GDP	5	5,5	3	4	2	-1	0,5	-0,5	2	2	3	1,5	1	2,5	3,5	3,5	1	-1	-0,5	2	1
	Priv. Cons.	3	4,5	3	3,5	2	1	-0,5	-0,5	0	1,5	3,5	3	3	2	3,5	3,5	2	0	-1,5	0,5	2
	Publ. Cons.	2	2,5	3	2,5	2,5	1	1	0,5	0	1	1,5	2	1,5	1	1,5	3	1,5	2	1	1	1,5
	Export	8,5	9,5	4	5	3	-1,5	4	0	4,5	6,5	4,5	0	0	5,5	6,5	6	3,5	-0,5	1,5	7,5	1
	Import	9,5	9	6	8	3,5	-1	2	0	3	4,5	5,5	4	2,5	5	6,5	6,5	6	1,5	0,5	5	2
	Cons. Prices	5,5	4	4	3	5	4	5	3,5	3	2	1,5	1	1	2,5	3	3	3,5	4	3	2,5	2
Ifo	GDP	4	4,5	3	4	2	-1	1	-0,5	2,25	2,25	3	2,25	1	2,25	3	3,25	1,5	-0,5	1	3	1,75
	Priv. Cons.	2,5	4,5	3	3,5	1,5	1	0	-0,5	1	1,5	3	3,5	2,5	2,5	4	3	2	0	-1	0,5	2,5
	Publ. Cons.	1,5	2,5	3	2,5	2,5	2	1	0	0	1,5	2	2	1,5	0,5	2	1	1,5	0,5	1	1,5	2
	Export	6,5	9	4,5	5,5	4	-0,5	5,5	-1	4,5	6	4	1	2	5	6	7,5	4	1,5	3	8,5	4
	Import	7	9	6	7	4	-1,5	3	0	4	4	5,5	5	3,5	4,5	7,5	9	5	1,5	2,5	4,5	3
	Cons. Prices	5	4	3,5	3,5	5	4	5	3,8	3	2	2	0,75	1,5	2,5	3,5	3,5	4	3,5	3	2,5	2
IfW	GDP	4	5	3,5	4	1,5	-1,5	-1	0	2,5	2	3	3	2,5	2,5	3	3,5	1,5	0	1	3,5	1,7
	Priv. Cons.	3	4,5	3,5	3,5	2	0,5	-0,5	-0,5	1,5	1	3	4,5	3	2,5	3,5	3	1,5	0	-1	1,5	2,1
	Publ. Cons.	2	2,5	3	2,5	2,5	2	1	0,5	0,5	1	1,5	2,5	1,5	0,5	1,5	1,5	1	0,5	0,5	1	1,6
	Export	7	10	6	6,5	2,5	-2	2	1,5	4	6	5,5	3	3	4	5,5	8	5	1,5	2,5	8	4,3
	Import	8	10	7	7,5	3,5	-3	2,5	1	3,5	4	5,5	6	5	5	7	9,5	5	1	2	6	3,9
	Cons. Prices	5	4	4	3,5	4,5	4	4,5	3,5	2,5	2	2	1	1,5	3	3,5	3,5	4	3,5	3	2,5	2
AWF	GDP	4	5,5	3	4	2,5	0	1	0	2	2	3	3	2	2	3	3	2	0,5	1	3	2,5
	Priv. Cons.	2	4	2,5	3,5	2	1,5	0,5	-0,5	1	1,5	3	4	3	2	3,5	2,5	2	1	-1	0,5	3
	Publ. Cons.	1,5	2,5	2,5	2,5	2,5	2	1	0,5	0	0,5	1,5	2	1,5	0	1,5	1,5	2	1,5	1	1	1,5
	Export	7,5	11	5,5	6,5	4,5	0	5	1	3,5	6	5	3	3,5	4	5,5	8	5,5	2	3,5	8	5
	Import	7,5	10,5	6	6,5	4	-0,5	3	1	3	4	5,5	5,5	5	4	6	9	6	3	4	5	4,5
	Cons. Prices	5,5	4,5	4	3,5	4,5	4	4,5	3,5	3	2	2	1,5	2	2,5	3	4	4	3,5	3	2,5	2
SVR	GDP	4,5	4,5	3,5	3,5	2,75	0,5	0,5	1	2,5	3	3	2	1,5	2,5	3	3,5	2,5	0	0	3	2
	Priv. Cons.	3	5	3	3,5	2	2	-0,5	-1	1,5	2,5	3	3	2,5	2,5	3,5	3	2	0	-1,5	1,5	2,5
	Publ. Cons.	1,5	2,5	3	2,5	2	2	1	0,5	0,5	1	2	2	1,5	0,5	1,5	-0,5	2	-1	-1	0,5	1,25
	Export	7,5	9	5	4	5	0,5	6	2,5	4	6,5	4,5	1,5	2,5	5	6	7,5	4,5	3	2,5	5,5	3,5
	Import	9	9	5	6	5	0,5	3	2	4,5	6,5	6	5,5	4	4,5	7	8,5	3,5	4,5	0,5	4	2,75
	Cons. Prices	3	5	3,5	2,5	4,5	4	5,5	4	3	2	1,5	1	1,5	2	3	3,5	4	3,5	3	2,5	2,25
WSI	GDP	3,5	6	3,5	3,7	2,4	0,1	0,8	0,5	1,3	2,1	3	2,2	1,6	1,7	2,8	2,7	2,2	0,9	0,5	3,84	2,5
	Priv. Cons.	3	5	3	3	2	1,5	0,5	-1,5	0	2,5	2,5	3	2	1,5	3	2,5	1,5	1,5	-1	0,2	2,5
	Publ. Cons.	2	5,5	3,5	3	2,5	1	0	-1	-0,5	1	1,5	2	1,5	1,5	1	1	0,5	2	-1	0,8	1,5
	Export	7	13	6	5	5	1,5	5	2,5	3,5	4,5	5,5	2,5	2,5	4,5	5,5	8	6,5	4	3,5	10,4	4
	Import	6	14	6	5,5	5,5	1,5	3	1,5	3	4,5	6	5	3	5	6	8	6	3,5	2	5,6	3
	Cons. Prices	5,5	5	4	3	4	4	5,5	4,5	3	2	2	1,5	2	2,5	3	3,5	4	3,5	2,5	2,1	2
OECD	GDP	3,25	3,5	3,25	4	2,25	-0,3	1,25	-0,3	2	2,75	3,25	3	1,5	2,5	3,2	3	1,8	0,5	0,4	2,8	2,4
	Priv. Cons.	3	3,5	3,25	4,25	1,75	1	0,25	-0,3	0,5	1,5	3	4,25	2,75	2	3,5	3,5	1,4	1,2	-1,2	1,2	2,4
	Publ. Cons.	2,5	2	3	2,75	2,75	1,25	0,5	0,5	0	0,75	1,5	2,5	1,75	0,75	1	1	0,8	1	0,9	0,4	2
	Export	3,75	7,5	5	5	5	2,5	8,5	1	4,75	7,5	5	1,5	1	6,75	7	8,8	5,2	2,2	2,9	8,4	6,5
	Import	5,5	7,5	5	8	4,25	-0,3	3,5	0,5	3	5	5	5,5	4	6	7,5	9,9	5,4	2,5	1,9	6,4	6,7
	Cons. Prices	4,75	4	4	2,75	5	4	4,25	3,75	3,25	2	1,25	0,75	1,75	2,25	2,6	3,9	4	3,5	3	2,1	2
Realization (End of previous year)	GDP	5,6	2,8	3,2	4,6	1,9	0,1	-1	1	2,6	2,6	2,6	1,9	3,7	3,3	4,7	3,7	1,6	-1,7	2,4	1,9	1,4
	Priv. Cons.	3,6	3,1	3,4	3,2	1,7	-1,2	-2,2	1,1	0,6	1,8	4,3	3,5	2,7	1,7	4,7	3,6	1,7	0,2	0,6	1,8	1,3
	Publ. Cons.	2,4	1	3,6	2,9	2,6	2	-1,1	0	2,4	2,2	2,3	1,6	2,2	-0,9	2,1	0,5	3,2	-1,2	1,4	2	1,8
	Export	11,1	3,9	4,4	5,4	5,9	8,5	3,7	-1,3	8	7,3	-0,2	0,8	5,8	11,5	9,9	12,7	3,7	-3,2	7,7*	5,9	4,5
	Import	10,6	4,3	5,9	11,2	5,8	1,2	0,8	0,5	5,5	4,7	3,5	4,9	6,3	8,8	11,9	12,2	3,9	-5,3	7,1*	6,4	2,2
	Cons. Prices	4,4	3,8	2,6	3,9	5,4	6	5,3	2,9	2,5	2,1	-0,5	0,5	1,2	3,2	2,6	3,8	4	3,2	2,7	1,9	2

Appendix D

Formulas:

$$MAE = \frac{1}{F}\sum_{t=1}^{F}\left|f_t - y_t\right|$$

$$RMSE = \sqrt{\frac{1}{F}\sum_{t=1}^{F}\left(f_t - y_t\right)^2}$$

$$MAPE = \frac{1}{F}\sum_{t=1}^{F}\frac{\left|f_t - y_t\right|}{y_t} * 100$$

$$TU1 = \sqrt{\frac{1}{F}\sum_{t=1}^{F}\left(f_t - y_t\right)^2 / \frac{1}{F}\sum_{t=1}^{F} y_t^{\,2}}$$

$$TU2 = \sqrt{\frac{1}{F}\sum_{t=1}^{F}\left(f_t - y_t\right)^2 / \frac{1}{F}\sum_{t=1}^{F}\left(y_t - y_{t-1}\right)^2}$$

$$TU3 = \sqrt{\frac{1}{F}\sum_{t=1}^{F}\left(f_t - y_t\right)^2 / \frac{1}{F}\sum_{t=1}^{F}\left(\left(y_{t-5} + y_{t-4} + y_{t-3} + y_{t-2} + y_{t-1}\right)/5 - y_t\right)^2}$$