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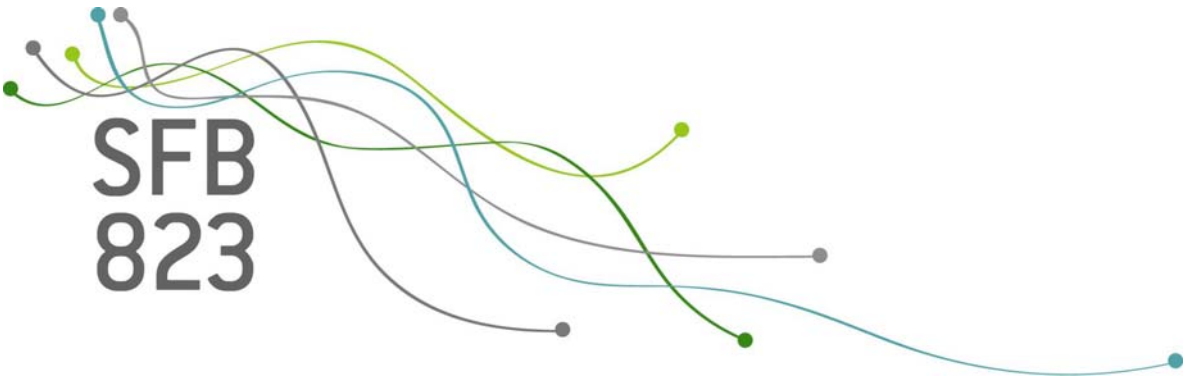

When Outcome Heterogeneously matters for Selection: a Generalized Selection Correction Estimator

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Abstract

The classical Heckman (1976, 1979) selection correction estimator (heckit) is misspecified and inconsistent if an interaction of the outcome variable and an explanatory variable matters for selection. To address this specification problem, a full information maximum likelihood estimator and a simple two-step estimator are developed. Monte-Carlo simulations illustrate that the bias of the ordinary heckit estimator is removed by these generalized estimation procedures. Along with OLS and the ordinary heckit procedure, we apply these estimators to data from a randomized trial that evaluates the effectiveness of financial incentives for weight loss among the obese. Estimation results indicate that the choice of the estimation procedure clearly matters.

JEL codes: C24, C93.
Keywords: selection bias, interaction, heterogeneity, generalized estimator.

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1 Introduction

The Heckman (1976, 1979) selection correction (heckit) estimator is a workhorse of applied econometrics, commonly used for removing possible bias due to selection on unobservables.[1] In many applications, selection into that subsample of observations, for which the outcome variable is observed, may be affected by the value of the outcome variable itself. Think, for instance, of estimating a wage equation. Here, wages are only observed for individuals who have accepted a wage offer. Yet, the likelihood of accepting the offer increases with the offered wage. Since in the regular heckit estimator, typically, all exogenous variables enter the selection part of the model, the selection equation can be interpreted as a reduced form representation that implicitly captures such impact of outcome on selection.[2]

However, the offered pay may be of differential relevance to different individuals, e.g., men and women. This renders the effect of the outcome variable on selection heterogeneous with respect to an explanatory variable. In technical terms, this means that not only the outcome variable but also an interaction with the relevant regressor enters the selection model. This, unlike the case of the outcome exerting a homogeneous effect on selection, is not accommodated by the regular heckit model. Ignoring the deferential effects of outcome on selection renders the econometric model misspecified and, in turn, renders the regular heckit estimator biased and inconsistent.

The present paper develops generalizations of the regular heckit estimator that allows for differential effects of outcome on selection. Besides full information maximum likelihood (FIML), we suggest a computational very simple two-step approach. Overcoming the inconsistency of the ordinary heckit model, the FIML allows for identifying the differential effect of the outcome variable on selection. The simpler two-step approach is also consistent. However, the coefficient of the interaction term cannot be identified.

We test the performance of the suggested estimators by the means of Monte Carlo simulations. We also apply them to data gathered from a randomized experiment, which was conducted to examine the effectiveness of financial for making obese

[1] Though it has been criticized for being very vulnerable to various kinds of misspecification (e.g. Puhani, 2000; Grasdal, 2001), and less restrictive semi-parametric alternatives have been proposed (e.g. Ichimura and Lee, 1991; Ahn and Powell, 1993); see Vella (1998) for a survey.

[2] The commonly used tobit (type 1) (Tobin, 1958) model represents an extreme case with selection *exclusively* depending on the outcome.

individuals reduce body weight. Here, the incentive scheme makes favorable outcomes more likely to be reported than unfavorable ones, simply by setting stronger incentives to report them. Thus, the suggested link between outcome and the probability of observing the outcome may only exist for the experimental group that is rewarded for success in losing weight.

The remainder of the paper is organized as follows. Section 2 develops the generalized heckit estimators. Section 3 compares the performance of the different estimators using a Monte Carlo experiment. Section 4 provides a real data application and Section 5 concludes.

2 A Generalized Heckit Model

Consider a familiar linear regression model, where the focus of the econometric analysis is on estimating the coefficient vector β:

$$Y_i = \beta' X_i + \varepsilon_i. \tag{1}$$

Here i indexes observations, and Y_i, ε_i, and X_i denote the outcome variable, a random error, and the vector of exogenous explanatory variables, respectively. The latter includes the variable D_i, which is of special relevance to the analysis, e.g., a treatment indicator.

However, Y_i is observed only for a subsample of observation. Selection into this subsample, indicated by $S_i = 1$, is modeled as suggested by Heckman (1979). Yet, besides a K-dimensional vector Z_i that includes X_i and some further exogenous variables (instruments), Y_i as well as the interaction term $Y_i D_i$ are allowed to enter the selection equation:

$$S_i = \begin{cases} 1 & \text{if} \quad \theta' Z_i + \tau Y_i + \gamma Y_i D_i + v_i > 0 \\ 0 & \text{else.} \end{cases} \tag{2}$$

As in the ordinary heckit model, joint normality $N(0, 0, \sigma_\varepsilon^2, \sigma_v^2, \sigma_{\varepsilon v})$ is assumed for the error terms ε_i and v_i. $\theta_1, \ldots, \theta_K$, τ, and γ denote unknown coefficients. Substi-

tuting Y_i by (1) and rearranging terms leads to

$$S_i = \begin{cases} 1 & \text{if} \quad \tilde{v}_i > -\alpha' Z_i - \gamma \beta' X_i D_i \\ 0 & \text{else} \end{cases} \tag{3}$$

$$\tilde{v}_i = v_i + (\tau + \gamma D_i)\, \varepsilon_i, \tag{4}$$

where $\alpha_k = \theta_k + \tau\beta_k$ holds for any regressor k that is shared by X_i and Z_i and $\alpha_k = \theta_k$ holds for the instruments. Evidently, the coefficient τ has no impact on the general structure of the model.[3] For the special case $\gamma = 0$, (1), (3), and (4) represent the standard Heckman (1979) selection model.

For $\gamma \neq 0$, however, the model deviates from the standard case for two reasons: (i) a full set of interaction terms $X_i D_i$ enters the selection equation and (ii), more important, D_i enters the error $\tilde{v}_i$, rendering the the error variance-covariance structure heterogeneous with respect to D_i:

$$\begin{aligned} \operatorname{var}(\tilde{v}_i | D_i) &= \sigma_v^2 + 2\,(\tau + \gamma D_i)\,\sigma_{\varepsilon v} + (\tau + \gamma D_i)^2\,\sigma_\varepsilon^2 & (5) \\ \operatorname{cov}(\varepsilon_i, \tilde{v}_i | D_i) &= \sigma_{\varepsilon v} + (\tau + \gamma D_i)\,\sigma_\varepsilon^2. & (6) \end{aligned}$$

Ignored heteroscedasticity in the probit and, hence, in the selection part of the heckit, is well known to render probit estimation inconsistent (Wooldridge, 2002; Harvey, 1976). Thus, a generalized estimator is required.

2.1 FIML Estimation

In oder to develop an estimable FIML estimator that accounts for the model structure, with no loss of generality, we introduce the normalization

$$\sigma_v^2 + 2\tau\sigma_{\varepsilon v} + \tau^2\sigma_\varepsilon^2 = 1. \tag{7}$$

That is, we assume standard normality for $\tilde{v}_i$ conditional on $D_i = 0$. This is equivalent to the familiar normalization required for identifying the coefficients of any probit model. We re-parameterize as follows:

$$\rho \equiv \operatorname{cor}(\varepsilon_i, \tilde{v}_i | D_i = 0) = \frac{\sigma_{\varepsilon v}}{\sigma_\varepsilon} + \tau\sigma_\varepsilon. \tag{8}$$

[3] Effectively, τ only changes the unknown error variance-covariance structure, which is subject to estimation. Hence, τ is not identified.

Then the individual log-likelihood l_i reads as

$$l_i = \begin{cases} \log \Phi \left(\frac{-\alpha' Z_i - \gamma \beta' X_i D_i}{\sqrt{1 + 2\rho\sigma_\varepsilon \gamma D_i + \sigma_\varepsilon^2 \gamma^2 D_i^2}} \right) & \text{if} \quad S_i = 0 \\ \log \Phi \left(\frac{\alpha' Z_i + \gamma \beta' X_i D_i + (Y_i - \beta' X_i)\left(\frac{\rho}{\sigma_\varepsilon} + \gamma D_i\right)}{\sqrt{1-\rho^2}} \right) & \\ \quad - \frac{1}{2}\left(\frac{Y_i - \beta' X_i}{\sigma_\varepsilon}\right)^2 - \log\left(\sigma_\varepsilon \sqrt{2\pi}\right) & \text{if} \quad S_i = 1. \end{cases} \tag{9}$$

See Appendix A.1 for how (9) is derived from the log-likelihood function of the ordinary heckit model. Besides the coefficient vectors α and β, the scalar parameters γ, σ_ε, and ρ are subject to estimation.[4] Note that D_i may either be continuous, a count, or binary.

The model is straightforwardly transferred to the case where the effect of Y_i on selection differs across $M + 1$ mutually exclusive groups, indexed by $m = 0, \ldots, M$. For group membership being indicated by a set of binary indicators $D_{0i}, \ldots, D_{Mi}$, the log-likelihood conditional on $D_{mi} = 1$ is identical to (9), besides D_i is substituted by the value one and γ is replaced by γ_m.[5] Here, γ_0 has to be restricted to zero in order to render the model identified.

2.2 Two-Step Estimation

The model (9) is, however, difficult to fit and may cause problems in the optimization procedure. Yet, for a binary variable D_i and, more general, group-wise heterogeneity, a computationally very simple two-step estimator is available. Here, the heterogeneity in the selection mechanism is accounted for by estimating group-wise probit models at the first stage. For each group m, a specific coefficient vector α_m is estimated, where the coefficients attached to $D_{1i}, \ldots, D_{Mi}$ need to be restricted to the value of zero. At the second stage a vector of group-specific inverse Mills-rations $\lambda(\cdot)$ enter as additional regressors

$$Y_i = \beta' X_i + \sum_{m=0}^{M} \delta_m \lambda(\hat{\alpha}_m' Z_i) D_{mi} + \tilde{\varepsilon}_i \qquad \text{if} \quad S_i = 1. \tag{10}$$

[4] Technically, $\operatorname{atanh}(\rho)$ and $\log(\sigma_\varepsilon)$ are estimated in the optimization procedure in order to avoid a bounded valid parameter space.

[5] Typically, all dummies D_{mi}, except for D_{0i} indicating the reference category, enter X_i and Z_i.

The attached coefficients δ_m, subject to estimation, capture $\sigma_\varepsilon \operatorname{cor}(\varepsilon_i, \tilde{v}_i | D_{mi} = 1)$. Two-step estimation comes, however, to the cost of efficiency loss. In the present case, it is not only genuinely less efficient than FIML, like, e.g., the two-step estimator for the ordinary heckit model. It also ignores many parameter restrictions that stem from the structural model. For this reason, the two-step approach inflates the number of parameters subject to estimation by $M(K-1) - M^2$. Moreover, (10) may suffer from near-collinearity of correction terms and group indicators. On the other hand, two-step estimating involves less assumptions about the selection mechanism than FIML and, hence, also accommodates types of heterogeneity in selection that render (9) misspecified.

3 Monte Carlo Analysis

In order to illustrate the performance of the FIML and the two-step estimators and to compare it with those of ordinary heckit and simple OLS estimation, we run a Monte-Carlo (MC) experiment, where the endogenous variables Y_i and S_i are generated according to (1) and (2). The exogenous variables, i.e. the vector Z_i, are drawn once and then kept fixed. We draw the binary indicator D_i from the $B(1, 0.5)$ distribution and two continuous control variables from the uniform $U(-1, 1)$ distribution. One of the latter is excluded from the vector X_i, while D_i enters (2) not only trough Z_i but also interacted with Y_i. For all coefficients β_k and θ_k, we choose the value of one, except for the constant terms, which both are set to zero. With respect to the variance-covariance matrix of the normal errors, we choose $\sigma_\varepsilon^2 = 2$, $\sigma_v^2 = 1$, and $\sigma_{\varepsilon v} = 0.75$. We vary the experimental setup with respect to: (i) γ, for which we try the values -1, 0, and 1; and (ii) τ, for which we try the two values consistent with (7), i.e., -0.75 and 0. The sample size is 10 000 and the size of the simulations is 2 000 repetitions. Our focus is on the estimators' performance in estimating the coefficients β. Hence, for each estimator, we report estimates for bias$(\hat{\beta})$ and MSE$(\hat{\beta})$.

As predicted by theory, MC-results (Table 1) display no significant (warranted by simulation based tests on joint unbiasedness of $\hat{\beta}$) bias for the FIML and the Two-Step estimator, while OLS is biased in any simulation. Furthermore, the ordinary heckit estimator does not exhibit a significant bias for $\gamma = 0$, while it is severely biased for $\gamma \neq 0$. Focussing on the coefficient attached to D_i, depending on the sign of γ, an upward or an downward bias may occur. Interestingly, for $\gamma \neq 0$, the ordi-

Table 1: **Monte-Carlo Simulation Results**

	FIML		Two-Step		Ordinary Heckit		OLS	
	Bias	MSE	Bias	MSE	Bias	MSE	Bias	MSE
simulation (i): $\gamma = -1$; $\tau = -0.75$								
D	0.001	0.002	0.005	0.024	−0.496	0.248	−0.756	0.573
control	0.000	0.001	0.001	0.001	−0.035	0.002	0.927	0.861
constant	−0.001	0.003	−0.001	0.004	0.272	0.077	−0.506	0.257
simulation (ii): $\gamma = -1$; $\tau = 0$								
D	0.001	0.004	0.001	0.009	−1.223	1.496	−1.225	1.501
control	0.000	0.001	0.000	0.001	−0.197	0.040	0.841	0.709
constant	0.000	0.002	0.001	0.004	0.612	0.379	0.494	0.244
simulation (iii): $\gamma = 0$; $\tau = -0.75$								
D	−0.002	0.002	0.000	0.008	−0.003	0.001	0.082	0.008
control	0.001	0.001	0.001	0.001	0.001	0.001	1.082	1.172
constant	0.002	0.002	0.000	0.004	0.002	0.002	−0.512	0.263
simulation (iv): $\gamma = 0$; $\tau = 0$								
D	0.000	0.002	0.002	0.004	0.001	0.002	−0.268	0.073
control	0.002	0.001	0.002	0.002	0.002	0.001	0.737	0.545
constant	0.000	0.002	−0.001	0.004	0.000	0.002	0.515	0.266
simulation (v): $\gamma = 1$; $\tau = -0.75$								
D	−0.001	0.003	−0.002	0.006	0.724	0.526	0.811	0.659
control	0.000	0.001	0.000	0.001	−0.209	0.045	0.842	0.709
constant	0.001	0.002	0.002	0.004	−0.336	0.117	−0.501	0.251
simulation (vi): $\gamma = 1$; $\tau = 0$								
D	−0.001	0.002	−0.002	0.004	0.238	0.058	−0.099	0.011
control	−0.002	0.001	−0.001	0.002	−0.058	0.005	0.600	0.361
constant	0.002	0.002	0.001	0.004	−0.102	0.013	0.544	0.297

Notes: results based on 2 000 replications; sample size $N = 10\,000$; exogenous variables drawn once and then kept fixed; true coefficient values: $\beta_D = 1$, $\beta_{control} = 1$, and $\beta_{const} = 0$.

nary heckit does not perform much better than OLS in terms of the estimated bias. In simulation (vi), it even perform worse. This means, correcting parametrically for selection bias but misspecifying the selection mechanism may not be an improvement compared to simply ignoring selectivity. As expected, Two-Step estimation performs worse compared to FIML, in terms of the estimated MSE.[6] Even for $\gamma = 0$ (simulations iii and iv), FIML exhibits an MSE that just marginally exceeds the MSE of the ordinary heckit model.

[6]For simulations based on a small sample ($N = 400$), this shortcoming of two-step estimation becomes even more prominent. There, in terms of the MSE, two-step estimation may even be outperformed by the biased ordinary heckit estimator.

4 Real Data Application

We apply the estimators discussed above to data from a randomized trial; see Augurzky et al. (2012) for a detailed description and a comprehensive empirical analysis. This experiment aims at analyzing the effectiveness of financial incentives for assisting obese individuals in losing bodyweight. By the end of a rehab hospital stay, 698 over-weight individuals were set an individual weight-loss target (6 to 8 percent of current body weight), which they were prompted to realize within four months. Participants were then randomly assigned to two incentive groups and one control group. While, contingent on success, a reward of up to €150 and €300, respectively, was offered to members of the incentive groups, the control group received no financial incentive. Rewards were offered as a function of the degree of target achievement, i.e., participants who lost some weight but failed to realize the weight-loss target received less than the maximum reward. After four months, participants were requested to visit an assigned pharmacy for verifying actual weight-loss. Yet, a substantial number of participants failed to show up at the weigh-in. More precisely, 178 individuals selected themselves out of the trial, while 520 complied and attended the weigh-in. The compliance rate varied substantially between groups. While for the control group it was 66.5 percent, it was 72.9 and 84.3 percent for the €150 and the €300 group, respectively. This nicely meets our earlier argument that the probability of reporting weight is affected by the interaction of actual weight-loss and group membership, as only those who were both successful and members of one of the incentive groups had an financial incentive to attend the weigh-in.

In the present empirical analysis, the degree of target achievement, i.e., actual weight-loss divided by targeted weight-loss, serves as dependent variable. Indicators for group membership are the key explanatory variables, with the control group serving as reference. Besides these, age and indicators for being female and being born in Germany enter the regression equation as controls. A further dummy indicating that a participant had to visit a nearby pharmacy, i.e., one within the same zip-code area as the place of residence, exclusively enters the selection equation. This exclusion restriction is justified by travel time representing a likely determinant for the decision whether or not to show up at the weigh-in. Yet, there is no obvious link to success.

Table 2 displays regression results for FIML, two-step, ordinary heckit, and OLS.

Table 2: **Results for Weight-Loss Experiment**

	FIML		**Two-Step**		**Ordinary Heckit**		**OLS**	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Main equation:								
€150	0.215	0.143	0.238	0.373	0.429**	0.091	0.408**	0.088
€300	0.470**	0.154	0.272	0.392	0.506**	0.104	0.452**	0.086
age	−0.001	0.004	−0.001	0.006	−0.003	0.004	−0.005	0.004
female	−0.160**	0.079	−0.147*	0.085	−0.172**	0.077	−0.178**	0.076
native	0.020	0.088	0.020	0.091	0.013	0.087	0.008	0.087
δ_{control}	-	-	0.059	0.687	-	-	-	-
$\delta_{\in 150}$	-	-	0.470	0.425	-	-	-	-
$\delta_{\in 300}$	-	-	0.762	0.903	-	-	-	-
constant	0.340	0.255	0.409	0.577	0.445	0.302	0.655**	0.201
Selection equation:								
€150	−0.077	0.206	-	-	0.200	0.124	-	-
€300	0.521*	0.277	-	-	0.595**	0.133	-	-
age	0.020**	0.005	-	-	0.019**	0.005	-	-
female	0.115	0.128	-	-	0.076	0.116	-	-
native	0.039	0.143	-	-	0.057	0.129	-	-
nearby pharmacy	0.332**	0.122	-	-	0.309**	0.108	-	-
$\gamma_{\in 150}$	1.206**	0.484	-	-	-	-	-	-
$\gamma_{\in 300}$	0.142	0.495	-	-	-	-	-	-
constant	−0.817**	0.326	-	-	−0.741**	0.278	-	-
Selection equation control group:								
age	-	-	0.016**	0.008	-	-	-	-
female	-	-	−0.042	0.198	-	-	-	-
native	-	-	0.075	0.216	-	-	-	-
nearby pharmacy	-	-	0.300*	0.179	-	-	-	-
constant	-	-	−0.569	0.489	-	-	-	-
Selection equation €150 group:								
age	-	-	0.024**	0.008	-	-	-	-
female	-	-	0.023	0.195	-	-	-	-
native	-	-	−0.018	0.208	-	-	-	-
nearby pharmacy	-	-	0.537**	0.183	-	-	-	-
constant	-	-	−0.836**	0.424	-	-	-	-
Selection equation €300 group:								
age	-	-	0.019**	0.009	-	-	-	-
female	-	-	0.308	0.231	-	-	-	-
native	-	-	0.175	0.274	-	-	-	-
nearby pharmacy	-	-	−0.056	0.215	-	-	-	-
constant	-	-	−0.077	0.522	-	-	-	-
σ_ε	0.836**	0.034	-		0.802**	0.036	0.795	
ρ	0.199	0.252	-		0.260	0.271	-	

Notes: ** significant at 5%; * significant at 10%; total number of obs. is 698; for 178 obs. weight-loss information is missing.

Test results do not clearly argue for selection bias being an issue since neither for ordinary heckit nor for FIML the estimate for ρ significantly deviates from zero. This equivalently holds for the two-step approach, where the group-specific Mill's ratios are jointly insignificant. Yet, conditional on selection correction, both FIML and two-step are clearly favored over ordinary heckit (p-values 0.03 and 0.01).

Focussing on estimated incentive effects, the choice of estimation method clearly matters. OLS and ordinary heckit, both, yield the result that receiving a financial

incentive increases the success rate by roughly 40 to 50 percentage points. Yet, the amount of the financial reward seems to be immaterial for either model. FIML and two-step estimation of the generalized model, however, yield a different picture. For the latter, no significant incentive effect is seen whatsoever. Here, the inefficiency of two-step estimation is underpinned by rather large standard errors. For FMIL, the estimated incentive effect for the €300 group is similar to ist counterpart from OLS and ordinary heckit estimation. Yet, the estimated effect for the €150 group is substantially smaller and even becomes statistically insignificant. Hence, on basis of FIML, one concludes that the amount of the reward matters for weight loss. The estimates for $\gamma_{\in 150}$ and $\gamma_{\in 300}$ bear the expected positive sign, however – contrary to expectations – $\gamma_{\in 300}$ is much smaller than $\gamma_{\in 150}$ and is accompanied by a relatively large standard error, rendering it statistically insignificant. This may be explained by the small number of dropouts in the €300 group, rendering the identification of $\gamma_{\in 300}$ difficult.

5 Conclusions

In this article we demonstrate that the classical Heckman (1976, 1979) selection correction estimator is misspecified and inconsistent when an interaction of the outcome and an explanatory variables matters for selection. Randomized trials assessing the effects of incentive scheme, may serve as a typical example for such kind of sample selection mechanism. An FIML and a simple two-step estimator that address this specification problem are developed. Monte-Carlo simulations illustrate that the bias of the ordinary Heckman (1976, 1979) estimator is cured by these generalized estimation procedures. Finally, the suggested estimators are applied to data from a randomized trial that evaluates the effectiveness of financial incentives for assisting obese in their attempt for losing bodyweight. Estimation results indicate that the choice of the estimation procedure clearly matters.

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A Appendix

A.1 Generalizing the Log-Likelihood Function

In order to generalize the log-likelihood function of the ordinary heckit model (see e.g. Amemiya, 1985, p. 386), we augment the index function $\alpha' Z_i$ by $\gamma\beta' X_i D_i$ and replace the scalar parameters σ_v^2 and $\sigma_{\varepsilon v}$ by the functions (5) and (6), respectively:

$$l_i = \begin{cases} \log\Phi\left(\frac{-\alpha' Z_i - \gamma\beta' X_i D_i}{\sqrt{\operatorname{var}(\tilde{v}_i|D_i)}}\right) & \text{if} \quad S_i = 0 \\ \log\Phi\left(\frac{\alpha' Z_i + \gamma\beta' X_i D_i + (Y_i - \beta' X_i)\left(\frac{\operatorname{cov}(\varepsilon_i, \tilde{v}_i|D_i)}{\sigma_\varepsilon^2}\right)}{\sqrt{\operatorname{var}(\tilde{v}_i|D_i)\left(1 - \frac{\operatorname{cov}(\varepsilon_i, \tilde{v}_i|D_i)^2}{\sigma_\varepsilon^2 \operatorname{var}(\tilde{v}_i|D_i)}\right)}}\right) & \text{if} \quad S_i = 1. \\ -\frac{1}{2}\left(\frac{Y_i - \beta' X_i}{\sigma_\varepsilon}\right)^2 - \log\left(\sigma_\varepsilon\sqrt{2\pi}\right) \end{cases} \tag{11}$$

Then we apply the normalization (7) to (5), and eliminate τ and $\sigma_{\varepsilon v}$ by entering (8) into the equation, yielding

$$\begin{aligned} \operatorname{var}(\tilde{v}_i|D_i) &= 1 + 2\gamma(\sigma_{\varepsilon v} + \tau\sigma_\varepsilon^2)D_i + \sigma_\varepsilon^2\gamma^2 D_i^2 \\ &= 1 + 2\rho\sigma_\varepsilon\gamma D_i + \sigma_\varepsilon^2\gamma^2 D_i^2, \end{aligned} \tag{12}$$

which is nonnegative, by ρ being bounded to the $[-1, 1]$ interval. Further, using (6) and, once more, eliminating τ and $\sigma_{\varepsilon v}$ by entering (8) into the equation yields

$$\frac{\operatorname{cov}(\varepsilon_i, \tilde{v}_i|D_i)}{\sigma_\varepsilon^2} = \frac{\sigma_{\varepsilon v} + (\tau + \gamma D_i)\,\sigma_\varepsilon^2}{\sigma_\varepsilon^2} = \frac{\rho}{\sigma_\varepsilon} + \gamma D_i. \tag{13}$$

Finally, using (13) and (12) we simplify

$$\begin{aligned} \operatorname{var}(\tilde{v}_i|D_i)\left(1 - \frac{\operatorname{cov}(\varepsilon_i, \tilde{v}_i|D_i)^2}{\sigma_\varepsilon^2 \operatorname{var}(\tilde{v}_i|D_i)}\right) &= \operatorname{var}(\tilde{v}_i|D_i) - \sigma_\varepsilon^2\left(\frac{\rho}{\sigma_\varepsilon} + \gamma D_i\right)^2 \\ &= \operatorname{var}(\tilde{v}_i|D_i) - \rho^2 - 2\rho\sigma_\varepsilon\gamma D_i - \sigma_\varepsilon^2\gamma^2 D_i^2 \\ &= 1 - \rho^2, \end{aligned} \tag{14}$$

and substitute (12), (13), and (14) into (11), yielding (9).