A Study of Uncertain Wind Power in Active-Reactive Optimal Power Flow

E. Mohagheghi, A. Gabash, and P. Li
Department of Simulation and Optimal Processes
Institute of Automation and Systems Engineering, Ilmenau University of Technology
98693 Ilmenau, Germany
erfan.mohagheghi@tu-ilmenau.de, aouss.gabash@tu-ilmenau.de, pu.li@tu-ilmenau.de

Abstract—Wind power fluctuates with time and it is reasonable to regard it as a random variable. Recently, an active-reactive optimal power flow (A-R-OPF) method in active distribution networks with wind stations has been developed to handle the problem of wind power curtailment (WPC). Since the mentioned method is deterministic, it may fail to handle uncertain wind power (UWP). Therefore, our study in this paper will firstly discuss the issue of UWP and secondly develop a new strategy which can improve the A-R-OPF by considering UWP. The new strategy can be distinguished from the original so that: 1) it considers shorter time intervals, i.e., 15 minutes instead of one hour and 2) it can handle both UWP and WPC simultaneously. The effectiveness of the new strategy is shown by using a real case medium-voltage distribution network.

Keywords—active-reactive optimal power flow (A-R-OPF); medium-voltage; uncertain wind power (UWP); wind power curtailment (WPC).

I. INTRODUCTION

Uncertain wind power (UWP) is one of many challenges that power system operators face when ensuring optimal and reliable operations. It is a well-known fact that wind power production varies with wind speed and cannot be controlled except by curtailing it [1]. Therefore, wind energy is a partially dispatchable generation source, and consequently curtailment investigation must be considered within a probabilistic rather than deterministic study context [2].

Recently, a method which is based on so-called chance constrained optimal power flow (OPF) was proposed in [3]. In [3], load power uncertainties were considered as multivariate random variables with a correlated normal distribution, but renewable energy generation (REG) was not considered. In contrast, the forecasted active power outputs of REG units were regarded as normally distributed random variables [4]. Moreover, the uncertainty of load was neglected because the REG, such as wind energy, has more variability than load [5]. It is noted in [4] that no variables were used for curtailing of REG.

More recently, a deterministic active-reactive optimal power flow (A-R-OPF) method in active distribution networks (ADNs) with REG and battery storage systems was developed [6][7][8]. One of the remarkable abilities of the A-R-OPF method is its capability to ensure feasible solutions even with a high penetration of wind power. This is achieved by incorporating a curtailment factor. Note that the problem of A-R-OPF in ADNs [6-8] (even with a deterministic formulation) represents a large-scale and complex optimization problem. Therefore, it is expected that considering any uncertain operating conditions [9] will further complicate the problem.

The principle of wind power curtailment (WPC) was also used in [10], where an OPF was formulated as a nonlinear chance constrained optimization problem under non-Gaussian uncertainties. Note that the formulated problem in [10] (even with a single snapshot) was solved with a considerable computational time. In addition, the expected wind speed and its variance were assumed to be the same at all wind stations (WSs).

Based on the above literature and distinguished from the previous works in the research area of ADNs [11][12][13], a new strategy to handle UWP and WPC in an ADN is proposed in this paper. The contributions of this work can be summarized as follows:

- Using a shorter time interval (ti) in A-R-OPF, i.e., ti = 15 minutes instead of one hour.
- Introducing a center for environment data which can provide forecasted and actual wind power production for WSs at different locations in the ADN.
- Developing a new strategy to handle UWP and WPC simultaneously. This ensures the feasibility when considering actual wind power production in the deterministic A-R-OPF.

II. DETERMINISTIC A-R-OPF WITH UWP

A. Deterministic A-R-OPF

In [6], an A-R-OPF problem was formulated with forecasted or deterministic input wind power and demand profiles in the time frame of optimization. However, the inaccuracies in these forecasts were not considered as given in [8]. A deterministic A-R-OPF problem can reduce the computational effort on the one hand, but on the other hand it cannot handle the inaccuracies in these profiles. Therefore, our aim here is to overcome such problems by focusing on UWP.
B. Uncertain Wind Power

The idea behind considering UWP in this work can be explained in Fig. 1. Here, it is assumed that there are two WSs with same installed capacities at different locations in an ADN. The produced wind power $P_w$ from each WS can be forecasted from the expected wind speed at each WS. Note that $P_w$ can be different during $t_i$ even with a relatively small distance between WSs. This is because the forecasted wind speed can be different at each WS.

From another perspective, the forecasted value of $P_w$ during $t_i$ can also be different from actual wind power production based on, e.g., a probability density function (PDF), as seen in Fig. 2. Here, $F$ stands for forecast (a forecasted value), $H$ for high-side (values higher than forecasted) and $L$ for low-side (values lower than forecasted). Note that $P_w$ in Fig. 2 is assumed normally distributed. The values of $F$, $H$ and $L$ are also illustrated with time for the two WSs, as shown in Fig. 3.

In Fig. 1, active $P_d$ and reactive $Q_d$ power demand are assumed to follow IEEE-RTS winter season’s days [8]. In contrast, active $P_{S1}$ and reactive $Q_{S1}$ power at slack bus $S_1$ (see Fig. 1) are allowed to be either positive or zero to avoid any possible generation rejections from reverse power flows (see details in [11]). This means that active and reactive power can be imported (in the case of low wind power), but not exported (in the case of high wind power) [14][15]. Note that due to system constraints, a wind power curtailment factor ($0 \leq \beta_{c.w} \leq 1$) at each WS is used as a control variable [6], where $\beta_{c.w} = 1$ when no curtailment and $\beta_{c.w} < 1$ otherwise, as seen in Fig. 1.

The objective of A-R-OPF in Fig. 1 is to maximize the total revenue from the wind energy and meanwhile to minimize the total costs of active energy losses in the grid and the total costs of active and reactive energy at bus $S_1$. Now, the problem of UWP in A-R-OPF is to find a strategy to ensure the feasibility when considering actual instead of forecasted wind power production.

III. PROPOSED STRATEGY

The proposed strategy in this paper requires a center for environment data as shown in Fig. 4. This center should be able to provide forecasted and actual wind power production of WSs at different locations in an ADN for a given $t_i$. 

![Fig. 1. Illustration of a meter-based method for charging and remunerating different entities connected to a power system [18]. Here, M stands for meter, TR for transformer, S_0 and S_1 for TR primary and secondary side, respectively.]

![Fig. 2. Illustration of a PDF of wind power. Here, $\xi$ is the random variable, $\mu$ is the mean or expectation and $\sigma$ is the standard deviation.]

![Fig. 3. Illustration of wind power profiles during an hour. Here, (a) and (b) stand for wind power profiles from two different WSs.]}
In this work, $ti$ is taken 15 minutes ahead, as seen in Fig. 3. In addition, two WSs are considered to be located at different buses, i.e., bus 2 and bus 16, as seen in Fig. 4. Based on the above considerations, the solution strategy of UWP in A-R-OPF can be summarized by the following steps:

1) Provide system parameters, energy prices and demand as inputs to the A-R-OPF with UWP, as seen in Fig. 4.

2) Provide $F(i,ti)$: forecasted wind power of WS at bus $i$ during $ti$. For example, $F(2,1) = P_w(2,1) = 9$ MW and $F(16,1) = P_w(16,1) = 8$ MW, as seen in Fig. 5.

3) Calculate $H(i,ti)$: wind power higher than forecasted of WS at bus $i$ during $ti$. In this paper, we use the following simple formula $H(i,ti) = F(i,ti) + \Delta P_w(i)$. Here, $\Delta P_w(i)$ is defined as a constant power at bus $i$ to get wind power values around the forecasts. Let $\Delta P_w(2) = \Delta P_w(16) = 1$ MW. Then, one can simply get $H(2,1) = P_w(2,1) = 10$ MW and $H(16,1) = P_w(16,1) = 9$ MW, as seen in Fig. 5. Note that if $H(i,ti) > P_w(i)$, then $H(i,ti) = P_w(i)$. Here, $P_w(i)$ is the rated power of WS at bus $i$ (see Table IV in Appendix).

4) Calculate $L(i,ti)$: wind power lower than forecasted of WS at bus $i$ during $ti$. In this paper, we use the following simple formula $L(i,ti) = F(i,ti) - \Delta P_w(i)$. Let $\Delta P_w(i)$ as defined above, then one can simply get $L(2,1) = P_w(2,1) = 8$ MW and $L(16,1) = P_w(16,1) = 7$ MW, as seen in Fig. 5. If $L(i,ti) < 0$, then $L(i,ti) = 0$.

5) Solve A-R-OPF for $ti$ and all main nine possible scenarios shown in Fig. 5. Then, save obtained results in a lookup table, as Table I. Here, $\beta_{c.w}(i,ti)$ is the optimal curtailment factor at bus $i$ during $ti$ for all nine scenarios.

It is to note that the required computational time to fill in the lookup table should be small enough in order to ensure the applicability of the proposed strategy.

Now, provide actual wind power (AWP) for WS at bus $i$ during $ti$ from the center in Fig. 4. Then, compare it with wind power scenarios (i.e., H-H, H-F, H-L, ...) in the lookup table, and choose the optimal control variables $\beta_{c.w}(i,ti)$ based on the following simple rules:

- Rule 1: If ($H \geq AWP > F$) then consider (AWP = H)
- Rule 2: If ($F \geq AWP > L$) then consider (AWP = F)
- Rule 3: If ($L \geq AWP \geq 0$) then consider (AWP = L)

The application of the above strategy will be further explained in section V.
IV. OPTIMIZATION PROBLEM FORMULATION

Here, we adapt the extended objective function $F$ (1) [15] by changing the discretization from (1-hour) to (15-minutes) as follows:

$$
\text{max } F = F_1 - F_2 - F_3 - F_4
$$

where

$$
F_1 = C_{pp}(t_i) \sum_{i=1}^{N} P_w(i,t_i) \beta_{c_w}(i,t_i)
$$

$$
F_2 = C_{pp}(t_i) P_{loss}(t_i)
$$

$$
F_3 = C_{pp}(t_i) P_{S1}(t_i)
$$

$$
F_4 = C_{pp}(t_i) Q_{S1}(t_i).
$$

It is aimed in (1) to maximize the total revenue from the wind energy $F_1$, and meanwhile to minimize the total costs of active energy losses in the grid $F_2$, the cost of active energy at slack bus $F_3$, and the cost of reactive energy at slack bus $F_4$, respectively. Here, $C_{pp}(t_i)$ is the active energy price during $t_i$, $C_{pr}(t_i)$ is the reactive energy price during $t_i$, $P_{loss}(t_i)$ is the active power losses during $t_i$, $N$ is the total number of buses, $P_w(i,t_i)$ is the active power of WS at bus $i$ during $t_i$ while $l$ stands for the set of WSs, $P_{S1}(t_i)$ and $Q_{S1}(t_i)$ are the active and reactive power injected at slack bus $S_1$ during $t_i$, respectively. The control variable of WSs is $\beta_{c_w}(i,t_i)$, which represents the curtailment factor of wind power at WS $i$ during $t_i$.

The optimization problem is solved with the general algebraic modeling system (GAMS) [16] where the equality (6) and inequality (7) constraints for the A-R-OPF are as follows (mathematical details can be found in [6]):

- Active power balance at each bus
- Reactive power balance at each bus

\[
\begin{align*}
P_{c}(2,1) & \quad \text{(MW)} & P_{s}(16,1) & \quad \text{(MW)} & \beta_{c_w}(2,1) & \quad \text{---} & \beta_{c_w}(16,1) & \quad \text{---} & P_{S1} & \quad \text{(MW)} & Q_{S1} & \quad \text{(Mvar)} \\
1 & \text{H-H} & 10 & 9 & 0.475 & 0.278 & 0 & 2.53 \\
2 & \text{H-F} & 10 & 8 & 0.475 & 0.313 & 0 & 2.53 \\
3 & \text{H-L} & 10 & 7 & 0.475 & 0.357 & 0 & 2.53 \\
4 & \text{F-H} & 9 & 9 & 0.528 & 0.278 & 0 & 2.53 \\
5 & \text{F-F} & 9 & 8 & 0.528 & 0.313 & 0 & 2.53 \\
6 & \text{F-L} & 9 & 7 & 0.528 & 0.357 & 0 & 2.53 \\
7 & \text{L-H} & 8 & 9 & 0.594 & 0.278 & 0 & 2.53 \\
8 & \text{L-F} & 8 & 8 & 0.594 & 0.313 & 0 & 2.53 \\
9 & \text{L-L} & 8 & 7 & 0.594 & 0.357 & 0 & 2.53 \\
\end{align*}
\]
The results in Table I show that the active power at bus $S_1$ is always zero for all scenarios because of total active power demand (7.234 MW), high wind active power production of WSs (see Table I), and limits on exported active power. It can be clearly seen in Table I that high wind power production leads to low values of curtailment factors to ensure a feasible solution. In contrast to active power, the reactive power at bus $S_1$ is always (2.53 Mvar) for all scenarios (see Table I). This is because of using unity power factors (PFs) of all WSs (see Table IV), total reactive power demand (2.62 Mvar) and the reactive power compensation of feeder capacitive susceptance [18].

The benefits of the proposed strategy can be clearly seen in Table II where the deterministic A-R-OPF fails to achieve feasible solutions during the first (i.e., $t_{i1}$) and fourth (i.e., $t_{i4}$) time periods. This is because the actual wind power production (see Fig. 6(a)) is higher than forecasted at both WSs. It means the new strategy uses the lowest curtailment factors, i.e., $\beta_{c.w}(2,1) = 0.475$ and $\beta_{c.w}(16,1) = 0.278$ (i.e., H-H) instead of $\beta_{c.w}(2,1) = 0.528$ and $\beta_{c.w}(16,1) = 0.313$ (i.e., F-F) (see Table I and Fig. 6). Of course the new strategy requires more CPU time. Note that the deterministic A-R-OPF can achieve a feasible solution during, e.g., the second time period (i.e., $t_{i2}$) because the actual wind power production from WS at bus 2 is much lower than the production from WS at bus 16.

From economical point of view, the new strategy requires almost a higher active energy import at slack bus $S_1$ in comparison with the deterministic A-R-OPF as seen in Fig. 7.
For example, the active energy import during the second time period (i.e., \( t2 \)) is the highest value because the actual wind power production from WS at bus 2 is too lower than expected. The numerical results of the objective function are given in Table III. In Fig. 7, it is clearly seen that the reactive power import is in a weak relation with wind active power curtailment.

VI. CONCLUSIONS

This paper discussed the issue of uncertain wind power (UWP) in an active distribution network and proposed a new strategy which can handle both UWP and wind power curtailment (WPC) simultaneously. The new strategy improves the deterministic active-reactive optimal power flow so that on one hand it uses shorter time intervals, i.e., 15 minutes instead of one hour and on the other hand it ensures not only optimal but also feasible solutions.

The benefits of the proposed strategy in handling both WPC and UWP are shown by using a real case medium-voltage network. Beside the advantages of the new strategy, there are some drawbacks in terms of computational effort. Therefore, our future research will focus on tackling such problems by exploring more efficient computational mechanisms.

APPENDIX

<table>
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<tr>
<th>TABLE IV. RATED POWER OF WSS (Pw) AND POWER FACTORS (PFs)</th>
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<tbody>
<tr>
<td>Wind stations</td>
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<tr>
<td>Bus</td>
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<tr>
<td>( P_w ) (MW)</td>
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<td>PFs</td>
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<th>TABLE V. DATA OF ENERGY PRICES DURING TIME INTERVALS [8]</th>
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<tr>
<td>( ti )</td>
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<tr>
<td>( n1 = 15 ) min</td>
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<td>( n2 = 15 ) min</td>
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<td>( n3 = 15 ) min</td>
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<td>( n4 = 15 ) min</td>
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REFERENCES


Erfan Mohagheghi (S’14) received his M.Sc. degree from University of Greenwich, UK, in 2012. He is currently working toward the Ph.D. degree at the Institute of Automation and Systems Engineering, Ilmenau University of Technology, Germany. His current research interests include distributed generation and power system optimization under uncertainty.

Aouss Gabash (S’11-A’12-M’12) received his M. Eng. degree from Aleppo University, Syria, in 2008, and the Ph.D. degree from Ilmenau University of Technology, Germany, in 2013. His current research interests include power system planning, analysis, operation, control, optimization techniques, artificial intelligence, and distributed generation and storage. Dr. Gabash is now a postdoctoral researcher at TU Ilmenau.

Pu Li received M. Eng. from Zhejiang University, China, in 1989 and Ph.D. from Technical University of Berlin, Germany, in 1998. He was a senior researcher at TU Berlin from 1998 to 2005. Since 2005 he has been a full professor at the Ilmenau University of Technology. His research interest is process systems engineering, i.e., modeling, simulation, optimization and control of industrial processes.