An empirical study on investors’ preferences for liquid assets

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Abstract
The present study seeks to analyze the demand for liquid assets under the assumption that holdings of money, U.S. Treasuries, and corporate debt securities directly contribute to investors’ utility. Specifically, the representative agent’s utility function includes a separate argument denoted as "liquidity services" which depends on the level of liquid asset holdings. First, this paper presents results from nonparametric tests of investors’ behavior - namely tests for utility maximization, and tests for weak separability. I find that monthly per capita data on consumption, money balances, and holdings of U.S. Treasuries, and corporate debt securities are consistent with utility maximization and weak separability. The second part of the empirical analysis employs Generalized Method of Moments to estimate coefficients of Euler equations which are derived from a variety of specifications of the representative agent’s modified utility function. Further, to find the most suitable functional form, parameter estimates are compared across different specifications and different data sets. However, only the most restrictive utility specification yields parameter estimates which are relatively robust to the choice of data, while in some cases the estimates differ from those known from the literature.

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\textit{Keywords:} nonparametric analysis, revealed preference, asset pricing, liquidity premium, euler equation.

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1 Introduction

Recent empirical studies on determinants of corporate-U.S. Treasury bond yield spreads find that Investors value the liquidity of U.S. Treasury bonds. At the same time U.S. Treasuries’ degree of liquidity, which might be perceived as an inherent feature, or being driven by changing market conditions, is found to be priced separately from commonly studied spread determinants which are implied by the standard Asset Pricing Theory’s Consumption Capital Asset Pricing Model (CCAPM).³

A recent study by Krishnamurthy and Vissing-Jorgensen (2012) (KVJ) finds a significant negative association between the aggregate supply of U.S. Treasuries and corporate-U.S. Treasury bond yield spreads. They argue that this reflects a demand function for what they call U.S. Treasury-specific liquidity services or "convenience yields". Therefore, the high level of liquidity services offered by Treasuries would drive down their yields compared to assets that do not to the same extent share this feature. Further, when the supply of Treasuries is low, the value that investors assign to the liquidity services offered by Treasuries is high, implying increasing (decreasing) Treasury prices (yields) and in turn, increasing corporate-Treasury bond yield spreads. The same argument applies in the opposite direction when the supply of treasuries is high. Moreover, Niestroj (2012) (NIE) finds evidence that the notion of priced liquidity services is not only U.S. Treasury-specific. In particular, it is shown that investors value liquidity services which can be provided by a variety of assets - while each asset might be featuring this attribute to a different degree. This does not only support the view that investors in general value the attribute of liquidity services provision when pricing assets, but also this points to the existence of a demand function for liquidity.

However, both approaches derive asset pricing models under the ad hoc assumption that asset holdings directly contribute to investors’ utility. Specifically, liquidity services are derived via an unknown aggregator function which is a separate argument of investors’ utility. Neither approach provides a complete specification of the underlying preference and aggregator functions but defines a set of requirements to them.

This paper seeks to fill this gap by providing a complete specification and parameterization of a representative agent’s utility function which can rationalize the investors’ behavior observed by KVJ and NIE. For that purpose I first use nonparametric testing routines to examine whether a preference maximization model cannot be rejected where liquidity services directly contribute to investors utility. Specifically, I check Varian’s (1982) necessary and sufficient revealed preference conditions for monthly data on consumption, money holdings, Treasury holdings and prices, and on corporate debt securities holdings and prices. Consistency of the data with these conditions means non-rejection of the hypothesis that investors

are maximizing a utility function which is nonsatiated, continuous, concave and monotonic.\textsuperscript{4} Further, I test whether the data satisfy necessary and sufficient revealed preference conditions for weak separability between several groupings of the liquidity services providing assets and consumption. This is done by applying the procedure proposed by Fleissig and Whitney (2003). The reason for employing this second nonparametric test is that KVJ and NIE implicitly assume weak separability for their analyses. As pointed out by Swofford and Whitney (1987) weak separability is a convenient feature as it keeps the subsequent theoretical analysis analytically tractable and for the empirical part of the study, it reduces data requirements and conserves statistical degrees of freedom. If both hypotheses are not rejected, the question for a suitable specification of the investor’s utility function arises. As the nonparametric testing routines applied in this study do not provide much guidance for that, Generalized Method of Moments (GMM) is employed to estimate coefficients of Euler equations which are derived from the investors’ optimization problem under several proposed utility specifications. This further poses an indirect test of the asset pricing models employed by KVJ and NIE. This is due to the fact that only a subset of the proposed utility function specifications meets the requirements which are imposed on investors’ demand behavior by their modified asset pricing model.

This paper provides evidence from the nonparametric testing routines that necessary and sufficient conditions for utility maximization and weak separability are obtained for the dataset. However, results from GMM estimations imply rejection of almost all proposed utility specifications. Only the model based on the specification proposed by Poterba and Rotemberg (1986) is not rejected. Estimation results however, imply parameter values which indicate misspecification.

Estimating parameters of utility functions which include consumption and an aggregator function of near monies holdings, which is denoted as "liquidity services", goes back to Poterba and Rotemberg (1986). The aim of this study was to examine the impact of open market operations on short-term interest rates. Analyzing the effects of nonstandard monetary policy operations such as large-scale asset purchase programs (LSAPs) has recently become an active field of macroeconomic research. These measures have been introduced to provide liquidity in exchange for private sectors’ assets. From a theoretical perspective, it is generally expected that such nonstandard open market operations in private assets do not exhibit an effect on real variables. In particular, as shown by Eggertsson and Woodford (2003), this irrelevance result by Wallace (1981) applies to the canonical New Keynesian macroeconomic model approach. Hence, the model framework for monetary policy analysis as summarized by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007)\textsuperscript{4}The data used for the present analysis are basically the same as in KVJ and NIE, while it is assumed that money, Treasuries, and corporate debt securities are providing liquidity services. The reason for the choice of the data set lies in the intention of this paper to carry forward the analysis of the previous authors.
does not provide a suitable approach for the evaluation of the effectiveness of this policy. Contributions like Chen, Curdia, and Ferrero (2012), Gertler and Karadi (2013), Gertler and Kiyotaki (2010), and Del Negro et al. (2013) rely on investors’ heterogenous preferences and on financial market frictions to make the relative supply of liquid assets have an effect on the equilibrium allocation. Schabert and Reynard (2009), Schabert and Hörmann (2011), and Christoffel and Schabert (2013) assume for this purpose liquidity constraints and model explicitly the central bank’s balance sheet policy options. Others like Krishnamurthy and Vissing-Jorgensen (2011), Swanson (2011), and Peersman (2011) employ econometric approaches by using event studies, and in the last case, a VAR analysis.

So far, macroeconomic models that analyze the effects of nonstandard monetary policy measures do not rely on household preferences which account for assets liquidity services. In particular, such an approach would be criticized as being ad hoc. The present study investigates whether there is evidence in favor of such an approach by deriving a first proposal for a suitable utility function. This is done by providing a set of microfoundations, starting with nonparametric hypothesis testing, and then by moving on to parameter estimations.

Varian’s (1982) Generalized Axiom of Revealed Preference (GARP) provides testable necessary and sufficient conditions for a finite number of observations on consumer behavior to be consistent with the preference maximization model. Varian (1982 and 1983) points out that the standard approach is to postulate a possible parametric form for demand functions and fitting them to observed data, and then to test for the hypothesis under consideration. In contrast to that, employing the test for GARP does not require an ad hoc specification of functional forms and is therefore completely nonparametric. Specifically, instead of testing the joint hypothesis that demand behavior can be described by some parametric form and the restriction one wants to test for, Varian (1982) provides a complete test of the hypothesis in question alone. As further pointed out by Varian (1982 and 1983), this test relies on algebraic conditions on a finite body of data to be consistent with the maximization hypothesis. These are denoted as "revealed preference" conditions and provide a complete test on the restriction imposed by maximizing behavior, in the sense that every maximizing consumer’s demand behavior must satisfy these conditions, and that all behavior that satisfies these conditions can be viewed as maximizing behavior.5

Under the assumption that assets are held for real nonpecuniary returns from such attributes as their perceived liquidity, it is a convenient feature of the utility function under consideration to be weakly separable in the arguments of consumption and so called liquidity services which are derived from liquid asset holdings. As pointed out by Swofford and Whit-
ney (1987), weak separability implies two-staged budgeting. Households first decide upon the allocation of expenditures between consumption and asset holdings. In the second stage households allocate expenditures among the goods within each subgroup based on the relative prices of the goods in that group. Weak separability has the necessary and sufficient condition that the marginal rate of substitution (MRS) between any two goods within a group is independent of the goods outside the group. Hence, for the determination of the utility’s functional form it can be assumed that liquidity services are derived by a yet unknown aggregator function of the liquid asset holdings, which represents a separate argument of the utility function. To test for weak separability I use the approach by Fleissig and Whitney (2003). Employing this procedure comes with the advantage that in case of nonrejection of the weak separability hypothesis one can easily revert to well known functional forms for the aggregator like constant-elasticity-of-substitution (CES) and Cobb-Douglas. If weak separability does not obtain, two stage budgeting procedure is not an accurate description of consumer behavior. Moreover, this would pose evidence against the asset pricing models proposed by KVJ and NIE.

Swofford and Whitney (1987) further point out, that the second stage of the two-staged budgeting process is the focus of mainly microeconomists examining how households allocate total consumption expenditure among various categories of goods and services. In particular, this involves estimating parameters of demand functions and utility functions. Monetary economists like Barnett (1980), Feldstein and Stock (1996), and Drake and Mills (2005) have used this approach to obtain estimates of elasticities of substitution between narrow transaction balances and less liquid near monies in attempts to clarify the appropriate definition for money balances. Nonparametric tests to evaluate if groups of monetary assets are weakly separable from other goods have been used, among others, by Barnett, Fisher, and Serletis (1992), Belongia (1996), Swofford and Whitney (1987, 1988), and Drake and Chrystal (1994, 1997) while the construction of the monetary aggregates is neglected.

With the revealed preference tests done the question for a suitable specification of the representative investor’s objective function arises. The approaches by KVJ and NIE impose several requirements to its unspecified underlying utility function, whereas the nonparametric revealed preference tests which are applied in this study provide support for a wide set of possible utility function specifications. To find the most suitable one, GMM is employed which is frequently used to estimate and test asset pricing models. In this paper I estimate coefficients of Euler equations which are derived from the investors’ optimization problem under different utility specifications. In particular, I consider Cobb-Douglas and CES aggregator functions for the liquidity services measure. The aggregator functions are nested in utility functions with each displaying constant relative risk aversion. As it can be shown,  

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6 For a theoretical treatment of this method see Hansen (1982). For recent empirical studies employing GMM on asset pricing models see e.g. Stock and Wright (2003), Yogo (2004), and Hall (2005).
the investors’ demand behavior observed by KVJ and NIE is consistent with the aggregator function to be CES, and the utility’s arguments of consumption and liquidity services to be not additively separable. Hence, this empirical analysis of different utility specifications can be regarded as an additional test of the models by KVJ and NIE.

The paper proceeds as follows: Section 2 describes the nonparametric testing routines which are applied in the present study and the data. Further, results from hypothesis testing are presented. Section 3 discusses the representative agent’s utility maximization problem and derives Euler equations under alternative specifications of the investor’s utility function. Results from GMM estimations are presented here as well. Section 4 concludes.

2 Nonparametric tests for utility maximization and weak separability

2.1 Testing the maximization hypothesis

This analysis employs Varian’s (1982 and 1983) nonparametric approach to demand analysis. Varian (1982) shows that observed demand behavior can be rationalized by a nonsatiated, continuous, concave, monotonic utility function if one of several equivalent conditions is met. The easiest one to test is Varian’s (1982) GARP.\footnote{The remainder of this subsection is taken from Varian (1982 and 1983).}

To recover investors’ preferences from a finite number of observations of \(k\)-vectors of prices and quantities \((p^i, x^i), i = 1, ..., n\), with \(p^i = (p^i_1, ..., p^i_k)\), and \(x^i = (x^i_1, ..., x^i_k)\), consider the following definitions:

**Definition (1):** A utility function rationalizes the data \((p^i, x^i), i = 1, ..., n\), if \(u(x^i) \geq u(x)\), for all \(x\) such that \(p^i x^i \geq p^i x\), for \(i \geq 1, ..., n\).

**Definition (2):** An observation \(x^i\) is directly revealed preferred to a bundle \(x\), written \(x^i R^0 x\), if \(p^i x^i \geq p^i x\). An observation \(x^i\) is revealed preferred to a bundle \(x\), written \(x^i Rx\), if there is some sequence of bundles \((x^j, x^k, ..., x^l)\) such that \(x^j R^0 x^j, x^j R^0 x^k, ..., x^j R^0 x\). Then \(R\) is the transitive closure of the resolution \(R^0\).

**Definition (3):** The data satisfies the Generalized Axiom of Revealed Preference (GARP) if \(x^i Rx^j\) implies \(p^i x^j \leq p^j x^i\).

Varian (1982) points out that the third definition demands that there are no cyclical inconsistencies if \(x^i\) is preferred to all other affordable bundles. Then \(x^i\) is better than any bundle \(x^j\) chosen at all prices \(p^j\). It is further pointed out, that the advantage of GARP is that it is an easily testable condition, and as Afriat’s Theorem demonstrates it is a necessary and sufficient condition for utility maximization:
**Afriat’s Theorem** (Afriat 1967, Diewert 1973, Varian 1982)

The following conditions are equivalent:

1. there exists a nonsatiated utility function that rationalizes the data.
2. the data satisfies GARP.
3. there exist numbers \( U^i, \lambda^i > 0, i = 1, \ldots, n \) that satisfy the Afriat inequalities: \( U^i \leq U^j + \lambda^i p^i (x^i - x^j) \), for \( i, j = 1, \ldots, n \).
4. there exists a concave, monotonic, continuous, nonsatiated utility function that rationalizes the data.

Where \( U^i \) is the utility level and \( \lambda^i \) the measure for marginal utility of income at observed demands. Varian (1982) points out, that the equivalence of conditions (1) and (4) shows that if some data can be rationalized by any nondegenerate utility function at all, this utility function has desirable properties. Or put differently, violations of continuity, concavity, and monotonicity cannot be detected within a finite number of observations. Further, conditions (2) and (3) give directly testable conditions that the data must satisfy if it is to be consistent with the maximization model. Condition (3) asks for a nonnegative solution to a set of linear inequalities. Condition (2) is more convenient for computation. As pointed out by Varian (1983), Afriat (1967) derived Afriat’s Theorem first with a different but equivalent version of condition (2) which demanded that data satisfies “cyclical consistency” and Diewert (1973) provided a different proof, omitting a consideration of condition (2). As further pointed out, Varian (1982) showed that GARP is equivalent to Afriat’s (1967) cyclical consistency condition. GARP is to be preferred as it is much easier to evaluate in practice.\(^8\)

To test whether GARP is satisfied Varian (1982) proposes to use the dataset to construct an \( n \) by \( n \) matrix \( M \), with the \( i - j \) entry is given by

\[
m_{ij} = \begin{cases} 
1, & \text{if } p^i x^i \geq p^j x^j, \text{ that is } x^i R^0 x^j \\
0, & \text{otherwise.}
\end{cases}
\]

The matrix \( M \) summarizes the relation \( R^0 \). Hence, once \( R \) - the transitive closure of the directly revealed preference relation \( R^0 \) - is known one can test whether GARP satisfied. For that purpose Varian (1982) proposes Warshall’s algorithm which operates on \( M \) to create the matrix \( M^T \), where

\[
mt_{ij} = \begin{cases} 
1, & \text{if } x^i R x^j \\
0, & \text{otherwise.}
\end{cases}
\]

\( MT \) represents the relation \( R \). Hence, to test for consistency with GARP one has to look at each element \( mt_{ij} = 1 \), and check if \( p^i x^i > p^j x^j \), for some \( i \) and \( j \). If that is the case then

\(^8\) For a detailed discussion see Varian (1982).
there is a violation of GARP detected.

2.2 Testing the weak separability hypothesis

To check whether goods can be either combined or have to be studied independently, a test for weak separability is performed. Fleissig and Whitney (2003) propose a new method to evaluate the separability conditions from the revealed preference theory of Varian (1983). Following Varian (1983) partition the data in two sets of goods and prices \((p^i, x^i), (q^i, y^i)\) with \(i = 1, ..., n\).

\[ \text{Definition (4): A utility function } u(x) \text{ is (weakly) separable in the } y \text{ goods if there exists a subutility function } \nu(y) \text{ and a macro function } u^*(x, \nu(y)) \text{ which is continuous and monotonically strictly increasing in } \nu(y), \text{ such that } u(x, y) \equiv u^*(x, \nu(y)). \]

Varian (1983) points out that the necessary condition for weak separability demands that the subdata must satisfy GARP because each observation must solve the problem

\[
\begin{align*}
\max & \; \nu(y) \\
\text{s.t.} & \; q^iy \leq q^iy'.
\end{align*}
\]

The necessary and sufficient conditions for separability are summarized by the following theorem of Varian (1983):

\textbf{Varian’s Separability Theorem (Varian 1983)} The following conditions are equivalent:

(i) There exists a weakly separable concave, monotonic, continuous nonsatiated utility function that rationalizes the data.

(ii) There exist numbers \(U^i, V^i, \lambda^i, \mu^i > 0\), that satisfy separability inequalities for \(i, j = 1, ..., n\):

\[
\begin{align*}
U^i & \leq U^j + \lambda^j p^j \left(x^i - x^j\right) + \lambda^j / \mu^j p^j \left(V^i - V^j\right), \\
V^i & \leq V^j + \mu^j p^j \left(y^i - y^j\right).
\end{align*}
\]

(iii) The data \((q^i, y^i)\) and \((p^i, 1/\mu^i, x^i, V^i)\) satisfy GARP for some choice of \((V^i, \mu^i)\) that satisfies the Afriat inequalities.

Fleissig and Whitney (2003) point out that condition (ii) provides a direct way for testing the necessary and sufficient conditions. However, one would need to check for a solution to

\textsuperscript{9}The remainder of this subsection is taken from Fleissig and Whitney (2003).
a system of $2n (n - 1)$ equations of which half of them are nonlinear. Fleissig and Whitney (2003) assert that condition (iii) is equivalent to evaluating GARP with $1/\mu^i$ as a "sub group price index" and $V^i$ as a "sub group quantity index" for the separable goods $y^i$. Varian’s (1983) NONPAR algorithm is based on condition (iii), where indices are calculated to satisfy the inequality constraint (b). If the data pass the test for GARP fulfilling condition (iii), then from condition (i), it follows that the observed data are consistent with weakly separable preferences. Fleissig and Whitney (2003) point out, if NONPAR does not find that the data on the $y$-goods pass the GARP test, weak separability cannot be rejected since there might be other values for the quantity and price indices that may satisfy the inequalities of (b). Hence, Varian’s (1983) NONPAR approach tests the sufficient but not necessary conditions for weak separability.

Fleissig and Whitney (2003) propose a new approach to evaluate Varian’s separability condition (iii) by using an alternative method to estimate $V^i$ and $\mu^i$. The aim is to find an estimate of an unknown aggregator function $\nu(y, q)$ which is a function of prices and quantities of the $y$-goods. This approach relies on Diewert (1976, 1978) who shows that a certain class of statistical index numbers provides a second-order approximation to an arbitrary or unknown, twice-differentiable linear homogeneous aggregator function. This class of index number is denoted as "superlative index". Two of those Indices which posses the property of being superlative are for example the Fisher Index and the Törnqvist-Theil Index. Therefore, Fleissig and Whitney (2003) build on calculating superlative index numbers to obtain estimates for $V^i$ and a corresponding range of $\mu^i$ that satisfy Varian’s conditions.

The first step of the procedure proposed by Fleissig and Whitney (2003) uses a superlative index number $QV^i = f(q, y)$, which is a function of prices and quantities from the $y$-goods as an initial estimate for $V^i$ in the inequality (b). The objective is to find how close the superlative index $QV^i$ is to providing a solution. By adding a positive number $QV^i_p$ or a negative number $QV^i_n$ to $QV^i$ the superlative index number with error

$$QV^{i*} = QV^i + QV^i_p - QV^i_n,$$

will provide a solution if one exists. If $QV^i_p = 0$, and $QV^i_n = 0$, for $i = 1, ..., n$, the superlative index without error provides a solution. Under the assumption that the superlative index number with error gives a solution to the separability inequalities, (b) can be written as

$$QV^i + QV^i_p - QV^i_n \leq QV^j + QV^j_p - QV^j_n + \mu^i p^i (y^i - y^j).$$

(3)

In the next step the deviations around the superlative index $QV^i$ are minimized by making $QV^i_p$ and $QV^i_n$ as small as possible. If there exists a superlative index number with error $QV^{i*}$

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and corresponding $\mu^i > 0$, that satisfy (3) then $QV^{i*}$ can be regarded as the "group quantity index" and $1/\mu^i$ as the "group price index" for the separable goods $y$. These are used by Fleissig and Whitney (2003) to solve the separability conditions (iii) of Varian (1983). As a vector $QV^{i*}$ might give a large range of values for $\mu^i$ that satisfy the separability conditions, the budget constraint from the separable $y$-data is used to find values for $\mu^i$ without restricting the solution. Taking the group quantity index $QV^i$ and group price index $1/\mu^i$, it is required that

$$(1/\mu^i) \ QV^i = inc^{y,i},$$

where $inc^{y,i}$ is the expenditure on the $y$-goods in period $i$. See equations (1) and (2). Solving for $\mu^i$ gives

$$\mu^i = QV^i/inc^{y,i}. \quad (4)$$

Hence, the aim is to keep $\mu^i$ the inverse of the group price index as close as possible to $QV^i/inc^{y,i}$ and thus to minimize deviations from adding up

$$\mu^i = QV^i/inc^{y,i} + \mu^i_p - \mu^i_n,$$

where $\mu^i_p$ are positive deviations and $\mu^i_n$ are negative deviations around $QV^i/inc^{y,i}$. When $QV^i_p$, $QV^i_n$, $\mu^i_p$, $\mu^i_n$ are close to 0, then the superlative index (with some error) provides a solution to the Afriat inequalities with adding up (closely approximated). Fleissig and Whitney (2003) note that to find a solution to the problem, Varian’s (1983) separability theorem requires the inverse of the group price index to be nonnegative. Additionally the group quantity indices with and without errors are required to be nonnegative to retain an economic interpretation of the solution.

Fleissig and Whitney (2003) use linear programming to find a solution to the problem described above. Under the constraints (3) and (4) as well as the respective nonnegativity constraints the objective is to minimize the deviations of $QV^{i*}$ for the calculated superlative index $QV^i$ and to minimize the deviations of $(1/\mu^i) QV^i$ from the expenditures on the $y$-separable goods. These equations can be represented in the form of a linear program (LP) model

$$\min \{ c^t x \ Ax \leq b, x \geq 0 \},$$

where $A$ is the coefficients matrix, $c$ is the objective vector and $b$ is the right-hand side vector. Note that this approach requires conversion of nonnegativity constraints to weak inequalities. The LP model by Fleissig and Whitney (2003) finds a solution to the Afriat inequalities, if it exists, by minimizing the deviations around $QV^i$ and $QV^i/inc^{y,i}$.11

\[\text{11 See Fleissig and Whitney (2003), pp. 135 - 136, 141.}\]
2.3 Data description

Aggregate time series data on four categories of goods are used for the present study. These categories cover data on consumption, money balances, corporate debt securities, and Treasury debt securities. The latter two are divided into two subclasses, namely short-term securities and long-term securities. In this study I use 474 monthly per capita observations from January 1969 to June 2008. The goods and assets which are subject to the testing procedures are labeled as:

- **CnDUR**: real average personal consumption of nondurables.
- **M**: money balances, currency component of M1 plus demand deposits.
- **TrBi**: holdings of Treasury bills.
- **TrBo**: holdings of Treasury bonds.
- **CP**: holdings of commercial paper, P1 rated.
- **CB**: holdings of private sector issued bonds, Aaa rated.

The data series are deflated using a 2005 price index and are calculated as per capita values by dividing through total population. As pointed out by Swofford and Whitney (1987), the way per capita asset holdings are calculated here, might be subject to some criticism. The "household sector" in this study includes asset holdings by institutional investors, personal trusts and nonprofit organizations. These organizations probably hold little cash and a small amount of demand deposits but holdings of corporate debt securities and Treasuries are substantial. Further, deriving per capita data by dividing through total population may also cause problems. Of course consumption is done by individuals under the age of eighteen acting on their own behalf. However, here the six year old is treated as an independent agent.

Securities’ prices are derived from returns on Treasury bills, Treasury bonds, commercial paper, and corporate bonds. These are treated as holding period returns of zero coupon discount bonds. Returns on equity are derived from Standard and Poors’ S&P 500 Index.

2.4 Nonparametric tests: results

First, the tests to check for the utility maximization hypothesis are performed. The data are not consistent with utility maximization if they violate GARP. The approach I pursue in this study, is to first check which grouping of goods can be rationalized by a well behaved utility function and then to test the feasible groupings for weak separability. I apply the proposed test for violations of GARP by Varian (1982) on the following groupings of goods:

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\(^{12}\)See Appendix A for data and variables description.
A. \textit{CnDUR, M, TrBi, CP},  
B. \textit{CnDUR, M, TrBo, CB},  
C. \textit{CnDUR, M, TrBi, TrBo}.

To keep the present study close to the framework of KVJ and NIE in each grouping only three liquid assets are included. Set A includes Treasury bills and commercial paper which have a rather short maturity length compared to the Treasury bonds and corporate bonds grouped in B. Analyzing set A separately from set B reflects the assumption that investors have a different valuation of short-term and long-term liquidity services. Set C captures the notion that an assets-in-the-utility specification might be suitable to explain a connection between changes in the slope of the yield curve and household demand behavior.

Results from testing A, B, and C for consistency with GARP are presented in Table 1. For grouping A, 97.5 percent of the data, for B, 96.7 percent of the data, and for C, 96.3 percent of the data satisfy GARP. As violations of GARP make up a very low share among the 474 observations, I regard the testing results for all groupings as not rejecting the utility maximization hypothesis for the data sample.\footnote{Varian (1991) proposes a statistical test for the size and number of violations of GARP. However, due to the very low share of violations in this study I refrain from further investigation.} Hence, per capita data on nondurable consumption and money balances, combined with U.S. Treasury holdings and corporate debt security holdings can be regarded as rationalized by a well-behaved utility function. Violations of GARP among data on asset prices and asset holdings might be found in times when unforeseen price movements are strong enough to make the holding of a once preferable portfolio to suddenly contradict the utility maximization principle.\footnote{During the Great Moderation there might have been few situations where financial markets saw such strong price movements. However, events like the Oil crisis, the Volcker disinflation, or Black Monday 1987 might have had an effect on securities prices which was forceful enough.}

With the maximization hypothesis being not rejected for sets A, B, and C, next is to test for weak separability using the procedure by Fleissig and Whitney (2003). Under the assumption that Treasury bills, Treasury bonds, commercial paper and corporate bonds yield nonpecuniary returns to the investor, such as liquidity services, I test the hypotheses whether the following sets of money and asset holdings can be regarded as an argument of investors’ utility which is weakly separable from consumption:

\begin{equation}
U_1 = U [CnDUR; V (M, TrBi, CP)], \tag{5}
\end{equation}

\begin{equation}
U_2 = U [CnDUR; V (M, TrBo, CB)], \tag{6}
\end{equation}

\begin{equation}
U_3 = U [CnDUR; V (M, TrBi, TrBo)]. \tag{7}
\end{equation}

Results are presented in Table 2. For the case of assumed 1 percent measurement error, among all of the groupings (5), (6), and (7) no violations of weak separability conditions are
found. For the case of 5 percent measurement error only for the most liquid grouping (5), no violations of the separability conditions are detected. Testing (6) at 5 percent measurement error finds violations of separability conditions for 28 percent of the observations, and for testing (7) at 5 percent measurement error the share of violations makes up 16 percent. However, I neglect the results for assumed 5 percent measurement errors as I regard U.S. data as being measured accurately enough for the purposes of the present study. Hence, for the purpose of the further empirical analysis I regard necessary and sufficient conditions for weakly separable utility maximization as being obtained for nondurables consumption and money balances, along with liquidity services associated with Treasury bills and commercial paper holdings, Treasury bonds and corporate bonds holdings, and Treasury bills and Treasury bonds holdings. Further, note that the procedure by Fleissig and Whitney (2003) evaluates separability conditions by using a superlative index to aggregate quantities of separable goods. For the necessary and sufficient separability conditions being met, this implies that the underlying unknown aggregator function can be regarded as linearly homogeneous and twice-differentiable. Hence, for the second part of the empirical analysis it can be assumed that CES or Cobb-Douglas aggregator functions might be suitable candidates for the separate argument within the utility function which captures liquidity services.

<table>
<thead>
<tr>
<th>Goods category</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
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<tbody>
<tr>
<td>CnDUR</td>
<td>X</td>
<td>X</td>
<td>X</td>
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<tr>
<td>M</td>
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<td>CP</td>
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<tr>
<td>CB</td>
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<td>X</td>
</tr>
</tbody>
</table>

| Share of violations | 0.025 | 0.033 | 0.037 |
| Share consistent with GARP | 0.975 | 0.967 | 0.963 |

Notes: This table presents results for testing sets A, B, and C for consistency with GARP.
Table 2: Test for weak separability

<table>
<thead>
<tr>
<th>Goods category</th>
<th>(1)</th>
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<tbody>
<tr>
<td>CnDUR</td>
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<td>M</td>
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<tr>
<td>CB</td>
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<td>X</td>
</tr>
</tbody>
</table>

| Measurement error | 1%  | 5%  | 1%  | 5%  | 1%  | 5%  |
| Share of violations | 0   | 0   | 0   | 0.28| 0   | 0.16|
| Share consistent with GARP/Sep | 1   | 1   | 1   | 0.72| 1   | 0.84|

Notes: This table presents results for the Fleissig and Whitney (2003) test for weak separability.

3 Determination of the utility’s parametric representation

With nonrejection of the hypotheses of utility maximization and weak separability, this study now turns to estimating preference parameters for a set of proposed utility functions. Non-parametric testing routines and evidence from KVJ and NIE support a liquidity services-in-the-utility formulation for empirical work on households’ asset demand. For the further proceeds of the analysis utility functions are assumed which include as arguments both, consumption, and liquidity services while the latter capture non-pecuniary returns to the investor by holdings of a certain group of assets. Liquidity services are derived by an aggregator function which has holdings of money, Treasuries and corporate debt securities as arguments. It is required that those assets are not perfect substitutes in terms of utility. Otherwise there would be no need for an aggregator function - a simple sum aggregate would suffice.

3.1 Modified asset pricing model

Following KVJ and NIE assume that under the premise that investors value liquidity services a representative agent’s utility function which shall fulfill the Inada conditions is of the form:
where \( c_t \) is the agent’s consumption at date \( t \) and \( \nu(\cdot) \) denotes the measure for liquidity services which is an (unknown) aggregator function of the real holdings of money \( m_t \), Treasuries \( b_t \), and corporate debt securities \( s_t \). The liquidity services function \( \nu(\cdot) \) is assumed to capture unique services provided by liquid assets which are valued by investors, where \( \nu'(\cdot) > 0 \), and \( \nu''(\cdot) < 0 \). The liquidity services function is concave as it is assumed that \( \nu'(\cdot) \) is increasing in \( m_t, b_t \) and \( s_t \), but the marginal benefit derived from liquidity services is decreasing in \( m_t, b_t \) and \( s_t \). Further, it has the property of \( \lim_{m_t,b_t,s_t \to \infty} \nu'(\cdot) = 0 \). This reflects the assumption that holding more liquidity services providing assets reduces the marginal value of an extra unit of such an assets. Further, this marginal value approaches zero if the agent is holding a sufficiently large amount of liquidity services providing assets. Moreover, under the assumption that investors value liquidity, holding one more unit of an asset that is the most liquid should c.p. generate more utility than holding one more unit of the least liquid i.e.

\[
\frac{\partial \nu(\cdot)}{\partial m_t} > \frac{\partial \nu(\cdot)}{\partial b_t} > \frac{\partial \nu(\cdot)}{\partial s_t}.
\]

From the first order conditions of the household’s utility maximization problem moment conditions for the GMM estimation are derived. These will be used in section 3.3 to estimate parameters of the implied demand functions. The representative household is further assumed to maximize the expected sum of a discounted stream of utilities

\[
E_0 \sum_{t=0}^{\infty} \beta^t u \left( c_t, \nu \left( \frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t} \right) \right),
\]

subject to the budget constraint

\[
P_t c_t + M_t + \frac{B_t}{R_t^b} + \frac{S_t}{R_t^s} + \frac{D_t}{R_t^d} \leq P_t y_t + M_{t-1} + B_{t-1} + S_{t-1} + D_{t-1},
\]

where \( E_0 \) is the expectation operator conditional on the information set in the initial period and \( \beta \in (0, 1) \), is the subjective discount factor. The price of one unit of consumption at date \( t \) is denoted by \( P_t \). The household gains a real endowed income \( y_t \) and carries wealth into the next period by investing in nominal holdings of money \( M_t \), Treasuries \( B_t \), and corporate debt securities \( S_t \). Nominal equity holdings \( D_t \) provide the numeraire asset in defining preferences. Further, the agent is assumed to hold only zero coupon discount bonds which pay out one unit of currency when being held to maturity. The gross returns on money, Treasuries, corporate debt securities, and equity are \( R_t^m \), \( R_t^b \), \( R_t^s \), and \( R_t^d \). Maximizing the objective (9) subject to the budget constraint (10) leads for given initial values and non-negativity constraints to the following first order conditions for real consumption and real holdings of money, equity,
Treasuries, and corporate debt securities:

\[ \frac{\partial u}{\partial c_t} = \lambda_t, \]  
\[ \frac{\partial u}{\partial m_t} + \beta E_t \left( \frac{P_t \lambda_{t+1}}{P_{t+1}} \right) = \lambda_t, \]  
\[ \beta E_t \left( \frac{P_t \lambda_{t+1}}{P_{t+1}} \right) = \lambda_t R^d_t, \]  
\[ \frac{\partial u}{\partial b_t} + \beta E_t \left( \frac{P_t \lambda_{t+1}}{P_{t+1}} \right) = \lambda_t R^b_t, \]  
\[ \frac{\partial u}{\partial s_t} + \beta E_t \left( \frac{P_t \lambda_{t+1}}{P_{t+1}} \right) = \lambda_t R^s_t, \]  

and (10) holding with equality and the accordant transversality conditions.\(^{15}\) Define the stochastic discount factor for nominal payoffs as \( M_{t+1} = \beta \frac{\partial u_m}{\partial u_{c_t}} \frac{P_t}{P_{t+1}} \), so that from (12), (14), and (15) the optimality conditions for holdings of money, Treasuries, and corporate debt securities can be derived which can be interpreted as pricing equations:

\[ \frac{\partial u}{\partial m_t} \frac{\partial u}{\partial c_t} + E_t [M_{t+1}] = 1, \]  
\[ \frac{\partial u}{\partial b_t} \frac{\partial u}{\partial c_t} + E_t [M_{t+1}] = 1 R^b_t, \]  
\[ \frac{\partial u}{\partial s_t} \frac{\partial u}{\partial c_t} + E_t [M_{t+1}] = 1 R^s_t. \]  

The first term on the left hand sides of equations (16), (17), and (18) captures the modification of the standard asset pricing model by the assumption of liquidity services.\(^{16}\) The marginal utility from holding money \( m_t \), Treasuries \( b_t \), and corporate bonds \( s_t \) induces a liquidity services premium on each asset’s price. Increasing the investors’ holdings of \( m_t, b_t \), and \( s_t \) should decrease liquid assets’ prices which is due to the assumption of \( \nu (\cdot) \) being concave. Hence, equations (16) - (18) reflect that under the assumption of liquidity services being an argument of the investor’s utility function, increasing the amount of liquid assets held will lower the investor’s willingness to pay for another unit of such assets.

Note that the assumed functional form of the aggregator \( \nu (\cdot) \) is crucial. KVJ and NIE do not fully specify the functional form of \( \nu (\cdot) \) but implicitly define a set of requirements to it. It can be shown that employing a CES aggregator nested in a CRRA utility would match those requirements.\(^{17}\) Specifically, in this case each asset’s liquidity services premium is not only driven by the level of holdings of the respective asset, but in addition to that, it is driven by the total holdings of liquidity services providing assets. Moreover, by assuming \( \frac{\partial \nu (\cdot)}{\partial m_t} > 0 \) and \( \frac{\partial \nu (\cdot)}{\partial b_t} > 0 \) and \( \frac{\partial \nu (\cdot)}{\partial s_t} > 0 \), the transversality conditions for holdings of Treasuries, corporate debt securities, and equity are given by: \( \lim_{j \to -\infty} \beta^j E_t \left( \lambda_{t+j} b_{t+j} / R^b_{t+j} \right) = 0 \), \( \lim_{j \to -\infty} \beta^j E_t \left( \lambda_{t+j} s_{t+j} / R^s_{t+j} \right) = 0 \), and \( \lim_{j \to -\infty} \beta^j E_t \left( \lambda_{t+j} d_{t+j} / R^d_{t+j} \right) = 0 \).

\(^{15}\)The transversality conditions for holdings of Treasuries, corporate debt securities, and equity are given by:

\[ \lim_{j \to -\infty} \beta^j E_t \left( \lambda_{t+j} b_{t+j} / R^b_{t+j} \right) = 0, \]  
\[ \lim_{j \to -\infty} \beta^j E_t \left( \lambda_{t+j} s_{t+j} / R^s_{t+j} \right) = 0, \]  
\[ \lim_{j \to -\infty} \beta^j E_t \left( \lambda_{t+j} d_{t+j} / R^d_{t+j} \right) = 0. \]

\(^{16}\)Please note that for simplicity default risk is neglected.

\(^{17}\)See Appendix B.
\[ \frac{\partial u(c)}{\partial m_t} > \frac{\partial u(c)}{\partial s_t} \], increasing the holdings of \( m_t \) should c.p. decrease asset prices to a larger extent than increasing the holdings of \( b_t \) and \( s_t \). Analogously, the same applies c.p. for increasing the holdings of \( b_t \) compared to increasing holdings of \( s_t \). This requirement can be fulfilled by making use of the CES aggregator. In contrast to that, employing a Cobb-Douglas aggregator nested in a CRRA utility implies that each asset’s liquidity services premium is driven by the level of holdings of the respective asset but not by the total holdings of liquidity services providing assets. E.g. the liquidity premium on \( R_t^s \) would be a function of \( s_t \), and not of \( m_t \) and \( b_t \). This implication however, is not in line with KVJ and NIE. Further, assuming additive separability between the utility function’s arguments of consumption and liquidity services, as well yields implications which are not in line with the investors’ behavior observed by KVJ and NIE. Specifically, for the case of a CES aggregator together with additive separability, each asset’s liquidity services premium would not be a decreasing function in total holdings of liquid asset. For the Cobb-Douglas aggregator, the liquidity premium would be decreasing in holdings of the asset under consideration but increasing in overall asset holdings. Note that the CES function degenerates to a Cobb-Douglas aggregator if the elasticity of substitution is unity. One could estimate the model under a parameter restriction on the elasticity of substitution. However, the present paper’s approach is to first estimate a broad variety of possible model specifications and then to select the most suitable one.

### 3.2 Moment conditions

Set (13) equal to (12), (14), and (15). Then plug in (11) for the shadow price of income \( \lambda_t \geq 0 \), then

\[
1 = E_t \left[ \frac{\partial u}{\partial c_t} - \beta \frac{P_t (1 + r^d_t)}{P_{t+1}} \frac{\partial u}{\partial c_{t+1}} \right],
\]

(19)

\[
0 = E_t \left[ \frac{\partial u}{\partial m_t} - \beta \frac{P_t r^d_t}{P_{t+1}} \frac{\partial u}{\partial c_{t+1}} \right],
\]

(20)

\[
0 = E_t \left[ \frac{\partial u}{\partial b_t} - \beta \frac{P_t (r^d_t - r^b_t)}{P_{t+1}} \frac{\partial u}{\partial c_{t+1}} \right],
\]

(21)

\[
0 = E_t \left[ \frac{\partial u}{\partial s_t} - \beta \frac{P_t (r^d_t - r^s_t)}{P_{t+1}} \frac{\partial u}{\partial c_{t+1}} \right],
\]

(22)

which yields Euler equations and implied demand functions for consumption (19), money (20), Treasuries (21), and corporate debt securities (22). Here I already used the representation of the equations as conditional moment conditions. Equations (19) - (22) are now written in terms of excess returns. Note that \( r^d_t, r^b_t, \) and \( r^s_t \) denote net returns on equity, Treasuries, and corporate debt securities. The Euler equation for consumption has the well known interpretation like in the standard case without assets in the utility. The Euler equation for money holdings (20) requires that in equilibrium utility cannot be increased by holding
one unit of money less at time $t$, investing it in equities, and consuming the payoff at time $t+1$. The forgone utility associated with a one unit reduction in money holdings is $\frac{\partial u}{\partial m} R_t$. Transferring one unit of money to equities at time $t$ increases real wealth at $t+1$ by $r_d^t$, since money yields no nominal return while equity does. The gain in utility if these higher proceeds are consumed in period $t+1$ is $\beta E_t \left[ \frac{r^t}{r_{t+1}} \frac{\partial u}{\partial c_{t+1}} \right]$. Equating this to foregone utility yields (20). Analogously Euler equations (21) and (22) equate the costs and benefits in terms of utility of transferring one unit of currency from Treasuries or corporate debt securities into equities for one period.

### 3.3 GMM estimation models

Methodologically, this paper follows previous authors like Holman (1998). For each proposed specification of utility function (8) correspondent sets of Euler equations are derived from (19) to (22) which are then estimated by using GMM. The Euler equations state that in equilibrium the representative agent’s expectations are orthogonal to all of the variables within the information set at the time predictions are made. Specifically, they imply population orthogonality conditions which are a function of the observed data and the preference parameters. The GMM estimator is a nonlinear instrumental variable estimator of the population parameters that tries to make the sample orthogonality conditions close to zero by minimizing a distance function.\(^{18}\) According to Verbeek (2000) there are several advantages of this method when estimating asset pricing models. One is that GMM does not require distributional assumptions or assumptions regarding data generating processes. Further it can allow for heteroskedasticity of unknown form which is a convenient feature when working with data on asset returns. Importantly it can estimate parameters even if the model cannot be solved analytically from the first order conditions. This is especially useful for the present asset pricing models which are comprised of the nonlinear Euler equations (19) to (22).

I use the same time series data on consumption and holdings of money, Treasuries and corporate debt securities, as well as the same data on prices and returns as in Section 2.3 of this paper. Further, three different sets of instruments are employed, depending on whether Treasury bills and commercial paper holdings, Treasury bills and Treasury bond holdings, or Treasury bond and corporate bond holdings, are arguments of (8). Following the approach of Hall (2005) each set of instruments includes a constant term, two lagged values of the real returns on equity, and the real returns on the correspondent assets included in each of the estimation models, as well as the past two growth rates of real per capita consumption and real per capita asset holdings. The reason for this choice is that lagged values and lagged growth rates can be assumed to be uncorrelated with current innovations.

Note that there are more instruments than parameters. Hence, the system of Euler

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\(^{18}\)Hansen (1982) provides the conditions under which the GMM estimator is consistent, asymptotically normal, and efficient.
equations is overidentified. The \( J \)-test of overidentifying restrictions by Hansen (1982) and Hansen and Singleton (1982) is used to conduct a joint test of the specification of the asset pricing model and the validity of the instrument set.

The following six specifications of investor’s utility are proposed:

I. Poterba Rotemberg Utility: Poterba and Rotemberg (1986) use a nested CES preferences specification:

\[
\begin{align*}
u(c_t, \nu \left( \frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t} \right)) &= \frac{1}{\sigma} \left[ \left( \frac{L_t}{P_t} \right)^\gamma + \left( \frac{M_t}{P_t} \right)^\gamma + (1 - \delta_M - \delta_B) \left( \frac{S_t}{P_t} \right)^\gamma \right]^{\frac{1}{\gamma}},
\end{align*}
\]

where \( L_t \) captures liquidity services derived from a CES aggregator function:

\[
L_t = \delta_M \left( \frac{M_t}{P_t} \right)^\gamma + \delta_B \left( \frac{B_t}{P_t} \right)^\gamma + (1 - \delta_M - \delta_B) \left( \frac{S_t}{P_t} \right)^\gamma.
\]

This utility function exhibits constant relative risk aversion. Further, utility is Cobb-Douglas in consumption and liquidity services, ensuring that more consumption raises the marginal utility of liquidity services and vice versa. The liquidity measure is a CES function of real money balances, Treasuries holdings and corporate debt securities holdings. It must be pointed out that these preferences are quite restrictive. In particular, they impose homogeneity and require separability between its arguments. Further, the elasticity of substitution between consumption and liquidity services is assumed to be equal to one.

With these preferences, from equations (19) to (22) the following moment conditions can be derived:

\[
\begin{align*}
1 &= E_t \left[ \beta \left( \frac{1 + \nu^d}{P_{t+1}} \right) \left( \frac{c_{t+1}}{c_t} \right)^{\sigma\alpha - 1} \left( \frac{L_{t+1}}{L_t} \right)^{\sigma(1-\alpha)} \right], \\
0 &= E_t \left[ \alpha \beta \left( \frac{P_t}{P_{t+1}} \right)^{\gamma - 1} \left( \frac{M_t}{P_t} \right)^\gamma \left( \frac{L_t}{P_t} \right)^{\sigma(1-\alpha)} \right], \\
0 &= E_t \left[ \alpha \beta \left( \frac{P_t}{P_{t+1}} \right)^{\gamma - 1} \left( \frac{B_t}{P_t} \right)^\gamma \left( \frac{L_t}{P_t} \right)^{\sigma(1-\alpha)} \right], \\
0 &= E_t \left[ \alpha \beta \left( \frac{P_t}{P_{t+1}} \right)^{\gamma - 1} \left( \frac{S_t}{P_t} \right)^\gamma \left( \frac{L_t}{P_t} \right)^{\sigma(1-\alpha)} \right].
\end{align*}
\]

I report estimates of the parameters \( \{\sigma, \beta, \alpha, \gamma, \delta_M, \delta_B\} \) in Section 3.4. For a first round of estimations I constrain the utility function parameters. I require that \( \delta_M, \delta_B, \) and \( (1 - \delta_M - \delta_B) \) be positive and sum up to one. Further, I require \( \alpha, \beta, \) and \( \gamma \) to be positive between zero and one, and \( \sigma \) to be less than zero. The validity of the model and the restrictions are checked by using the J-test of overidentifying restrictions. The constraints are successively relaxed if the J-test rejects the model together with the restrictions in place. If \( \alpha \) was equal to one,
convenience yields and liquidity services can not be regarded as a source of utility. Note that if \( \gamma \) was estimated to be exactly zero, \( L_t \) would degenerate to a Cobb-Douglas aggregation. If the elasticity of substitution \( \gamma \) is close to one, linear aggregation would be implied. In the latter case money, Treasuries, and corporate debt securities would be close substitutes in terms of utility.

II. Nested CES Liquidity Services: In contrast to Poterba and Rotemberg (1986), in this case utility is not Cobb-Douglas in consumption and liquidity services:

\[
u \left( c_t, \nu \left( \frac{M_t}{P_t} \frac{B_t}{P_t} \frac{S_t}{P_t} \right) \right) = \frac{1}{\sigma} \left\{ c_t L_t \left( \frac{M_t}{P_t} \frac{B_t}{P_t} \frac{S_t}{P_t} \right) \right\}^\sigma,
\]

where \( L_t \) captures liquidity services derived by the CES aggregator function

\[
L_t = \left[ \delta_M \left( \frac{M_t}{P_t} \right)^\gamma + \delta_B \left( \frac{B_t}{P_t} \right)^\gamma + (1 - \delta_M - \delta_B) \left( \frac{S_t}{P_t} \right)^\gamma \right]^\frac{1}{\gamma}.
\]

This utility function as well exhibits constant relative risk aversion. These preferences are less restrictive compared to Poterba and Rotemberg (1986) as a unitary elasticity of substitution between consumption and liquid assets is not demanded. Following (19) to (22) moment conditions are then given by

\[
1 = E_t \left[ \frac{\beta}{P_t} \frac{(1 + r_t^d)}{P_{t+1}} \left( \frac{c_{t+1}}{c_t} \right)^{\sigma-1} \left( \frac{L_{t+1}}{L_t} \right)^\sigma \right],
\]

\[
0 = E_t \left[ c_t^\sigma L_t^{\sigma-1} \delta_M \left( \frac{M_t}{P_t} \right)^{\gamma-1} - \beta \frac{P_t r_t^d}{P_{t+1}} c_t^{\sigma-1} L_t^{\sigma} \right],
\]

\[
0 = E_t \left[ c_t^\sigma L_t^{\sigma-1} \delta_B \left( \frac{B_t}{P_t} \right)^{\gamma-1} - \beta \frac{P_t (r_t^d - r_t^b)}{P_{t+1}} c_t^{\sigma-1} L_t^{\sigma} \right],
\]

\[
0 = E_t \left[ c_t^\sigma L_t^{\sigma-1} (1 - \delta_M - \delta_B) \left( \frac{S_t}{P_t} \right)^{\gamma-1} - \beta \frac{P_t (r_t^d - r_t^b)}{P_{t+1}} c_t^{\sigma-1} L_t^{\sigma} \right].
\]

I report estimates of the parameters \( \{ \sigma, \beta, \gamma, \delta_M, \delta_B \} \) in Section 3.4. For the estimation I constrain the utility function parameters. I require that \( \delta_M, \delta_B, \) and \( (1 - \delta_M - \delta_B) \) be positive and sum up to one. Further, I require \( \beta \) and \( \gamma \) to be positive and between zero and one, and \( \sigma \) to be less than zero.

III. Nested CES Liquidity Services, additively separable: Utility is not Cobb-Douglas in consumption and liquidity services but assumed to be additively separable in its arguments:

\[
u \left( c_t, \nu \left( \frac{M_t}{P_t} \frac{B_t}{P_t} \frac{S_t}{P_t} \right) \right) = \frac{1}{\sigma} \left\{ c_t + L_t \left( \frac{M_t}{P_t} \frac{B_t}{P_t} \frac{S_t}{P_t} \right) \right\}^\sigma,
\]
where $L_t$ captures liquidity services derived by the CES aggregator function

$$L_t = \left[ \delta_M \left( \frac{M_t}{P_t} \right)^\gamma + \delta_B \left( \frac{B_t}{P_t} \right)^\gamma + (1 - \delta_M - \delta_B) \left( \frac{S_t}{P_t} \right)^\gamma \right]^{\frac{1}{\gamma}}.$$

Here equations (19) to (22) become

1. $1 = E_t \left[ \beta \frac{P_t}{P_{t+1}} \left( \frac{c_{t+1} + L_{t+1}}{c_t + L_t} \right)^{\sigma-1} \right]$,  
2. $0 = E_t \left[ (c_t + L_t)^{\sigma-1} L_t^{\frac{1}{\gamma}} \delta_M \left( \frac{M_t}{P_t} \right)^{\gamma-1} - \frac{P_t r^d_t}{P_{t+1}} \left( c_{t+1} + L_{t+1} \right)^{\sigma-1} \right]$,  
3. $0 = E_t \left[ (c_t + L_t)^{\sigma-1} L_t^{\frac{1}{\gamma}} \delta_B \left( \frac{B_t}{P_t} \right)^{\gamma-1} - \frac{P_t (r^d_t - r^b_t)}{P_{t+1}} \left( c_{t+1} + L_{t+1} \right)^{\sigma-1} \right]$,  
4. $0 = E_t \left[ (c_t + L_t)^{\sigma-1} L_t^{\frac{1}{\gamma}} \left( 1 - \delta_M - \delta_B \right) \left( \frac{S_t}{P_t} \right)^{\gamma-1} - \frac{P_t (r^d_t - r^b_t)}{P_{t+1}} \left( c_{t+1} + L_{t+1} \right)^{\sigma-1} \right]$.

I report estimates of the parameters $\{\sigma, \beta, \gamma, \delta_M, \delta_B\}$ in section 3.4. For the estimation I constrain the utility function parameters. I require that $\delta_M$, $\delta_B$, and $(1 - \delta_M - \delta_B)$ be positive and sum up to one. Further, I require $\beta$ and $\gamma$ to be positive between zero and one, and $\sigma$ to be less than zero.

**IV. Cobb-Douglas Utility with Cobb-Douglas liquidity services:** Utility is assumed to be Cobb-Douglas in consumption and liquidity services.

$$u \left( c_t, \nu \left( \frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t} \right) \right) = \frac{1}{\sigma} \left\{ c_t^\sigma \left( \frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t} \right)^{1-\alpha} \right\}^{\sigma},$$

where $L_t$ denotes liquidity services which are as well derived by a Cobb-Douglas aggregator function

$$L_t = \left( \frac{M_t}{P_t} \right)^{\delta_M} \left( \frac{B_t}{P_t} \right)^{\delta_B} \left( \frac{S_t}{P_t} \right)^{(1-\delta_M-\delta_B)}.$$

Again these preferences are quite restrictive. The utility function exhibits constant relative risk aversion. It imposes homogeneity and separability. Further, a unitary elasticity of substitution between consumption and liquidity services is assumed.
From (19) to (22) moment conditions are then given by
\[
1 = E_t \left[ \beta \frac{P_t (1 + r_t^d)}{P_{t+1}} \left( \frac{c_{t+1}}{c_t} \right)^{\sigma - 1} \left( \frac{L_{t+1}}{L_t} \right)^{\sigma (1 - \alpha)} \right],
\]
\[
0 = E_t \left[ c_t^{\sigma_0} \delta_M \left( \frac{M_t}{P_t} \right)^{\delta_M (1 - \alpha)} \left( \frac{B_t}{P_t} \right)^{\delta_B (1 - \alpha)} \left( \frac{S_t}{P_t} \right)^{(1 - \delta_M - \delta_B)\sigma (1 - \alpha)} \right],
\]
\[
0 = E_t \left[ c_t^{\sigma_0} \delta_B \left( \frac{B_t}{P_t} \right)^{\delta_B (1 - \alpha)} \left( \frac{S_t}{P_t} \right)^{(1 - \delta_M - \delta_B)\sigma (1 - \alpha)} \right],
\]
\[
0 = E_t \left[ c_t^{\sigma_0} \delta_M \left( \frac{M_t}{P_t} \right)^{\delta_M (1 - \alpha)} \left( \frac{B_t}{P_t} \right)^{\delta_B (1 - \alpha)} \left( \frac{S_t}{P_t} \right)^{(1 - \delta_M - \delta_B)\sigma (1 - \alpha)} \right].
\]

I report estimates of the parameters \( \{\sigma, \beta, \alpha, \delta_M, \delta_B\} \) in Section 3.4. For the estimation I constrain the utility function parameters. I require \( \delta_M, \delta_B, \) and \( (1 - \delta_M - \delta_B) \) to be positive and sum up to one, and \( \alpha \) and \( \beta \) to be positive between zero and one, and \( \sigma \) to be less than zero.

V. Nested Cobb-Douglas Liquidity Services: Utility is not Cobb-Douglas in consumption and liquidity services
\[
u \left( c_t, \nu \left( \frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t} \right) \right) = \frac{1}{\sigma} \left[ c_t L_t \left( \frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t} \right) \right]^{\sigma},
\]
where \( L_t \) captures liquidity services derived from a Cobb-Douglas aggregator function
\[
L_t = \left( \frac{M_t}{P_t} \right)^{\delta_M} \left( \frac{B_t}{P_t} \right)^{\delta_B} \left( \frac{S_t}{P_t} \right)^{(1 - \delta_M - \delta_B)}.
\]

Then equations (19) to (22) imply
\[
1 = E_t \left[ \beta \frac{P_t (1 + r_t^d)}{P_{t+1}} \left( \frac{c_{t+1}}{c_t} \right)^{\sigma - 1} \left( \frac{L_{t+1}}{L_t} \right)^{\sigma} \right],
\]
\[
0 = E_t \left[ c_t^{\sigma_0} \delta_M \left( \frac{M_t}{P_t} \right)^{\delta_M (1 - \alpha)} \left( \frac{B_t}{P_t} \right)^{\delta_B (1 - \alpha)} \left( \frac{S_t}{P_t} \right)^{(1 - \delta_M - \delta_B)\sigma} - \beta \frac{P_t L_t}{P_{t+1}} L_t^{\sigma - 1} \right],
\]
\[
0 = E_t \left[ c_t^{\sigma_0} \delta_B \left( \frac{B_t}{P_t} \right)^{\delta_B (1 - \alpha)} \left( \frac{S_t}{P_t} \right)^{(1 - \delta_M - \delta_B)\sigma} - \beta \frac{P_t (r_t^d - r_t^s)}{P_{t+1}} L_t^{\sigma - 1} \right],
\]
\[
0 = E_t \left[ c_t^{\sigma_0} \delta_M \left( \frac{M_t}{P_t} \right)^{\delta_M (1 - \alpha)} \left( \frac{B_t}{P_t} \right)^{\delta_B (1 - \alpha)} \left( 1 - \delta_M - \delta_B \right) \left( \frac{S_t}{P_t} \right)^{(1 - \delta_M - \delta_B)\sigma} - \beta \frac{P_t (r_t^d - r_t^s)}{P_{t+1}} L_t^{\sigma - 1} \right].
\]

I report estimates of the parameters \( \{\sigma, \beta, \delta_M, \delta_B\} \) in Section 3.4. For the estimation I constrain the utility function parameters. I require that \( \delta_M, \delta_B, \) and \( (1 - \delta_M - \delta_B) \) be positive.
and sum up to one. Further, I require $\beta$ to be positive between zero and one, and $\sigma$ to be less than zero.

VI. Nested Cobb-Douglas Liquidity Services, additively separable: Utility is additively separable in consumption and liquidity services:

$$\bar{u}(c_t, \nu\left(\frac{M_t}{P_t}, B_t, S_t\right)) = \frac{1}{\sigma} \left\{ \frac{c_t + L_t}{P_t} \left(\frac{M_t}{P_t}, B_t, S_t\right) \right\}^\sigma,$$

where $L_t$ captures liquidity services derived from a Cobb-Douglas aggregator function

$$L_t = \left(\frac{M_t}{P_t}\right)^{\delta_M} \left(\frac{B_t}{P_t}\right)^{\delta_B} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)}.$$

This utility function exhibits constant relative risk aversion. Equations (19) to (22) imply the following moment conditions

$$1 = E_t \left[ \beta \frac{P_t (1 + r^d_t)}{P_{t+1}} \left(\frac{c_{t+1+L_{t+1}}}{c_t + L_t}\right)^{\sigma-1} \right],$$

$$0 = E_t \left[ (c_t + L_t)^{\sigma-1} \beta M_t \left(\frac{M_t}{P_t}\right)^{\delta_M-1} \left(\frac{B_t}{P_t}\right)^{\delta_B} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)} \right],$$

$$0 = E_t \left[ (c_t + L_t)^{\sigma-1} \beta B_t \left(\frac{M_t}{P_t}\right)^{\delta_M} \left(\frac{B_t}{P_t}\right)^{\delta_B-1} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)} \right],$$

$$0 = E_t \left[ (c_t + L_t)^{\sigma-1} (1-\delta_M - \delta_B) M_t \left(\frac{M_t}{P_t}\right)^{\delta_M} \left(\frac{B_t}{P_t}\right)^{\delta_B} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)-1} \right].$$

I report estimates of the parameters $\{\sigma, \beta, \delta_M, \delta_B\}$ in Section 3.4. For the estimation I constrain the utility function parameters. I require that $\delta_M$, $\delta_B$, and $(1-\delta_M-\delta_B)$ be positive and sum up to one. Further, I require $\beta$ to be positive between zero and one, and $\sigma$ to be less than zero.

3.4 GMM estimation results

In the following subsections of this paper GMM estimation results are discussed for the moment condition sets I to VI. For each Table presenting estimation results, in the first column estimates are shown for the dataset including nondurables consumption, money balances, and data on returns and holdings of Treasury bills and commercial paper. Columns 2 and 3 present results for the sets including data on Treasury bills and Treasury bonds, and on Treasury bonds and corporate bonds.
I. Poterba Rotemberg Utility: Table 3 presents the estimates of the parameters from specification I. with corresponding moment conditions. Three sets of estimates corresponding to each dataset are reported. For all of the three data sets the J-test of overidentifying restrictions indicates rejection of model I. at the 5 percent level with all constraints on $\sigma$, $\beta$, $\alpha$, $\gamma$, $\delta_M$, and $\delta_B$ in place. The model is rejected as well at the 5 percent level for relaxing the restriction on $\beta$ and subsequently relaxing the restriction on $\gamma$. The model is not rejected at the 5 percent significance level if the restrictions on $\sigma$, $\delta_M$, $\delta_B$, and $(1 - \delta_M - \delta_B)$ are in place for all data sets. The estimated coefficient values are remarkably similar across the three model estimations. Further, in the fourth column I report results from Poterba and Rotemberg (1986). Note that in this case liquidity services are derived from a different set of assets. For their estimation the authors use quarterly data from 1959:Q1 to 1981:Q3 on nondurables consumption, money, time deposits, and Treasury bills. Still, it is notable that estimation results of the present study for the utility specification I. are close to the results of Poterba and Rotemberg (1986). This is found in spite of employing different data sets, different instrument sets, and a different data frequency. For the present model the intertemporal elasticity of substitution $\sigma$ is expected to be larger than zero in absolute terms. If $\sigma = 0$, then the nested function degenerates to a logarithmic function of consumption and liquidity services. Poterba and Rotemberg (1986) estimate a range of $\sigma$ between $-6.5$ and $-5.6$, whereas the present study finds a range between $-7.7$ and $-6.0$. Across all three datasets the estimated discount rate $\beta$ is greater than zero and below one. However, unity is not excluded from the 95 percent confidence intervals in columns 1 and 2. Still the point estimates are slightly smaller than those which are found in other studies. The estimated share of expenditures devoted to consumption $\alpha$ is for all estimations significantly greater than zero at the 1 percent level and lies between 0.68 and 1.07. For the estimations presented in columns 1 and 2 of Table 3, results for $\alpha$ suggest that liquidity services are not a direct source of utility. Only for the estimation model including data on Treasury bonds and corporate bonds, the size of $\alpha$ implies that convenience yield is a source of utility. However, the size of the point estimate suggests that an implausibly large share of households expenditures is devoted to Treasury bond and corporate bond holdings. The main backdraw of the present study is that the nonparametric testing procedures provide little guidance about the functional form of the household’s utility and about the complete set of liquidity services providing assets. Hence, the three data groupings considered for this study might leave out

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19Poterba and Rotemberg (1986) further provide estimation results for tax adjusted data and different sets of instruments. In Table 3 results are taken from an estimation which is closest to the setting which is analyzed in this paper.

20Hansen and Singleton (1983) estimate that $\beta$ lies between 0.995 and 1.096. Poterba and Rotemberg (1986) find that for their utility specification the discount factor is larger than unity.
further assets which yield liquidity services.\textsuperscript{21} The inverse of the elasticity of substitution between liquidity services yielding assets $\gamma$ is significantly larger than zero and lies between 0.5 and 0.58. If $\gamma$ was estimated to be 1 a linear aggregator function would be implied. Assets would be one for one substitutes in terms of utility. If $\gamma$ was equal to zero, the aggregator would reduce to the Cobb-Douglas specification. The hypothesis that $\gamma = 1$ is rejected for all three estimations. Within the convenience yield aggregator the coefficients $\delta_M$, $\delta_B$, and $(1 - \delta_M - \delta_B)$ are estimated with relatively wide confidence intervals. Further, for the estimation model presented in the second column of Table 3, $\delta_B$ is not significantly different from zero at the 5 percent level. However, for all three estimations the following pattern of point estimates is observed: $\delta_M > \delta_B > (1 - \delta_M - \delta_B)$. Assuming that real asset holdings were of equal size, this result implies that marginal utility of another unit of real money balances would exceed that from another unit of Treasuries or corporate debt securities.

\textbf{II. Nested CES Liquidity services} The estimates of the parameters from specification II are shown in Table 4. Notably the J-test only does not reject the validity of the model which is estimated for data on Treasury bills and Treasury bonds. For this estimation result presented here, only the restrictions on $\sigma$, $\delta_M$, $\delta_B$, and $(1 - \delta_M - \delta_B)$ are in place. Note that the estimated share of expenditures devoted to consumption $\alpha$ is larger that unity implying that liquidity services are not a direct source of utility. The estimation model including data on Treasury bills and commercial paper as well as the estimation model including data on Treasury bonds and corporate bonds are rejected. Further, point estimates for the three datasets are not as similar among each other as for utility specification I. The most striking difference is that estimates for $\sigma$ range between $-3.19$ and $-11.83$ and estimates for $\gamma$ range between $0.09$ and $1.00$, implying perfect substitutability of liquid assets for the latter case. Specification II. compared to specification I. does not require the elasticity of substitution between consumption and the liquid assets’ aggregate to be unity. As this requirement seems to be quite restrictive and there is no theoretical guidance about the size of the elasticity of substitution for this model, one would not a priori expect that estimating model II yields J-tests of overidentifying restrictions which indicate rejection of two of the three estimation models.

\textbf{III. - VI. Specifications} The GMM estimation routine does not find a solution for the minimization problem. This is found for all of the three different data sets with the corresponding instrument sets.

\textsuperscript{21}Note that for the present study three datasets were employed which are intended to closely match the datasets used in KVJ and NIE.
4 Conclusion

This paper presents a model of aggregate demand for consumption, money balances, U.S. Treasuries, and corporate debt securities where the asset holdings directly contribute to the investors’ utility. The first part of the analysis applies Varian’s (1982) nonparametric testing procedure on monthly per capita data on nondurable consumption, money balances, U.S. Treasuries, and corporate debt securities. As violations of GARP can only be detected for a very low share of the observations, results for the nonparametric testing routines can be seen as not rejecting the utility maximization hypothesis. Therefore, all of the three groupings can be regarded as being rationalized by a well-behaved utility function. Additionally, necessary and sufficient conditions for weakly separable utility maximization are obtained for monthly per capita data on nondurables consumption and money balances, along with liquidity services derived from Treasury holdings and corporate debt securities holdings. In the second part of the analysis Euler equations implied by the modified asset-pricing model are estimated under alternative utility specifications. Surprisingly, only the restrictive utility specification proposed by Poterba and Rotemberg (1986) yields parameter estimates which are relatively robust to the choice of data. Estimation results however, imply parameter values which indicate misspecification.

In the presence of many assets which might provide liquidity services, a more complete modelling of the financial sector is needed. The paper makes a step in that direction. However, the analysis suffers from several shortcomings. These are primarily limitations of the particular functional form and parameterization of the utility function and the data choice. Eventually, the approach should be extended to incorporate a broader range of assets. However, the nonparametric testing routines provide little guidance about the true functional form and the true set of liquidity services providing assets. A second issue is that the menu of important assets changes over time as Poterba and Rotemberg (1986) noted before. E.g. financial innovations like the increasing importance of money market mutual funds allow assets to be repackaged to yield different degree of liquidity services.
References


A Data sources

CnDUR: This variable is constructed as the monthly real per capita consumption expenditures on nondurable goods. Monthly data on aggregate expenditures on nondurable consumption goods are from the Federal Reserve’s FRED database (series PCEND). Monthly real values are obtained by using a deflator calculated from a chain-type price index for personal consumption expenditures (FRED series PCEPI) with 2005 = 100. Further, per capita data are derived by dividing through monthly total population (FRED series POP).

M: Measures monthly real per capita money balances. This is proxied by the data series on the currency component of M1 measure plus demand deposits from the FRED database (series CURRDD). As for CnDUR, real per capita balances are derived by using the same price index deflator and by dividing through total U.S. population.

TrBi: This variable is intended to proxy for the real per capita holdings of Treasury bills (4-week to 52-week maturity). Here, data on the face value of outstanding marketable U.S. Treasury bills is taken from Datastream (series name: U.S. Federal Debt - Marketable Securities Treasury Bills Curn, Id: USSECTRBA). Note that these data do not contain non-marketable Treasuries i.e. as held in a TreasuryDirect account. Unfortunately I do not have data on the distribution of maturities among non-markethables. So I can not quantify the shares of bills, notes, and bonds. Therefore, data on the face value of non-marketable bills is left out. The time series is as well transformed to real per capita values in the same way as CnDUR. Prices are calculated from the 3-Month Treasury Bill Secondary Market Rate (FRED series TB3MS) which is assumed to be a holding period return of a zero coupon bill. Further, from the raw data returns are calculated on monthly basis and deflated by the gross growth rate of the price index PCEPI. Quantities held are then derived by dividing correspondent real per-capita values by the implied prices. This is done for all groups of assets (See TrBo, CP, and CB).

TrBo: This variable is intended to proxy for the real per capita holdings of Treasury bonds (20 to 30 years maturity). Data is taken from Thomson Reuters Datastream (series name: U.S. Federal Debt - Marketable Securities Treasury Bonds Curn, Id: USSECTRDA). The time series is transformed to real per capita values in the same way as CnDUR. Prices and monthly net returns are calculated in the same way as for the variable TrBi. Correspondent data is taken from the yields on Long-Term U.S. Government Securities (FRED series LTGOVTBD for the period 1969 - 2000 and series GS20 from 2000 - 2008).

CP: Proxies the per capita holdings of commercial paper. Here data on the face value of outstanding commercial paper issued by nonfarm and nonfinancial corporate business is taken from FRED (series CPLBSNNCB). As before the time series is converted to real per
capita values. Prices monthly net returns are derived from commercial paper yields in the same way as for the variable TrBi. Prior to 1971 I use the commercial paper yields series for prime commercial paper, 4-6 month maturity, from Banking and Monetary Statistics (Table 12.5 for 1941-1970). For 1971-1996 it is the 3-Month Commercial Paper Rate from FRED (series CP3M) and for 1997-2008 the 3-Month AA Nonfinancial Commercial Paper Rate from FRED (series CPN3M).

CB: Proxies the per capita holdings of corporate bonds. Here data on the face value of outstanding corporate bonds issued by nonfarm and nonfinancial corporate business is taken from FRED (series CBLBSNNCB). As before the time series is converted to real per capita values. Prices monthly net returns for corporate bonds are calculated in the same way as for the variable TrBi. Data are taken from Moody’s Seasoned Aaa Corporate Bond Yield Index (FRED series AAA).

Further, for the GMM estimation model monthly returns on the numeraire asset \( r_t \) are proxied by returns calculated on the S&P 500 Stock Price Index (FRED series SP500). Further returns are deflated by the gross growth rate of the price index PCEPI.

B Pricing equations

i. Assume that the aggregator function \( \nu(\cdot) \) is CES, and is further nested in a CRRA utility function (this is similar to Poterba and Rotemberg (1986)):

\[
u \left( c_t, \nu \left( \frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t} \right) \right) = \frac{1}{\sigma} \left\{ c_t L_t \left( \frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t} \right) \right\}^\sigma,
\]

\[
L_t = \left[ \delta_M \left( \frac{M_t}{P_t} \right)^\gamma + \delta_B \left( \frac{B_t}{P_t} \right)^\gamma + (1 - \delta_M - \delta_B) \left( \frac{S_t}{P_t} \right)^\gamma \right]^{\frac{1}{\gamma}},
\]

where \( \sigma < 0, 0 < \gamma < 1 \), and \( 0 < \delta_M, \delta_B < 1 \). Then from equations (16) - (18) it follows that

\[
\frac{c_t L_t}{L_t} \delta_M \left( \frac{M_t}{P_t} \right)^{\gamma-1} + E_t [M_{t+1}] = 1,
\]

\[
\frac{c_t L_t}{L_t} \delta_B \left( \frac{B_t}{P_t} \right)^{\gamma-1} + E_t [M_{t+1}] = \frac{1}{R_t^B},
\]

\[
\frac{c_t L_t}{L_t} (1 - \delta_M - \delta_B) \left( \frac{S_t}{P_t} \right)^{\gamma-1} + E_t [M_{t+1}] = \frac{1}{R_t^S}.
\]

For the case of a nested CES aggregator each liquidity services premium on the asset’s prices is not only determined by the level of holdings of the respective asset but also determined by the total holdings of liquidity services providing assets, aggregated by \( L_t \). Hence, prices decrease with additional asset holdings as well as in \( L_t \).
ii. Now assume that the aggregator function $\nu(\cdot)$ is Cobb-Douglas, which is nested in a CRRA utility function:

$$u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right) = \frac{1}{\sigma} \left\{ c_t L_t \left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right) \right\}^\sigma,$$

$$L_t = \left(\frac{M_t}{P_t}\right)^{\delta_M} \left(\frac{B_t}{P_t}\right)^{\delta_B} \left(\frac{S_t}{P_t}\right)^{(1-\delta_M-\delta_B)},$$

with $\sigma < 0$, and $\delta_M, \delta_B, (1-\delta_M-\delta_B)$ summing up to 1. Then from equations (16) - (18)

$$c_t \delta_M \left(\frac{M_t}{P_t}\right)^{-1} + E_t [M_{t+1}] = 1,$$

$$c_t \delta_B \left(\frac{B_t}{P_t}\right)^{-1} + E_t [B_{t+1}] = \frac{1}{R^t_t},$$

$$c_t (1-\delta_M-\delta_B) \left(\frac{S_t}{P_t}\right)^{-1} + E_t [S_{t+1}] = \frac{1}{R^t_s}.$$

For the case of the nested Cobb-Douglas aggregator each asset’s liquidity services premium is driven by the current level of holdings of the respective asset but not by the total holdings of liquid assets. This implication is not in line with KVJ and NIE.

iii. Next assume again that the utility function is CRRA with a nested CES aggregator. However, utility is in this case additively separable in its arguments:

$$u\left(c_t, \nu\left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right)\right) = \frac{1}{\sigma} \left( c_t + L_t \left(\frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t}\right) \right)^\sigma,$$

$$L_t = \left[ \delta_M \left(\frac{M_t}{P_t}\right)^\gamma + \delta_B \left(\frac{B_t}{P_t}\right)^\gamma + (1-\delta_M-\delta_B) \left(\frac{S_t}{P_t}\right)^\gamma \right]^{\frac{1}{\gamma}},$$

where $\sigma < 0$, $0 < \gamma < 1$, and $0 < \delta_M, \delta_B < 1$. Then from equations (16) - (18)

$$L_t^{\frac{\gamma-1}{\gamma}} \delta_M \left(\frac{M_t}{P_t}\right)^{-1} + E_t [M_{t+1}] = 1,$$

$$L_t^{\frac{\gamma-1}{\gamma}} \delta_B \left(\frac{B_t}{P_t}\right)^{-1} + E_t [B_{t+1}] = \frac{1}{R^t_t},$$

$$L_t^{\frac{\gamma-1}{\gamma}} (1-\delta_M-\delta_B) \left(\frac{S_t}{P_t}\right)^{-1} + E_t [S_{t+1}] = \frac{1}{R^t_s}.$$

As for a CES function it is assumed that $0 < \gamma < 1$, it implies for this model specification that increasing the holdings of one of the three assets under consideration increases asset prices. This implication is not in line with the assumptions by KVJ and NIE.
iv. Assume that the utility function is CRRA with a nested Cobb-Douglas aggregator and that it is additively separable in its arguments:

\[
u \left( c_t, \nu \left( \frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t} \right) \right) = \frac{1}{\sigma} \left( c_t + L_t \left( \frac{M_t}{P_t}, \frac{B_t}{P_t}, \frac{S_t}{P_t} \right) \right)^\sigma,
\]

\[
L_t = \left( \frac{M_t}{P_t} \right)^{\delta_M} \left( \frac{B_t}{P_t} \right)^{\delta_B} \left( \frac{S_t}{P_t} \right)^{(1-\delta_M-\delta_B)},
\]

with \( \sigma < 0 \), and \( \delta_M, \delta_B, (1-\delta_M-\delta_B) \) summing up to 1. Then from equations (16) - (18)

\[
\delta_M \left( \frac{M_t}{P_t} \right)^{\delta_M-1} \left( \frac{B_t}{P_t} \right)^{\delta_B} \left( \frac{S_t}{P_t} \right)^{(1-\delta_M-\delta_B)} + E_t [M_{t+1}] = 1,
\]

\[
\delta_B \left( \frac{M_t}{P_t} \right)^{\delta_M} \left( \frac{B_t}{P_t} \right)^{\delta_B-1} \left( \frac{S_t}{P_t} \right)^{(1-\delta_M-\delta_B)} + E_t [M_{t+1}] = \frac{1}{R_t},
\]

\[
(1-\delta_M-\delta_B) \left( \frac{M_t}{P_t} \right)^{\delta_M} \left( \frac{B_t}{P_t} \right)^{\delta_B} \left( \frac{S_t}{P_t} \right)^{(1-\delta_M-\delta_B)-1} + E_t [M_{t+1}] = \frac{1}{R_t}.
\]

As \( 0 < \delta_M < 1 \), and \( 0 < \delta_B < 1 \), the premium on each asset’s price decreases with the level of the respective asset’s holdings. However, the premium on each asset’s price increases with additional holdings of each of the respective other two assets. This implication is not in line with KVJ and NIE.
Table 3: Parameter estimates of specification I. Poterba and Rotemberg (1986) utility

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<td>Parameter</td>
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<td>(3)'</td>
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<td>$-6.743^{**}$</td>
<td>$-7.734^{**}$</td>
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<td>$0.999^{**}$</td>
<td>$0.997^{**}$</td>
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<tr>
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<tr>
<td>$[\begin{pmatrix} \ \ \ \ 0.446; 0.552 \end{pmatrix}]$</td>
<td>$[\begin{pmatrix} \ \ \ \ 0.536; 0.620 \end{pmatrix}]$</td>
<td>$[\begin{pmatrix} \ \ \ \ 0.556; 0.574 \end{pmatrix}]$</td>
<td>$0.269$</td>
</tr>
<tr>
<td>STD</td>
<td>$1.074^{**}$</td>
<td>$1.046^{**}$</td>
<td>$0.681^{**}$</td>
</tr>
<tr>
<td>$[\begin{pmatrix} \ \ \ \ 1.071; 1.077 \end{pmatrix}]$</td>
<td>$[\begin{pmatrix} \ \ \ \ 1.043; 1.049 \end{pmatrix}]$</td>
<td>$[\begin{pmatrix} \ \ \ \ 0.679; 0.682 \end{pmatrix}]$</td>
<td>$0.965$</td>
</tr>
<tr>
<td>$\delta_M$</td>
<td>$0.532^{**}$</td>
<td>$0.496^{**}$</td>
<td>$0.316$</td>
</tr>
<tr>
<td>$[\begin{pmatrix} \ \ \ \ 0.405; 0.670 \end{pmatrix}]$</td>
<td>$[\begin{pmatrix} \ \ \ \ 0.454; 0.611 \end{pmatrix}]$</td>
<td>$[\begin{pmatrix} \ \ \ \ 0.437; 0.554 \end{pmatrix}]$</td>
<td>$0.316$</td>
</tr>
<tr>
<td>$\delta_B$</td>
<td>$0.307$</td>
<td>$0.429^{**}$</td>
<td>$0.515$</td>
</tr>
<tr>
<td>$[\begin{pmatrix} \ \ \ \ 0.122; 0.427 \end{pmatrix}]$</td>
<td>$[\begin{pmatrix} \ \ \ \ 0.055; 0.559 \end{pmatrix}]$</td>
<td>$[\begin{pmatrix} \ \ \ \ 0.355; 0.503 \end{pmatrix}]$</td>
<td>$0.515$</td>
</tr>
<tr>
<td>$1 - \delta_M - \delta_B$</td>
<td>$0.157^{*}$</td>
<td>$0.160$</td>
<td>$0.075^{**}$</td>
</tr>
<tr>
<td>$[\begin{pmatrix} \ \ \ \ 0.122; 0.427 \end{pmatrix}]$</td>
<td>$[\begin{pmatrix} \ \ \ \ 0.055; 0.559 \end{pmatrix}]$</td>
<td>$[\begin{pmatrix} \ \ \ \ 0.355; 0.503 \end{pmatrix}]$</td>
<td>$0.168$</td>
</tr>
<tr>
<td>J-Test (p-val)</td>
<td>$0.426$</td>
<td>$0.381$</td>
<td>$0.380$</td>
</tr>
<tr>
<td>N</td>
<td>474</td>
<td>474</td>
<td>474</td>
</tr>
</tbody>
</table>

Notes: ' indicates that only restrictions on $\sigma$, $\delta_M$, $\delta_B$, and $1 - \delta_M - \delta_B$ are in place.

** Significant at the 1 percent level.

* Significant at the 5 percent level.

Confidence intervals are provided within the brackets.

STD denotes "Small Time Deposits".
Table 4: Parameter estimates of specification II. nested CES liquidity services

<table>
<thead>
<tr>
<th>Period</th>
<th>Jan 1969 - Jun 2008</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Parameter</td>
<td>M, TrBi, CP</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>(-3.189^{**})</td>
</tr>
<tr>
<td></td>
<td>([-3.197; -3.180])</td>
</tr>
<tr>
<td>(\beta)</td>
<td>(0.995^{**})</td>
</tr>
<tr>
<td></td>
<td>([0.993; 0.996])</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>(1.001^{**})</td>
</tr>
<tr>
<td></td>
<td>([1.001; 1.001])</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>(1.074^{**})</td>
</tr>
<tr>
<td></td>
<td>([1.071; 1.077])</td>
</tr>
<tr>
<td>(\delta_M)</td>
<td>(0.743^{**})</td>
</tr>
<tr>
<td></td>
<td>([0.707; 0.781])</td>
</tr>
<tr>
<td>(\delta_B)</td>
<td>(0.256^{**})</td>
</tr>
<tr>
<td></td>
<td>([0.248; 0.289])</td>
</tr>
<tr>
<td>(1 - \delta_M - \delta_B)</td>
<td>(0^*)</td>
</tr>
<tr>
<td>J-Test (p-val)</td>
<td>(0.038)</td>
</tr>
</tbody>
</table>

Notes: * indicates that only restrictions on \(\sigma\), \(\delta_M\), \(\delta_B\), and \(1 - \delta_M - \delta_B\) are in place.
** Significant at the 1 percent level.
* Significant at the 5 percent level.
Confidence intervals are provided within the brackets.