CLASSIFICATION OF WEST GERMAN BUSINESS CYCLES[1]

by

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Summary

This paper applies linear discriminant analysis to classify West German business cycles from 1955 to 1994 into a four phase scheme (upswing, downswing, and upper/lower turning point phases). It describes the scheme as well as the selection of the classifying variables, and presents classification results for various sample periods. Special attention is given to changes of the explanatory power of the variables and its implication for changes of West German cycle patterns.

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The *Age of Uncertainty* is looking for certitudes and at least economists seem to find them by returning home from the concepts of shock and chaos to the more seasoned ones of cycle and trend. Since the mid 1990s the *Council of Economic Advisers* (1994, pp. 57ff.) has been giving cyclical considerations, comparisons, and analogies an increasing weight in its macroeconomic analysis and forecasts, German empirical research is slowly following. There are a number of good reasons for this renewed interest, such as theoretical innovations or new empirical developments as suggested by *Gordon* (1986, p. 12).[3]

Though the changes listed did not pass by unnoticed, their cyclical relevance was often examined for only a few variables, or for short periods. More comprehensive studies such as those possible in an econometric model framework, or by employing the *Burns/Mitchell* (1947) reference cycle scheme, are still rare.[4] A much more surprising fact is that the basically univariate or recursive understanding of the cycle phenomen as implied by this method has hardly been challenged. Though, to be fair, there is much evidence that the univariate character of the scheme was a result of the methodical and computational limits of that time. Anyway, it is surprising that a pioneering study of changes in the U.S. cycle applying multivariate linear (discriminant) analysis (LDA) by *Meyer/Weinberg* (1975a, b; 1976) not only failed to draw much attention, but also remained without successors or further updates.[5] Of course, the Meyer/Weinberg-approach also has its limitations. But compared, for example, with the econometric model approach[6], its data requirements are often smaller and thus can be better focused on cyclical changes.[7]

This paper examines and classifies West German business cycles using LDA, broadening a previous study by the same authors (Heilemann/Muench 1996). The paper explores the potential of a four stage classification scheme derived from a traditional two

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3 For an early 1970s perspective on such shifts, focusing mainly on changes in the price/wage sector and in "public policy in economic affairs", see, for example, Meyer, Weinberg 1975a, pp.169ff. For a broader view addressing "Sources of Greater Postwar Stability", "The Size of Government and Its Role as a Buffer", "The Full Employment Commitment and the Role of Stabilization Policy", "Structural Changes" in Various Markets, "Greater Wage and Price Stickiness: Causes and Consequences", "Impulses and Propagation in Components or Spending", see Gordon 1986, p.12, closely following Burns 1960, Gordon (ed.) 1986; more recently: Belongia, Garfinkel (eds.) 1992.

4 For an excellent study of both the subject and the possibilities and limitations of the reference cycle approach, see Zarnowitz, Moore 1986 and the following discussion. For a recent application to West German data, see Lucke 1995.

5 It is somewhat startling that even the rather comprehensive NBER "Studies in Business Cycles" volume edited by Gordon (ed.) 1986 does not mention the study, although it was an NBER study in that is evaluated the classification record of forecasts, too, and the NBER's 55th Annual Report gives a long summary of it. - For an examination and a modified update of the Meyer/Weinberg results for the U.S., see Heilemann 1982.

6 It does without refering to the peculiar understanding and treatment of cyclical factors in macroeconometric models, cf. on this e.g., Gordon 1986, p.27f.

7 For an attempt at classification of survey data following the Meyer/Weinberg (1975b) scheme for West Germany by employing factor analysis as well as cluster analysis, see Hartmann 1992.

phase scheme for West Germany. To shed some light on changes in the explanatory variables and their relative importance, the classifications are reiterated for various sample periods.

Though the Meyer/Weinberg results for the U.S. and for West Germany have been encouraging, at least for the time span covered by their studies (for the U.S.: 1948 to 1973, for West Germany: 1951 to 1967), the exploration of the subject and the use of the method for this purpose are still in their infancy. As to the method, the present study will concentrate on estimating classification functions for a set of cyclical relevant variables for several sample periods.

The paper is organized as follows. The next section (I) briefly discusses classification schemes and presents a new four phase scheme for the business cycle. Section II presents a short survey of the elements of LDA, the data, and the variables employed in the study. Section III discusses the results. The final section (IV) summarizes the findings and presents some conclusions for further work.

I. The classification scheme

Classification of business cycles in Germany looks back on a long tradition (for a comprehensive study of the problems involved, see Koerber-Weik 1983): *Spiethoff* (1925) published his highly elaborated six/nine phase scheme as early as the mid 1920s, but after WW II the replacement of phenomenological methods by more analytical ones in business cycle/short term analysis and forecasting reduced its use and influence considerably. Attempts to adjust classification schemes to the then emerging new reality of "growth cycles" (Wagner 1972; Tichy 1976, pp. 76ff.; Zarnowitz 1995, pp. 262ff.) did not alter this trend. Today, references to cycle classifications in Germany are to be found, for instance, in the presentation of cyclical indicators for manufacturing by the *Deutsche Bundesbank* (1995, p. 85), or in the reports of the Council of Economic Experts (Sachverstaendigenrat zur Begutachtung der gesamtwirtschaftlichen Entwicklung 1996, p. 286). The schemes used are simple two stage patterns, discriminating between upswing and downswing. Their main function is to support data assessment; references to the classifications in the proper analyses start only in the mid 90s and are still few. As mentioned above, Meyer/Weinberg (1975b) developed what was then a "modern" four phase scheme (Recession, Recovery, Demand Pull, and Stagflation) and tested it with some success for West Germany and other countries over the period 1951 to 1967. Their scheme and their results remained nearly unnoticed.

The establishment of a scheme to classify business cycles requires two steps: Firstly, the separation *between* the (complete) cycles, and secondly the separation of the various phases *within* the cycles (with, of course, implications for the prior separation between the cycles, as will be shown below). The separations of the cycles were made by computing a trend for industrial production. Intersections of trend and actual development were taken as *a priori* beginnings of upswings or downswings, respectively. The resul-

ting timing of cycles (table 1) does not differ much from that revealed by other classification techniques, e.g. GDP/GNP rates of change.

Table 1

A priori and final classification of West German business cycles into a four-stage scheme

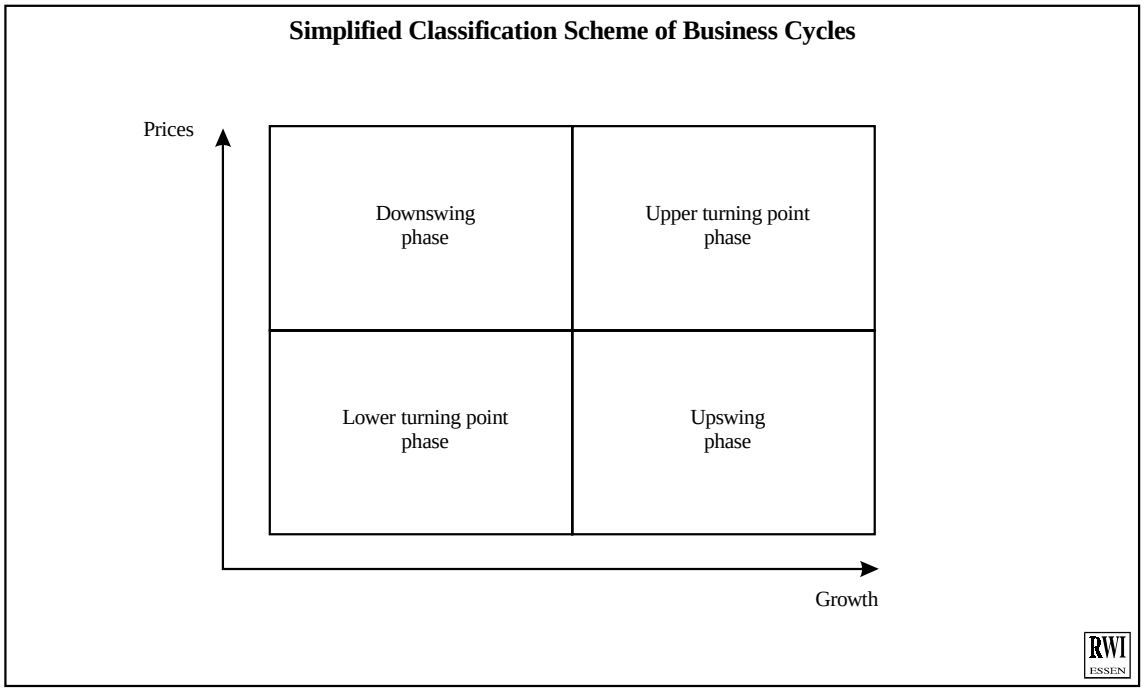
Cycle[1]	Starting quarter[2]			
	LTP	UP	UTP	DOWN
1 a) ... to 1953-1 ( )	...	...	1951-1 (2)	1951-3 (7)
b) ... to 1953-1 ( )	...	...	1951-1 (2)	1951-3 (7)
2 a) 1953-2 to 1958-2 (21)	1953-2 (4)	1954-2 (4)	1955-2 (4)	1956-2 (9)
b) 1953-2 to 1958-2 (21)	1953-2 (4)	1954-2 (4)	1955-2 (4)	1956-9 (9)
3 a) 1958-3 to 1962-2 (16)	1958-3 (4)	1959-3 (4)	1960-3 (4)	1961-3 (4)
b) 1958-3 to 1962-4 (18)	1958-3 (4)	1959-3 (3)	1960-2 (5)	1961-3 (6)
4 a) 1962-3 to 1966-4 (18)	1962-3 (4)	1963-3 (4)	1964-3 (4)	1965-3 (6)
b) 1963-1 to 1966-4 (16)	1963-1 (1)	1963-2 (6)	1964-4 (3)	1965-3 (6)
5 a) 1967-1 to 1971-1 (17)	1967-1 (4)	1968-1 (4)	1969-1 (4)	1970-1 (5)
b) 1967-1 to 1971-1 (17)	1967-1 (4)	1968-1 (6)	1969-3 (2)	1970-1 (5)
6 a) 1971-2 to 1974-2 (13)	1971-2 (4)	1972-2 (2)	1972-4 (4)	1973-2 (3)
b) 1971-2 to 1974-1 (12)	1971-2 (4)	1972-2 (2)	1972-4 (2)	1973-2 (4)
7 a) 1974-3 to 1982-2 (32)	1974-3 (4)	1975-3 (14)	1979-1 (4)	1980-1 (10)
b) 1974-2 to 1982-1 (32)	1974-2 (7)	1976-1 (13)	1979-2 (4)	1980-2 (8)
8 a) 1982-3 to 1994-1 (47)	1982-3 (4)	1983-3 (29)	1990-4 (4)	1991-4 (10)
b) 1982-2 to 1994-1 (48)	1982-2 (6)	1983-4 (27)	1990-3 (6)	1992-1 (9)
9 a) 1994-2 to ...	1994-2 (3)	-	-	-
b) 1994-2 to ...	1994-2 (1)	1994-3 (2)	-	-
All				
a) 1951-1 to 1994-4 (176)	(31)	(61)	(30)	(54)
b) 1951-1 to 1994-4 (176)	(31)	(63)	(28)	(54)

Authors' computations. - LTP: Lower Turning Point; UP: Upswing; UTP: Upper Turning Point; DOWN: Downswing. - 1) a: a priori classification, b: final classification. - 2) Cycle/phase lengths quarters in parentheses.

The within cycle classification started from a two phase classification, periods above the trend being labelled as "Upswing" (UP), the remaining as "Downswing" (DOWN). Then the two quarters preceding/following the upper and the lower turning points were earmarked as Upper Turning Point/Lower Turning Point (UTP/LTP) phases (cf. figure 1 and section III).

The differentiations between and within the cycles are the result of *a priori* classifications and their dates have to be tested and, if necessary, revised in the subsequent empirical analysis. Furthermore, while the between cycle discrimination is based on a well established practice, the within cycle classification relies to some degree on heuristic principles, though there are, of course, also good theoretical and empirical reasons for employing the variables (Tichy 1994, pp. 70ff.). Periods around the lower turning point

Figure 1



also mark the end, or the start of cycles here. In a theoretical sense this is, of course, not correct - cycles start where, in the upswing, the equilibrium stage is passed (Schumpeter 1961, pp. 158ff.). Unfortunately, such dates are hard to locate - even in an univariate framework.

II. Method, data, and variables

Method[8]

Modern discriminant analysis comprises a multitude of procedures for separation and classification of groups and objects. In the present analysis we restrict ourselves to the oldest (and most simple) technique, to multivariate linear discriminant analysis (LDA). The reasons for doing so are (see Erb 1990, p. 5):

- *Robustness*. In spite of comparatively restrictive conditions LDA has proved to be a robust method even in those cases where some assumptions (variables from multivariate normal distributions and equal covariance matrices for all groups) are violated.

8 For a broader portrayal of the literature on the method and recent developments, see, e.g. *Sonderforschungsbereich* 475 (1996), pp. 230ff.; Roehl/Weihs (1998).

- *Applicability*. In contrast to most other discriminant techniques restricted to classification of new individuals, LDA also allows an analysis and explanation of group differences. For that reason, it is often employed in those cases where an optimal separation of groups is the dominant goal.

- *Clarity*. Because of the linear character of discriminant functions, the results of LDA are easy to interpret and to present.

- *Availability*. LDA is part of all major statistical packages like BMDP, SAS, and SPSS - though with different computing routines.[9]

The main objective of LDA (and, of course, any other discriminant method) is to classify objects by a set of independent variables x_1, ..., x_m into g given groups,

$$y_i = c_1 x_1 + ... + c_m x_m \qquad (1)$$

where

y_i: dependent (grouping) variable, with i = 1, ..., g (number of groups with $g \geq 2$);

x_j: independent variables, j = 1, ..., m;

c_j: coefficients.

For n cases, the observations $x_1,...,x_m$ of the m-dimensional criterion are given. The observations of the (n,m)-matrix

$$x = \begin{bmatrix} x_1' \\ \cdot \\ \cdot \\ \cdot \\ x_n' \end{bmatrix} = \begin{bmatrix} x_{11} & \cdot & \cdot & \cdot & x_{1m} \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ x_{n1} & \cdot & \cdot & \cdot & x_{nm} \end{bmatrix} \qquad (2)$$

arise from g different groups or classes, and so x can be partitioned into g (n_k,m)-submatrices (with $n = n_1 + ... + n_g$):

$$x = \begin{bmatrix} x_1 \\ \cdot \\ x_k \\ \cdot \\ x_g \end{bmatrix} \qquad (3)$$

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9 The present study employed the Multivariate Discriminant Analysis Routine of SPSS (1993).

with $x_k = \left(x_{k1},...,x_{ki},...,x_{kn_k}\right)'$ containing the observations from group G_k (k = 1, ..., g)

In the simple case of two groups, (3) reduces to

$$x_1^{'} = \left(x_{11},...,x_{1n_1}\right) \text{ and } x_2^{'} = \left(x_{21},...,x_{2n_2}\right) \tag{4}$$

By a linear transformation of the m-dimensional vector of observations x to a scalar, the m-dimensional problem becomes a 1-dimensional one:

$$y_i = c_1 x_1 + c_2 x_2 \tag{5}$$

In LDA, the coefficients (c_j) are estimated in such a way that the values of the discriminant function (5) differ as much as possible *between* the groups, or so that for the discriminant scores the ratio

$$\frac{\text{between-groups sum of squares}}{\text{within-groups sum of squares}} \tag{6}$$

is a maximum.

In the general case of $g \geq 2$ groups, a maximum number of min(m,g-1) discriminant functions can be derived. The first function has the largest ratio of between-groups to within-groups sums of squares. The second function is orthogonal to the first and has the next largest ratio, and so on. Because the coefficients of the different discriminant functions are derived from a classic eigenvalue problem (Erb 1990, p.36), special norming conditions have to be set up to achieve unique solutions.

The main questions to be answered by LDA are:

- How well do the variables discriminate between given groups?
- Which variables are good discriminators?
- What decision rule should be used for classifying (new) objects?

From a methodical perspective, using LDA for business cycle classification can be seen as a mix of the reference cycle approach as established by Burns/Mitchell (1947), the composite indicator approach, and regression analysis.[10] Classification is based on a

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10 An important difference between the reference cycle approach and the usual time series analysis is, of course, the role played by time. Multivariate discriminant analysis seems to share some limitations with the reference cycle approach, so the difficulties of both techniques are still disputed, see Zarno-

multivariate scheme instead of, as in the reference cycle approach, on an univariate, more or less production oriented variable (rates of change of real GDP). Therefore, the argument against employing the phenomenon to be explained ("the cycle") for classification is not of much relevance here. The variable necessary for the a priori classification (real GDP) is only one of a multitude of variables denoting the cycle.

Multivariate discriminant analysis as employed here, however, has one important flaw, though it shares it with the reference cycle approach: it depends on the mean values of the classifying variables and the distance from them as the classification criterion, thereby ignoring the "volatility" of these variables (for which variance may not always be a good indicator). This could be particularly harmful in the present context where an increased volatility is seen in the literature as a main attribute of recent cycles.

Data

The data used are quarterly data, though for some variables shorter periods would have been possible. The data are not seasonally adjusted. This is different from Meyer/Weinberg (1975a, b), but here this is in most cases balanced by frequent use of rates of change against previous year (Heilemann/Barabas 1996, p. 404), which has been confirmed by initial results with seasonally adjusted data.

The National Account (NA) data base covers the period 1951-1 to 1994-4. It should be kept in mind that the pre-1960 data are not official data but produced by Deutsches Institut fuer Wirtschaftsforschung 1972) excluding Berlin (West) and the Saarland, and that German unification and statistical consequences of the Single European Market spoil some post 1990 NA data. Tests within the framework of the present study indicate that this does not prevent the use of the post 1989 data in question. The monetary data (money supply and interest rates) are those of the *Deutsche Bundesbank*. Unfortunately long term interest rates and short term interest rates have only been available since 1955. In addition, up to the middle of the 60s they were not market data but controlled by the Deutsche Bundesbank.

Variables

The selection of the classifying variables is, of course, of crucial importance. It can be directed by cognitive and policy criteria or tradition ("stylized facts"). We followed a mixed strategy and started with a set of variables traditionally used in macroeconomic short term analysis. However, we restricted the selection to NA variables and other "objective" variables such as monetary aggregates, following more the route of macroeconometric modelling than that of composite indicators.

The detailed selection of variables was made in the following way. In a first step, more than 20 variables were chosen on the basis of literature and experience. The list included GNP, Private consumption, Government consumption, GNP price deflator, Consumer

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witz, Moore 1986, p. 578 (discussion).

price deflator, Fixed investment, Investment in machinery, Construction outlays, Housing construction, Hourly wages, Unit labour cost, Short term interest rate, Long term interest rate, Money supply M1, M2, M3, Unemployment rate, Net exports, and Government deficit (on this list see also Meyer, Weinberg 1975a, table 3, p.179). Most variables were used in nominal and real terms and as a percentage of GNP. Formal selection criteria as listed in Brosius (1989) together with the idea of having all the important fields of economic activity (supply/demand, labour market), prices, monetary sphere (money aggregates, fiscal deficit, interest rates)) represented, resulted in the following list of twelve variables: Wage and salary earners, Real GNP, Government deficit as percent of GNP, Net exports as percent of GNP, Money supply M1, Unit labor cost, GNP price deflator, Consumer price index, Real long term interest rate, Real private consumption, Real investment in commercial construction and Short term interest rate (see figure 1* (Appendix)). From an economic point of view, the list has a number of deficiencies - e.g. the exclusion of Investment in machinery, or, of Changes in stocks - which definitely requires future research.

III. Classification Results

Our scheme (classification, variables) was tested over the period 1955-4 to 1994-4, the (present) lack of adequate pre 1955 interest rate data prohibiting an earlier start. In general, the results corroborate the scheme. But they also suggest that in the early sixties West Germany's cyclical behaviour underwent considerable changes. The presentation of results will therefore be split: first, we render the rather stable results for the period 1963-1 to 1994-4, then we report on deviations from this pattern in the 1955 to 1962 and the 1963 to 1994 periods.

The 1963-1 to 1994-4 period

The discriminant functions and the classification results for this period confirm the classification scheme remarkably well. The "final" cycle classification (table 1) is generally met, no phases are missing (though, of course, this is in no way a necessary condition of the course of business cycles - neither in other schemes nor in our scheme (Meyer/Weinberg 1975b, pp.7ff., or, e.g., Adams 1925, pp.206f.). While the a priori fixed 4 quarter lengths of the LTP/UTP-phases are modified and now range between 1 and 7 and 2 and 6 quarters, respectively, the average value in the period studied is about 4 quarters for the LTP-stage and 3 quarters for the UTP-stage. The relative duration of these phases varies only between 6 and 12 percent of the full cycle length (LTP phase) and 12 and 25 percent (UTP phase). From a full cycle perspective, but also when compared to the duration of UPs and DOWNs (average values 9 and 6 periods respectively), both phases are in general of lesser importance, though the unusually short sixth cycle (1971-2 to 1974-1) is notable also in this respect. A short duration of the post war DOWNs in industrialized countries is common "cyclical" wisdom.

Table 2

Average values of classifying variables 1953-2 to 1994-1

	Cycle	LTP	UP	UTP	DOWN	Complete Cycle
		Wage and salary earners[1]				
2:	1953-2 to 1958-2	3.9	4.6	5.5	2.5	3.7
3:	1958-3 to 1962-4	1.1	2.5	2.7	1.6	1.9
4:	1963-1 to 1966-4	0.9	1.2	1.4	0.4	0.9
5:	1967-1 to 1971-1	-3.4	1.4	2.6	2.4	0.7
6:	1971-2 to 1974-1	1.3	1.0	1.5	1.34	1.3
7:	1974-2 to 1982-1	-1.9	0.8	2.4	0.5	0.
8:	1982-2 to 1994-1	-1.4	1.1	3.0	-0.5	0.7
	All	-1.4	1.0	2.4	1.0	0.7
		Real GNP[1]				
2:	1953-2 to 1958-2	7.3	9.2	11.8	5.1	7.6
3:	1958-3 to 1962-4	6.2	8.8	7.6	4.0	6.3
4:	1963-1 to 1966-4	-1.4	5.7	5.4	3.6	4.4
5:	1967-1 to 1971-1	-0.3	6.3	7.2	5.7	4.7
6:	1971-2 to 1974-1	2.7	3.6	6.0	3.5	3.7
7:	1974-2 to 1982-1	-0.8	3.8	4.3	-0.1	1.9
8:	1982-2 to 1994-1	0.1	3.0	5.4	-0.3	2.3
	All	0.2	3.8	5.4	1.9	2.9
		Real private consumption[1]				
2:	1953-2 to 1958-2	10.0	7.0	11.0	6.2	8.0
3:	1958-3 to 1962-4	6.7	4.6	8.0	5.8	6.4
4:	1963-1 to 1966-4	2.7	4.0	6.4	4.5	4.6
5:	1967-1 to 1971-1	1.1	5.8	7.9	7.5	5.4
6:	1971-2 to 1974-1	5.5	4.3	4.0	2.0	3.9
7:	1974-2 to 1982-1	2.1	4.0	3.8	-0.3	2.5
8:	1982-2 to 1994-1	-0.1	2.8	5.7	1.3	2.5
	All	1.9	3.5	5.4	2.6	3.2
		Real investment in (commercial) construction[1]				
2:	1953-2 to 1958-2	15.6	13.7	19.5	3.3	10.7
3:	1958-3 to 1962-4	11.6	10.0	5.2	2.0	6.4
4:	1963-1 to 1966-4	-22.1	6.2	5.0	3.1	3.0
5:	1967-1 to 1971-1	-15.9	7.7	12.5	14.5	4.7
6:	1971-2 to 1974-1	5.0	-0.4	2.7	-3.9	0.7
7:	1974-2 to 1982-1	-9.7	3.1	9.3	-0.9	0.1
8:	1982-2 to 1994-1	1.4	3.1	1.6	-1.6	1.8
	All	-5.5	3.7	5.4	1.7	1.7
		Net exports as percent of GNP				
2:	1953-2.to 1958-2	3.8	2.8	2.2	3.7	3.3
3:	1958-3 to 1962-4	3.8	3.5	2.8	1.3	2.7
4:	1963-1 to 1966-4	0.9	1.6	0.5	1.0	1.1
5:	1967-1 to 1971-1	3.5	3.4	3.1	2.1	3.0
6:	1971-2 to 1974-1	1.8	1.5	3.0	3.7	2.6
7:	1974-2 to 1982-1	3.5	2.6	0.5	0.6	2.1
8:	1982-2 to 1994-1	2.4	5.1	6.6	7.1	5.3
	All	3.0	3.9	3.3	3.2	3.5
		Government deficit as percent of GNP				
2:	1953-2 to 1958-2	9.1	5.2	5.3	3.4	5.2
3:	1958-3 to 1962-4	1.5	2.5	3.5	1.9	2.4
4:	1963-1 to 1966-4	1.9	0.8	-0.1	-0.4	0.3
5:	1967-1 to 1971-1	-1.4	-0.2	1.3	0.4	-0.2
6:	1971-2 to 1974-1	-0.3	-1.2	0.8	0.6	0.0
7:	1974-2 to 1982-1	-3.8	-2.8	-2.6	-3.7	-3.2
8:	1982-2 to 1994-1	-2.5	-1.4	-3.4	-2.6	-2.0
	All	-2.0	-1.4	-1.6	-1.6	-1.6

Table 2, continued

	Cycle	LTP	UP	UTP	DOWN	Complete Cycle
			GNP price deflator			
2:	1953-2 to 1958-2	-0.4	0.3	2.3	3.6	2.0
3:	1958-3 to 1962-4	1.1	2.2	3.4	4.3	3.0
4:	1963-1 to 1966-4	4.2	2.7	4.1	3.5	3.4
5:	1967-1 to 1971-1	1.6	2.7	5.0	7.4	4.1
6:	1971-2 to 1974-1	7.2	5.1	5.6	6.3	6.3
7:	1974-2 to 1982-1	6.5	3.9	4.1	4.6	4.7
8:	1982-2 to 1994-1	3.8	2.3	3.7	3.7	2.9
	All	4.8	2.8	4.2	4.8	3.8
			Consumer price index[1]			
2:	1953-2 to 1958-2	-1.3	1.1	1.7	2.7	1.5
3:	1958-3 to 1962-4	0.5	1.6	1.9	3.1	2.0
4:	1963-1 to 1966-4	3.3	2.5	2.5	3.7	3.0
5:	1967-1 to 1971-1	1.6	1.7	2.1	3.8	2.3
6:	1971-2 to 1974-1	5.3	5.4	5.8	6.8	5.9
7:	1974-2 to 1982-1	6.5	3.4	4.7	6.0	4.9
8:	1982-2 to 1994-1	4.2	1.5	3.5	3.5	2.5
	All	4.5	2.3	3.7	4.6	3.4
			Unit labor cost[1]			
2:	1953-2 to 1958-2	3.1	0.7	2.1	5.2	3.3
3:	1958-3 to 1962-4	0.7	-0.2	5.4	6.5	3.8
4:	1963-1 to 1966-4	6.5	2.6	4.8	5.2	4.2
5:	1967-3 to 1971-1	0.3	2.2	6.1	12.4	5.2
6:	1971-2 to 1974-1	8.9	6.1	6.5	8.8	8.0
7:	1974-2 to 1982-1	8.1	3.5	3.8	6.3	5.3
8:	1982-2 to 1994-1	2.2	1.5	2.3	3.5	2.1
	All	4.9	2.2	4.0	6.6	4.0
			Short term interest rate			
3:	1958-3 to 1962-4	3.2	3.9	4.7	3.5	3.8
4:	1963-1 to 1966-4	3.4	4.0	4.7	6.4	5.0
5:	1967-1 to 1971-1	4.3	4.0	7.2	8.9	5.9
6:	1971-2 to 1974-1	6.4	4.8	7.9	12.7	8.5
7:	1974-2 to 1982-1	6.8	4.1	7.8	10.8	6.8
8:	1982-2 to 1994-1	6.9	5.4	9.1	8.1	6.6
	All	6.1	4.8	7.6	9.2	6.5
			Real long term interest rate			
3:	1958-3 to 1962-4	4.9	3.8	2.7	4.9	3.1
4:	1963-1 to 1966-4	2.0	3.5	2.4	4.0	3.4
5:	1967-1 to 1971-1	5.4	3.9	2.4	0.4	3.1
6:	1971-2 to 1974-1	1.0	3.2	3.0	3.6	2.5
7:	1974-2 to 1982-1	3.1	3.0	4.0	5.1	3.7
8:	1982-2 to 1994-1	4.6	4.5	5.2	3.4	4.4
	All	3.5	4.0	3.8	3.5	3.8
			Money supply M1[1]			
2:	1953-2 to 1958-2	10.1	9.1	11.2	10.2	10.2
3:	1958-3 to 1962-4	12.7	13.8	7.8	11.3	11.1
4:	1963-1 to 1966-4	8.0	7.7	9.0	5.7	7.2
5:	1967-1 to 1971-1	3.4	7.6	8.6	6.9	6.5
6:	1971-2 to 1974-1	13.1	13.3	13.6	2.8	9.8
7:	1974-2 to 1982-1	11.1	10.6	5.2	1.7	7.8
8:	1982-2 to 1994-1	7.7	6.9	18.5	7.7	8.6
	All	9.1	8.3	12.0	5.1	8.1

Authors' computations. - LTP: Lower Turning Point; UP: Upswing; UTP: Upper Turning Point; DOWN: Downswing. - 1) Rates of change.

For a further economic evaluation, table 2 presents the average values for the classifying variables. They more or less confirm common expectations about their cyclical behaviour. In a simplified form, the phases might be portrayed as follows:

- The *LTP-phase* is characterized by stagnating demand, negative Investment in commercial construction, shrinking Employment, declining Unit labour cost (compared to DOWN), Short term interest rates going down and Money supply already expanding;

- the *UP-phase* is the period in which demand strongly expands, inflation is low, Short term Interest rates are declining (compared to DOWN) and Employment begins to increase;

- the *UTP-phase* is marked by a further expansion of demand - induced internally as well as externally, rising Employment despite inflating Unit labour cost and prices, declining Real long term interest rates and rising Short term interest rates, and increasing Money supply;

- the *DOWN-phase*, finally, reflects declining demand, further increases of Unit labour cost and prices, low Employment expansion, declining Real long term interest rates, a sharp decline in Money supply, and a further increase in Short term interest rates.

A detailed distinction between pure cyclical factors and policy in this picture is beyond the scope of this paper. As table 2 suggests, policy is a latecomer: Money supply shows its biggest expansion in the UTP-stage with Real long term interest rates declining and Government deficit rising again. Unfortunately, the average values do not reveal an unequivocal picture of the West German business cycle. As rather clear tendencies evolve that, first, the duration of the phases is relatively constant; second, the cyclical movement of the various variables is rather stable; third, and not too surprisingly, the cyclical strength (as measured by "average growth" of the variables) varies considerably, with the sixth cycle (1971-2 to 1974-1) being the strongest. As to the length of the cycles/phases, one could object that the LTP/UP-phase of recent cycles did not become shorter as sometimes stipulated for the downward phase of the *Kondratieff*-cycle (Zarnowitz, Moore 1986, p. 523), but for a discussion of this our evidence is too brief. The shorter duration of the DOWNs is in accordance with the general cyclical experience (Tichy 1994, p. 51, Zarnowitz 1996, p. 77). But all in all, most of these findings are rather difficult to interpret and deserve much more consideration of the subject-matter than can be given here.

Table 3

Estimation records for various standardized canonical discriminant functions[1]
1955 to 1994

Variable		Coefficients Function 1	2	3	F-Value to enter
Wage and salary earners	a	-.75	-.70	.00	33.2
	b	.89	-.66	.12	32.9
	c	1.17	-.38	.28	34.8
Real GNP	a	-.69	-.50	-.34	20.0
	b	.52	-.52	-.47	20.3
	c	.49	-.57	-.35	25.6
Real private consumption	a	.80	-.12	.87	10.8
	b	-.76	-.09	.98	9.6
	c	-.82	-.09	.63	9.3
Real investment in commercial construction	a	.16	.23	.04	6.3
	b	-.19	.20	-.08	5.2
	c	-.18	-.02	-.35	11.7
Net exports as percent of GNP	a	-.36	.02	-.25	1.8
	b	.50	.06	-.15	1.9
	c	.51	.15	-.41	1.2
Government deficit as percent of GNP	a	.73	.46	.15	2.7
	b	-.37	.50	.37	0.8
	c	-.13	.49	.20	0.3
GNP price deflator	a	-1.36	1.98	1.67	12.6
	b	1.83	2.18	1.41	13.9
	c	1.33	2.46	1.25	16.8
Consumer price index	a	.51	-.58	-.26	13.6
	b	-.43	-.60	-.10	16.0
	c	-.25	-.38	-.10	22.9
Unit labor cost	a	.50	-.17	-.93	18.6
	b	-.55	-.07	-.73	18.3
	c	-.23	-.37	-1.13	15.4
Short term interest rate	a	1.49	-1.04	-.07	17.8
	b	-1.87	-1.14	-.03	23.5
	c	-1.85	-1.30	.18	44.8
Real long term interest rate	a	-.77	1.41	1.06	2.4
	b	1.30	1.65	1.09	2.0
	c	1.28	1.58	.58	0.9
Money supply M1	a	.53	.12	.47	5.6
	b	-.46	.21	.50	7.0
	c	-.30	.04	.61	10.4

Table 3, continued

Eigenvalues

	Function[2]	Eigen-val.	% of variance	cum. %	canonical correl.		after function	Wilks' λ	χ^2	df	Significance
						:	0	.2	273.9	36	.00
	1*	1.4	52.0	52.0	.77	:	1	.4	141.9	22	.00
a	2*	1.1	38.7	90.6	.72	:	2	.8	34.2	10	.00
	3*	.3	9.4	100.0	.45	:					
						:	0	.1	288.7	36	.00
	1*	1.9	56.3	56.3	.81	:	1	.4	142.6	22	.00
b	2*	1.2	35.0	91.2	.74	:	2	.8	35.6	10	.00
	3*	.3	8.8	100.0	.48	:					
						:	0	.1	341.3	36	.00
	1*	3.5	61.5	61.5	.88	:	1	.3	162.8	22	.00
c	2*	1.8	31.0	92.5	.80	:	2	.7	42.3	10	.00
	3*	.4	7.5	100.0	.55	:					

Authors' computations. Eigenval: Eigenvalues of the discriminant functions in declining order. % of variance: % importance of the discriminant functions. cum %: Cumulative importance in relative terms. df: degrees of freedom. For a detailed description of the statistics see Brosius (1989). - 1) a: Results for period 1955-4 to 1994-4, b: 1958-3 to 1994-4, c: 1963-1 to 1994-4. - 2) * marks the 3 canonical discriminant functions remaining in the analysis.

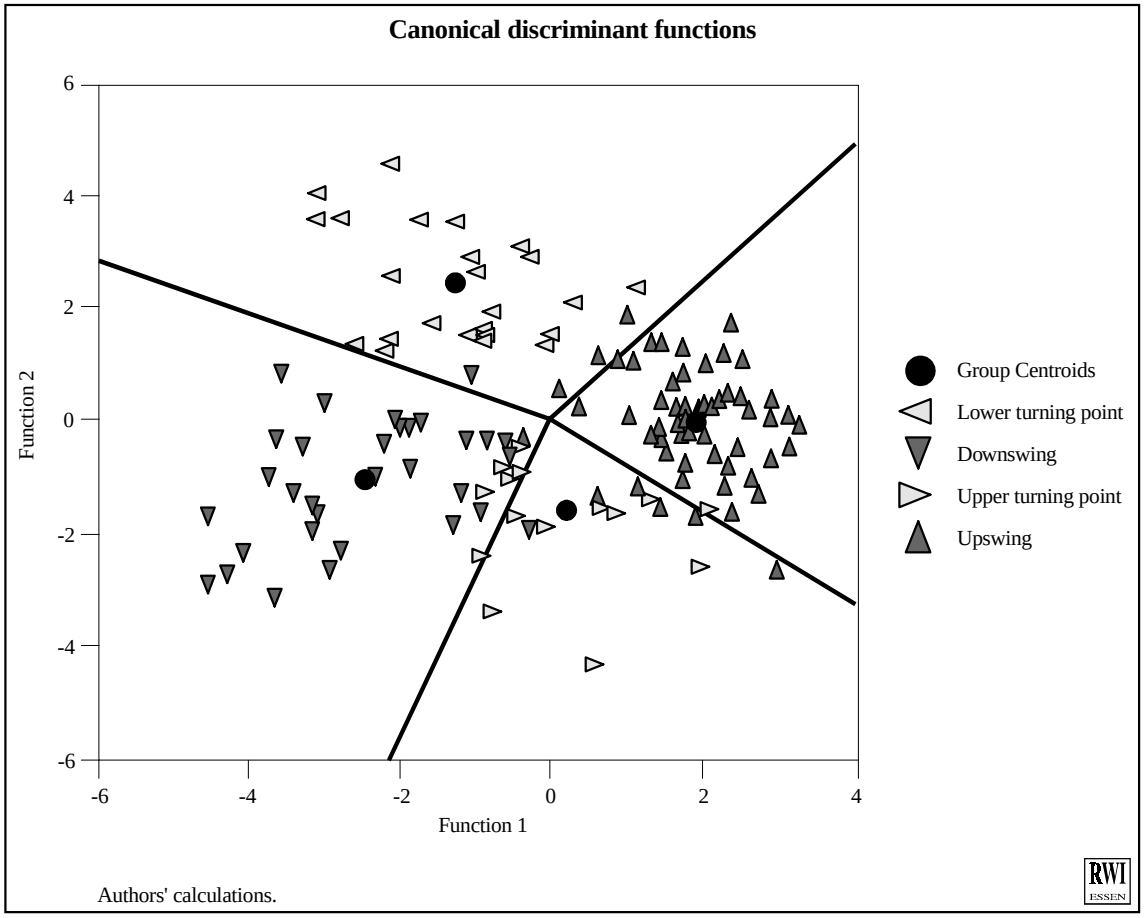
The twelve variables selected for the analysis were rather successful in separating the established classification. Using all the observations available, table 3 shows that the explanatory power of the first discriminant function, measured by the eigenvalue ratio, is not too impressive at 61 % and not much ahead of the second (31 %). The third function with about 8 % is considerably lower, but Wilk's λ and the corresponding χ^2 value indicate that the function still contributes significantly to the separation of groups. The F-values disclose, however, that the means of the four groups - when tested separately - are statistically different for only nine of the twelve variables.

It may be interesting to note that a classification based solely on the rates of change of real GNP (as assumed by conventional schemes) led to only 43 % of correct classications, with particularly disappointing results for the UPs (43 %) and the DOWNs (16 %).

Before looking at the weights given by the first two canonical discriminant functions to the classifying variables it should be noted that the ranking of the F-ratio values (table 3) shows some similarities with the Meyer/Weinberg (1975a, table 5, p.182) results for the U.S. This holds in particular for the more important classifying variables such as employment, real GNP, GNP deflator - despite the structural differences of the two countries (e.g. export ratios/behavior), their different performances and the different sets of variables (in the U.S. analysis e.g., stock exchange data, money aggregates), their periodicity (monthly data in the U.S. analysis) and the different periods covered by the two

analyses.[11] While the rank of the coefficients of the classifying variables in the first and second standardized canonical discriminant functions of the two studies seems to be not too different (rank correlation coefficients are > 0.6), their weights differ significantly.

Figure 2



Considerable differences of the first and the second standardized canonical discriminant function must be noted for GNP-deflator, Wage and salary earners, Private consumption, and Short term interest rates. This suggests that the (first) function differentiates between UP and DOWN. The classification behaviour implied by the discriminant functions can be illustrated by plotting the discriminant functions' scores and their group centroids (figure 2). The first canonical function is plotted against the horizontal axis and the second against the vertical axis. As already indicated by the standardized coefficients (and by the average values of the classifying variables), the picture is not so clear-cut as one would like to have it. Nonetheless, it should be noted that, first, the group centroids are well separated; second, the UP and DOWN phases as well as the in-between phases face each other as expected.

11 Even a standardization of the sample periods of the two studies is not yet possible.

Table 4

Classification results for different samples[1]
within the 1963-1 to 1994-4 period

Actual group	No. of cases	Predicted group membership LTP	UP	UTP	DOWN
		Whole sample period			
LTP	23	23 100.0%	0 .0%	0 .0%	0 .0%
UP	56	1 1.8%	53 94.6%	2 3.6%	0 .0%
UTP	17	0 .0%	1 5.9%	16 94.1%	0 .0%
DOWN	32	1 3.1%	0 .0%	1 3.1%	30 93.8%
Total error rate: 4.7%					
		Leave one out			
LTP	23	22 95.7%	1 4.3%	0 .0%	0 .0%
UP	56	2 3.6%	51 91.1%	3 5.3%	0 .0%
UTP	17	0 .0%	1 5.9%	12 70.6%	4 23.5%
DOWN	32	1 3.1%	0 .0%	5 15.6%	26 81.3%
Total error rate: 13.7%					
		Without cycle 4 (1963-1 to 1966-4)			
LTP	1	0 .0%	1 100.0%	0 .0%	0 .0%
UP	6	0 .0%	6 100.0%	0 .0%	0 .0%
UTP	3	0 .0%	1 33.3%	2 66.7%	0 .0%
DOWN	6	1 16.7%	2 33.3%	2 33.3%	1 16.7%
Total error rate: 43.8%					

Table 4, continued

Actual group	No. of cases	Predicted group membership: LTP	UP	UTP	DOWN
	Without cycle 5 (1967-1 to 1971-1)				
LTP	4	1 25.0%	2 50.0%	0 .0%	1 25.0%
UP	6	0 .0%	6 100.0%	0 .0%	0 .0%
UTP	2	0 .0%	0 .0%	2 100.0%	0 .0%
DOWN	5	0 .0%	0 .0%	1 20.0%	4 80.0%
Total error rate: 23.6%					
	Without cycle 6 (1971-2 to 1974-1)				
LTP	4	4 100.0%	0 .0%	0 .0%	0 .0%
UP	2	1 50.0%	1 50.0%	0 .0%	0 .0%
UTP	2	0 .0%	0 .0%	2 100.0%	0 .0%
DOWN	4	0 .0%	0 .0%	0 .0%	4 100.0%
Total error rate: 8.2%					
	Without cycle 7 (1974-2 to 1982-1)				
LTP	7	7 100.0%	0 .0%	0 .0%	0 .0%
UP	13	2 15.4%	10 76.9%	0 .0%	1 7.7%
UTP	4	0 .0%	2 50.0%	0 .0%	2 50.0%
DOWN	8	0 .0%	0 .0%	0 .0%	8 100.0%
Total error rate: 21.9%					

Table 4, continued

Actual group	No. of cases	Predicted group membership LTP	UP	UTP	DOWN
	Without cycle 8 (1982-2 to 1994-1)				
LTP	6	4 66.7%	0 .0%	1 16.7%	1 16.7%
UP	27	0 .0%	25 92.6%	2 7.4%	0 .0%
UTP	6	0 .0%	1 16.7%	5 83.3%	0 .0%
DOWN	9	5 55.6%	1 11.1%	1 11.1%	2 22.2%

Total error rate: 25.0%

Authors' computations. - LTP: Lower Turning Point; UP: Upswing; UTP: Upper Turning Point; DOWN: Downswing. - 1) For detailed results see table 1* and 2* in the appendix.

The discriminant analysis over the period 1963-1 to 1994-4 - in econometric modelling terms: an ex post analysis within the sample period - correctly classifies more than 95 % of all cases (table 4). Somewhat surprisingly, there are no great differences in this respect between the traditional UPs and DOWNs on the one hand and UTP and LTP stages on the other (table 1*, table 2*, Appendix). Difficulties in matching the final classification come up only occasionally. Three out of the six misclassifications occur in the fourth cycle (1963-1 to 1966-4)[12], the remaining three in the last three cycles. The sometimes suggested (premature) end of the seventh cycle in 1977/78 and of the eighth in 1987 are rejected by these results.

Because of the upward bias that results from classifying the same data which were used to calculate the discriminant functions, *Lachenbruch* (1967) suggested the application of the leave-one-out method, where an element (period) is classified by discriminant functions which have been estimated on the basis of the remaining n-1 periods. This procedure is repeated as long as all periods have been classified, generating a nearly unbiased estimation of the ex post forecast outside the sample period. The percentage of correctly classified cases is reduced from 95 to 87 percent, or, in absolute figures, the number of not correctly classified cases increases from 6 to 17. Table 2*, appendix, illustrates that this deterioration is mostly due to misclassifications in borderline periods.

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12 More on this below. It should be noticed that Meyer/Weinberg (1975b, pp.7f.) also had difficulties with this cycle. They could not identify the "stagflation phase" which should have happened around early 1965 - exactly about the same time at which our scheme also encounters difficulties.

Table 5

Classification errors for selected sample periods[1]

	Final[2]	1962-4	from 1955-4 to 1966-4	1971-1	1974-1
1956-2	DOWN				UTP
1957-1	DOWN		UTP	UP	UP
1957-2	DOWN		UTP	UTP	LTP
1957-3	DOWN	UTP			UTP
1961-2	UTP				DOWN
1961-4	DOWN				LTP
1962-2	DOWN				UTP
1962-3	DOWN		UTP	UP	UP
1962-4	DOWN			UTP	
1963-1	UTP			DOWN	DOWN
1964-4	UTP			UP	LTP
1965-1	UTP			UP	UP
1966-4	DOWN			DOWN	LTP
1968-1	UP			LTP	
1969-1	UP			UTP	UTP
1970-2	DOWN				UTP
1972-2	UP				
1972-3	UP				DOWN
1972-4	UTP				DOWN

Author's computations. - 1) LTP: Lower Turning Point; UP: Upswing; UTP: Upper Turning Point; DOWN: Downswing. The shaded areas present the sample periods. - 2) Final classification to enter the discriminant analysis.

Changes

As stated before, the question of changes in West German cyclical behavior arises here on two levels. Firstly, we have to deal with the question of changes in the overall cycle pattern; secondly, we have to look for changes within the system. Of course, changes *between* cycles may be consequences of changes *within* cycles.

Missing interest rate data restrict - for the time being - the start of our analysis to 1955-4. In addition, there are no data available for commercial construction in the 1950s so this variable was replaced by total investment in construction.

The estimation results of the discriminant functions over the full sample period (1955/1994) and for 1958/1994, covering only complete cycles, are presented in table 3. Though an interpretation of the coefficients is difficult, the results seem to indicate mainly a shift in the importance of the Short term interest rate and Unit labor cost. The

explanatory power of the functions point at a greater complexity of the cyclical classification in the fifties, since over the complete cycle the first discriminant function explains only 50.7 % of the variance against 62 % for the two other samples. The classificatory or explanatory power of the discriminant functions estimated over the complete sample period was very poor in the 1955/1962 period: 15 out of 29 quarters were misclassified and even for the period 1963-1 to 1994-4 period classification errors increase from 6 to 12. Table 5 indicates these fundamental changes in West Germany's cyclical behavior at the beginning of the full employment period in the late fifties by presenting increasing sample periods all starting in 1955-4. These difficulties reach well into the sixties, as table 4 reflects.

To look for changes within the 1963/1994 time span, classifications were repeated, subsequently excluding the fourth cycle, the fifth, etc. from the data base (table 4). The results in this ex post analysis outside the sample period are rather straightforward: noteworthy shifts are only experienced for the classifications deleting the fourth (1963/66) and the eighth cycle (1982/94), respectively, expressing an exceptional character of the two cycles.[13] This is not too surprising in the case of the eighth cycle: Given the extraordinary effects caused by German unification (revealed here by the values of Government deficit, Net exports, Unit labour cost, etc.), classification difficulties could well be expected. As to the fourth cycle, such an easy explanation for missing the downturn phase in 1966-1 and in 1966-3 is unfortunately not at hand. Seen from the basis of the average values (table 2), there is some evidence that the real side (Wage and salary earners, Real GNP, Real investment in commercial construction, Net exports) showed an unusual behaviour. The long UPs and the imminent DOWNs obviously compressed the UTP-phase.

The average values also point at changes in pricing behaviour in the 1970s ("stagflation"). While previously Inflation rates had stagnated or had shown some decline during the downswing, they now markedly increased in the sixth (1971/74) and seventh cycle (1974/82) - as a consequence of the oil price shocks. A tendency which was, however, already reversed in the following cycle.

Changes in the classification importance of the variables can also be detected by looking at their F-values/significance. Excluding the eighth cycle (1983/94), the Deficit share and the Net export share are significant for no classification - which makes sense given their history. Other variables such as Unit labour cost or the Long term real interest rate occasionally lose their classificatory meaning. (Again, some of this can already be inferred from the behaviour and structure of average values.) However, it is difficult to discover a longer lasting or strong tendency - i.e. an increase in the importance of monetary variables or of policy variables as suggested for instance by Meyer/Weinberg (1975a, pp.169 for the post-WW II period), compared with the pre-WW II period, not to speak of *Borchardt*'s (1981) findings.

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13 This is supported by a closer look at the discriminant functions for the various sample periods as well as the average values of the classifying variables. The present framework, however, does not allow their presentation, but the material is available from the authors upon request.

IV. Summary and conclusions

The study demonstrates that the duration and stages of (West) German business cycles between 1963 and 1994 can be well described by a simple four phase multivariate cycle classification scheme. The statistical properties of the linear discriminant analysis (LDA) used for classification appear satisfactory, their twelve classifying variables and their weights (coefficients) plausible, and the explanatory power of the functions within and outside the sample period acceptable. Difficulties with the scheme/variables arise mainly in the downswing period 1965-3 to 1966-4 in the fourth cycle, problems also encountered by previous analyses. While for the 1963/94 period the results display all in all much stability, the 1955/62 period does not fit into this pattern, indicating a change of the regime which might even reach into the fourth cycle (1963 - 1966). Though LDA is much richer than this, it is, however, beyond the possibilities of the present exploration to exploit them more completely.

So further research in this field should therefore address three issues: Firstly (and least difficult to perform), how sensitive is the analysis with respect to the inclusion of further/different variables (e.g., "stylized facts")? Such an extension can be guided equally by cognitive or by policy interests. The inclusion of shorter period data could help to improve the indicator or early warning function of classification. Secondly, the taxonomy used here should be tested against other schemes for West Germany, e.g., that used by Meyer/Weinberg (1975b), or that by *Helmstaedter* (1992). Thirdly, the analysis of changes should be broadened by inclusion of the first two post war cycles of West Germany. This should help us to find more satisfying answers as to changes in the West German cycle patterns and would be another step towards an analysis of the cyclical behaviour and its changes.

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Appendix

Table 1*

Posterior probability classifications[1,2]

1963 - 1994

	Actual	Highest probability			2nd highest		Scores for function		
	group	group	P(D/G)	P(G/D)	group	P(G/D)			
							1	2	3
1963-1	LTP	LTP	.108	.710	UP	.287	1.119	2.403	.931
1963-2	UP	UP	.969	.996	UTP	.004	2.024	.324	-.671
1963-3	UP	UP	.881	.971	UTP	.028	1.496	-.539	-.783
1963-4	UP	UP	.343	.941	UTP	.058	1.426	-1.484	-1.265
1964-1	UP	UP	.493	.992	UTP	.009	2.256	-1.143	-1.284
1964-2	UP	UP	.828	.969	UTP	.031	2.458	-.428	.294
1964-3	UP	UP	.914	.898	UTP	.101	1.353	-.217	.126
1964-4	UTP	UTP	.404	.565	DOWN	.364	-.665	-.766	.195
1965-1	UTP	UP	.935	.957	UTP	.043	1.923	-.001	.338
1965-2	UTP	UTP	.785	.917	DOWN	.065	-.591	-.931	1.175
1965-3	DOWN	DOWN	.592	.741	UTP	.250	-1.222	-1.188	.052
1965-4	DOWN	DOWN	.383	.501	UTP	.497	-1.309	-1.747	.562
1966-1	DOWN	UTP	.261	.438	DOWN	.391	-.613	-.327	.083
1966-2	DOWN	DOWN	.273	.541	UTP	.270	-.559	-.506	-.455
1966-3	DOWN	LTP	.412	.766	DOWN	.181	-1.053	.889	-.046
1966-4	DOWN	DOWN	.306	.977	LTP	.020	-1.758	.080	-1.904
1967-1	LTP	LTP	.315	.994	DOWN	.006	-2.099	2.617	-1.304
1967-2	LTP	LTP	.226	1.000	DOWN	.000	-1.241	3.588	-1.398
1967-3	LTP	LTP	.441	.995	UP	.005	-.244	2.961	-.804
1967-4	LTP	LTP	.432	.918	UP	.080	.332	2.146	.494
1968-1	UP	UP	.397	.999	LTP	.001	2.269	1.202	-1.490
1968-2	UP	UP	.871	.982	UTP	.014	1.591	.731	.004
1968-3	UP	UP	.895	.996	UTP	.004	2.603	.234	-.134
1968-4	UP	UP	.037	.968	UTP	.032	2.959	-2.582	-1.109
1969-1	UP	UP	.385	.933	UTP	.067	2.878	-.635	.968
1969-2	UP	UP	.622	.956	UTP	.044	2.628	-.989	.160
1969-3	UTP	UTP	.928	.914	UP	.083	.625	-1.442	.947
1969-4	UTP	UTP	.523	.637	UTP	.348	-.915	-1.251	.4810
1970-1	DOWN	DOWN	.130	1.000	UTP	.000	-3.672	-3.032	-.704
1970-2	DOWN	DOWN	.460	.997	UTP	.003	-2.931	-2.531	-.465
1970-3	DOWN	DOWN	.822	1.000	UTP	.000	-3.402	-1.188	-.581
1970-4	DOWN	DOWN	.687	1.000	UTP	.000	-3.163	-1.860	-.996
1971-1	DOWN	DOWN	.332	.568	UTP	.313	-.887	-.261	.112
1971-2	LTP	LTP	.486	.979	UTP	.014	-.870	1.592	1.583
1971-3	LTP	LTP	.324	.950	DOWN	.045	-2.193	1.339	1.513
1971-4	LTP	LTP	.732	.993	DOWN	.006	-1.572	1.801	1.223
1972-1	LTP	LTP	.221	.846	UTP	.092	-.057	1.405	1.686
1972-2	UP	UP	.409	.962	LTP	.021	1.712	1.318	.756
1972-3	UP	LTP	.067	.609	UP	.351	.992	1.899	1.657
1972-4	UTP	UTP	.565	.840	DOWN	.089	-.517	-.383	1.029
1973-1	UTP	UTP	.626	.924	DOWN	.037	-.473	-.404	1.461
1973-2	DOWN	DOWN	.029	.992	UTP	.008	-4.072	-2.240	1.716
1973-3	DOWN	DOWN	.044	1.000	UTP	.000	-4.530	-2.831	.197
1973-4	DOWN	DOWN	.107	1.000	UTP	.000	-4.284	-2.616	-.923
1974-1	DOWN	DOWN	.678	.967	LTP	.027	-1.872	-.026	-.980
1974-2	LTP	LTP	.688	.999	UP	.000	-1.032	2.952	-.7160
1974-3	LTP	LTP	.175	1.000	DOWN	.000	-3.077	3.636	1.069
1974-4	LTP	LTP	.285	1.000	DOWN	.000	-2.799	3.613	-.136

Table 1*, continued

	Actual group	Highest probability group	P(D/G)	P(G/D)	2nd highest group	P(G/D)	Scores for function 1	2	3
1975-1	LTP	LTP	.131	1.000	DOWN	.000	-3.056	4.084	.270
1975-2	LTP	LTP	.083	1.000	DOWN	.000	-2.107	4.611	1.647
1975-3	LTP	LTP	.638	1.000	DOWN	.000	-1.731	3.650	.834
1975-4	LTP	LTP	.348	1.000	UP	.000	-.381	3.131	1.839
1976-1	UP	UP	.073	.706	LTP	.150	1.307	1.405	1.848
1976-2	UP	UP	.619	.998	UTP	.002	2.511	1.117	.074
1976-3	UP	UP	.320	.993	LTP	.006	2.345	1.751	.182
1976-4	UP	UP	.557	.999	UTP	.001	3.124	-.431	-.894
1977-1	UP	UP	.780	.991	UTP	.008	1.425	-.140	-1.239
1977-2	UP	UP	.479	.880	UTP	.060	.371	.274	-.671
1977-3	UP	UTP	.280	.484	DOWN	.247	-.377	-.298	.057
1977-4	UP	UP	.839	.905	UTP	.089	1.001	.052	-.054
1978-1	UP	UP	.722	.904	UTP	.095	1.652	.257	.790
1978-2	UP	UP	.846	.947	UTP	.052	1.902	.151	.578
1978-3	UP	UP	.517	.982	UTP	.015	2.056	1.042	.770
1978-4	UP	UP	.985	.993	UTP	.006	2.137	.300	-.289
1979-1	UP	UP	.648	.997	UTP	.003	3.092	.113	.101
1979-2	UTP	UTP	.714	.814	UP	.185	1.301	-1.319	1.182
1979-3	UTP	UTP	.638	.752	UP	.243	.825	-1.558	.316
1979-4	UTP	UTP	.069	.621	DOWN	.376	-.821	-3.302	-.285
1980-1	UTP	UTP	.373	.691	DOWN	.306	-.953	-2.313	.338
1980-2	DOWN	DOWN	.278	.953	UTP	.031	-.949	-1.540	-1.613
1980-3	DOWN	DOWN	.647	.998	UTP	.002	-2.776	-2.185	-.873
1980-4	DOWN	DOWN	.993	.996	UTP	.003	-2.337	-.700	-.763
1981-1	DOWN	DOWN	.771	.997	UTP	.003	-3.100	-1.536	.156
1981-2	DOWN	DOWN	.142	1.000	UTP	.000	-4.534	-1.619	.387
1981-3	DOWN	DOWN	.495	.999	UTP	.001	-3.718	-.914	.407
1981-4	DOWN	DOWN	.690	.952	LTP	.044	-2.069	.125	-.747
1982-1	DOWN	DOWN	.505	.964	LTP	.036	-3.017	.402	-.815
1982-2	LTP	LTP	.396	.831	DOWN	.168	-2.577	1.394	.180
1982-3	LTP	LTP	.445	.854	DOWN	.146	-2.116	1.474	-.562
1982-4	LTP	LTP	.155	.892	DOWN	.089	-1.072	1.557	-1.684
1983-1	LTP	LTP	.732	.967	DOWN	.020	-.915	1.548	-.088
1983-2	LTP	LTP	.882	1.000	UP	.000	-.980	2.687	1.123
1983-3	LTP	LTP	.721	.995	UTP	.002	-.793	2.007	1.305
1983-4	UP	UP	.263	.638	LTP	.270	.636	1.136	.764
1984-1	UP	UP	.795	.989	UTP	.010	1.439	-.268	-1.181
1984-2	UP	UP	.560	.910	LTP	.061	1.055	1.118	.061
1984-3	UP	UP	.917	.965	UTP	.031	1.438	.418	.066
1984-4	UP	UP	.421	.999	UTP	.001	2.170	-.556	-1.867
1985-1	UP	UP	.987	.989	UTP	.011	2.006	-.241	-.562
1985-2	UP	UP	.673	.998	UTP	.0020	1.930	-.115	-1.548
1985-3	UP	UP	.990	.992	UTP	.008	1.829	.107	-.636
1985-4	UP	UP	.882	.998	UTP	.001	2.322	.501	-.784
1986-1	UP	UP	.686	.999	UTP	.001	2.481	.421	-1.289
1986-2	UP	UP	.733	.976	UTP	.019	1.729	.845	.433
1986-3	UP	UP	.965	.996	UTP	.004	2.174	.394	-.502
1986-4	UP	UP	.691	.999	UTP	.001	2.901	.384	-.840
1987-1	UP	UP	.749	.999	UTP	.001	2.875	.050	-.791
1987-2	UP	UP	.715	.998	UTP	.002	1.779	.267	-1.444
1987-3	UP	UP	.901	.992	UTP	.008	1.711	-.209	-1.025
1987-4	UP	UP	.858	.995	UTP	.005	1.674	-.039	-1.161
1988-1	UP	UP	.252	.997	UTP	.003	2.741	-1.288	-1.607
1988-2	UP	UP	.512	.999	UTP	.001	3.276	-.067	-.907
1988-3	UP	UP	.889	.910	UTP	.090	1.753	-.7160	.000
1988-4	UP	UP	.758	.992	UTP	.008	2.320	-.767	-.939
1989-1	UP	UP	.395	.929	UTP	.071	1.917	-1.646	-.813
1989-2	UP	UP	.759	.862	UTP	.137	1.727	-1.030	-.034
1989-3	UP	UP	.558	.869	UTP	.128	1.136	-1.146	-.748

Table 1*, continued

	Actual group	Highest probability group	P(D/G)	P(G/D)	2nd highest group	P(G/D)	Scores for function 1	2	3
1989-4	UP	UTP	.402	.560	UP	.421	.615	-1.369	-.196
1990-1	UP	UP	.387	.843	UTP	.157	2.387	-1.577	.207
1990-2	UP	UP	.289	.632	UTP	.367	1.830	-.096	1.623
1990-3	UTP	UTP	.028	.995	UP	.005	2.040	-1.515	3.837
1990-4	UTP	UTP	.062	.998	UP	.002	1.904	-2.550	3.292
1991-1	UTP	UTP	.001	1.000	UP	.000	.546	-4.262	4.564
1991-2	UTP	UTP	.058	1.000	DOWN	.000	-.492	-1.668	4.097
1991-3	UTP	UTP	.961	.976	DOWN	.014	-.115	-1.829	1.099
1991-4	UTP	UTP	.665	.819	DOWN	.122	-.439	-.853	.615
1992-1	DOWN	DOWN	.131	.552	UTP	.330	-.316	-1.817	-1.067
1992-2	DOWN	DOWN	.447	.723	UTP	.196	-1.165	-.248	.130
1992-3	DOWN	DOWN	.898	.961	UTP	.034	-1.872	-.729	-.095
1992-4	DOWN	DOWN	.871	.999	UTP	.001	-3.154	-1.398	-.228
1993-1	DOWN	DOWN	.305	.999	LTP	.001	-3.641	-.243	-1.805
1993-2	DOWN	DOWN	.161	.898	LTP	.103	-3.569	.944	-.955
1993-3	DOWN	DOWN	.779	.998	LTP	.002	-3.298	-.365	-.618
1993-4	DOWN	DOWN	.113	.994	LTP	.005	-1.994	-.023	-2.698
1994-1	DOWN	DOWN	.908	.985	LTP	.010	-2.213	-.299	-.555
1994-2	LTP	LTP	.435	.831	UP	.157	-.003	1.578	-.087
1994-3	UP	UP	.524	.899	LTP	.084	.888	1.119	-.184
1994-4	UP	UP	.535	.957	LTP	.037	1.399	1.385	-.090

Authors' computations. - 1) LTP: Lower Turning Point; UP: Upswing; UTP: Upper Turning Point; DOWN: Downswing. Misclassified periods are flagged with asterisks. - 2) For details, see Brosius (1989).

Table 2*

Classification results for various sample periods[1]

	Final[2]	All[3]	One out[4]	Without cycle[5]				
				4	5	6	7	8
1963-1	LTP	LTP	UP	UP	UP	LTP	LTP	LTP
1963-2	UP	UP	UP	UP	UP	UP	UP	UP
1963-3	UP	UP	UP	UP	UP	UP	UP	UP
1963-4	UP	UP	UP	UP	UP	UP	UP	UP
1964-1	UP	UP	UP	UP	UP	UP	UP	UP
1964-2	UP	UP	UP	UP	UP	UP	UP	UP
1964-3	UP	UP	UP	UP	UP	UP	UP	UP
1964-4	UTP	UTP	DOWN	UTP	UTP	DOWN	DOWN	UTP
1965-1	UTP	UP	UP	UP	UP	UP	UP	UP
1965-2	UTP	UTP	UTP	UTP	UTP	UTP	UTP	UTP
1965-3	DOWN	DOWN	UTP	UTP	DOWN	DOWN	DOWN	DOWN
1965-4	DOWN	DOWN	UTP	UTP	DOWN	DOWN	DOWN	UTP
1966-1	DOWN	UTP	UTP	UP	DOWN	DOWN	DOWN	DOWN
1966-2	DOWN	DOWN	DOWN	UP	DOWN	DOWN	DOWN	DOWN
1966-3	DOWN	LTP	LTP	LTP	DOWN	LTP	LTP	DOWN
1966-4	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1967-1	LTP	LTP	LTP	LTP	DOWN	LTP	LTP	LTP
1967-2	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1967-3	LTP	LTP	LTP	LTP	UP	LTP	LTP	LTP
1967-4	LTP	LTP	LTP	LTP	UP	LTP	LTP	LTP
1968-1	UP	UP	UP	UP	UP	UP	UP	UP
1968-2	UP	UP	UP	UP	UP	UP	UP	UP
1968-3	UP	UP	UP	UP	UP	UP	UP	UP
1968-4	UP	UP	UP	UP	UP	UP	UP	UP
1969-1	UP	UP	UP	UP	UP	UP	UP	UP
1969-2	UP	UP	UP	UP	UP	UP	UP	UP
1969-3	UTP	UTP	UTP	UTP	UTP	UTP	UTP	UTP
1969-4	UTP	UTP	DOWN	UTP	UTP	UTP	UTP	UTP
1970-1	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1970-2	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1970-3	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1970-4	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1971-1	DOWN	DOWN	UTP	DOWN	UTP	DOWN	DOWN	UTP
1971-2	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1971-3	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1971-4	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1972-1	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1972-2	UP	UP	UP	UP	UP	UP	LTP	UP
1972-3	UP	LTP	LTP	UP	LTP	LTP	LTP	UP
1972-4	UTP	UTP	UTP	UTP	UTP	UTP	UTP	UTP
1973-1	UTP	UTP	UTP	UTP	UTP	UTP	UTP	UTP
1973-2	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1973-3	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1973-4	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1974-1	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1974-2	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1974-3	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1974-4	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1975-1	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1975-2	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP

Table 2* ,continued

	Final[2]	All[3]	One out[4]	Without cycle[5]				
				4	5	6	7	8
1975-3	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1975-4	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1976-1	UP	UP	LTP	UP	UP	UP	LTP	UP
1976-2	UP	UP	UP	UP	UP	UP	UP	UP
1976-3	UP	UP	UP	UP	UP	UP	UP	UP
1976-4	UP	UP	UP	UP	UP	UP	UP	UP
1977-1	UP	UP	UP	UP	UP	UP	UP	UP
1977-2	UP	UP	UP	UP	UP	UP	LTP	UP
1977-3	UP	UTP	UTP	UP	DOWN	UTP	DOWN	UTP
1977-4	UP	UP	UP	UP	UP	UP	UP	UP
1978-1	UP	UP	UP	UP	UP	UP	UP	UP
1978-2	UP	UP	UP	UP	UP	UP	UP	UP
1978-3	UP	UP	UP	UP	UP	UP	UP	UP
1978-4	UP	UP	UP	UP	UP	UP	UP	UP
1979-1	UP	UP	UP	UP	UP	UP	UP	UP
1979-2	UTP	UTP	UTP	UTP	UTP	UTP	UP	UTP
1979-3	UTP	UTP	UTP	UTP	UTP	UTP	UP	UTP
1979-4	UTP	UTP	DOWN	UTP	UTP	UTP	DOWN	UTP
1980-1	UTP	UTP	DOWN	UTP	UTP	UTP	DOWN	UTP
1980-2	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1980-3	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1980-4	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1981-1	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1981-2	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1981-3	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1981-4	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1982-1	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1982-2	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1982-3	LTP	LTP	LTP	LTP	LTP	LTP	LTP	DOWN
1982-4	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1983-1	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1983-2	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1983-3	LTP	LTP	LTP	LTP	LTP	LTP	LTP	LTP
1983-4	UP	UP	UP	LTP	UP	UP	LTP	UTP
1984-1	UP	UP	UP	UP	UP	UP	UP	UP
1984-2	UP	UP	UP	UP	UP	UP	UP	UP
1984-3	UP	UP	UP	UP	UP	UP	UP	UP
1984-4	UP	UP	UP	UP	UP	UP	UP	UP
1985-1	UP	UP	UP	UP	UP	UP	UP	UP
1985-2	UP	UP	UP	UP	UP	UP	UP	UP
1985-3	UP	UP	UP	UP	UP	UP	UP	UP
1985-4	UP	UP	UP	UP	UP	UP	UP	UP
1986-1	UP	UP	UP	UP	UP	UP	UP	UP
1986-2	UP	UP	UP	UP	UP	UP	UP	UP
1986-3	UP	UP	UP	UP	UP	UP	UP	UP
1986-4	UP	UP	UP	UP	UP	UP	UP	UP
1987-1	UP	UP	UP	UP	UP	UP	UP	UP
1987-2	UP	UP	UP	UP	UP	UP	UP	UP
1987-3	UP	UP	UP	UP	UP	UP	UP	UP
1987-4	UP	UP	UP	UP	UP	UP	UP	UP
1988-1	UP	UP	UP	UP	UP	UP	UP	UP
1988-2	UP	UP	UP	UP	UP	UP	UP	UP

Table 2* ,continued

	Final[2]	All[3]	One out[4]	Without cycle[5]				
				4	5	6	7	8
1988-3	UP	UP	UP	UP	UP	UP	UP	UP
1988-4	UP	UP	UP	UP	UP	UP	UP	UP
1989-1	UP	UP	UP	UP	UP	UP	UP	UP
1989-2	UP	UP	UP	UP	UP	UP	UP	UP
1989-3	UP	UP	UP	UP	UP	UP	UP	UP
1989-4	UP	UTP	UTP	UTP	UP	UTP	UP	UP
1990-1	UP	UP	UP	UP	UP	UP	UP	UP
1990-2	UP	UP	UTP	UP	UP	UP	UP	UP
1990-3	UTP	UTP	UTP	UTP	UTP	UTP	UTP	UTP
1990-4	UTP	UTP	UTP	UTP	UTP	UTP	UTP	UP
1991-1	UTP	UTP	UTP	UTP	UTP	UTP	UTP	UTP
1991-2	UTP	UTP	UTP	UTP	UTP	UTP	UTP	UTP
1991-3	UTP	UTP	UTP	UTP	UTP	UTP	UTP	UTP
1991-4	UTP	UTP	UTP	UTP	UTP	UTP	UTP	UTP
1992-1	DOWN	DOWN	UTP	DOWN	UTP	DOWN	DOWN	UP
1992-2	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	LTP
1992-3	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	UTP
1992-4	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1993-1	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	LTP
1993-2	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	LTP
1993-3	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN
1993-4	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	DOWN	LTP
1994-1	DOWN	DOWN	LTP	DOWN	DOWN	DOWN	DOWN	LTP
1994-2	LTP	LTP	UP	LTP	LTP	LTP	LTP	LTP
1994-3	UP	UP	UP	UP	UP	UP	UP	UP
1994-4	UP	UP	UP	UP	UP	UP	UP	UP

Author's computations, for details see text. - 1) LTP: Lower Turning Point; UP: Upswing; UTP: Upper Turning Point; DOWN: Downswing. - 2) Final classification to enter the discriminant analysis. - 3) Complete sample period results. - 4) "Leave one out"-results. - 5) The shaded areas are classification results outside the sample period.

Figure 1*

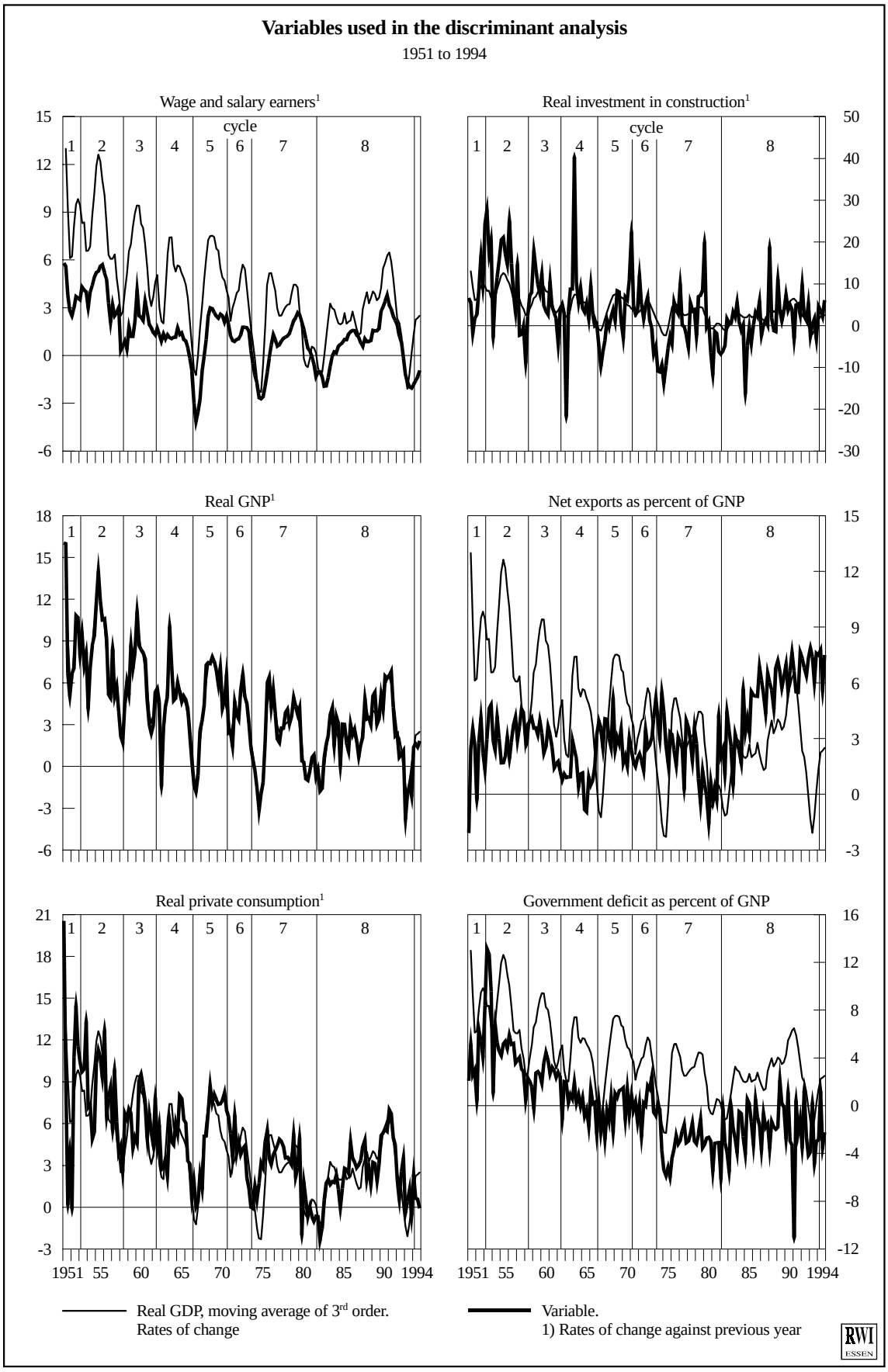
Variables used in the discriminant analysis
1951 to 1994
Wage and salary earners[1]
Real investment in construction[1]
Real GNP[1]
Net exports as percent of GNP
Real private consumption[1]
Government deficit as percent of GNP
cycle
1 2 3 4 5 6 7 8
1951 55 60 65 70 75 80 85 90 1994
Real GDP, moving average of 3rd order.
Rates of change
Variable.
1) Rates of change against previous year
RWI ESSEN

Figure 1*, continued