

The difficulties of the teaching of the analysis in the transition of the middle and a higher education at Kaposvár University

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I have been teaching for 25 years. I used to teach in the lower and higher classes of primary school and secondary school, too. I have been teaching at Kaposvár University for 4 years.

My students study:

- Agribusiness and agricultural rural development programme
- Agricultural engineering programme
- Finance and accountancy programme

We find that our students have little success in mathematics. But why is it so? In my opinion one of the reasons is that the higher education became multitudinous, and so even the ones with average ability get to into universities. On the other hand: As I see, the problem is that in the teaching-learning process the foundations of mathematics is left for higher education, but this way the acquisition of other subjects is hindered, too, because the “laying of foundations” is not finished yet.

The teaching-learning process is damaged on the different levels of the education.

How can we make up for the there differences in higher education?

I think this topic important because analysis of mathematics is a basic subject for our students and they have to know functional operations in order to be able to describe economic processes with the help of functions.

I have been dealing with the Bruner’s representational theory and I am trying to adapt it to my research. Bruner examined the codes with the help of which man stores the information arriving from the external world. All thought processes may happen on one of three kinds of level according to it:

- Material level (actual objective acts, activities)
- Iconic level (visual education, situation)
- Symbolic level

The 3 representation methods take part in each phase of the teaching process. In my opinion the visual education is very important, that is why I tried to provide everyday, lifelike illustrations to help the acquisition of the material.

Before starting studying the students are tested from in mathematics. Questions are about the number and function abstraction and about the model creation in the test. We reveal their deficiencies based on their solutions.

We found that they have deficiencies in the following:

- The order of doing operations on numbers (this is very important)
- The rules of the index laws
- Methods of fractions

These are the tests:

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Test A

(You may work with any calculator.)

1. Find the accurate value of the next expressions. Put them in increasing order.

$$\sqrt{9}; \log_3 \frac{1}{27}; \sqrt[3]{-\frac{1}{8}}; \frac{5^2 - 2^2}{3}; \left(\frac{1}{3}\right)^{-1}$$

2. Solve the next equations and inequalities.

a) $(x-2) \cdot (x^2-6) \cdot (x^2+2) = 0$

b) $5^x + 5^{x-1} + 5^{x+1} = 155$

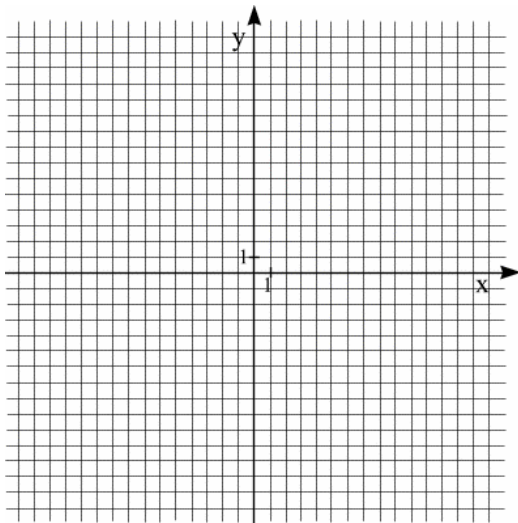
c) $|x-1| \leq 3$

d) $x^2 > 36$

e) $-2x + 4 > 1$

3. Graph the next functions.

$$f(x) = x^2 \quad g(x) = x^2 - 4 \quad h(x) = (x-4)^2$$



4. Define the domain.

$$a(x) = \frac{1}{8-4x} \quad b(x) = \sqrt{4x+8} \quad c(x) = \log_5(3-x)$$

5. The price of a shirt is 4500 Ft. Its price is decreased by 15%. How much is it now? What percentage is this of the original price?
6. Andrea has 500 Ft pocket money. She economizes by saving 300 Ft every month. Antal has 5 Ft, he doubles his previous monthly amount every month. Whose money will be more after a year?
7. Miklós with his son and Péter with his son went angling. Miklós caught as many fish as his son, Péter three times as many as his son. Altogether 35 fish were caught. Miklós revealed that his son was called Gergely. What is Péter's son called?

29.08.2007.

Test B

(You may work with any calculator.)

8. Find the accurate value of the next expressions. Put them in decreasing order.

$$\sqrt{\frac{1}{9}}; \log_3 81; \sqrt[3]{-8}; \frac{5^2 - 2^2}{7}; \left(\frac{1}{3}\right)^{-2}$$

9. Solve the next equations and inequalities.

a) $(x+2) \cdot (x^2+6) \cdot (x^2-2) = 0$

b) $2^{x+2} + 2^{x-1} - 2^{x+1} = 5$

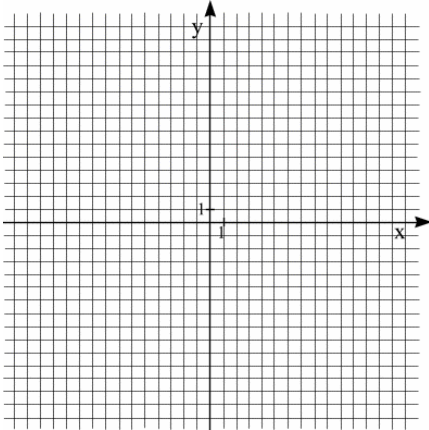
c) $|x-1| > 3$

d) $x^2 \leq 36$

e) $-2x + 4 < 3$

10. Graph the next functions.

$$f(x) = x^2 \quad g(x) = x^2 - 9 \quad h(x) = (x+3)^2$$

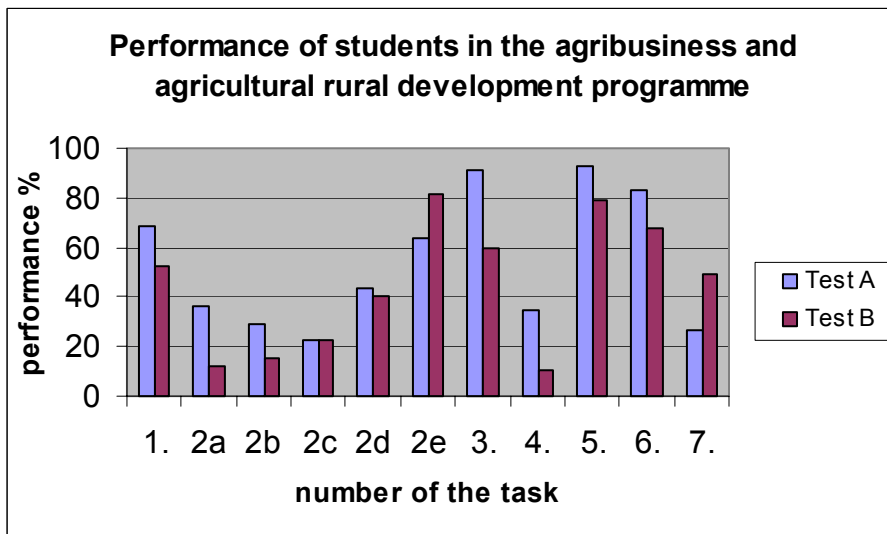
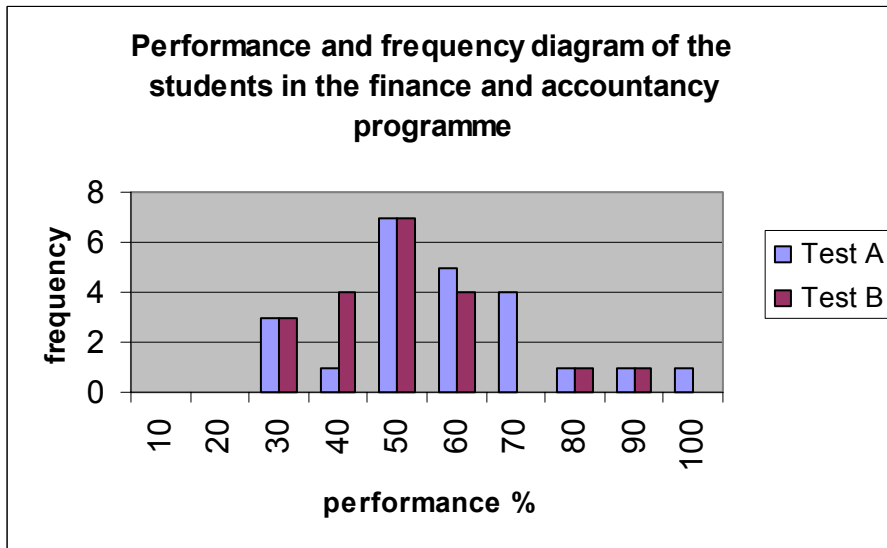
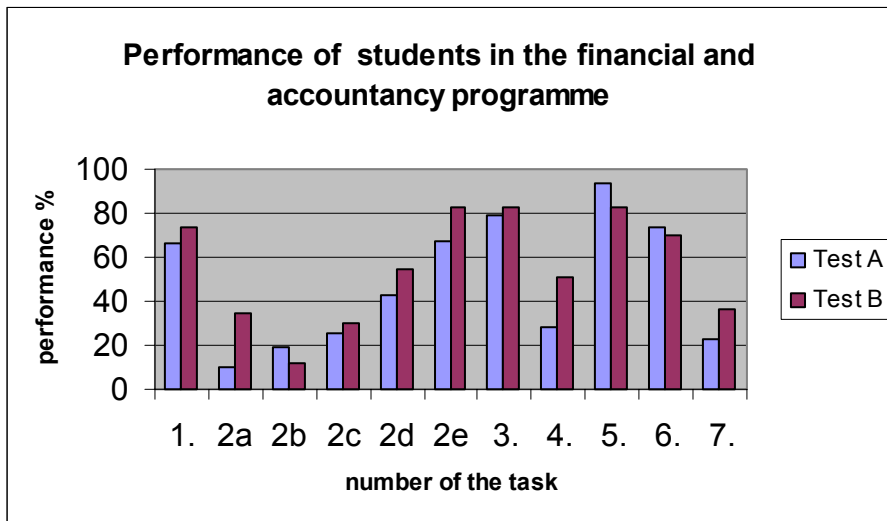


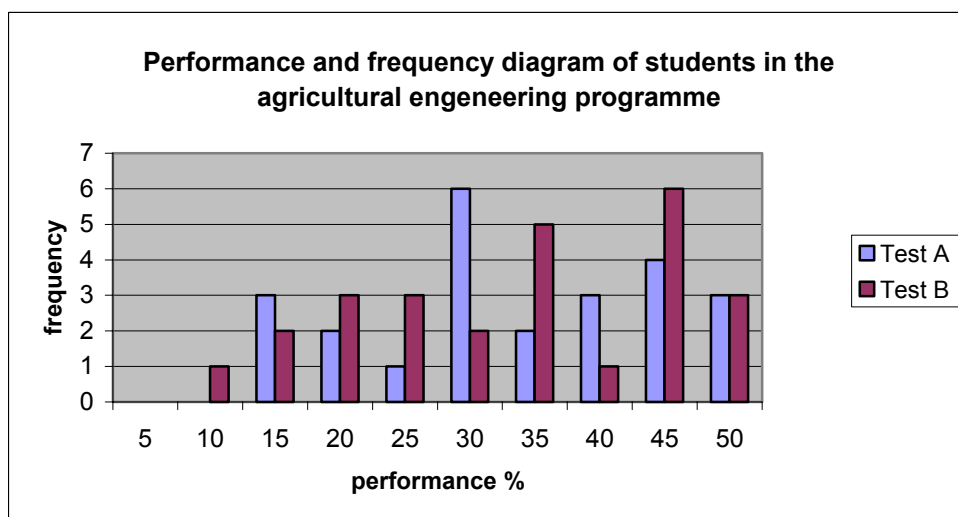
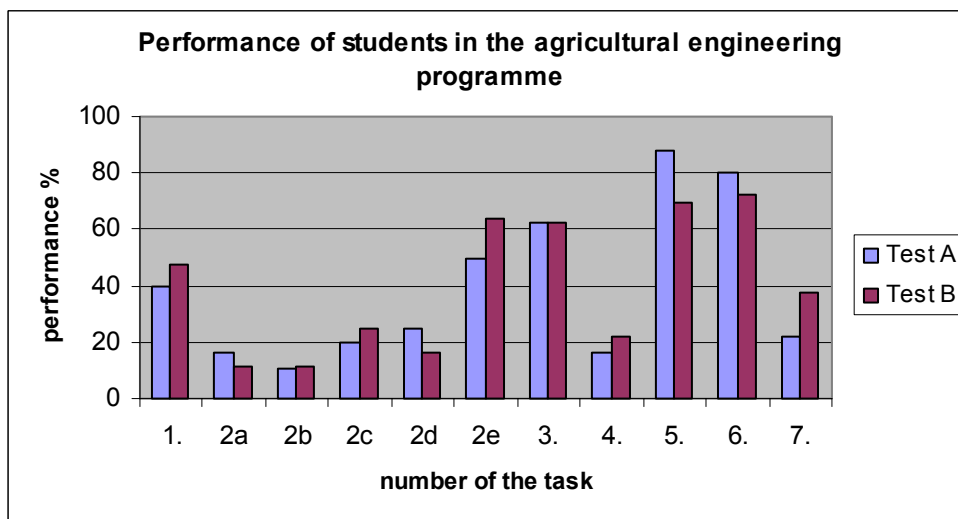
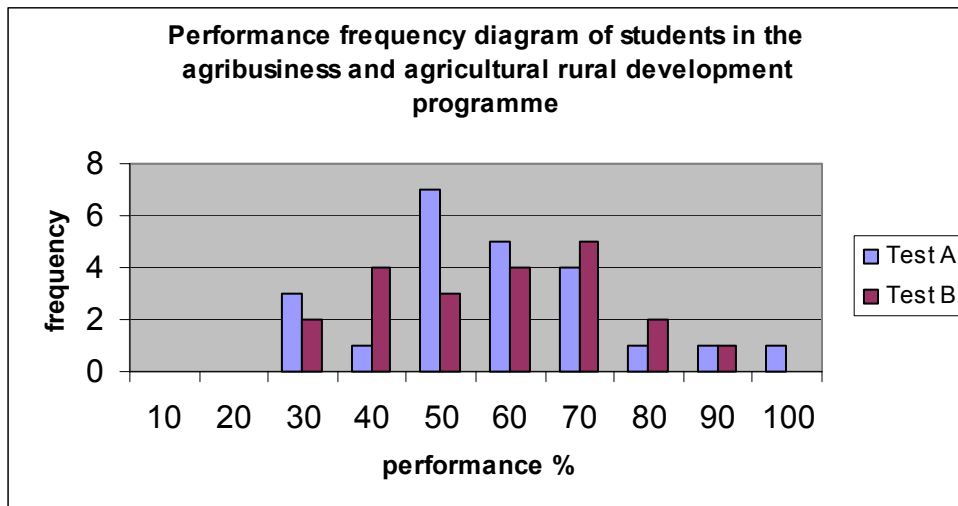
11. Define the domain.

$$a(x) = \frac{1}{4-8x} \quad b(x) = \sqrt{8x+4} \quad c(x) = \log_5(-3-2x)$$

12. We deposit 150000 Ft in a bank. The annual interest rate is 8%. How much money will we have one year later? What percentage is this of the original amount?
13. There are two pizzerias in the town. The first pizzeria says: buy one pizza on Monday and buy on each weekday one more than the previous day, then you get a free pizza at the weekend (On Saturday and Sunday too). The second pizzeria says: buy half a pizza on Monday and on each weekday double the amount of the previous day, then you get a free pizza at the weekend. Which pizzeria makes more profit? (The pizzas cost the same in both restaurants.)
14. András and Béla did not wait for the tram and they took the road to the next stop. When they were at one third of the road, the tram appeared behind them. András turned back, and the tram and he arrived into the stop at the same time. Béla went on, and the tram caught up with him in the next stop. How many times the velocity of the tram bigger than the pedestrians? (András's and Béla's velocity is equal.)

These are the results of students:





We saw the weakest result in the solution of the next tasks:

2. Solve the next equations and inequalities.

a) $(x-2) \cdot (x^2-6) \cdot (x^2+2) = 0$

b) $5^x + 5^{x-1} + 5^{x+1} = 155$

c) $|x-1| \leq 3$

d) $x^2 > 36$

e) $-2x + 4 > 1$

4. Define the domain.

$$a(x) = \frac{1}{8-4x} \quad b(x) = \sqrt{4x+8} \quad c(x) = \log_5(3-x)$$

In these tasks the students could achieve only 10-20%.

These types of tasks are really important in the course of the first six-month mathematics. We recommend an optional subject to the students. It was called Teaching of mathematics using computer. This course was going in parallel with the mathematics I. (calculus) subject.

85% of the students took the course. We taught with my colleague in 3 groups: finance, rural development and in a group with mixed combination.

The subject had a threefold aim:

- The development and conditioning of the basis
- To link it closely with higher mathematics
- To link it with the use of computers.

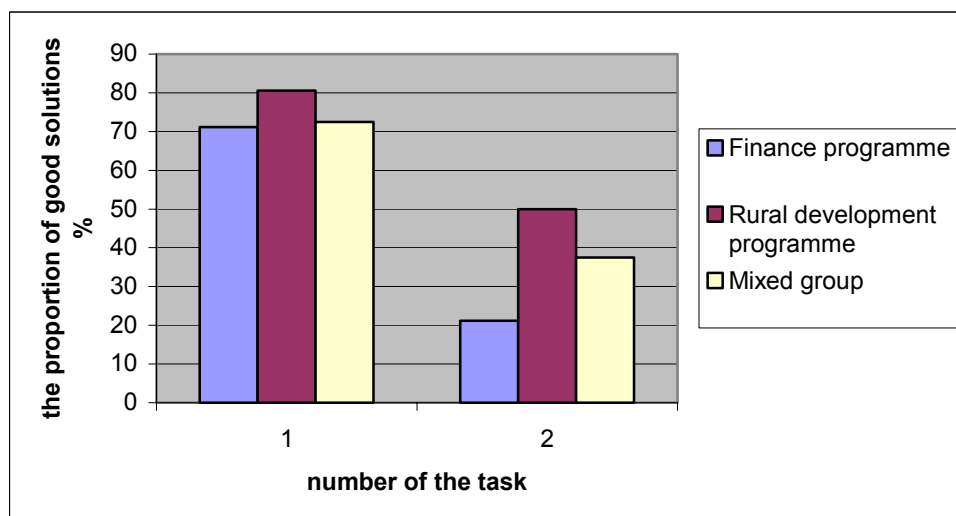
I first tried to development:

- solve of the equations and inequalities, because we have to use at the monotonic sequences and find the limits and define the domain, too

We dealt with the next types:

$$\frac{3x-5}{2x+3} < 1 \quad (1) \quad \left| \frac{3-4x}{9+2x} + 2 \right| < 1 \quad (2)$$

After 1 months of practice, we attained the next result:



I show some of the student's solved work.

a) $\frac{2x+3}{x-4} > 1 \quad | \cdot (-1)$ $\frac{2x+3}{x-4} - \frac{x-4}{x-4} > 0$ $\frac{2x+3-x+4}{x-4} > 0$ $\frac{x+7}{x-4} > 0$

2.) a) $\frac{2x+3}{x-4} > 1 \quad | \cdot (x-4)$

$$2x+3 > x-4$$

$$x+3 > -4$$

$$\underline{x > -7}$$

2.) $\frac{2x+3}{x-4} > 1$

$$2x+3 > x-4$$

$$2x > x-7$$

$$\underline{x > -7}$$

a) $\frac{3x-5}{2x+3} < 1$ $2x+3 > 0$

$$\frac{3x-5}{2x+3} - 1 < 0$$

$$\frac{3x-5-2x-3}{2x+3} < 0$$

$$\frac{x-8}{2x+3} < 0$$

$\frac{+}{+}$ $-x+8 < 0$ $2x+3 < 0$ $\underline{8 < x}$ $2x < -3$ $x < -\frac{3}{2}$	$\frac{+}{-}$ $x-8 < 0$ $\underline{-2x-3 < 0}$ $x < 8$ $-2x < 3$ $x > -\frac{3}{2}$
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a) $2x+3 \neq 0$
 $2x \neq -3$
 $x \neq -\frac{3}{2}$

$$\frac{3x-5}{2x+3} - 1 < 0$$

$$\frac{3x-5-(2x+3)}{2x+3} < 0$$

$$\frac{3x-5-2x-3}{2x+3} < 0$$

$$\frac{x-8}{2x+3} < 0$$

$\frac{+}{+}$ $x-8 < 0$ $2x+3 > 0$ $\underline{x < 8}$ $x > -\frac{3}{2}$	$\frac{+}{-}$ $x-8 > 0$ $2x+3 < 0$ $\underline{x > 8}$ $x < -\frac{3}{2}$
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2. a)

$$\frac{2x+3}{x-4} > 1$$

$$\frac{2x+3}{x-4} - 1 > 0$$

$$\frac{2x+3-x+4}{x-4} > 0$$

$$\frac{x+7}{x-4} > 0$$

$$1. \frac{+}{+} \quad 2. \frac{-}{-}$$

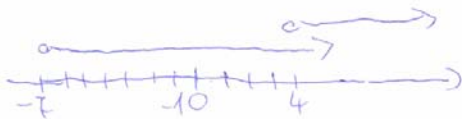
1. eset:

$$\frac{x+7}{x-4} > 0$$

$$x+7 > x-4$$

$$x+7 > 0 \quad \text{és} \quad x-4 > 0$$

$$x > -7 \quad \quad \quad x > 4$$



$$\boxed{x > 4}$$

2. eset

$$x+7 < 0 \quad \text{és} \quad x-4 < 0$$

$$x < -7 \quad \quad \quad x < 4$$



$$\boxed{x < -7}$$

~~megoldás:~~

$$\underline{\underline{x > 4 \quad \vee \quad x < -7}}$$

$$b) \left| \frac{3-4x}{9+2x} + 2 \right| < 1 \quad x \neq -4,5$$

$$\left| \frac{3-4x}{9+2x} + \frac{18+4x}{9+2x} \right| < 1$$

$$\left| \frac{21}{9+2x} \right| < 1$$

$$\frac{21-9-2x}{9+2x} < 0$$

$$\frac{12-2x}{9+2x} < 0$$



$$12 > 2x$$

$$\underline{6 > x}$$

$$9+2x < 0$$

$$9 < -2x$$

$$4,5 < -x$$

$$\underline{\underline{-4,5 > x}}$$

$$b_1 \left| \frac{3-4x+2}{9+2x} \right| < 1$$

$$\left| \frac{3-4x+2(9+2x)}{9+2x} \right| < 1$$

$$\left| \frac{3-4x+18+4x}{9+2x} \right| < 1$$

$$\left| \frac{21}{9+2x} \right| < 1$$

$$\frac{21}{9+2x} < 1$$

$$21 < 9+2x$$

$$12 < 2x$$

$$6 < x$$

$$u.) \left| \frac{n-1}{n+2} - 1 \right| < 10^{-3}$$

$$\left| \frac{n-1}{n+2} - \frac{n+2}{n+2} \right| < \frac{1}{1000}$$

$$\frac{n-1-n-2}{n+2} < \frac{1}{1000}$$

$$\frac{-3}{n+2} < 0,001 \quad | \cdot n+2$$

$$-3 < 0,001n + 0,002 \quad | +3 \quad | -0,001n$$

$$-0,001n < 3,002 \quad | : -0,001$$

$$n > 3,002$$

$$\text{Elt.: } \frac{3,002-1}{3,002+2} - 1 < \frac{1}{1000}$$

$$\frac{2,002}{5,002} - \frac{5,002}{5,002} < \frac{1}{1000}$$

$$\frac{-3}{5,002} < \frac{1}{1000}$$

Methods on expressions had helped by work using numbers; we had to substitute many numbers for the expressions.

They had many problems of the monotonic sequence to determine number $n+1$ in a series.
I show it:

① $a_n = \frac{3-n}{3n-5} \rightarrow -\frac{1}{3}$

1. $\frac{2-n}{3n-2}$ 2. $\frac{3-n}{3n-5}$ 3. 0

$\frac{2}{-2} = -1$ $\frac{1}{1} = 1$ 0

rigorously monoton a series

$$a_{n+1} < a_n$$

$$\frac{2-n}{3n-2} < \frac{3-n}{3n-5}$$

$$\frac{2-n}{3n-2} - \frac{3-n}{3n-5} < 0$$

$$\frac{(2-n)(3n-5) - (3-n)(3n-2)}{(3n-2)(3n-5)} < 0$$

$$\frac{(6n-10-3n^2+5n) - (9n^2-6-3n^2+2n)}{(3n-2)(3n-5)} < 0$$

$a^2 + a - 4$

sejt. szig. mon. sorozat

~~$-\frac{1}{3}$~~ 1 0 $-\frac{1}{7}$ $-\frac{2}{10}$

$$a_{n+1} < a_n$$

$$\frac{4-n}{3n-4} < \frac{3-n}{3n-5} \quad | - \frac{3-n}{3n-5}$$

$$\frac{4-n}{3n-4} - \frac{3-n}{3n-5} < 0$$

$$\frac{(4-n)(3n-5) - (3-n)(3n-4)}{(3n-4)(3n-5)} < 0$$

$$\frac{12n-20-3n^2+5n - (9n-12-3n^2+4n)}{(3n-4)(3n-5)} < 0$$

$$\frac{4n-8}{(3n-4)(3n-5)} < 0$$

$$a_n = \frac{4n-4}{5-2n}$$

$$\rightarrow a_{n+1} = \frac{4(n+1)-4}{5-2(n+1)}$$

$$a_{n+1} = \frac{4n}{5-2n-2}$$

$$a_{n+1} = \frac{4n}{3-2n}$$

This is a correct solution

I teach it in the following way:

$$1 \mapsto \frac{2 \cdot 1 - 3}{5 - 1} \quad 2 \mapsto \frac{2 \cdot 2 - 3}{5 - 2} \quad 3 \mapsto \frac{2 \cdot 3 - 3}{5 - 3}$$

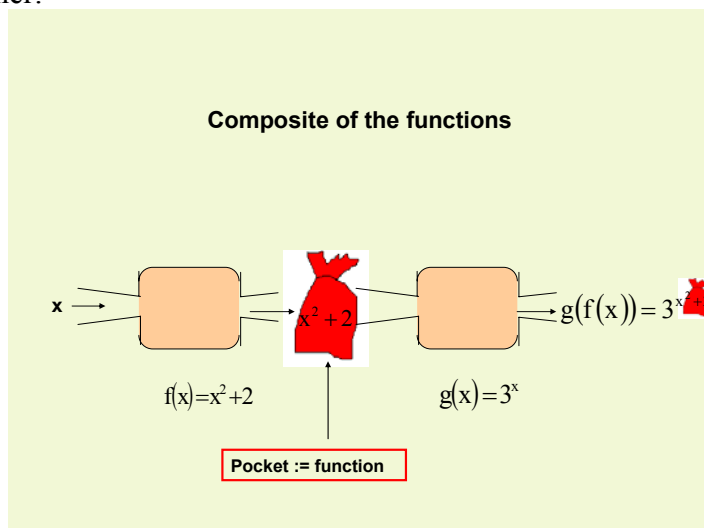
$$4 \mapsto \frac{2 \cdot 4 - 3}{5 - 4} \quad \circ \quad \circ \quad \circ \quad n \mapsto \frac{2 \cdot n - 3}{5 - n}$$

n := ()

$$\begin{matrix} \uparrow & & \uparrow & & \uparrow \\ (&) \mapsto & \frac{2 \cdot (&) - 3}{5 - (&)} \\ \uparrow & & \uparrow & & \uparrow \\ \mathbf{n+1} & & \mathbf{n+1} & & \mathbf{n+1} \end{matrix}$$

- another problem is the composite function, because we have to use at the derivative of a function is found

I teach it in this manner:



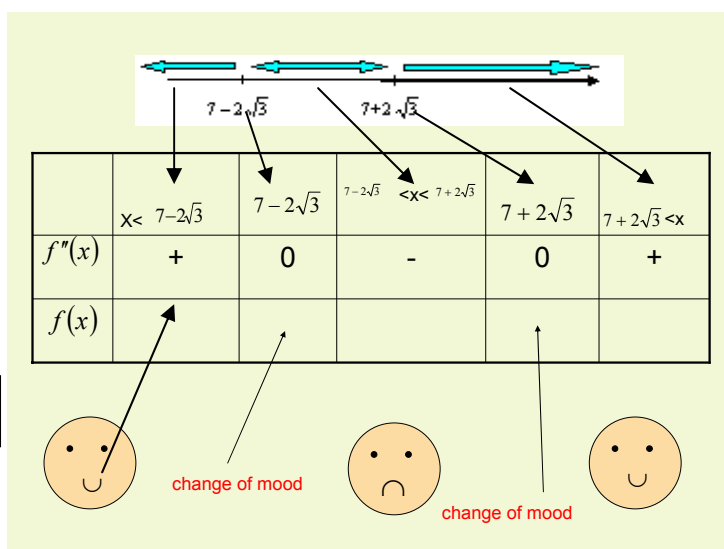
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I made the table about the **derived of the composite function** (paket= function):

$f(x)$	$f'(x)$
const	0
x	1
$x^n, n=\text{szám}$	$n \cdot x^{n-1}$
e^x	e^x
a^x	$a^x \ln a$
$\ln x$	$\frac{1}{x}$
$\lg_a x$	$\frac{1}{x \ln a}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\operatorname{tg} x$	$\frac{1}{\cos^2 x}$
$\operatorname{ctg} x$	$-\frac{1}{\sin^2 x}$
$\operatorname{arctg} x$	$\frac{1}{1+x^2}$
$\operatorname{arccot} x$	$-\frac{1}{1+x^2}$
$\operatorname{arcsin} x$	$\frac{1}{\sqrt{1-x^2}}$
$\operatorname{arccos} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\operatorname{sh} x$	$\operatorname{ch} x$
$\operatorname{ch} x$	$\operatorname{sh} x$

f^n	$n \cdot f^{n-1} \cdot f'$
$n=\text{szám}$	
e^f	$e^f \cdot f'$
a^f	$a^f \cdot \ln a \cdot f'$
$\ln f$	$\frac{1}{f} \cdot f'$
$\lg_a f$	$\frac{1}{f \cdot \ln a} \cdot f'$
$\sin f$	$\cos f \cdot f'$
$\cos f$	$-\sin f \cdot f'$
$\operatorname{tg} f$	$\frac{1}{\cos^2 f} \cdot f'$
$\operatorname{ctg} f$	$-\frac{1}{\sin^2 f} \cdot f'$
$\operatorname{arctg} f$	$\frac{1}{1+f^2} \cdot f'$
$\operatorname{arccot} f$	$-\frac{1}{1+f^2} \cdot f'$
$\operatorname{arc} \sin f$	$\frac{1}{\sqrt{1-f^2}} \cdot f'$
$\operatorname{arccos} f$	$-\frac{1}{\sqrt{1-f^2}} \cdot f'$
$\operatorname{sh} f$	$\operatorname{ch} f \cdot f'$
$\operatorname{ch} f$	$\operatorname{sh} f \cdot f'$

The next table says about convex or concave of the function:



We also discussed the functional representation, there was an opportunity to consider the domain, where it is north representing them, we could apply Excel for the multiplication and reciprocal of functions. It also helped in understanding limit value.

The illustration of fundamental conceptions is relatively good. However only few if any able to illustrate the functional diagram based on the features resulting in the means of differential calculus. Excel provided help in this problem.

I discovered that the students enjoyed using the computer for checking, after we had solved the tasks in a traditional way.

There are working of the students by hand without the Excel program:

$$\begin{aligned}
 1, \quad & y = (x-2)^2 + 1 \\
 & y = (3-2)^2 + 1 = 2 \\
 & y' = 2(x-2) \cdot 1 \\
 & y' = 2 \cdot (3-2) \cdot 1 = 2 \\
 & y - 2 = 2(x-3) \qquad y = 2(x-3) + 2 = 2x - 6 + 2 = \underline{2x - 4} \\
 y_{\text{tör}_1} &= 1(x-3) + 2 \qquad y \Rightarrow x - 3 + 2 = \underline{x - 1} \\
 y_{\text{tör}_2} &= 3(x-3) + 2 \qquad y \Rightarrow 3x - 9 + 2 = \underline{3x - 7} \\
 \\
 2, \quad & y = x^2 - 2x - 4 \text{ lü } x \\
 & y' = 2x - 2 - 4 \cdot \frac{1}{x} \\
 & y'' = 2 + 4 \cdot \frac{1}{x^2} \\
 \\
 & 2x - 2 - 4 \cdot \frac{1}{x} = 0 \quad / \cdot x \\
 & 2x^2 - 2x - 4 = 0 \\
 \\
 x_{1,2} &= \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 2 \cdot (-4)}}{2 \cdot 2} \begin{matrix} \rightarrow -1 \\ \downarrow 2 \end{matrix} \\
 \\
 2 + \frac{4}{x^2} &= 0 \\
 2x^2 + 4 &= 0 \\
 2x^2 &= -4 \\
 x^2 &= -2 \quad \text{mivel ilyen valós szám}
 \end{aligned}$$

Stützstellen mit. nachrechen

1. $w = \frac{f(5) - f(-1)}{5 - (-1)} = \frac{10 - 10}{5 + 1} = \frac{0}{6} = 0$ - kein Wert

$f' = 2(x-2) \cdot 1 = 2x - 4$ $x_0 = 3$ $2 \cdot 3 - 4 = 2$

$y - f(x_0) = f'(x_0)(x - x_0)$

$f(x_0) = 2$

$f'(x_0) = 2$

$y - 2 = 2(x - 3)$

$y - 2 = 2x - 6$

$y = 2x - 4$

2. $y = x^2 - 2x - 4 \ln x$

Et. Df: $x \in \mathbb{R}$ $\frac{1}{x} = 4 \cdot x^{-1}$

$f' = 2x - 2 - (0 \cdot \ln x + 4 \cdot \frac{1}{x})$ $2x - 2 - 4 \cdot \frac{1}{x} = 0$

$f'' = 2 - (0 \cdot x^{-1} + 4 \cdot (-1)x^{-2})$ $2(x - 1 - \frac{2}{x}) = 0$

$2x - 2 - 4 \cdot x^{-1}$ $x - 1 - \frac{2}{x} = 0$

$2 - (0 \cdot x^{-1} + 4 \cdot (-1)x^{-2})$ $x^2 - x - 2 = 0$

$f'' = 2 + 4x^{-2}$ $x_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot (-1) \cdot (-2)}}{2} = \frac{1 \pm 3}{2}$ $x_1 = 2$ $x_2 = -1$

$2 + 4x^{-2} = 0$

$2(1 + 2x^{-2}) = 0$

$1 + 2x^{-2} = 0$

$1 + \frac{2}{x^2} = 0$

$x^2 + 2 = 0$

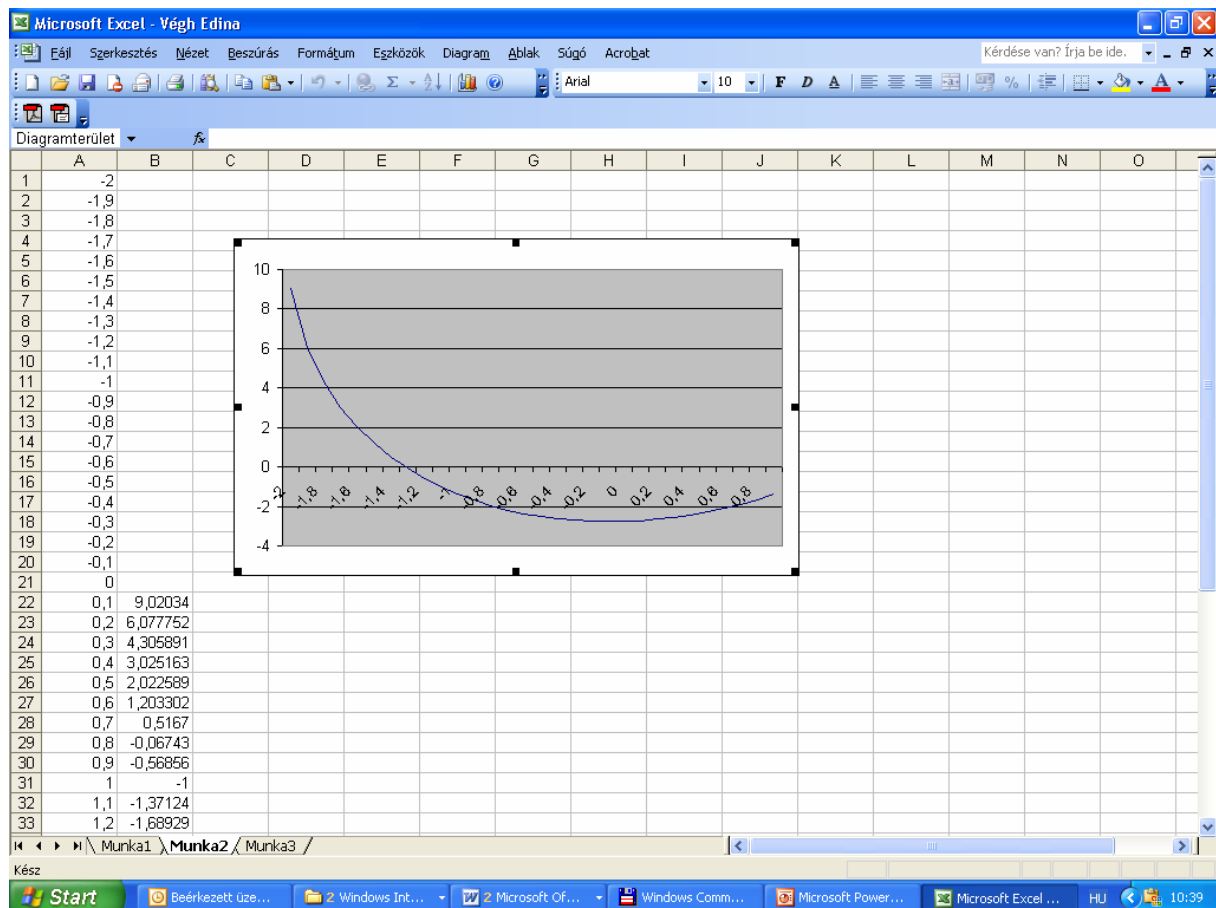
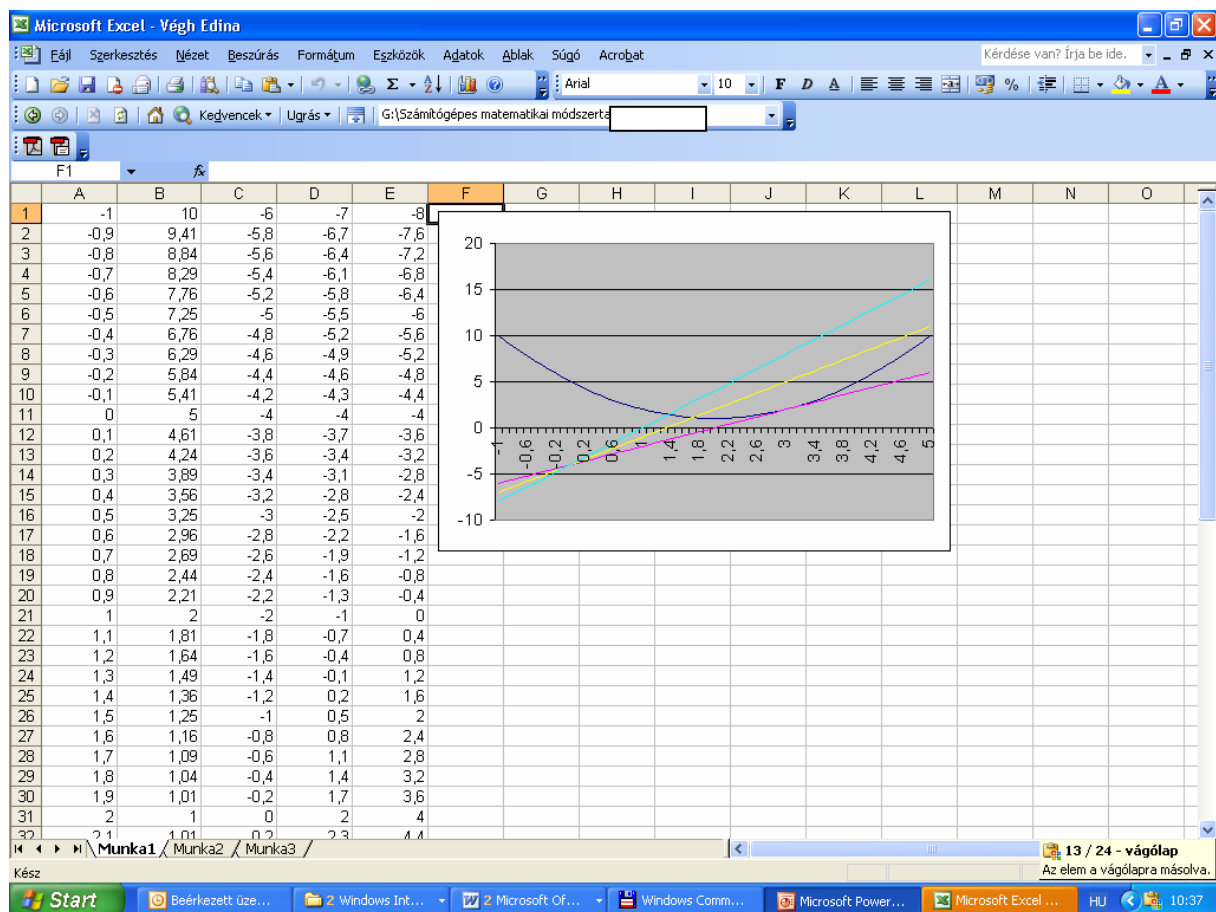
$x^2 = -2$

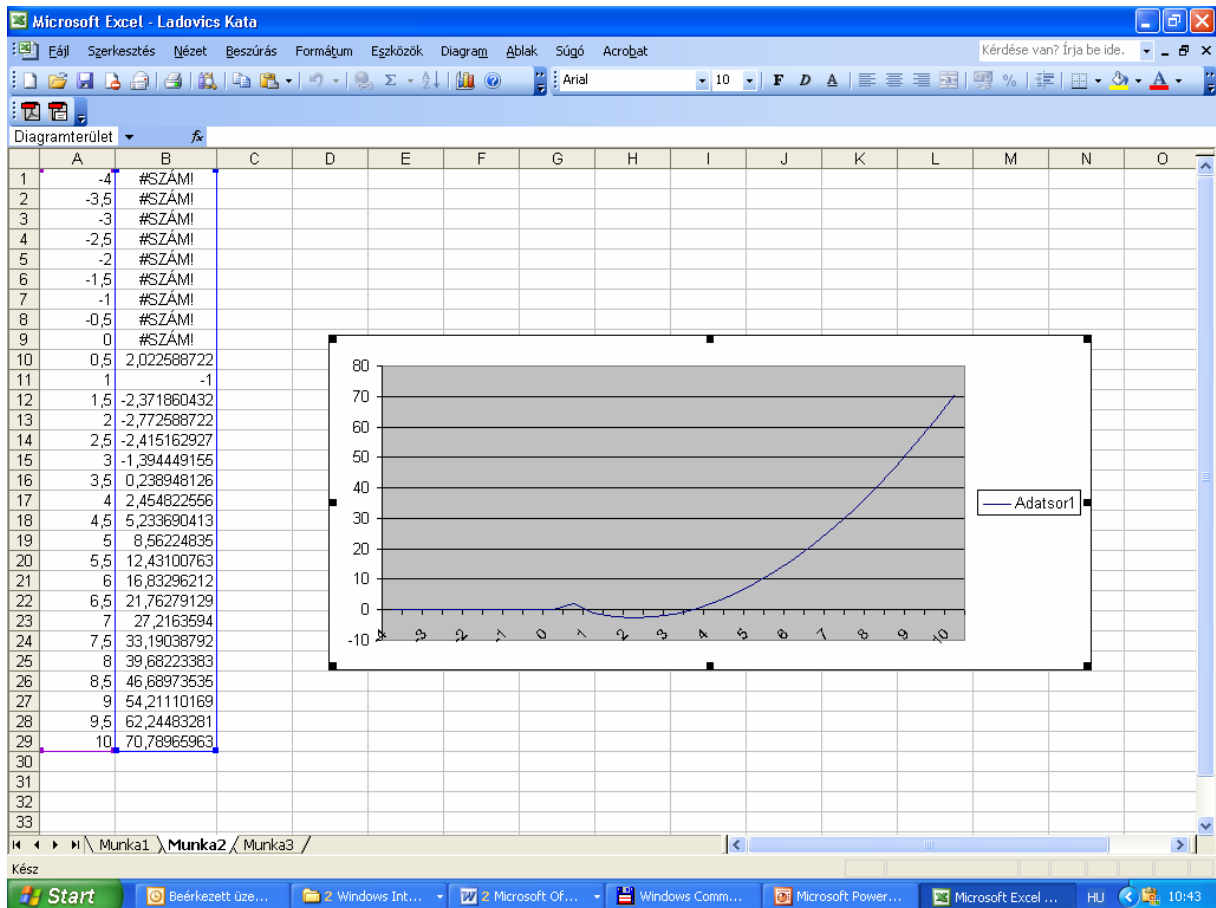
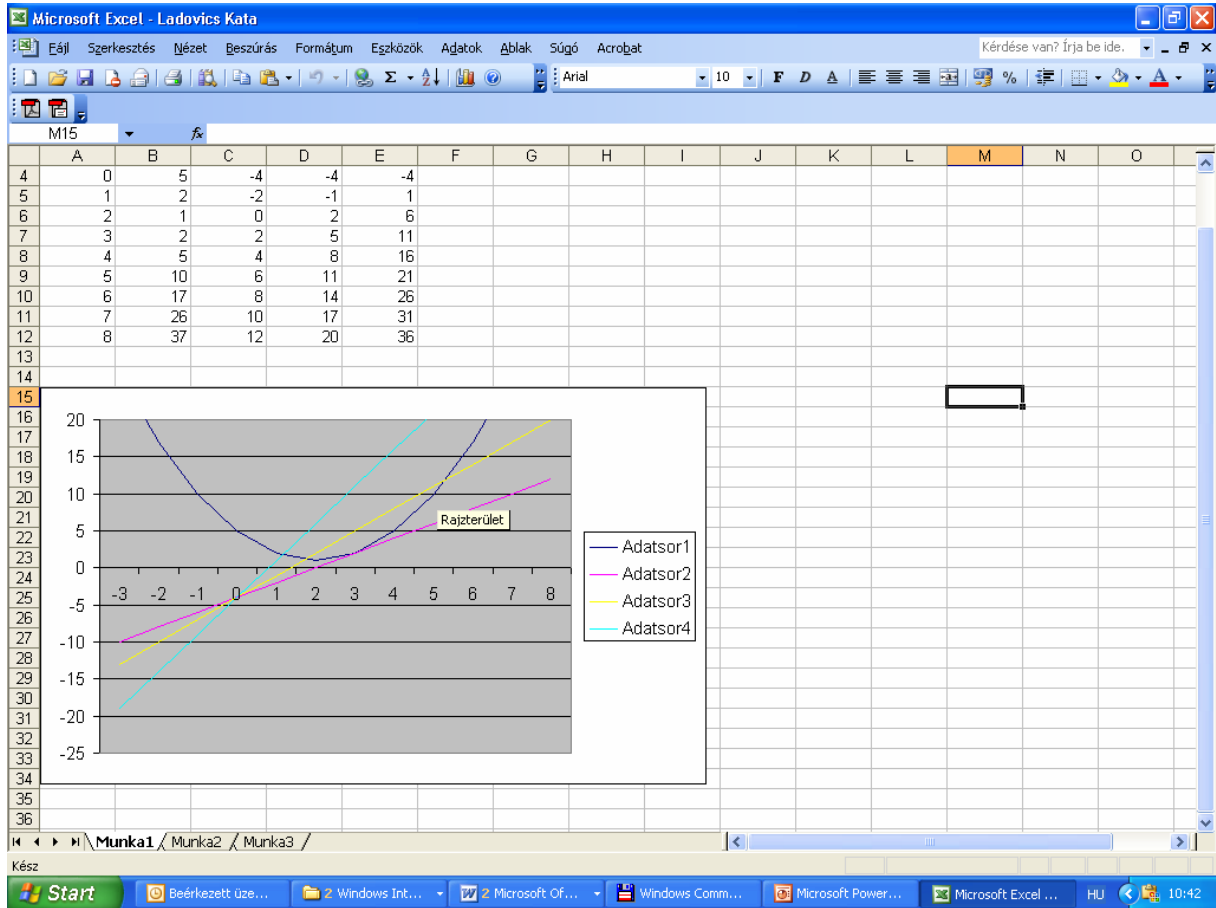
x	$x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
f'		0		0	
f''	↘		↗		↘

These are the exercises:

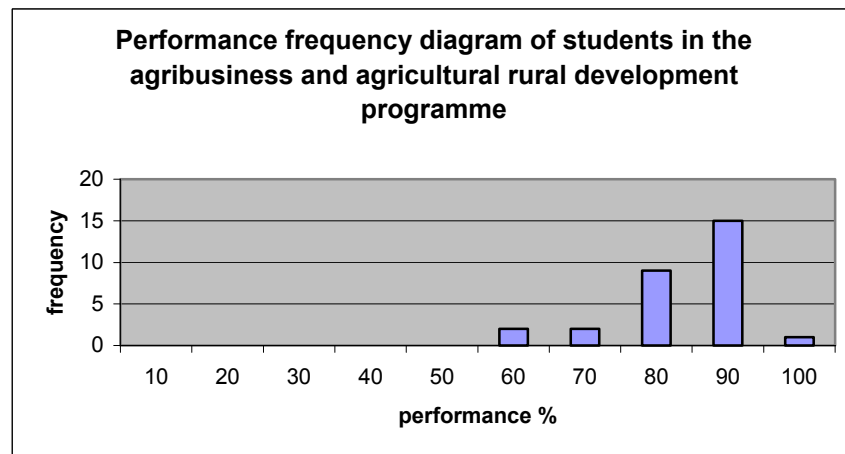
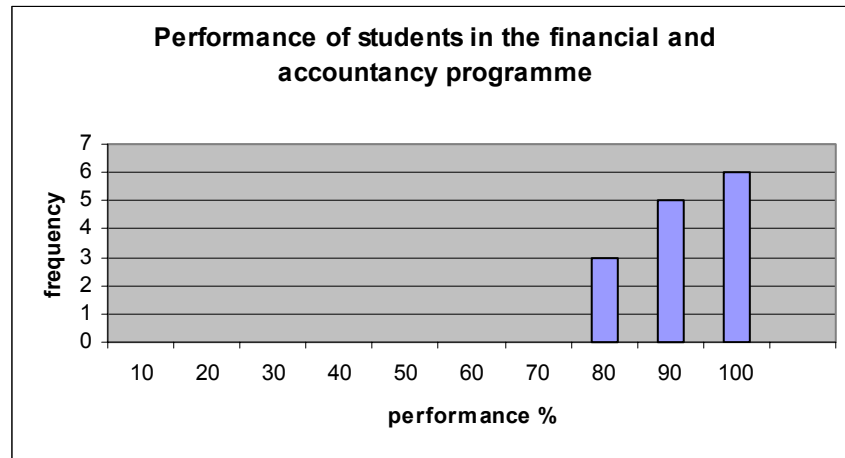
- Graph the next $y = (x - 2)^2 + 1$ function and its tangent in the point of tangency $x_0 = -1$! Draw two lines, what intersect this function in point of tangency!
- Graph the next function and find the characteristic properties!
 $y = x^2 - 2x - 4 \ln x$

I show the working of my students with the Excel program in the following:





At the beginning of December we made the students write the test again. The situation in December was the following:



I think the use of the computer is an opportunity to help the interaction between the cognitive levels listed above. In the last semester we collect positive feedback during teaching this subject. The use of Excel is effective, and it facilitated the formulation of the definitions of sequence and function. It was important to use a programme which is available for every student and they can use it during preparation, as well. Within the frames of this subject there is a possibility for development in my opinion.

After finishing the exams I enquired the students opinion concerning the subject:

Did the subject help in the revision of the relevant chapters of the secondary school teaching matter?

Yes 79%

Partly 21%

Did it facilitate to concentrate on the mathematical problem?

Yes 59%

No 12%

Partly 29%

Did you use Excel for checking at home?

Yes 36%

No 47%
Partly 17%

In your opinion would it be useful to connect the sequel to further mathematical studies?

Yes 68%
No 3%
Partly 29%

The opinions of our students:

- The lessons were held a pleasant and relaxed atmosphere; they helped me to understand new information, and enriched my knowledge.
- It was very useful and rewarding.
- It was a great help to make up for my deficiencies. I appreciated the relatively slow pace and the step-by-step solutions.
- The secondary school teaching material became more systematic, owing to the subject.
- It helped a lot to understand the subject-matter of the lectures.
- In my opinion it helped me in the practical acquisition of mathematics. The representation of functions in Excel supported the visualization of the tasks solved in an algebraic way.
- It was useful, it helped in learning.
- It would have been more enjoyable in a bigger classroom, with more working computers. Excel did not cause any problems, it is an excellent means for checking. It is a pity that due to schedule problems, I could not register for the subject any longer.

In the future we plan the application of computer methods in the further basic chapters of mathematics.