NEW SOLUTIONS FOR SURFACE RECONSTRUCTION FROM DISCRETE POINT DATA BY MEANS OF COMPUTATIONAL INTELLIGENCE

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Abstract: Surface reconstruction by means of triangulation of digitized point data leads to computational complex optimization problems. Here, deterministic algorithms often result in insufficient solutions or very long computation times. In this article, alternative methods of computational intelligence are discussed.
A comparative analysis of two evolutionary algorithms applied to four different smoothness criteria for the triangulation of sparse point data sets is presented.
Optimally triangulated surfaces are the basis for many practical applications. The results presented here cover the efficient implementation and the influence of different triangulations for an adequate touch probe radius compensation (TPRC).

Keywords: Triangulation, evolutionary algorithms, touch probe radius compensation

Introduction

In various practical endevours, triangulation has become a widely used method for representing surfaces that are given by three-dimensional discrete point data. In the field of surface reconstruction, these sets are generated via optical or tactile digitizing methods. These techniques generate a huge number of points that cannot be properly processed via CAD systems. Therefore, the number of points have to be reduced to a sparse set of point data. Any reduction of the number of points usually implies a loss of accuracy. Thus, a proper piecewise linear interpolation method is needed that gives best approximations of the original smooth surfaces.
Due to the fact that global triangulation of sparse data is a highly nonlinear optimization problem with a high computational complexity, nondeterministic optimization strategies have to be applied to gain optimal interpolation results.
Evolutionary algorithms show a great flexibility and robustness. Their ability to find solutions even in complex problem spaces surely is one of the reasons for their growing popularity.
The problem of finding an optimal function (metric) to describe optically sufficiently smooth triangulations has not been solved yet. Here, methods from Computational Intelligence (CI) give very promising means to solve this problem. The term CI subsumes numerical nature-motivated strategies like evolutionary algorithms, neural networks and fuzzy logic. The focus of this article lies on evolutionary algorithms, particularly on simulated annealing (SA) and an evolutionary multi-individual strategy (EA).
Here, four fitness functions will be compared using two evolutionary strategies. It will be shown that depending on the parameterization and the fitness function EAs provide a proper means to solve the computationally hard combinatorial topology problem of surface reconstruction.

Formal problem definition

Let $P = \{p_i | p_i \in \mathbb{R}^3, i = 1, \ldots, n\}$. Construct a piecewise linear C^0 interpolant (triangulation) through this data set. Here, the functional case, i.e. the data are supposed to originate from an unknown bivariate function defined over a domain $W \subset \mathbb{R}^2$, is considered. The data represent closed objects.

The objective is to find a quality criterion for triangulations with

- shape preserving properties (representing the original function at least "optically smooth"),
- applicability to 3D scattered data,
- high efficiency.

Another point is to present an efficient algorithm that generates optimal triangulations.

Complexity of approximations by triangulations

Aggarwal and Suri (1994) define the ϵ-*approximation* to be a piecewise-linear function $g(x, y)$ defined on a set P of sampled points in $\mathbb{R}^3$ from an unknown bivariate function $f(x, y)$, if, for every $p \in P$ $|f(x,y) - g(x,y)| \leq \epsilon$ halts. The problem of computing an ϵ-*approximation* with a minimum number of vertices is NP-hard (Aggarwal, 1994).

It will be shown that even simplified versions of the problem of surface approximation can be computationally very hard. Given a fixed number of vertices, the problem can be reduced to the characterization of triangulations using smoothness quality criteria.

Minimum weight triangulation (MWT)

Given a set of points $P = \{p_i | \ p_i \in \mathbb{R}^3, i = 1, \ldots, n\}$, not all being collinear, we define the set of possible triangulation $T_i(P)$ of the convex hull $Conv(P)$ of P

$$T_i(P) := \{t_1, .., t_m | \ t_k \text{ with vertices } v_j \in P\} \quad (1)$$

where the t_k triangles have disjoint and not empty interiors. The number m of triangles (w.l.o.g. in the projection onto the xy-plane) with h holes and n vertices is $m = 2n - k + 2h - 2$ where k is the number of points on the boundary of $Conv(P)$. The number of interior edges E_i is $E_i = 3n - 2k + 3h - 3$ (Schumaker, 1993).

Let $S_i(P)$ be the edges (line segments) of $T_i(P)$

$$S_i(P) = \{\sigma_j | \ \sigma_j \text{ line segment of } T_i(P)\} \quad , \quad (2)$$

the total length of all edges is

$$LOE_i(P) = \sum_{\sigma_j \in S_i(P)} \| \sigma_j \| \quad . \quad (3)$$

The minimum of the line segment lengths of all possible triangulations $T_i(P)$ is

$$MWT(P) = min_i\{LOE_i(P)\} \quad . \quad (4)$$

The minimum weight triangulation (MWT) follows the idea of giving the minimum distance neighbor from each point in P.

Complexity of MWT

Considering the projection of a three-dimensional graph onto the xy-plane, the NP-hardness or the existence of a polynomial-time algorithm remains open. However, the minimum weight triangulation for a simple polygon with n vertices can be computed by dynamic programming on time $O(n^3)$. There are several heuristics that attempt to find as small triangulations as possible. These include the Delaunay triangulation by Shamos and Hoey (1975) and the Greedy triangulation by Duppe and Gottschalk (1970). The conjecture that these triangulations lead to MWT has been disproven, e.g. by Lloyd (Preparata, 1985).

Minimum surface area (SURF)

Problem definition: Given a finite set P of arbitrary but fixed points in three-dimensional Euclidean space. Among all polyhedra $\mathcal{P}$ for a vertex set P, find one with the smallest surface area.

Let $P = \{p_i | \ p_i \in \mathbb{R}^3, i = 1, \ldots, n\}$ be a set of vertices and $T_i(P)$ a triangulation like in MWT.

The size $\triangle_k$ of the face of a triangle $t_k \in T_i(P)$ with vertices v_{k1}, v_{k2}, v_{k3}:

$$\triangle_k = \frac{1}{2} \| a \| \cdot \| b \| \cdot sin(\gamma) \quad (5)$$

$$a = v_{k1} - v_{k2}, b = v_{k3} - v_{k2} \quad (6)$$

$$\gamma = \frac{a \times b}{\| a \| \cdot \| b \|} \quad . \quad (7)$$

The total sum of the triangle areas $\triangle_k$ of each triangulation $T_i(P)$ is

$$SURF_i(P) = \sum_k \triangle_k \quad . \quad (8)$$

The optimal surface with the smallest surface area is defined by

$$SURF = min_i\{SURF_i(P)\} \quad . \quad (9)$$

Complexity of SURF

Following the proof of Fekete and Pulleyblank (1993), one can show that the problem of finding a minimum surface polyhedron can be described by a reduction of Hamiltonian Cycles in Grid Graphs (HCGG). Thus, the problem of SURF is NP-hard.

The authors show more generally that in any fixed dimension $1 < k < d$ and $2 \leq d$ it is NP-hard to minimize the volume of a simple non-degenerate d-dimensional polyhedron with a given set of vertices in d-dimensional space.

Minimum sum of angles between normals (ABN)

The directions of the surface normals provide implicit information about the relative orientations of the surface points making up part of any given centroid triangle. This directional information can be partially determined via calculations of the angle between the normals. Furthermore, Phong (1975) shading generates smoother surfaces when the angle between normals is very small.

Let $P = \{p_i | \, p_i \in \mathbb{R}^3, i = 1, \ldots, n\}$ be a set of vertices and $T_i(P)$ a triangulation like in MWT.

Let t_ν and t_μ be two triangles with two common vertices $v_{1\nu\mu}$ and $v_{2\nu\mu}$ and vertex v_ν belonging to triangle t_ν and v_μ belonging to triangle t_μ, respectively.

$$\alpha_{\nu\mu} = \frac{arccos(n_\nu \cdot n_\mu)}{\| n_\nu \| \cdot \| n_\mu \|} \tag{10}$$

$$n_s = \frac{a_s \times b_s}{\| a_s \times b_s \|} \tag{11}$$

$$a_s = v_{1\nu\mu} - v_s, b_s = v_{2\nu\mu} - v_s \tag{12}$$

with $s \in \{\nu, \mu\}$.

The total sum of the angles between normals $\alpha_{\nu\mu}$ of each triangulation $T_i(P)$ is

$$ABN_i(P) = \sum_{\nu\mu} \alpha_{\nu\mu} \quad . \tag{13}$$

We are searching for the minimum of the sums $ABN_i(P)$ of all possible triangulations $T_i(P)$

$$ABN(P) = min_i\{ABN_i(P)\} \quad . \tag{14}$$

Minimal total absolute curvature (TAC)

Classically, tightness is defined in terms of the total absolute curvature integral: a mapping of a surface into space is called tight if it has minimal total absolute curvature.
Given a mapping f of a surface M into space, the total absolute curvature is

$$\tau(f) = \frac{1}{2\pi} \int_M |K| \, dA \tag{15}$$

where K is the Gaussian curvature.

Given a triangulation $\triangle$ with some vertex ν with coordinate x_0, let $x_1, \ldots, x_{M_\nu}$ be the coordinates of the ordered (e.g. clockwise) neighboring vertices of ν. Then, the integral curvature at the vertex ν is

$$K(\nu) = 2\pi - \sum_{j=1}^{M_\nu} \alpha_j \quad . \tag{16}$$

The α_j denote the angles between the edges from the neighboring points on the convex hull of x_0 that meet in x_0 (van Damme and Alboul, 1995).

The definition of the total absolute curvature of a complete triangulation $T_i(P)$ is

$$TAC_i(P) = \sum_{\nu \in P} K(\nu) \quad . \tag{17}$$

The optimal surface is the minimum of all $TAC_i(P)$, i.e.

$$TAC(P) = min_i\{TAC_i(P)\} \quad . \tag{18}$$

Simulated Annealing

The simulated annealing (SA) process follows the model of natural self organization (e.g. building of crystal structures) of hot metal that is cooling down. A descriptive mathematical model that abstracts from local particle-to-particle interaction has been formulated by Boltzmann first. Metropolis et al. (1953) have implemented the model algorithmically first.
By means of a Monte-Carlo method new particle configurations are generated. Their free energy E_{new} is compared with that of the former state E_{old}. If the condition $E_{new} \leq E_{old}$ holds, the new configuration "survives" and forms the basis for the next generation. The new state may survive also if $E_{new} > E_{old}$, but only with a certain probability

$$w = \frac{1}{c} \, exp\left(\frac{E_{old} - E_{new}}{KT}\right), \tag{19}$$

where K denotes the Boltzmann constant and T the current temperature. c serves to normalize the probability distribution (e.g. $\sqrt{2\pi}$). An individual will survive, if $\theta \leq w$, with $\theta \in [0, 1]$ from a uniform distribution. The temperature T is lowered during the optimization process due to a given function, e.g. $T_k = \alpha^k \, T_0, k = 1, 2, \ldots$, and $0 < \alpha < 1$.

Applied to the optimization of triangulations, different quality criteria can be used. The states E_{old} and E_{new} represent different triangulations $T_i(P)$. The states can be compared or subtracted with the scalar operators '$-$' and '$<$' after the application of a quality criterion.

A state E_{new} is generated from E_{old} by flipping edges within a triangulation. An edge e is called *flippable*, if e is contained in the boundary of two triangles t_i and t_j and $C = t_i \cup t_j$ is a convex quadrilateral. Flipping e denotes the operation of removing e form a triangulation T and replacing it by the other diagonal of C.

Hurtado and Urritia (1996) show that any triangulation in the plane of an n point set contains at least $\frac{n-4}{2}$ edges that can be flipped. The transformation of a triangulation T of a polygon Q_n with k reflex vertices into a triangulation T' of Q_n can be performed by flipping at most $O(n+k^2)$ edges.

The number of edges flipped during each optimization step depends on the actual temperature T. Hence, the number of edges to be flipped with T_0 is at least $\frac{n-4}{2}$ and the maximum is $n-k$ where k is the number of points on the boundary of $Conv(P)$. The maximum number of flippable edges can be significant lower and depends on the actual triangulation.

Evolutionary Algorithms

Evolutionary algorithms are a class of heuristic strategies that follow Darwin's idea of *survival of the fittest*. Genetic algorithms (GA) (Holland, 1975), evolutionary strategies (ES) (Rechenberg, 1971; Schwefel, 1975) and evolutionary programming (EP) (Fogel, 1966) are subclasses of evolutionary algorithms. Simulated annealing (SA) and single trial versions of ES are closely related in case they are designed for optimization over continuous variables (Rudolph, 1994).
Following the definition of Bäck (1996), the general evolutionary algorithm can be defined as an 8-tuple

$$EA = (I, \Phi, \Omega, \Psi, s, \iota, \mu, \lambda) \quad , \qquad (20)$$

where $I = A_x \times A_s$ is the space of *individuals*, and A_x,A_s denote arbitrary sets. $\Phi = I \longrightarrow \mathbb{R}$ denotes a fitness function (quality criterion) assigning scalars to individuals.

$$\Omega = \{\omega_{\Theta_1}, \ldots, \omega_{\Theta_z} \,|\, \omega_{\Theta_i} : I^\lambda \rightarrow I^\lambda\} \cup \Omega_{\Theta_0} \qquad (21)$$

where

$$\Omega_{\Theta_0} = \{\omega_{\Theta_0} : I^\mu \rightarrow I^\lambda\} \qquad (22)$$

is a set of probabilistic *genetic operators* ω_{Θ_i}, each of which is controlled by specific parameters summarized in the sets $\Theta_i \subset \mathbb{R}$.

$$s_{\Theta_s} : (I^\lambda \cup I^{\mu+\lambda}) \longrightarrow I^\mu \qquad (23)$$

denotes the selection operator, which may change the number of individuals from λ or $\mu + \lambda$ to μ. μ is the number of "parent" individuals and λ denotes the number of "offspring" individuals. $\iota : I^\mu \longrightarrow \{true, false\}$ is a termination criterion for the EA, and the transition function $\Psi : I^\mu \longrightarrow I^\mu$ describes the complete process of transforming a population P into a subsequent one by applying genetic operators and selection:

$$\Psi = s \circ \omega_{\Theta_{i_1}} \circ \ldots \circ \omega_{\Theta_{i_j}} \circ \omega_{\Theta_0} \quad , \qquad (24)$$

where $\{i_1, \ldots, i_j\} \subset \{1, \ldots, z\}$, and $Q \in \{\emptyset, P\}$.

The probabilistic genetic operators $\omega_{\Theta_{i_j}}$ in a general EA are called *recombination* and *mutation*. Jacob (1997) describes further genetic operators like *inversion*, *deletion* and *duplication*.
The selection function s may contain random elements (e.g. fitness-proportional selection) or may use linear or exponential ranking schemes or tournament strategies (for a comparison of selection strategies used in GAs see, e.g. (Blickle and Thiele, 1995)).

The EA described here uses a mutation operator only. The mutation of an individual I_j (a triangulation) has been realized by randomly flipping edges. The number of edges changed depends on a mutation rate function that is motivated by discrete optimization strategies.

$$flips(i) = \frac{1}{a \cdot i + \frac{1}{S_{begin} - S_{end}}} + S_{end} \qquad (25)$$

where S_{begin} denotes the number of flips in generation 0, and S_{end} is a minimal number of edge-swapping operations.
Following the approach of ES, the operator s selects the μ best individuals (triangulations) from the λ offspring which are evaluated by *any* fitness function Φ that characterizes the smoothness of a triangulation.

The termination criterion ι yields *true*, if the actual number of generations exceeds a given limit.
The initial population can be generated using any deterministic triangulation scheme, like Delaunay, Greedy or Dijkstra (Weinert and Mehnen, 1997).

Optimization Results

The test point data set has been generated via a tactile line-oriented digitizing process of a milled workpiece. This object shows typical surface structures, like cylinder, cone and sphere segments. The original data set has 135000 points, which has been reduced by more than 99% to a very sparse point set of 1164 points. The points have been eliminated by a tangent approximation strategy (TDV) (Friedhoff, 1996) fitting a linear 2D approximation into the point data by choosing only the start and end point of each line segment and eliminating all points that are enclosed in a tube like ϵ-region around each line segment.

The initial individual for the SA and the EA has been generated by means of a local deterministic triangulation algorithm (Friedhoff, 1996).

Quality criteria

The surfaces generated via the SA and EA for the global surface reconstruction using MWT, SURF, ABN, and TAC are displayed in figures 1 and 2. Figures 3 and 4 show the wire frames of each triangulation, respectively.

The results of the optimization runs for EA and SA are very similar and are fairly sensitive to the choice of parameters.

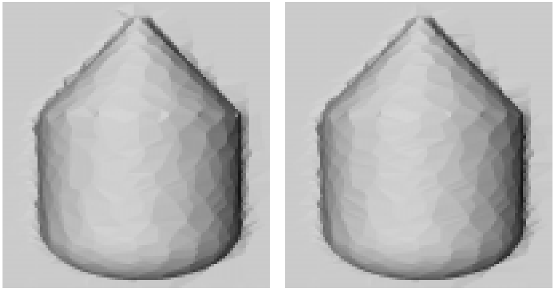

Figure 1: Surfaces generated using MWT (left), SURF (right)

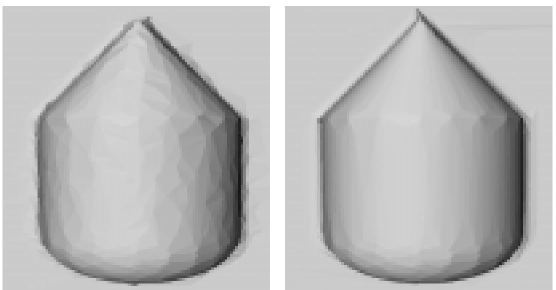

Figure 2: Surfaces generated using ABN (left), TAC (right)

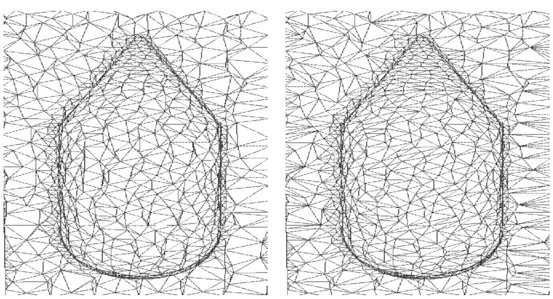

Figure 3: Wire frames generated using MWT (left), SURF (right)

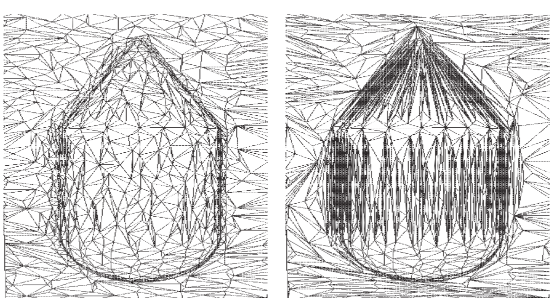

Figure 4: Wire frames generated using ABN (left), TAC (right)

The fitness function has a relevant influence on the proper parameter settings. Table 1 shows the best settings for the SA:

	MWT	SURF	ABN	TAC
T_0	50.0	0.89	100.0	0.98
α	0.95	0.95	0.97	0.95

Table 1: SA Parameter Settings.

Following the advice of Schumaker (1993), it is reasonable to choose T_0 to be twice the largest fitness function value of good edge swaps. The annealing parameters α have been found by discrete exploration of the interval $]0,1[$.

Table 2 shows the best settings for the EA-(5,50) and EA-(5+50) strategies. The parameters have been the same for the plus and the comma strategy.

	MWT	SURF	ABN	TAC
S_{begin}	600	600	1000	1000
S_{end}	5	5	5	100
a	0.0005	0.0005	0.0005	0.0005

Table 2: EA Parameter Settings.

The runs have been terminated after 3000 generations. The optimal number of parents μ and offspring λ has been found experimentally. Generally, the ratio of μ and λ should be chosen to be inversely proportional to the probability of success. The expectation values for the success probability can be calculated from a given fitness function by integrating only the probabilities of improvement occurrences over the success area. However, in practical applications, the calculation of these values can become rather difficult.

All criteria show good approximation abilities. Long flag-shaped triangles that reach out of the surface can be avoided. This problem is typical for deterministic strategies that follow the line structure of the digitized point data.

The MWT criterion typically generates triangulations with angles of 60 degree on the average. These triangulations are similar to Delaunay triangulations. Unfortunately, the surface structures generated usually do not show very high surface qualities because equilateral triangles usually do not fit a surface very well in areas with a high curvature (e.g. torus-like fillet radii) when only few points are given. Potential practical applications of MWT in mechanical engineering may also lie in the field of ultra-light constructions.

SURF typically generated quite good surface reconstructions. The advantage of this criterion is that it exploits the information that lies within the 3D structure of the points. MWT does not use this information and tries to minimize the mesh even in 2D. SURF is motivated by minimal surfaces like soap films. The properties of these differential geometrical objects can be adopted by SURF. Potential practical applications of least-area surfaces can also be seen in engineering problems, e.g., the design of minimal-weight constructions with huge volumina.

The ABN criterion reproduces the average curvature of a surface. The ABN strategy produces rather good results around saddle points but performs not too well on convex/concave areas. However, the ability of ABN to minimize the angles between the normals of the triangles leads to smooth rendering properties.

The TAC criterion usually generates best, i.e. optically smooth, surfaces. Experiments with other surface structures show similar results. The surface is approximated with optimally adapted triangles that show long edges when the curvature in one direction is nearly zero and positive or negative in the orthogonal direction.
Another advantage of TAC lies in the property of extreme vertices. A vertex v of a polyhedral surface M is called a locally extreme vertex if v is a vertex of the convex hull of the star of v, and it is a (globally) extreme vertex if it is a vertex of the convex hull of M. A locally extreme vertex is the local maximum for the height function on M in some direction, and a vertex that is not locally extreme is a saddle point for every height function on M. A polyhedral surface M is tight if, and only if, every locally extreme vertex of M is a globally extreme vertex of M. Thus, a local characterization of the vertices leads to a characterization of the global optimum.
The TAC criterion seems to generate no minimal surface in general. The application of SURF to a TAC solution leads to further reduction of the global triangle surface area (for nontrivial surfaces).

Each smoothness criterion shows a typical and different behavior. An EA produced typical and structural different triangulations after changing the criterion. This may allow the conclusion that all smoothness criteria presented here show an inherently different behavior on nontrivial sparse point sets in 3D.

Convergence

While SA and EAs provide a good means for global optimization, usually, it cannot be guaranteed that these strategies will find the global optimum. For $(1, \lambda)$ ES optimizations Rudolph (1997) showed that, for special preconditions of the fitness function and for discrete parameter spaces, a global optimum will be reached after a finite number of generations with probability one. A precondition for the reachability of and possibly convergence to a global optimum is the ability of the evolutionary algorithm to explore the complete search space. Dyn (1993) proved that, given any two triangulations T_1 and T_2 on the domain Ω with polygonal boundary, it is always possible to get from T_1 to T_2 by a finite sequence of edge swaps. This ensures the reachability of the global optimum by a (μ, λ) ES. In contrast, $(\mu + \lambda)$ strategies may get stuck in local optima if mutations are single edge swaps. In order to explore the complete search space, this EA should allow huge mutation steps, which can be represented by few random individuals that are included into the offspring population. Then, the reachability of the optimum and even convergence of the $(\mu + \lambda)$ strategy can be guaranteed.

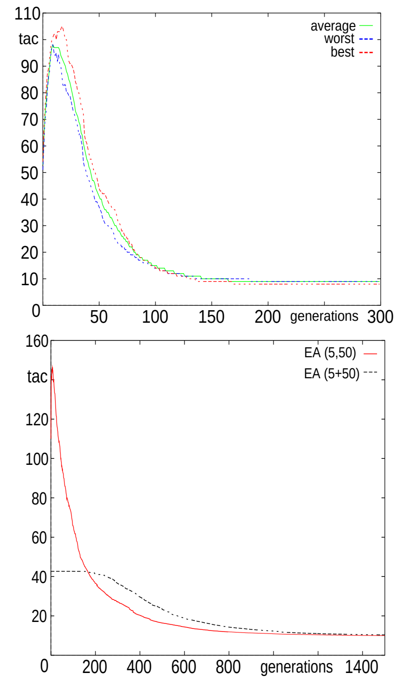

Figure 5: Objective-function values from SA (top) and EA (bottom) runs using TAC

Figure 5 displays the convergence behavior of SA, EA-$(5 + 50)$ and EA-$(5, 50)$ strategies using TAC. Each criterion has been applied five times with the parameter settings from table 1 and 2. The graphs of the objective functions MWT, ABN, SURF und TAC have been very similar. The values of the objective functions are exponentially decreasing.

In the interval $[25, \ldots, 300]$, the fitness values from the SA approximately follow the functions

$$v_{MWT}(x) = \frac{e^{25-0.04 \cdot x}}{2 \cdot 10^7} + 10^5 \quad , \qquad (26)$$

$$v_{SURF}(x) = \frac{e^{25-0.03 \cdot x}}{7 \cdot 10^8} + 8.57 \cdot 10^3 \quad , \qquad (27)$$

$$v_{ABN}(x) = \frac{e^{25-0.02 \cdot x}}{2 \cdot 10^6} + 1.62 \cdot 10^4 \quad , \qquad (28)$$

$$v_{TAC}(x) = \frac{e^{25-0.03 \cdot x}}{5 \cdot 10^8} + 9 \qquad (29)$$

for the MWT, SURF, ABN, and TAC criterion, respectively.

In the interval $[30, \cdots, 2000]$ of generations, the average convergence of the EA-$(5, 50)$ strategy can be approximated by the functions

$$v_{MWT}(x) = \frac{e^{30-0.008 \cdot \sqrt{140x}}}{1.8 \cdot 10^9} + 1.03 \cdot 10^4 \quad , \qquad (30)$$

$$v_{SURF}(x) = \frac{e^{30-0.020\cdot\sqrt{150x}}}{8.3\cdot 10^9} + 8.57\cdot 10^3 \quad , \quad (31)$$

$$v_{ABN}(x) = \frac{e^{30-0.015\cdot\sqrt{25x}}}{2\cdot 10^8} + 1.85\cdot 10^4 \quad , \quad (32)$$

$$v_{TAC}(x) = \frac{e^{30-0.05\cdot\sqrt{9x}}}{5\cdot 10^{10}} + 8.5 \quad . \quad (33)$$

In the interval $[300, \cdots, 2000]$ of generations, the average convergence of the EA-$(5+50)$ strategy can be approximated by the functions

$$v_{MWT}(x) = \frac{e^{30-0.01\cdot\sqrt{120x}}}{9.2\cdot 10^8} + 1.01\cdot 10^4 \quad , \quad (34)$$

$$v_{SURF}(x) = \frac{e^{30-0.020\cdot\sqrt{120x}}}{1.3\cdot 10^8} + 8.57\cdot 10^3 \quad , \quad (35)$$

$$v_{ABN}(x) = \frac{e^{30-0.015\cdot\sqrt{25x}}}{2.3\cdot 10^8} + 1.5\cdot 10^4 \quad , \quad (36)$$

$$v_{TAC}(x) = \frac{e^{30-0.014\cdot\sqrt{150x}}}{2\cdot 10^{10}} + 9 \quad . \quad (37)$$

Experiments with an SA ($T_0 = 0$ for all runs, thus allowing improvements only) with a successively increased number of digitized point data ($[1367, \cdots, 7222]$) from the same object using the TAC criterion show an exponential slowdown of the convergence velocity. However, optima with nearly the same fitness values have been found after at least 300 generations.
Edelsbrunner and Tan (1993) showed that the number of different triangulations of P in $\mathbb{R}^2$ depends on the number of vertices $n = |P|$ and on their relative location. If P is in convex position, then it admits $\frac{1}{n-1}\binom{2n-4}{n-2} \geq 2^{n-3}$ different triangulations. In order to choose an optimal triangulation under some criterion, it is thus not feasible to exhaustively search the set of all triangulations. Although the convergence velocity of EA depends on the number of vertices, only heuristical strategies like evolutionary algorithms will allow to find fairly good triangulations in acceptable time.

All experiments have been made on a SGI Power ONYX RE2 with two R10000 processors.

Application to touch probe radius compensation

Tactile digitization of workpiece surfaces is often preferred to optical methods due to its higher accuracy. One of its drawbacks, however, is that it provides so-called center data: the measuring equipment reports coordinates of the probe's center rather than those of the actual point of contact with the sampled workpiece. This nature of data must be taken into account in subsequent processing, usually in the form of an explicit step of touch probe radius compensation.

Friedhoff (1996) compiles different approaches to the solution of this problem. One of these solutions is based on the observation that the problem of touch probe compensation is related to simulation of 3-axis milling.

In milling simulation, a set M of line segments in 3D space defining the path of some reference point of the cutter during the process of milling and the shape of the cutter are given. The task is to calculate the surface cut into a solid block when the reference point, i.e., the cutter, is moved along the path M. Taking the touch probe centers to be points on some milling path, and the sensor tip as cutter, the surface calculated by milling simulation gives the same points like real tactile digitization.

The difficulty with this idea is to find "milling paths" in a way that the obtained surface represents the digitized surface reasonably. It turns out that a reasonable approximation can often be obtained by interpolating the measured center points of the probe by triangular surfaces. Finding a suitable triangulation is one of the main problems due to touch probe radius compensation. Here, the methods from optimal triangulation can serve as adequate tools.

Summary and Conclusion

It has been shown that evolutionary algorithms are useful for solving the highly complex (NP-hard) problem of 3D triangulation. Four different smoothness criteria have been compared. All four fitness functions lead to smooth piecewise linear interpolations for given nontrivial sparse point data sets. Each criterion lead to different characteristic solutions. The gained quality depends on the criterion, the parameter settings and the local curvature properties of the surface. Criteria that cover the average characteristics of a surface lead to better, i.e., tighter global surface reconstructions. Our experiments gave the impression that TAC and ABN generate the best global triangulations.

EAs show high efficiency for complex (multimodal, combinatorial and computationally hard) optimization problems. However, the proper parameterizing of an EA can be rather difficult. Due to the high combinatorial complexity of triangulation, the convergence velocity decreases depending on the number of vertices. Thus, for large numbers of vertices, the optimization may become slow although the convergence of the evolutionary algorithms towards a local optimum is exponential. However, large numbers of vertices imply large numbers of triangles. Usually, this leads to problems with CAD systems. Hence, we considered quality criteria and efficient optimization strategies for sparse digitized point data to generate more efficient CAD surface representations.

Future research will focus on sufficient and efficient characterizations of tight surface triangulations, on efficient segmentations of point sets, and on the parallelizations of EAs. This will yield further insight into the complex problem of surface reconstruction.

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References

Aggarwal, A. and Suri, S. (1994). Surface approximations and geometric partitions, *In: Proc. 5th Symposium on Discrete Algorithms*, pp. 24-33.

Alboul, L. and van Damme, R. M. J. (1995). *Polyhedral Metrics in Surface Reconstruction: Tight Triangulations*, Universiteit Twente, No. 1275.

Bäck, Th. (1996). *Evolutionary Algorithms in Theory and Practice*, Oxford University Press, pp. 63-89.

Blickle, T. and Thiele, L. (1995). A Comparison of Selection Schemes used in Genetic Algorithms, *TIK-Report*, ETH Zürich, Vol. 11.

Duppe, R.D. and Gottschalk, H.H. (1970). Automatische Interpolation von Isolinien bei willkürlichen Stützpunkten, *Allgemeine Vermessungsnachrichten*, pp. 77.

Dyn, N., Goren, I., and Rippa, S. (1993). Transforming Triangulations in Polygonal Domains, *Computer Aided Geometric Design*, Vol. 10, pp. 531-536.

Edelsbrunner, H. and Tan, T.S. (1993). A Quadratic Time Algorithm for the Minmax Length Triangulation, *In: Siam J. Comp.*, Vol. 22, No. 3, pp. 527-551.

Fekete, S. and Pulleybank, W. (1993). Area Optimization of Simple Polygons, *In: Ninth Annual Symposium on Computational Geometry*, San Diego, California, May 19-21, ACM Press, New York, pp. 173-182.

Fogel, L.J., Owens, A.J., and Walsh, M.J. (1966). *Artificial intelligence through simulated evolution*, John Wiley, New York.

Friedhoff, J. (1996). *Aufbereitung von 3D-Digitalisierdaten für den Werkzeug-, Formen- und Modellbau*, Vulkan Verlag, Essen.

Holland, J.J. (1975). Adaptation in natural and *artificial systems*, University of Michigan Press, Ann Arbor.

Hurtado, F., Noy, M., and Urritia, J. (1996). Flipping Edges in Triangulations, *In: Computational Geometry '96*, Philadelphia, USA, pp. 214-223.

Jacob, Ch. (1997). *Principia Evolvica: simulierte Evolution mit Mathematica*, dpunkt Verlag, Heidelberg.

Metropolis, N., Rosenbluth, A.W., and Teller, E. (1953). Equations of state calculations by fast computing machines, *In: J. Chem. Phys*, Vol. 22, pp. 1087-1092.

Phong, B.T. (1975). Illumination for Computer Generated Pictures, *In: Communications of the ACM*, Vol. 18, No. 6, pp. 311-317.

Preparata, F. (1985). *Computational Geometry,* Springer, pp. 227.

Rechenberg, I. (1971). *Evolutionsstrategie: Optimierung technischer Systeme nach Prinzipien der biologischen Evolution*, PhD Thesis, Technische Universität Berlin.

Rudolph, G. (1994). Massively Parallel Simulated Annealing and its Relation to Evolutionary Algorithms, *In: Evolutionary Computation*, Vol. 1, No. 4, pp. 361-382.

Rudolph, G. (1997). *Convergence Properties of Evolutionary Algorithms*, Kovač, Hamburg.

Schumaker, L. (1993). Computational Triangulation using Simulated Annealing, *In: Computer Aided Geometry Design*, Vol. 10, pp. 329-345.

Schwefel, H.-P. (1975). *Evolutionsstrategie und numerische Optimierung*, PhD Thesis, Technische Universität Berlin.

Shamos, M.I. and Hoey, D. (1975). Closest Point Problems, *In: Proceedings of the 16th Annual Symposium on the Foundations of Computer Science*, IEEE.

Weinert, K., Mehnen, J., and Prestifilippo, G. (1997). *Optimal Surface Reconstruction from Digitized Point Data using CI Methods*, University of Dortmund, Technical Report, CI-5/97, SFB 531.