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The sound of fractions - Interdisciplinary tasks between music and mathematics

Abstract

Teachers often don't educate children about the full creative potential of mathematics, because they didn't ever discover it themselves. (Newton, 2013) This leads to mathematics being mostly limited to calculus. This article will concentrate on two interdisciplinary forms of representation which could help to change this issue by letting children work creatively with new interdisciplinary concepts.

What research says

In most cases, the connection of mathematics and music is either highly misunderstood or overlooked completely. Lynn Newton, among others, validated this in a study in which she asked her students about their beliefs on the opportunity for being creative in different school subjects (Newton, 2013). Her study revealed the following mindset of the preservice teachers:

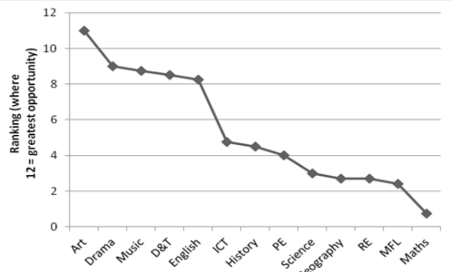

Figure 1

"The major conclusion is that these teachers hold the general notion that the arts (subjects like music and art) are creative while other 'non-arts' (subjects like science, mathematics or ICT) are not." (Newton, 2013, p. 37)

Motivated by these results I carried out my own survey, which delivered quite similar results (Nutzinger, 2015). In my study most of the preservice mathematics teachers and even staff members stated similar beliefs. As a consequence, I began to develop concepts on how to deal with the facts that came to light in those studies.

I found ideas about how to realize that knowledge in several researches, among them one of Pajares:

"Beliefs are unlikely to be replaced unless they prove unsatisfactory." (Pajares, 1992)

Figure 2

As we just found out, the beliefs concerning mathematics and music are highly conflictive. Considering this, I elaborated a constellation that could provide the necessary uncomfortable anomalies to change beliefs.

In order to impact learners' beliefs, we need to create interdisciplinary tasks for use in class, so that the conflictive information can be implemented in our daily work. My two ideas on how to do so both concentrate on the use of different forms of representation.

The sound of fractions

The first task is an experiment which has already been run by Pythagoreans. (Critchley, 2008) A string divided in different proportions on a monochord e.g., produces different musical tones.

C	C#	D	D#	E	F	F#	G	G#	A	A#	B	C'
$\frac{1}{1}$	$\frac{243}{256}$	$\frac{8}{9}$	$\frac{27}{32}$	$\frac{64}{81}$	$\frac{3}{4}$	$\frac{512}{729}$	$\frac{2}{3}$	$\frac{81}{128}$	$\frac{16}{27}$	$\frac{9}{16}$	$\frac{128}{243}$	$\frac{1}{2}$

Half the string gives us the 8^{th}. Two thirds give us a 5^{th}. The pattern continues that way.

Another possibility to generate tones is to use different weights (see figure 1), which would reassemble the reciprocal fraction to make the same tone audible.

This simple experiment can easily be carried out in class with pupils of grade 4 upwards, showing quite creative perspectives.

I am convinced we need to let children find their own scales and new intervals, allow them to reproduce a given scale or interval and find the fraction, encourage them to hear different intervals and the related fractions, and make it possible for them to create their own intervals. In this way they would effectively create their own music.

I am positive that it is important to give them the creative freedom to experiment with these ideas and to exchange their individual views about them.

On the one hand, this is a very active way of discovering how tones come to existence. On the other hand, this method offers children a new form of representation of several mathematical concepts: Fraction, Classification, Relation, Mathematical language, Formulas, definitions, etc.

A suitable example for the compatibility of the mathematical and the musical language was published by Wille:

> *A* ***pitch space*** *is an ordered pair (T,h), where T is a set and h is an injective mapping of T into the set R+ of all positive numbers. The elements of T are called* ***tones****, and for* $t \in T$ *h(t) is called the* ***pitch****. (Wille, 1976, p. 239)*

Here an example for the tempered pitch.

$$T := \{-48, -47, \ldots, 35, 36\}, h(t) := 440 \cdot 2^{\frac{t}{12}}$$

Inserting an element of the set T into the mapping h(t) gives you the exact frequency of a musical tone. The easiest is the number 0, which is element of T. The result of h (0) is evidently 440 and stands for the tone A (440 Hz).

Dmitri Tymozko's "A Geometry of Music" (Tymoczko, 2011)

Tymozko experimented with several geometric shapes one can apply to make music visible. One of the ideas he published is a visualization of musical tones on a circle. Connecting the three tones of a chord results in a triangle. Figure 3 shows the C-major chord.

Now we can see that only two geometrical transformations are needed to reach all possible harmonics in major and minor.

Rotating the triangle will result in another major chord, as visualized in figure 4. Figure 5 transforms the triangle from C-major to a-minor by a simple reflection on the displayed axis.

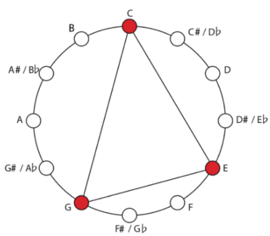

Figure 3: C major chord

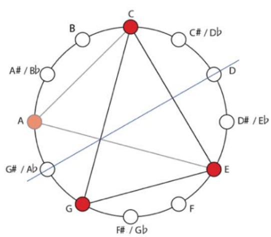

Figure 5: From C major to A

This new form of representation enables children to see the visualized connections between chords. It even shows that a major chord and its relative minor parallel are connected in an audible as well as in a visual way. Using this discovery in class enriches both mathematics and music.

In mathematics we can concentrate on all the geometric transformations.

- Transformation: e.g. how can we transform the c-major triangle to a f-major triangle?
- Similarity: e.g. similar figures sound similar.
- Congruency: e.g. compare the c-major and the d-major triangle.
- Polygons: e.g. a triangle represents a triad, a quadrilateral a 4-note chord
- Geometrical drawing
- Construction: e.g. construct a “major-triangle”.
- Definition: e.g. what are the characteristics of a “major-triangle”?
- Symmetries: e.g. what is the relation between a “major-“ and a “minor-triangle”?
- …

In music we can make harmonics visible and help students to reach a deeper understanding of the theory of harmony. This particular claim will have yet to be verified in future studies.

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Figure 1 by Newton L. 2013, Figure 2 see figure, Figures 3-5 by Hans Peter Nutzinger