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Research on the introduction of integration in Japanese High Schools

1. Introduction

It is important that high school students understand mathematical concepts; however, it is generally not the case that all the concepts of mathematics are fully understood. One such example is the concept of 'integration'. Although sectional mensuration is the basis of the concept of definite integrals, in Japanese high school mathematics, integration is introduced as the inverse operation of differentiation. The advantage in this method of introduction is that the students can make integration calculations easily. However, previous studies have shown this to have the disadvantage that students do not fully understand the concept of integration. To address this problem, we have been working on the teaching materials, methods, and practices that have been used so far to enable students to understand the concept of integration. The following historical facts were revealed in this process. In the high school curriculum of 1960 onwards, integration was being taught in two stages over the second and third grades of high school. From the 1970s onwards, the present method of introducing integration has been followed.

In this paper, we first re-examine the transformation in the teaching of integration in Japanese high school mathematics textbooks. We then identify problem areas, and finally, show an example of a teaching practice that facilitates students' understanding of the concept of integration.

2. The transformation in the teaching of integration and current issues

Japanese high school textbooks are prepared based on government curriculum guidelines, and are subject to verification by the Ministry of Education, Culture, Sports, and Science. Therefore, the content of different textbooks can be almost the same. Further, the course curriculum is revised every 10 years.

We first discuss the transformation in the introduction of integration in Japanese high school mathematics textbooks, and then identify the current challenges.

2-1. Transformation in the teaching of integration

Integration came to be taught in the second and third grades of high school, in accordance with government curriculum guidelines introduced in 1960. Subsequently, integration was taught in two stages. Currently, integration is taught in the second and third grades, mainly to benefit those who wish to

go to science universities. For those who wish to study humanities at university, integration is only taught during the second grade.

There are several ways in which integration was introduced in the textbooks of this period. For example, 'sectional mensuration → definite integrals → indefinite integrals →…', 'indefinite integrals → sectional mensuration → definite integrals →…', and 'sectional mensuration → indefinite integrals → definite integrals →…'. Whatever the method of introduction, it was always based on sectional mensuration. During this time, there were several discussions about whether to introduce definite integrals through sectional mensuration, or whether it might be more convenient to introduce the indefinite integral as the inverse operation of differentiation.

Post the 1970 curriculum guideline notification, sectional mensuration came to be taught in the third grade and omitted in the second grade. This was because infinite series, which until then had been taught in the second grade, were now being taught in the third grade. Hence, it became difficult to adopt the introduction method of 'sectional mensuration → definite integrals' in the second year, and the indefinite integral came to be introduced as the inverse operation of differentiation. Thus, the teaching method of 'indefinite integrals → definite integrals' came into being.

However, to enable students to understand the concept of integration, sections such as 'the definite integral as a quantity', which explained the relation between the definite integral and sectional mensuration, were provided. This can be seen, for example, in 'Revised Mathematics II B' (Keirinkan, 1976). In this text, the introduction method adopted was 'speed and distance → indefinite integrals → definite integrals → area → volume → definite integral as a quantity'. Subsequently, with each revision of curriculum guidelines in line with the social climate, the learning content of textbooks with regard to integration was carefully selected and streamlined. For example, present textbooks teach not only 'speed and distance' and 'definite integral as a quantity' in the third grade, but also cover 'volume' over this period.

Currently, second grade students use textbooks based on the high school curriculum introduced in 1999. The curriculum guidelines were revised in 2009, and so these textbooks will change from next year, but the part relating to the introduction of integration has not changed. For example, in 'Revised Mathematics II' (Suken Shuppan, 2007) and 'Mathematics II' (Suken Shuppan, 2012), the method of introducing integration is 'indefinite integrals → definite integrals → area'.

2-2. Current challenges in the teaching of integration

As shown in 2-1, integration is introduced in the second grade as 'the inverse operation of differentiation'. Hence, there is no need to know the concept of limits or the computation of sectional mensuration. For this reason, it has the following advantages:

• Students can easily compute integrals, and can even obtain the area of irregular shapes using definite integrals;

• A sense of accomplishment in being able to solve problems.

However, the following problems may also arise:

• As students in the second grade do not know the historical background of definite integrals, or the reason why it is represented by $\int_a^b f(x)dx$, the meaning of the integration symbol is not clear to them, and hence they do not fully understand the concept of integration;

• Although they learn sectional mensuration in the third grade, when it comes to problems such as 'Evaluate the limiting value $S = \lim_{n\to\infty}\left(\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots\cdots + \frac{1}{2n}\right)$' (Revised Mathematics III, Suken Shuppan, 2008, p.163), students are strongly inclined to solve the problem without fully understanding the concept of the definite integral. When the degree of comprehension of the concept of integration among university students (science department) was investigated, it became clear that many do not fully understand the concept as being based on the quadrature method. This problem had also been identified in previous studies.

3. Example of a teaching practice that addresses this problem

An overview of the teaching practice adopted in the third grade in order to overcome the problem outlined in 2-2 is now given. This method does not appear in any textbook, but has been found to be effective in allowing students to understand the concept and merit of integration.

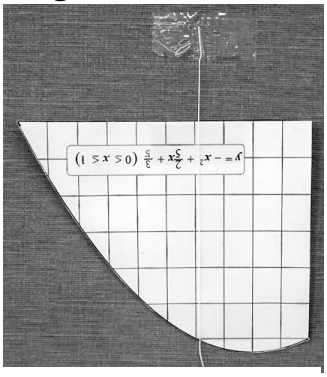

Figure 1. Model created by high school students. (Balanced even when turned upside down)

Teaching Example is as follows.

After explaining sectional mensuration, students were asked to consider problems replacing '$\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n} f\left(\frac{k}{n}\right)$' by a definite integral, or the reverse problem. The students were then encouraged to create and solve similar problems among themselves.

Further, they were led to the formula '$x_g = \frac{\int_a^b xf(x)dx}{\int_a^b f(x)dx}$' for determining the centre of gravity of plane figures using the concept of sectional mensuration. They then determined the coordinates of the centre of gravity of a plane figure, such as that shown in Figure 1, and verified their answers by actual practice.

4. Conclusion

Transformations in the method of introducing integration in Japanese high schools have been discussed, and the current challenges this practice faces have been clearly identified. The government curriculum guidelines are revised every 10 years, in accordance with social conditions, and thus, the method by which integration is introduced has also changed. There are two main ways in which integration is introduced, namely starting with sectional mensuration or as the inverse operation of differentiation. The advantages and disadvantages of both methods have been discussed.

By adding materials that are not given in textbooks, we were able to facilitate an understanding of the concept of integration in students. It is believed that the use of models, which is not very common in senior high schools in Japan, is especially effective. Sectional mensuration, which is taught in the third grade, helps give a better understanding of the concept of definite integrals, and it is necessary for teachers to realise this and devise suitable teaching methods. In other words, it is necessary for teachers to have leadership qualities and creative ingenuity.

We would like to examine issues how to teach students the concept of the integration in the second grade.

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