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The modeling dimension in mathematics and sciences in Geneva lower secondary education curriculum

A new curriculum in French speaking Switzerland

A new curriculum (known as *Plan d'Etude Romand*, PER) for all compulsory education (grade -2 to 9) has recently been adopted for all French speaking Switzerland and launched in Geneva, since 2011/2012 for the first grades of all cycles. The PER divides the school disciplines into five domains, one being *Mathematics and Sciences of the Nature* (MSN), for which modeling as a common federating theme for the whole domain: “Representing to oneself, questioning (*problématiser*) and modeling situations and solving problems by building and mobilizing notions, concepts, approaches (*démarches*) and reasoning specific to Mathematics and Sciences in the area of natural and technical phenomena, from living species, environment, as well as numbers and space.” (PER, introduction to MSN).

Moreover, in the lexical index of the domain MSN, modeling is defined as “the idea of associating to a complex situation a model that makes it comprehensible by reducing it to its essential elements”. In parallel with the new PER an inter-cantonal commission has produced a new mathematics textbook for grades 7-8-9, in which modeling is defined by: “Create a simplified representation of a problem (schema, sketch, table, graph, simulation, etc.), in order to understand and solve it”. Thus the term modeling is taken in a broad meaning, is not only applied to a ‘real concrete’ situation, whose complexity is to be reduced, in order to be treated mathematically. Indeed, it is clearly said that modeling can be intra-mathematical. In this sense, it is closer to a view shared by several researchers in mathematics education, especially within Chevallard’s *Anthropological Theory of Didactics*, which claims that “most of the mathematical activity can be identified (...) with a mathematical modeling activity” (Chevallard, Bosch and Gascón, 1997, p.51 quoted by García & Ruiz Hiugeras, 2005, p.1647, see also Artaud, 2007). Moreover, in the PER modeling is seen as a way of translating a situation in another system of representation; the fact that the complexity is reduced may not be as central as it is in a more restricted version of modeling. On another hand, more than one model can be involved and a discussion about the values of the different models can be as important as solving a problem. In sciences, modeling may also involve only

non-mathematical models. We will show how we addressed this issue in our training-course in our involvement in the European project PRIMAS¹.

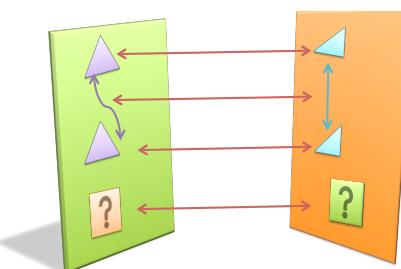
A training course within the European project PRIMAS

The new PER and the new textbook for grade 7 led the educational authorities in Geneva to organize a one-day (compulsory) training course for all mathematics teachers of grade 7-8-9 (around 350 teachers, half of them teaching also some science). As part of the European project PRIMAS our team of 15 teachers, teachers' trainers, researchers in mathematics and science education was contacted to organize this training. One day training is short, too short to be able to change things radically. Moreover, experience with in-service teachers' training has shown that valuable training must be practical, close to teachers' concerns and aware of their potential of evolution without trying to be too radical. In our case, we tried to take these concerns into account by setting up a training-course to show that investigation and modeling can be taught in a somehow modest version without consuming too much time and necessitating other resources than what is offered in the official textbook. In this sense, we tried to be realistic since the training was addressed to all teachers of Geneva and a command from the institution.

A typology for modeling activities

Our first concern was to have a definition of modeling that could be both broad enough to respect the PER and, at the same time, operational in order to analyze the different activities that could be seen as modeling. In the literature one can find several definitions of what is modeling, we decided to adopt a very broad one, which is not very far from the one proposed by Niss, Blum and Galbraith (2007, p. 4)

Modeling means setting up/discussing/tackling a correspondence (mapping) between two (or more) systems including objects, relations between these objects and questions.



¹ The project PRIMAS, *Promoting inquiry in mathematics and science across Europe*, has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement n° 244380. This text reflects only the author's views and the European Union is not liable for any use that may be made of the information contained herein.

There is no condition that one domain is non mathematical and the other one is mathematical. In this sense our definition can involve two mathematical domains or, on the contrary, two non-mathematical domains (like this is used in biology teaching quite often). In the “traditional” mathematical modeling, the first system is extra-mathematical, while the second one is mathematical. We preferred the term of system to domain as used by Niss, Blum and Grabaith. One reason is that the term ‘domain’ may induce the fact that one is extra-mathematical and the other one mathematical, while system can be seen as more neutral. Another reason is to point out the fact that we deal with objects and relations between them. The most challenging way of engaging students in modeling is to give them a problematic question in a system and to leave them build another system (the model) in which this question can be solved. In this case, students have the responsibility of reducing the complexity of the first system to some significant elements, choose the second system and build the mapping, solve the problem in the second system and interpret the solution in terms of the first system. All this is described in many papers about modeling, in particular in the introduction of the ICMI study quoted above. However, in some teaching situations, the two systems may be given and what is expected from the students is either to interpret part of one system in terms of the other or to discuss the validity of the correspondence in relation to a certain type of question (in particular more than two systems can be given and the correspondences should be compared in terms of consistency). These types of activities, even if less challenging, may still offer a good opportunity to engage students into a rich reflection about modeling, in particular it is often the case in science teaching in Primary school. Finally in some situations, the two systems are given but most of the students’ task is to solve the question in the second system, without real reflection about the relation with the first one. Most of the time, this type of activity does not present much challenge in terms of modeling, the initial system is only evoked, the mapping with the other system is transparent and the modeling acts, at most, as an extra motivation for students.

Based on these remarks, we proposed a typology of modeling activities in 3 levels:

- *Level 1* – The two systems are given but the task given to students involves only the second system
- *Level 2* – The two systems are given and the task involves the two systems
- *Level 3* – Only one system is given and students have to choose the other(s)

In our training course we presented this typology to the trainees and we ask them to use it to classify a selection of 12 activities taken from the new

textbook for grade 7. Then we made a more specific analysis on three of them focusing on the teacher's work to realize a maximum of the potential of modeling and investigation for the students. In our talk we present some examples of the use of this tool, that we do not have space to develop here

Conclusion

The aim of the training-course is to help teachers taking at best into account the requirement of the curriculum concerning modeling and investigation. Our categorization in 3 levels is a tool in order to analyze the potential of modeling in an activity. In the training-course, we have also worked on the actual realization of such an activity in class and discuss some ingredients of teacher's work. We have used videos of real class activities, especially concerning the divided square. Indeed however the activity is a good modeling activity the question of how it is implemented in the class and how much the students are involved in a real investigation is crucial. In everyday teaching, teachers need some tools to be able to regulate their way of piloting such an activity. Example like what we presented very briefly above about the dialectical use of milieu and didactical contract in the activity of the divided square is one of our leading orientation in our actions within PRIMAS in order to promote investigation in mathematics and sciences especially through modeling.

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