

**From design experiments to
innovation at scale:
potential and challenges
for research in mathematics
education**

Professor Dr Celia Hoyles O.B.E

London Knowledge Lab, Institute of Education,
University of London

Director of the National Centre for Excellence in
the Teaching of Mathematics

Vielen Dank für die Einladung

Ich liebe Münster

structure of my talk

1. design experiments in mathematics education: some general issues
2. design experiments that embed use of digital technologies
3. some ongoing research
4. factors in scaling up

different 'types' of research
'science' & 'engineering'

design research

starts with “big ideas” in mathematics that are identified by research (theory or empirical experiment) as hard to teach and learn (they are ‘obstacles’)

theoretically & empirically-driven task design & task progression to

- address these obstacles
- align with ‘local’ curriculum & assessment practices

iterative design experiments to test with teachers as co-designers

Design research often reports ‘dramatic’ impact on student outcomes

need more caution about reporting results of design experiments?

- innovations in and of themselves do not innovate
- maybe other options explain positive results of implementing the innovation or design experiment that do not privilege one part of the innovation

Temptation to pick one area as reason for change///

XXXXX has demonstrated that attainment can be dramatically improved by using XXXXXXXXX to improve students' learning

But many caveats

intervention groups relatively high attaining in relation to matched comparison

different timings of the student tests for intervention and for the comparison group

teachers involved in the interventions self-selected volunteers and very enthusiastic

the intervention involved professional development for teachers

professional development undertaken by the design team

BUT cumulative evidence of effects of design research is dramatic

Design research: a framework?

Some conjectures:

1. recognise importance of design & complexity of implementation
2. make explicit the
 - epistemological basis of the intervention
 - and the design decisions made
4. be clear about the need for evidence-based, expert **teacher professional development & materials**

Design research: a framework?

And recognise the limitations of what can be achieved by teaching: e,g

“difficulty of multiplicative reasoning and mastering proportionality is not simply engendered by teaching--- huge conceptual step to be taken by students in order to master proportional (or, more generally, functional) reasoning”. (Nunes & Bryant)

need to revisit in spiral way?

and bridge to ‘standard’ curriculum?

A note about context?

- mathematics carries considerable social capital
- But in a world of digital images, animations, easy information retrieval & communication students can dismiss mathematics as irrelevant, boring and hard
- importance of tapping into youth culture to motivate students to sustain their engagement with mathematics

We try to build **engaging** environments for learning mathematics where mathematics is

- **needed** for students to achieve the goals of the activity
- **compelling** for students
- the **actions in** the environment help students engage in mathematical thinking: shift from pragmatic to epistemic use of digital tools

Design research that embeds digital technology

We exploit **explicitly** the functionalities of the technology **for mathematics teaching & learning**

- its dynamic & visual nature ...plus multi-media, connectivity
- its potential to
 - **present & link different representations dynamically**
 - provide feedback designed so students **learn** from the results of their actions
 - leave a trace of **how solution is constructed**, not just the answer
 - provide tools so students can **share** and discuss

Design research that embeds digital technology

Again dramatic success in
design experiments
But not just the technology of
course.....
and problems of scale

Some examples

**Designing software for mathematical
engagement through modelling, 17th ICMI study
book** Hoyles & Lagrange (2009)

Theoretical framework of Constructionism

ICMI Study 1: the influence of computers & informatics on mathematics & its teaching 1985

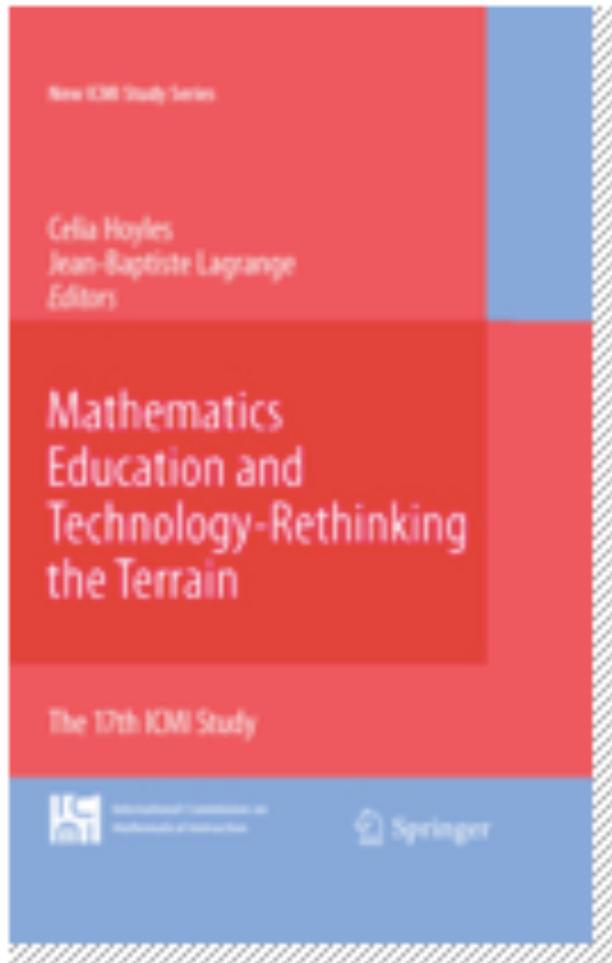
Churchhouse, R. F., Cornu, B., Howson, A. G. Kahane, J.-P., van Lint. J. H., Pluvinage, F., Ralston, A. & Yamaguti, M. (Editorial Board). (1986). *The influence of computers and informatics on mathematics and its teaching*. Cambridge: Cambridge University Press, ICMI Study series.

reprinted 1992

The 17th ICMI Study

Technology revisited

Hoyles, C & Lagrange J-B (eds) (2009) *Mathematics Education and Technology- Rethinking the terrain*



Five perspectives

- theory: (wealth of perspectives)
- the role of the teacher
- curricula framing
- geographical diversity
- design

'new' issues

Many new issues compared to first ICMI study

- equity
- diversity
- Papert's 10% - new types of mathematical knowledge and practice
-
- connectivity

And now...?
Mobile technology
applets & tablets
and...



Design research that embeds digital technology

top down or bottom up?

Need balance of

- teacher autonomy & supportive CPD
- pragmatic tool use & epistemic tool use
-and top down support aligned with bottom up growth

Need to research the mechanism of scaling up

Starting points for successful scaling of design research?

1. enhance teacher capacity for professional learning in mathematics
2. make mathematics more visible: opening up the 'black box'
3. build teacher capacity 'bottom up' through communities of teachers

1. Enhance teacher capacity for professional learning in mathematics

The National Centre for Excellence in the Teaching of Mathematics

Set up in 2006 I have been Director since 2007

Vision to develop a sustainable national infrastructure for subject-specific professional development of teachers of mathematics

Aims:

to meet the professional aspirations of all teachers of mathematics

to realise the potential of learners

NCETM: blended activities

face-to-face events & network meetings

interactions with the NCETM portal

www.ncetm.org.uk

news, resources, tools & group discussions

Be here: **Resources**

View a different section

Personal Learning Space

TOYLES

- My Details
- My Communities
- My Files & Folders
- My Favourites & Notes

- My Self-evaluation
- My Online CPD
- My Learning Journal
- My Career Portfolio

- Sharing & Contacts



The NCETM 'essentials' collection: Guiding you to the right solution

The NCETM online resources number many thousands of pages, and it can sometimes be a little daunting to find exactly what you want. Our guidance pages contain links to relevant resources from across the NCETM, and include links to discussion forums, offering the opportunity to examine and record your own preferences and thoughts.

Featured Projects



Mathematics resources for Teachers in Training

Resources, guidance and reflective activities for ITE students and tutors, aiming to support all aspects of the course that relate to mathematics.



Teaching Assistants and Other Adults in the Mathematics Classroom

If you work with groups of children in the classroom and are not the classroom teacher, this microsite aims to help support workers develop the wide variety of professional skills needed.

Online Magazines



- ▶ Early Years
- ▶ Primary
- ▶ Secondary
- ▶ FE

Online Learning



Online CPD Modules: guided activities to develop your thinking and reflection.

NCETM portal micro-sites

Teachers Talking Theory: In Action - Chris Slaughter

07 May 2008 by admin

Teachers Talking Theory: In Action is a set of professional development resources which comprise of video clips and associated activities. The materials can stimulate discussion and experimentation and are designed to be used in many different scenarios.

Tim
The importance of considering actions in word problems

Teachers Talking Theory in Action

Chris Slaughter, Kingsbridge Primary School

Themes

- The place of the bead bar and the empty number line in supporting children's calculation skills
- The role of imagery including the importance of children drawing their own pictures

Chris is interested in helping children to use a number line with understanding an image to think about calculations with. She is working to develop a clear progression from the use of the concrete apparatus of the bead bar through drawings and jottings to the efficient and effective use of an empty number line.



Learning Maths Outside the Classroom - Monuments

30 April 2008 by admin

Learning Maths outside the classroom

Built Environment and Heritage
The DCSF published its manifesto 'Learning Outside the Classroom' in 2006. This article explores a 'Built Environment and Heritage' project, one of the environments which the 'Learning Maths Outside the Classroom' microsite explores.

Monuments

Year Four children at Corbridge Primary School, began their project with a visit to Newcastle and Gateshead to view some examples of public art along with guide, sculptor Richard Broderick.

On an extremely windy day in January children visited The Angel of the North, Vulcan, the giant hand entitled 'Reaching for the stars' alongside the older monuments of The Mucky Angel in the Haymarket and Greys monument in the centre of the city.



Children were asked to estimate and measure using a variety of equipment, circumferences, and perimeters and to attempt to calculate the heights of the various edifices. They were encouraged to have a go and come up with their own solutions as to the best way, working in teams to solve the different challenges.

Video Clip 'Sculpture with Richard Broderick'

In the Laing Art gallery they were introduced to an interesting painting where a sculpture is seen to break through the roof of a gallery space. Children were asked to create their own pieces of public art and to calculate the space that would be needed to properly exhibit them.

School Grounds

Places of Worship and Charities

Natural Environment

Museums and Art Galleries

Farming and Countryside

Built Environment and Heritage

Sports and Adventurous Activities

Family Learning

Learning Outside the Classroom

Topic of the month



Maths of Wimbledon

NCETM Reports

- Researching Effective CPD in Mathematics Education (RECME)

www.ncetm.org.uk/recme-report

- Mathematics Matters

[The NCETM publishes Mathematics Matters Final Report – NCETM](#)



- ▶ What Makes A Good Resource
- ▶ Early Years Sector
- ▶ Primary Sector
- ▶ Secondary Sector
- ▶ Further Education Sector
- ▶ Special Educational Needs Sector
- ▶ Using the Materials

What makes a good classroom resource?: Changes in my classroom

Classroom resources alone cannot guarantee effective learning and teaching. But they do have the potential to trigger new approaches and behaviours – i.e. changes in our classroom. In this way good resources can provide a great source of professional development.

The activities have been written and devised by real teachers determining teaching mathematical ideas and topics.



Sections on

What I did

Reflection

Comments

www.ncetm.org.uk/goodresource

Collaborative Teacher Projects www.ncetm.org.uk/ctp

- Funding of up to £5000 for projects in schools for teachers to work collaboratively with the support of an improvement agent
- **Why?**

Why Collaborative Teacher Projects? www.ncetm.org.uk/ctp

giving status to teachers' work

“NCETM funding was hugely valuable but the really big thing was that it meant we had a name for the project, we had status within the schools and we had a reason for meetings...and that was really good and really big and really important”

building a community

“I was surprised at how it welded my staff together doing this type of learning in this particular way, how it helped develop the ethos of the school, it sounds very dramatic but it did ... really thinking together and sharing thoughts and ideas and reflecting together really helped to bind my staff together as a very strong team”

taking risks

“People actually taking things they were not sure about, trying them in their class and coming back the next time saying that they couldn't believe how much the children were talking about mathematics...and there was a lot of enthusiasm for getting children talking about mathematics ... all of us got quite excited ...”

Over 100,000 registered
users.....

End of March 2013



we encourage collaborative projects that involved the use of digital technologies

But this is not sufficient for scaling ☹️



2. Make mathematics more visible: opening up the 'black box'



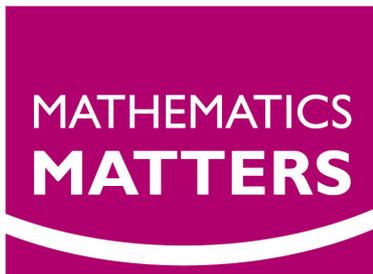
Research Case Studies of impact
of contemporary research in
mathematics on society

Mobile Phones: improving signal transmission

Climate Change

Human Genome

Advancing the Digital Arts



http://www.ima.org.uk/i_love_maths/mathematics_matters.cfm

2. Make mathematics more visible: opening up the 'black box'

**Getting mathematicians to provide a
glimpse of the underlying mathematics**

Hoyles., C. Noss, R. Kent, P. Bakke, r A. *Mathematics in the Workplace: Issues and Challenges* in Eds Alain Damlamian, Jose Francisco Rodrigues , Rudolf Straesser (eds.): *Educational Interfaces between Mathematics and Industry - A joint ICMI/CIAM study*. Springer (Heidelberg 2013.)

3. build teacher capacity ‘bottom up’ through communities of teachers

around **specific** aspects of core knowledge

- with **different** levels of support & engagement
- **spiral & iterative** with teachers (& schools) taking control at different levels
- CPD by ‘experts’ ...who can balance the support of **instrumentation** with a focus on epistemic tool use ----

Example 1



an intelligent exploratory learning environment for supporting the construction of mathematical generalisations & their expression as rules

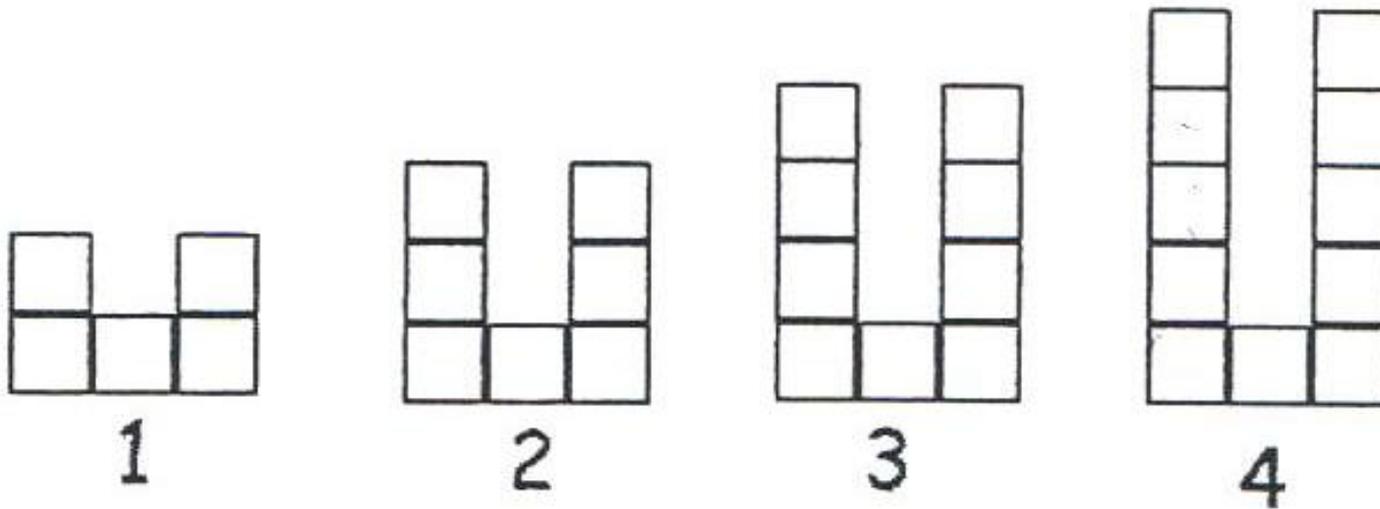
What is the problem?

expressing generality is a well-known challenging topic...

- students might identify patterns but are not necessarily able to articulate the generality
- algebra is often viewed as an endpoint & disconnected from other means of expressing generality

UK Curriculum for grade 4/5

7 Draw the next 2 diagrams in this sequence:

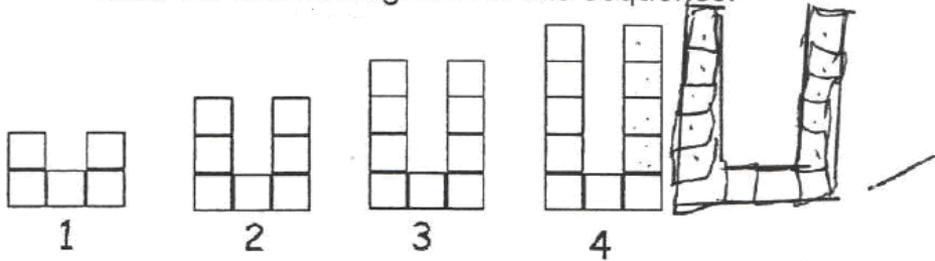


- Write down the number of squares in each term:
- How many squares are needed for the 10th term?
- How many squares are needed for the 50th term?

Show your working out.

an epistemological obstacle

7 Draw the next 2 diagrams in this sequence:



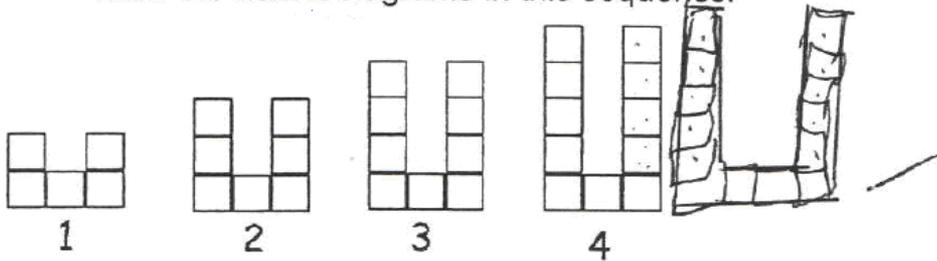
- a) Write down the number of squares in each term $5, 7, 9, 11$ ✓
b) How many squares are needed for the 10th term? $50 \times$
c) How many squares are needed for the 50th term? $250 \times$
Show your working out.

'our' epistemology

- the special case is a way of thinking about the general case
 - How many here? How many there? How many *in general*? Count and at same time recognise structure of the pattern, **apply the pattern** to any 'given number'.

an epistemological obstacle

7 Draw the next 2 diagrams in this sequence:



- a) Write down the number of squares in each term $5, 7, 9, 11$ ✓
b) How many squares are needed for the 10th term? $50 \times$
c) How many squares are needed for the 50th term? $250 \times$
Show your working out.

students' epistemology

- the answer is the number of tiles – i' ll count them then spot **any** pattern
- what is this other thing i' m supposed to do and why am I supposed to be doing it?

Migen: Design decisions in response to the epistemological challenges

- dynamically presented tasks
- ‘unlocked numbers’ as a model for constants and variables ...as bridge to algebra
- synchronous view of the general case alongside any actions on special cases..... theory building....
- some intelligent support & teacher tools

MiGEN: Intelligent Support for Mathematical Generalisation

Hoyles, Noss, Mavrikis, Geraniou & computer science team 2007- 2011

EPSRC/ESRC, TEL research programme

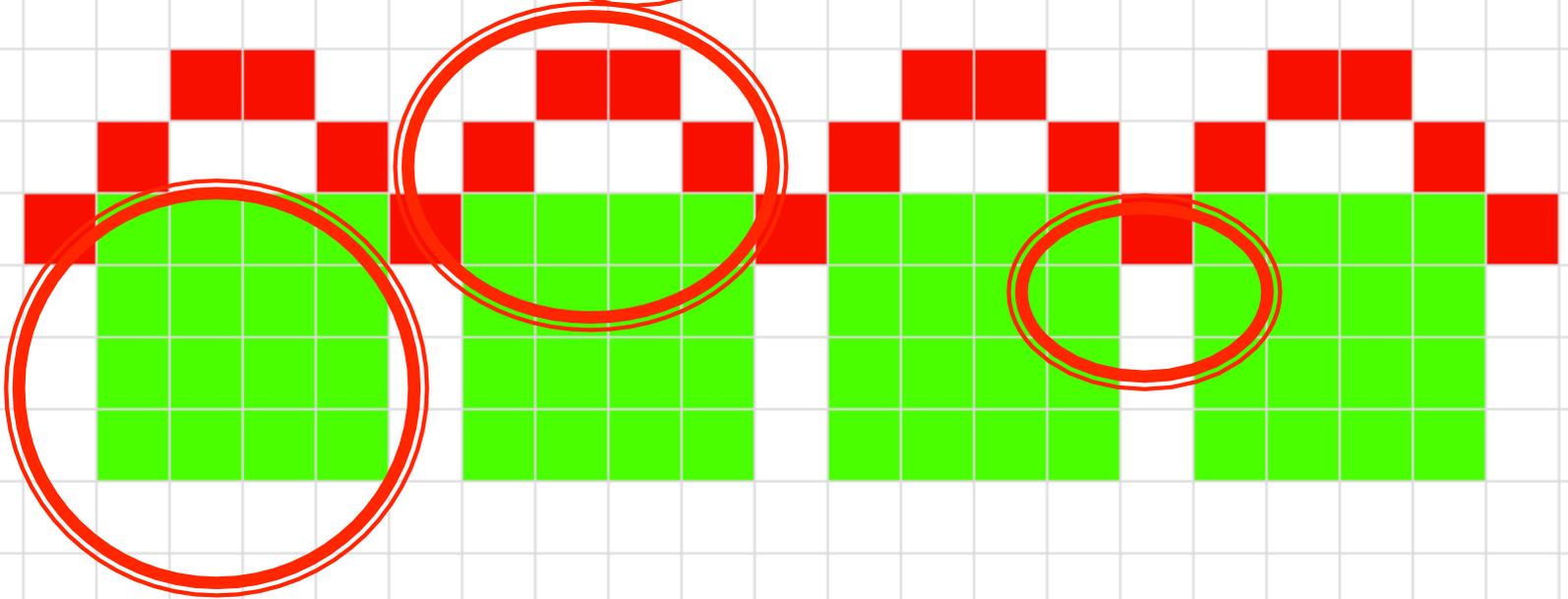
My Model



1



**Building a model & rule
for the number of tiles in
our microworld eXpresser**



Model rules



$$\frac{n}{4} \times 16$$

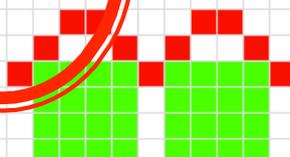
$$\frac{n}{4} \times 6$$

$$\frac{n}{4} - 1$$

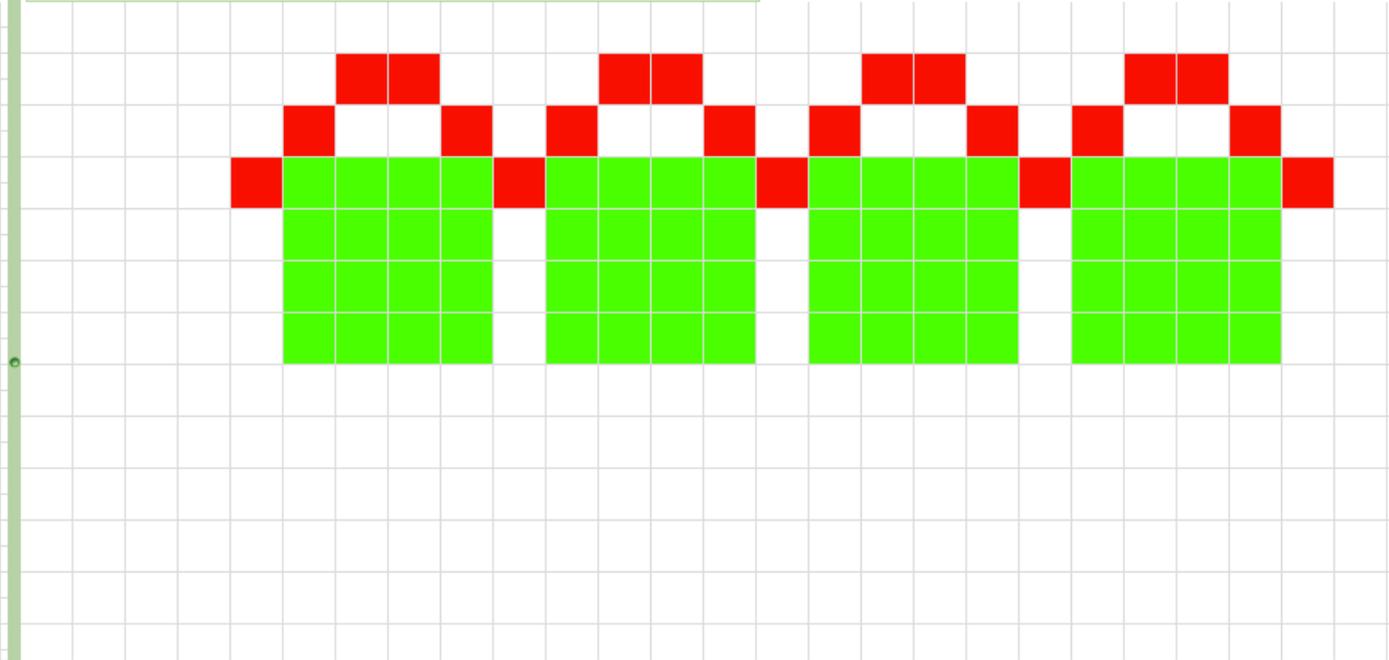


'looking at' the general

Computer's Model 



My Model    

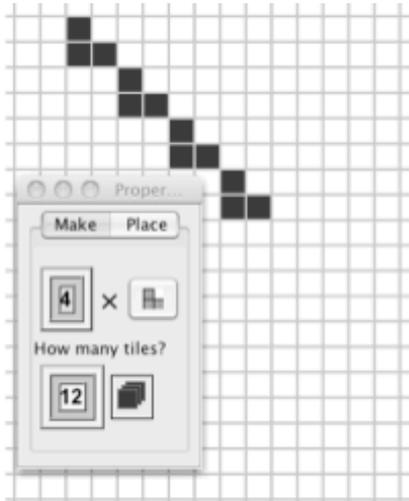


Unlocked numbers
n is 2
Rule for total number of  tiles:
 $(n \times 16 + n \times 6) - (n - 1)$ ✓

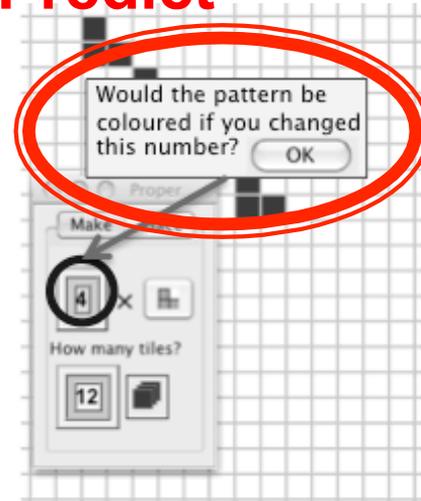
Model rules  $\frac{n}{4} \times 16 + \frac{n}{4} \times 6 - \frac{n}{4} - 1$ ✓

Sequence of prompts from intelligent support

Predict

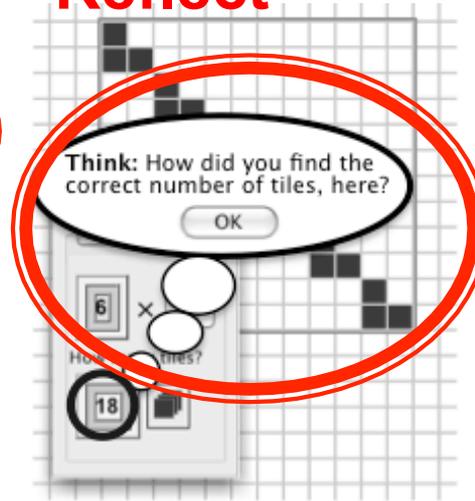


(a)



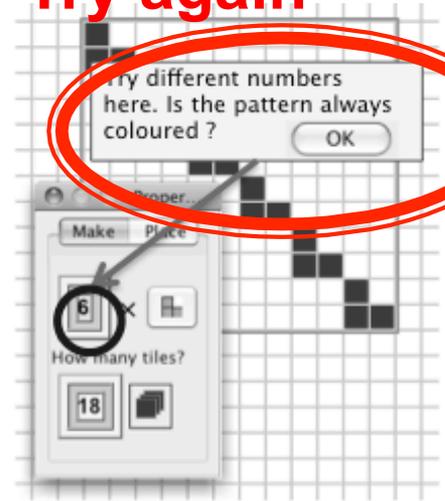
(b)

Reflect

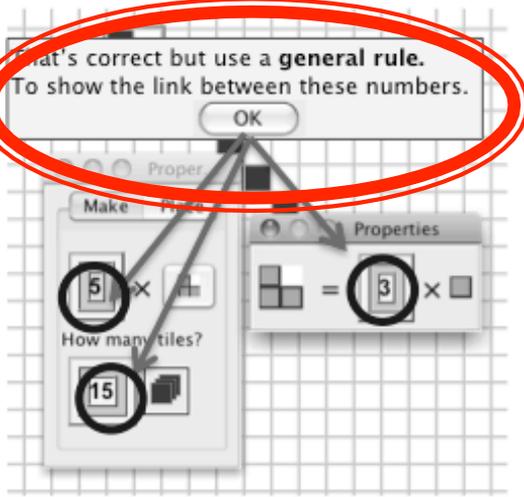


(c)

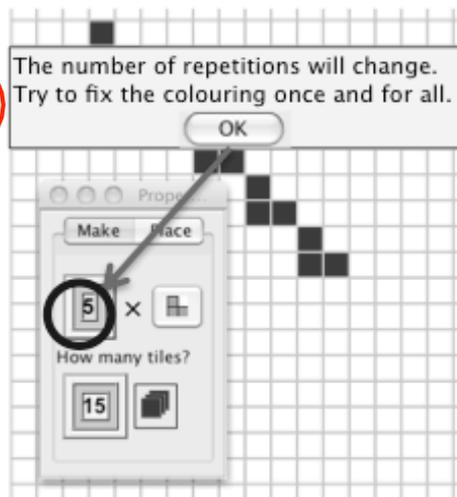
Try again



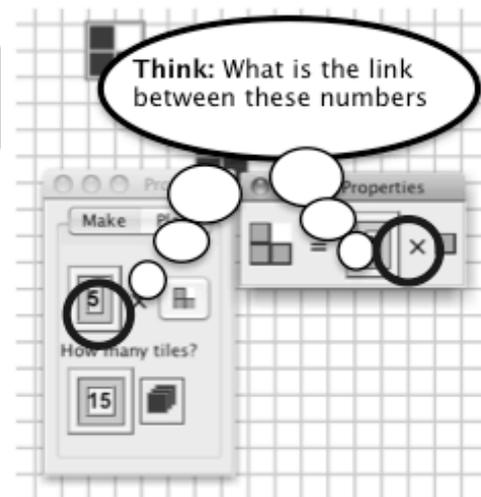
(d)



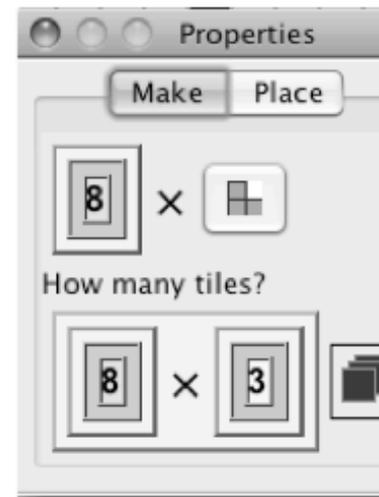
(e)



(f)



(g)



(i)

focus attention on links

Next design challenge....

- mapping new learning trajectories
- designing collaborative bridging tasks

And to scale ...

Massive 'new' interest.. Why ?.....

Why all our kids should be taught how to code

There is a growing consensus that the way children in schools are being taught information technology is in need of a radical overhaul. Here John Naughton explains the problem and offers a manifesto for revolutionary action



John Naughton

The Observer, Saturday 31 March 2012 20.15 BST

 [Jump to comments \(288\)](#)



Everybody
needs to
program:
re-mixing

And – inevitably -- -

calls for **programming to be part
of mathematics** 😊

3. Example 2: a design experiment ‘with an eye to scaling’

Cornerstone Maths

**An approach to technology-enhanced
curriculum innovation at scale**

Celia Hoyles & Richard Noss LKL

Jeremy Roschelle & SRI international



Leading education
and social research
Institute of Education
University of London



Four Elements of Cornerstone Maths

‘Big research-based mathematical ideas’: explicit & evidence-based reasons for using ICT for mathematics learning: multiple & dynamic representations

Teacher professional development
+ Teacher community

```
graph TD; A["'Big research-based mathematical ideas': explicit & evidence-based reasons for using ICT for mathematics learning: multiple & dynamic representations"] --> B["Replacement curriculum units  
Package of teacher & student Activities (aligned to National Curriculum), trialled"]; C["Teacher professional development  
+ Teacher community"] --> B; B --> D["Scaling"]
```

Replacement curriculum units
Package of teacher & student Activities (aligned to National Curriculum), trialled

Scaling

Why replacement unit strategy ?

- one unit is large enough to observe the hoped for classroom change and meaningful learning, but small enough that teachers & schools will take risk of experiment
- unit explicitly connects the innovation (and researchers' interests) to teachers' priorities for curriculum coverage and improving student outcomes on hard-to-teach topics. (Roschelle et al, 2010)



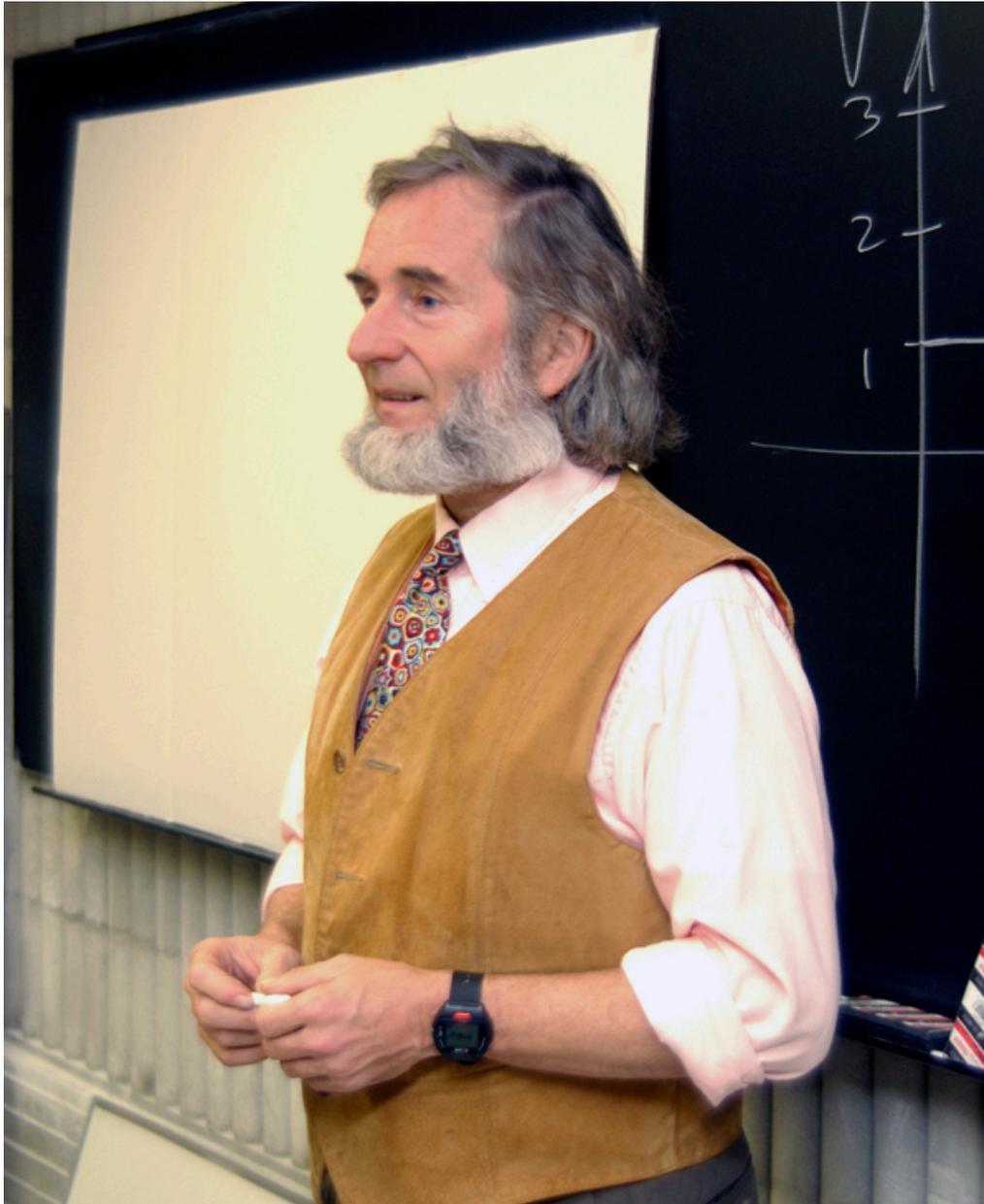
Unit 1

Designing Mobile Games

A module on
linear functions

Derived from
ground breaking
work of Simcalc
Kaput et al

Democratizing Access to Advanced Mathematics



Jim Kaput
“SimCalc”

Unit 1 Linear Functions

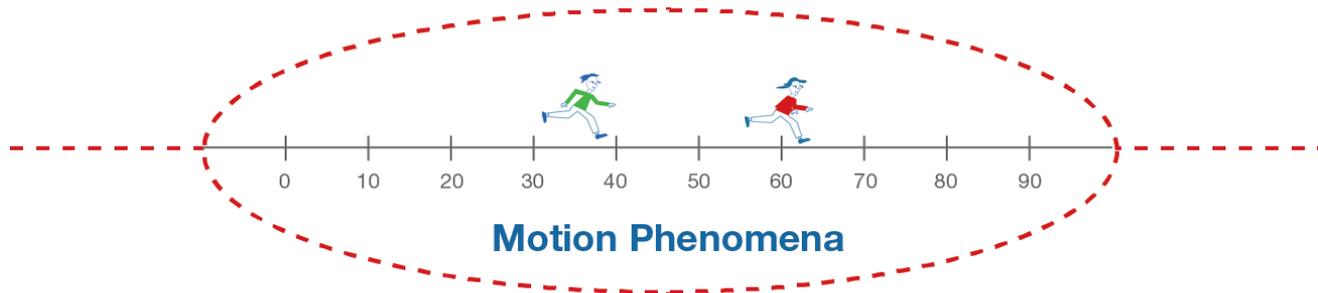
Narrative

Red Reynaldo started off ahead, but was running slowly. Green Geneva started off behind, but ran faster and was ahead by the end.

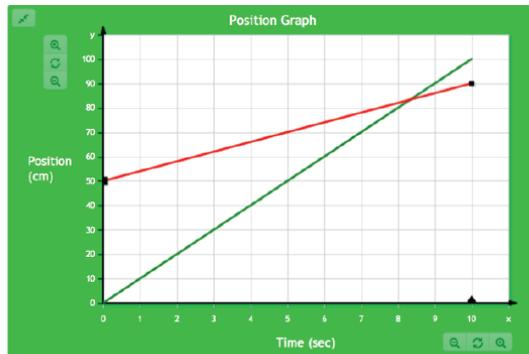
Algebraic Expression

$$y = 4x + 50$$

$$y = 10x$$



Graph



Table

Time (sec)	Reynaldo	Geneva
00.00	50.00	0.00
01.00	54.00	10.00
02.00	58.00	20.00
03.00	62.00	30.00
04.00	66.00	40.00
05.00	70.00	50.00
06.00	74.00	60.00
07.00	78.00	70.00
08.00	82.00	80.00
09.00	86.00	90.00
10.00	90.00	100.00

Time (sec)	Reynaldo	Geneva
05.00	70.00	50.00

Big 'mathematical' ideas

- **Coordinating** algebraic, graphical, and tabular representations
- $y = mx + c$ as a model of constant velocity motion – the **meaning of m and c in the motion context**
- **Velocity** as speed with direction
- **Average velocity**

Pilot: Design experiment over 18 months with
9 schools, 19 teachers

very **diverse** sample in terms of

- school contexts
- prior student achievement
- teacher experience

Massive success as evident in robust evaluation

- pre- and post- tests of student responses to core mathematical items
- teacher, student interviews & classroom observation
- external evaluation

we believe this is sufficiently robust to scale but...we shall see ..we are scaling to 100 schools from Jan 2013

Unit 1: Results from pilot (pre and post test)

Learning gains were as good as those observed in previous Texas study

Gains predominantly in “complex” conceptions (light blue)

(TEST SCORES:

mean pre = 10.7

mean post= 18.5,

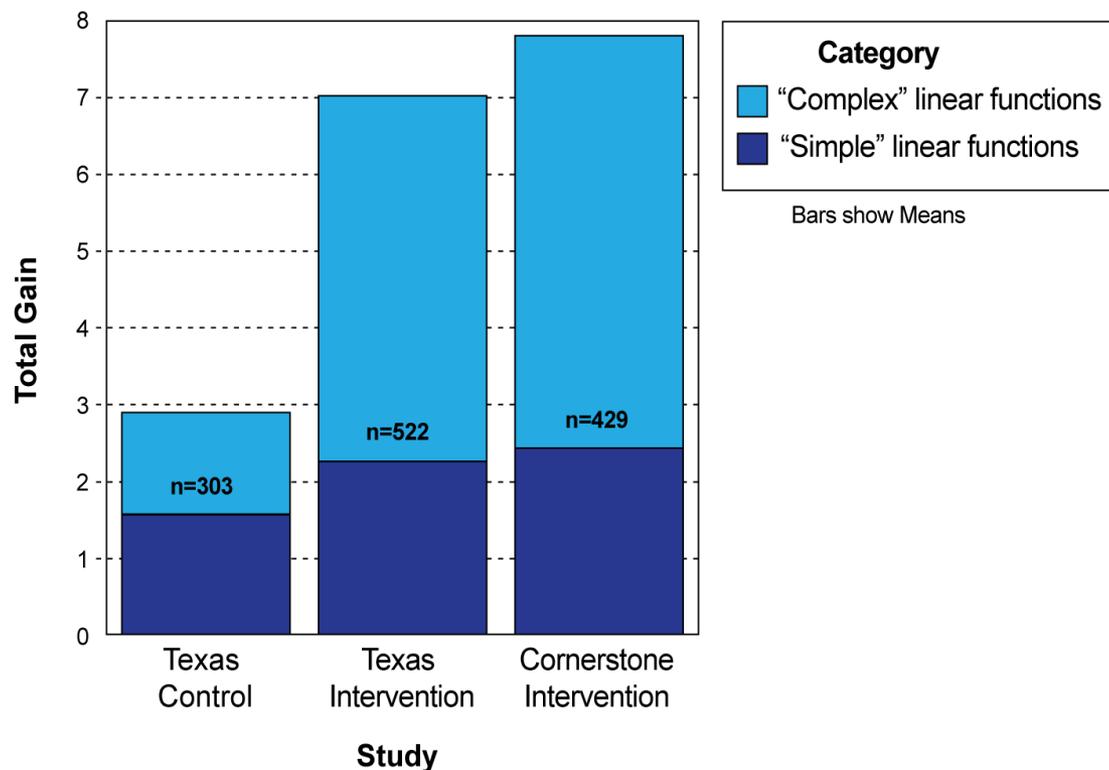
Maximum of 32)

STUDENTS (N=429)

(Y7) 179 (42%)

(Y8) 227 (53%)

(Y9) 23 (5%)



Our plans for scaling

<http://tinyurl.com/csmaths>

1. move to web based offer (including on Ipads) that embeds
 - software
 - curriculum materials
 - formative assessment
2. growing a team of expert CPD providers
- 3 building teacher capacity & leadership in regional hubs
- 4 moving from central to hub-based offer of professional support
 - face to face
 - through web-based in collaborative **online** teacher communities

Cornerstone units

- linear functions (Unit 1)
- geometric similarity (Unit 2): building on research with dynamic geometry
- algebraic generalisations (Unit 3): building on specifically on MiGen research
- ratio and proportion (Unit 4)

Final remarks

1. Be clear (again) that hardware alone doesn't work
“XXXXX to buy \$50 million of tablets”
2. If we have an effect, what is it an effect of?
 - An *integration* of dynamic representations, curriculum workbooks & teacher professional development
3. What it is an effect on?
 - In Unit 1 Conceptual understanding of rate, expressed through multiple representations; weaker effects on more numeric proportionality problems
4. Always more challenges
 - New National Curriculum in England
 - move away from paper-based materials?
 - New platforms

Broader Closing Remarks

It took a long time –15 years - and large multidisciplinary teams

Many experiments at scale may be failing not because the technology is bad, but because of poor integrations:

- Leaving it to teachers to connect with curriculum
- Not providing focused PD with expert PD team
- Measuring the wrong outcomes
- More????

But it is worth it.....

One teacher 18months after pilot:

“ There was a distinct demarcation in my year 8 class between half students who did Cornerstone last year and half who didn't. I can categorically say that Cornerstone pupils' retention a year on is significantly better than pupils who had done similar things out of a textbook.”

Reflection from an experienced CPD provider:

“ important point is that the teachers are fairly every day teachers ...it is being part of the research and development project that has developed them into teachers who could present clear reflections about their participation and what they and the pupils gained. **Wonderful**”.

Danke für Ihre Aufmerksamkeit

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Mathematics Matters case studies: Institute of Mathematics and its Applications
http://www.ima.org.uk/i_love_maths/mathematics_matters.cfm

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