From design experiments to innovation at scale: potential and challenges for research in mathematics education

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Vielen Dank für die Einladung

Ich liebe Münster
structure of my talk

1. design experiments in mathematics education: some general issues
2. design experiments that embed use of digital technologies
3. some ongoing research
4. factors in scaling up

different ‘types’ of research
‘science’ & ‘engineering’
design research

starts with “big ideas” in mathematics that are identified by research (theory or empirical experiment) as hard to teach and learn (they are ‘obstacles’)

theoretically & empirically-driven task design & task progression to

- address these obstacles
- align with ‘local’ curriculum & assessment practices

iterative design experiments to test with teachers as co-designers
Design research often reports ‘dramatic’ impact on student outcomes

need more caution about reporting results of design experiments?

• innovations in and of themselves do not innovate

• maybe other options explain positive results of implementing the innovation or design experiment that do not privilege one part of the innovation
Temptation to pick one area as reason for change///

XXXXX has demonstrated that attainment can be dramatically improved by using XXXXXXXXXXX to improve students’ learning

But many caveats
intervention groups relatively high attaining in relation to matched comparison
different timings of the student tests for intervention and for the comparison group
teachers involved in the interventions self-selected volunteers and very enthusiastic
the intervention involved professional development for teachers

professional development undertaken by the design team

BUT cumulative evidence of effects of design research is dramatic
Design research: a framework?

Some conjectures:
1. recognise importance of design & complexity of implementation
2. make explicit the
   • epistemological basis of the intervention
   • and the design decisions made
4. be clear about the need for evidence-based, expert teacher professional development & materials
Design research: a framework?

And recognise the limitations of what can be achieved by teaching: e.g.

“difficulty of multiplicative reasoning and mastering proportionality is not simply engendered by teaching---huge conceptual step to be taken by students in order to master proportional (or, more generally, functional) reasoning”. (Nunes & Bryant)

need to revisit in spiral way?

and bridge to ‘standard’ curriculum?
A note about context?

- mathematics carries considerable social capital

- But in a world of digital images, animations, easy information retrieval & communication students can dismiss mathematics as irrelevant, boring and hard

- importance of tapping into youth culture to motivate students to sustain their engagement with mathematics
We try to build **engaging** environments for learning mathematics where mathematics is

- **needed** for students to achieve the goals of the activity
- **compelling** for students
- the **actions in** the environment help students engage in mathematical thinking: shift from pragmatic to epistemic use of digital tools
Design research that embeds digital technology

We exploit explicitly the functionalities of the technology for mathematics teaching & learning

- its dynamic & visual nature …plus multi-media, connectivity ….

- its potential to
  - present & link different representations dynamically
  - provide feedback designed so students learn from the results of their actions
  - leave a trace of how solution is constructed, not just the answer
  - provide tools so students can share and discuss

Theoretical framework of Constructionism
Design research that embeds digital technology

Again dramatic success in design experiments
But not just the technology of course……..
and problems of scale

Some examples
Designing software for mathematical engagement through modelling, 17th ICMI study book  Hoyles & Lagrange (2009)

Theoretical framework of Constructionism
ICMI Study 1: the influence of computers & informatics on mathematics & its teaching 1985


reprinted 1992
The 17th ICMI Study
Technology revisited


Five perspectives

- theory: (wealth of perspectives)
- the role of the teacher
- curricula framing
- geographical diversity
- design
‘new’ issues

Many new issues compared to first ICMI study

- equity
- diversity
- Papert’s 10% - new types of mathematical knowledge and practice
- ........
- connectivity

And now…?
Mobile technology applets & tablets and…
Design research that embeds digital technology

top down or bottom up?

Need balance of

• teacher autonomy & supportive CPD
• pragmatic tool use & epistemic tool use
• ....and top down support aligned with bottom up growth

Need to research the mechanism of scaling up
Starting points for successful scaling of design research?

1. enhance teacher capacity for professional learning in mathematics
2. make mathematics more visible: opening up the ‘black box’
3. build teacher capacity ‘bottom up’ through communities of teachers
Enhance teacher capacity for professional learning in mathematics

The National Centre for Excellence in the Teaching of Mathematics

Set up in 2006 I have been Director since 2007

Vision to develop a sustainable national infrastructure for subject-specific professional development of teachers of mathematics

Aims:

to meet the professional aspirations of all teachers of mathematics
to realise the potential of learners
NCETM: blended activities

face-to-face events & network meetings

interactions with the NCETM portal
www.ncetm.org.uk

news, resources, tools & group discussions
The NCETM ‘essentials’ collection: Guiding you to the right solution

The NCETM online resources number many thousands of pages, and it can sometimes be a little daunting to find exactly what you want. Our guidance pages contain links to relevant resources from across the NCETM, and include links to discussion forums, offering the opportunity to examine and record your own practice and thoughts.

Featured Projects

Mathematics resources for Teachers in Training
Resources, guidance and reflective activities for ITE students and tutors, aiming to support all aspects of the course that relate to mathematics.

Teaching Assistants and Other Adults in the Mathematics Classroom
If you work with groups of children in the classroom and are not the classroom teacher, this microsite aims to help support workers develop the wide variety of professional skills needed.
NCETM portal micro-sites

Teachers Talking Theory in Action

Learning Outside the Classroom

Maths of Wimbledon
NCETM Reports

• Reseaching Effective CPD in Mathematics Education (RECME)
  www.ncetm.org.uk/recme-report

• Mathematics Matters
  The NCETM publishes Mathematics Matters Final Report – NCETM
What makes a good classroom resource?: Changes in my classroom

Classroom resources alone cannot guarantee effective learning and teaching. But they do have the potential to trigger new approaches and behaviours – i.e. changes in our classroom. In this way good resources can provide a great source of professional development.

The activities have been written and devised by real teachers determining teaching mathematical ideas and topics.

Quicklinks
- What Makes A Good Resource
- Early Years Sector
- Primary Sector
- Secondary Sector
- Further Education Sector
- Special Educational Needs Sector
- Using the Materials

Sections on
What I did
Reflection
Comments

www.ncetm.org.uk/goodresource
Collaborative Teacher Projects  www.ncetm.org.uk/ctp

• Funding of up to £5000 for projects in schools for teachers to work collaboratively with the support of an improvement agent

• Why?
Why Collaborative Teacher Projects?  

**giving status to teachers’ work**

“NCETM funding was hugely valuable but the really big thing was that it meant we had a name for the project, we had status within the schools and we had a reason for meetings...and that was really good and really big and really important”

**building a community**

“I was surprised at how it welded my staff together doing this type of learning in this particular way, how it helped develop the ethos of the school, it sounds very dramatic but it did ... really thinking together and sharing thoughts and ideas and reflecting together really helped to bind my staff together as a very strong team”

**taking risks**

“People actually taking things they were not sure about, trying them in their class and coming back the next time saying that they couldn’t believe how much the children were talking about mathematics...and there was a lot of enthusiasm for getting children talking about mathematics ... all of us got quite excited ...”
Statistics

Over 100,000 registered users........

End of March  2013
Through the NCETM I have a sense that a real mathematical community is starting to be developed, nurtured and appreciated. As a maths teacher for over 25 years I now have access to external support and dialogue, peer support, opportunities for learning and to build on my own expertise as a leader of CPD within my department.

NCETM community

we encourage collaborative projects that involved the use of digital technologies

But this is not sufficient for scaling 😞
2. Make mathematics more visible: opening up the ‘black box’

Research Case Studies of impact of contemporary research in mathematics on society
Mobile Phones: improving signal transmission
Climate Change
Human Genome
Advancing the Digital Arts

http://wwwIMA.org.uk/i_love_maths/mathematics_matters.cfm)
2. Make mathematics more visible: opening up the ‘black box’

Getting mathematicians to provide a glimpse of the underlying mathematics

3. build teacher capacity ‘bottom up’ through communities of teachers

around specific aspects of core knowledge

- with different levels of support & engagement
- spiral & iterative with teachers (& schools) taking control at different levels
- CPD by ‘experts’…who can balance the support of instrumentation with a focus on epistemic tool use ----
Example 1

an intelligent exploratory learning environment for supporting the construction of mathematical generalisations & their expression as rules
What is the problem?

expressing generality is a well-known challenging topic...

- students might identify patterns but are not necessarily able to articulate the generality

- algebra is often viewed as an endpoint & disconnected from other means of expressing generality
7. Draw the next 2 diagrams in this sequence:

1 2 3 4

a) Write down the number of squares in each term:
b) How many squares are needed for the 10th term?
c) How many squares are needed for the 50th term?

Show your working out.
A well-known problem

7. Draw the next 2 diagrams in this sequence:

1.  
2.  
3.  
4.  

a) Write down the number of squares in each term: 5, 7, 9, 11
b) How many squares are needed for the 10th term? 50

Show your working out.
an epistemological obstacle

'our' epistemology

- the special case is a way of thinking about the general case
  - How many here? How many there? How many in general? Count and at same time recognise structure of the pattern, apply the pattern to any 'given number'.

Show your working out.
an epistemological obstacle

Draw the next 2 diagrams in this sequence:

1  2  3  4

7  8  9  10

a) Write down the number of squares in each term:
   5  7  9  11

b) How many squares are needed for the 10th term?
   50

c) How many squares are needed for the 50th term?
   250

Show your working out.

students’ epistemology

- the answer is the number of tiles – I’ll count them then spot any pattern

- what is this other thing I’m supposed to do and why am I supposed to be doing it?
Migen: Design decisions in response to the epistemological challenges

- dynamically presented tasks
- ‘unlocked numbers’ as a model for constants and variables …as bridge to algebra
- synchronous view of the general case alongside any actions on special cases……. theory building….  
- some intelligent support & teacher tools
Building a model & rule for the number of tiles in our microworld eXpresser
‘looking at’ the general
Sequence of prompts from intelligent support

Predict

Would the pattern be coloured if you changed this number? OK

Reflect

Think: How did you find the correct number of tiles, here?

Try again

Try different numbers here. Is the pattern always coloured?

What's correct but use a general rule. To show the link between these numbers.

The number of repetitions will change. Try to fix the colouring once and for all.

Think: What is the link between these numbers

focus attention on links
Next design challenge....

- mapping new learning trajectories
- designing collaborative bridging tasks

And to scale …

Massive ‘new’ interest.. Why ?.....
Everybody needs to program: re-mixing
And – inevitably -- -
calls for **programming to be part of mathematics** 😊
3. Example 2: a design experiment ‘with an eye to scaling’

Cornerstone Maths

An approach to technology-enhanced curriculum innovation at scale

Celia Hoyles & Richard Noss LKL
Jeremy Roschelle & SRI international
Four Elements of Cornerstone Maths

‘Big research-based mathematical ideas’: explicit & evidence-based reasons for using ICT for mathematics learning: multiple & dynamic representations

Teacher professional development + Teacher community

Replacement curriculum units
Package of teacher & student Activities (aligned to National Curriculum), trialled

Scaling
Why replacement unit strategy?

• one unit is large enough to observe the hoped for classroom change and meaningful learning, but small enough that teachers & schools will take risk of experiment

• unit explicitly connects the innovation (and researchers' interests) to teachers' priorities for curriculum coverage and improving student outcomes on hard-to-teach topics. (Roschelle et al, 2010)
Unit 1
Designing Mobile Games

A module on linear functions

Derived from groundbreaking work of Simcalc Kaput et al
Democratizing Access to Advanced Mathematics

Jim Kaput
“SimCalc”
Red Reynaldo started off ahead, but was running slowly. Green Geneva started off behind, but ran faster and was ahead by the end.

**Algebraic Expression**

- $y = 4x + 50$
- $y = 10x$

**Graph**

**Table**

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Big ‘mathematical’ ideas

• **Coordinating** algebraic, graphical, and tabular representations

• $y = mx + c$ as a model of constant velocity motion – the **meaning of $m$ and $c$ in the motion context**

• **Velocity** as speed with direction

• **Average velocity**
Pilot: Design experiment over 18 months with 9 schools, 19 teachers
very diverse sample in terms of
• school contexts
• prior student achievement
• teacher experience

Massive success as evident in robust evaluation
• pre- and post- tests of student responses to core mathematical items
• teacher, student interviews & classroom observation
• external evaluation

we believe this is sufficiently robust to scale but...we shall see ..we are scaling to 100 schools from Jan 2013 ....
Quantitative results

Bar chart showing the total gain in different studies:
- Texas Control: n=303
- Texas Intervention: n=522
- Cornerstone Intervention: n=429

The chart includes bars for "complex" and "simple" linear functions, with bars indicating means. The study groups are Texas Control, Texas Intervention, and Cornerstone Intervention.
Unit 1: Results from pilot (pre and post test)

Learning gains were as good as those observed in previous Texas study

Gains predominantly in “complex” conceptions (light blue)

(TEST SCORES: mean pre = 10.7 mean post= 18.5, Maximum of 32)

STUDENTS (N=429)
(Y7) 179 (42%)
(Y8) 227 (53%)
(Y9) 23  (5%)
Our plans for scaling
http://tinyurl.com/csmaths

1. move to web based offer (including on Ipads) that embeds
   - software
   - curriculum materials
   - formative assessment

2. growing a team of expert CPD providers

3. building teacher capacity & leadership in regional hubs

4. moving from central to hub-based offer of professional support
   - face to face
   - through web-based in collaborative online teacher communities
Cornerstone units

- linear functions (Unit 1)
- geometric similarity (Unit 2): building on research with dynamic geometry
- algebraic generalisations (Unit 3): building on specifically on MiGen research
- ratio and proportion (Unit 4)
Final remarks

1. Be clear (again) that hardware alone doesn’t work “XXXXX to buy $50 million of tablets”

2. If we have an effect, what is it an effect of?
   - An integration of dynamic representations, curriculum workbooks & teacher professional development

3. What it is an effect on?
   - In Unit 1 Conceptual understanding of rate, expressed through multiple representations; weaker effects on more numeric proportionality problems

4. Always more challenges
   - New National Curriculum in England
   - move away from paper-based materials?
   - New platforms
Broader Closing Remarks

It took a long time –15 years - and large multidisciplinary teams

Many experiments at scale may be failing not because the technology is bad, but because of poor integrations:

– Leaving it to teachers to connect with curriculum
– Not providing focused PD with expert PD team
– Measuring the wrong outcomes
– More????
One teacher 18months after pilot:
“ There was a distinct demarcation in my year 8 class between half students who did Cornerstone last year and half who didn’t. I can categorically say that Cornerstone pupils’ retention a year on is significantly better than pupils who had done similar things out of a textbook.”

Reflection from an experienced CPD provider:
“ important point is that the teachers are fairly every day teachers …it is being part of the research and development project that has developed them into teachers who could present clear reflections about their participation and what they and the pupils gained. Wonderful”.

But it is worth it........
Danke für Ihre Aufmerksamkeit
Some References


Some References (ctd)


Some References (ctd)


Some References (ctd)


