

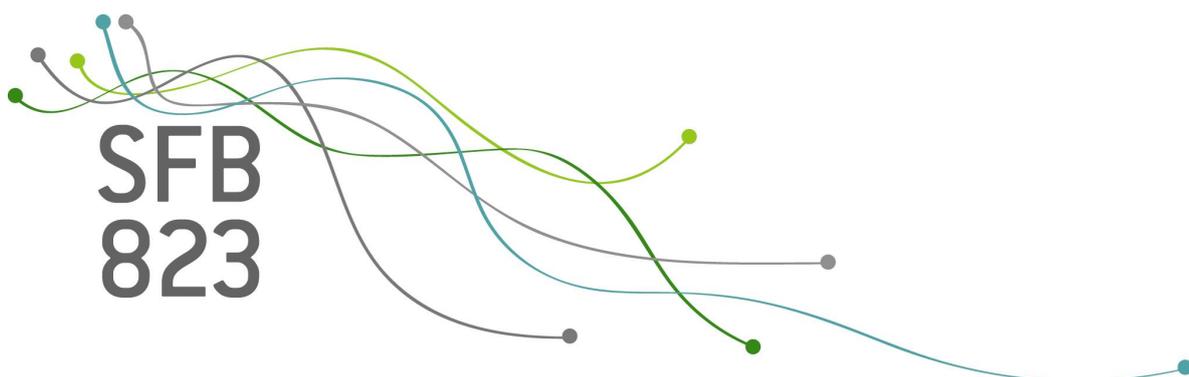
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# Block-recursive non-Gaussian structural vector autoregressions

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Discussion Paper





# Block-recursive non-Gaussian structural vector autoregressions

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This study combines block-recursive restrictions with higher-order moment conditions to identify and estimate non-Gaussian structural vector autoregressions. The estimator allows to impose a block-recursive structure on the SVAR and for a given block-recursive structure we derive a conservative set of assumptions on the dependence and Gaussianity of the shocks to ensure identification. We use a Monte Carlo simulation to illustrate the advantages of the proposed block-recursive estimator compared to unrestricted, purely data driven estimators in small samples. The block-recursive estimator is used to analyze the interdependence of monetary policy and the stock market. We find that a positive stock market shock contemporaneously increases the nominal interest rate, while contractionary monetary policy shocks lead to lower stock returns on impact.

*JEL Codes:* C32, E52, E44

*Keywords:* SVAR, identification, non-Gaussianity, block-recursive, stock market, monetary policy

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# 1 Introduction

Identification of structural vector autoregressions (SVARs) requires to assume an a priori structure of the model. Traditionally, identification is based on imposing structure on the interaction of the variables, ideally derived from macroeconomic theory (e.g., short-run restrictions Sims (1980) or long-run restrictions Blanchard and Quah (1993)). However, theoretical restrictions are rare and oftentimes debatable. More recently, data driven approaches allow to identify the SVAR without imposing any restrictions on the interaction. Instead, identification is achieved by imposing structure on the stochastic properties of the shocks (e.g., time-varying volatility as discussed in Rigobon (2003), Lanne et al. (2010), Lütkepohl and Netšunajev (2017), and Lewis (2021) or non-Gaussian and independent shocks as discussed in Gouriéroux et al. (2017), Lanne et al. (2017), Lanne and Luoto (2021), Keweloh (2021b), or Guay (2021)). These data-driven approaches typically show a significant performance decline in larger applications and small samples. So the state of the art approaches for SVARs are two extreme cases which either fully rely on sometimes debatable restrictions, or abstain from any restrictions at the cost of less reliable estimates.

In many applications we may derive some, but not sufficiently many restrictions to ensure identification. Therefore, with a traditional purely restriction based approach, even the most plausible restrictions can be worthless if there are not sufficiently many. The generalized method of moments estimator proposed in this study combines the traditional identification approach based on restrictions with the more recent data driven approach based on non-Gaussianity. Our estimator allows the researcher to rely on restrictions if possible and, thereby, increase the finite sample performance. However, if no plausible restrictions are available, the researcher can be agnostic on the interaction of the variables and rely on non-Gaussianity.

In particular, our approach allows to impose a block-recursive order, meaning shocks in a given block only influence variables in the same block or blocks ordered below. Identification of the shocks within a certain block is based on higher-order moment conditions derived from mean independent shocks in the given block. However, the impact of the shocks in one block on variables in another block is identified based only on second-order moments and requires only uncorrelated shocks between the blocks. If each block contains exactly one shock such that

the SVAR is recursive, the proposed estimator is equal to the recursive estimator obtained by applying the Cholesky decomposition and if there is only one block containing all shocks, the proposed estimator is equal to the unrestricted SVAR GMM estimators based on higher-order moment conditions proposed by Lanne and Luoto (2021) or Keweloh (2021b).

Guay (2021) also proposes to combine the restriction and non-Gaussian identification approaches. In particular, he constructs a test to determine, which part of the SVAR is identified based on non-Gaussianity and only uses restrictions to identify the remaining part of the SVAR. This approach implies that if all shocks are non-Gaussian, no restrictions have to be used and the SVAR can be estimated solely by higher-order moment conditions. In consequence, the identification approach relies as heavily on non-Gaussianity as possible and as little on restrictions as necessary. In contrast to that, our approach relies as heavily on restrictions as possible and as little on non-Gaussianity as necessary. Therefore, the more block-recursive restrictions the researcher applies, the less the estimator depends on moments beyond the variance. If we are only concerned about identification, relying on restrictions appears unnecessarily restrictive in a non-Gaussian SVAR with independent shocks. However, in a Monte Carlo simulation we demonstrate that the performance of a purely data driven estimator based on non-Gaussianity deteriorates substantially with a decreasing sample size and an increasing model size. Additionally, the simulation illustrates that exploiting the block-recursive order can stop this performance decline. The estimator proposed in this study increases the finite sample performance of the estimator by incorporating those restrictions, which are well grounded by economic theory.

A good example for the advantages of our estimator can be seen when analyzing the interaction between monetary policy and the stock market. Beaudry and Portier (2006) argue that an efficient stock market should react immediately to all available information and with a quarterly data frequency, one would expect that also the FED is able to react to stock market shocks immediately. As short-run restrictions are not available, Bjørnland and Leitemo (2009) propose to use a long-run zero restriction on the effect of monetary policy shocks on stock prices, motivated by the concept of long-run neutrality of monetary policy. They find an immediate negative effect of monetary policy on stock prices and a positive effect of stock prices on the nominal interest rate. However, recent studies of for instance Moran and Queralto (2018), Bianchi et al. (2019) or Jordà et al. (2020) show that monetary policy shocks seem to have a much more persistent effect on

the real economy, which casts some doubt on the validity of a long-run restriction. Additionally, committing to long-run neutrality of monetary policy essentially boils down to the assumption of a specific model, for example an exogenous growth model, and the dismissal of various endogenous growth models. Alternatively, Lütkepohl and Netšunajev (2017) use a fully data-driven approach based on variations in the volatility of the shocks to estimate the interaction of monetary policy and the stock market without restrictions on the short- or long-run interaction. They find that a tightening of monetary policy leads to a simultaneous negative response of the stock market, but also to an initial increase of inflation and output. Due to the counterintuitive response of output and inflation, the authors admit that labeling the shock as a monetary policy shock in a "conventional" sense may be misleading. Additionally, Lanne et al. (2017) use non-Gaussian and independent shocks to identify the interaction of monetary policy and the stock market and find that an unexpected tightening of monetary policy has an immediate negative impact on financial conditions. However, both data driven approaches are unable to label a stock market shock and hence, it remains unclear how monetary policy reacts to stock market shocks.

In our application, we argue that price rigidities and adjustment costs can justify a block-recursive order and we restrict the response of inflation, investments and output such that these variables cannot respond immediately to stock market and monetary policy shocks. However, the responses of interest rates and stock returns remain unrestricted, such that both variables can simultaneously respond to all shocks. We find that stock prices immediately decrease in response to a tightening of monetary policy. Moreover, output, investment and stock prices show a persistent negative reaction to monetary policy shocks. However, due to large confidence bands around the long-run responses, we cannot reject the null hypothesis that monetary policy has no long-run impact on stock prices. In contrast to the fully data driven approaches in the literature we are able to label a stock market shock. We find that positive stock market shocks behave like news shocks and indicate a future business cycle expansion and the central bank reacts immediately with a tightening of monetary policy.

The remainder of this article is structured as follows: Section 2 reviews the SVAR and different identification schemes. Section 3 introduces the block-recursive SVAR GMM and CUE estimator and conducts a Monte Carlo experiment, illustrating the improved performance of the block-recursive SVAR CUE estimator. In Section 4 we use the proposed block-recursive estimator to

analyze the interaction of the stock market and monetary policy. Section 5 concludes.

## 2 Overview SVAR

This section briefly explains the identification problem and common identification approaches of SVAR models. A detailed overview can be found in Kilian and Lütkepohl (2017). Consider the SVAR

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + B_0 \varepsilon_t, \quad (1)$$

with parameter matrices  $A_1, \dots, A_p \in \mathbb{R}^{n \times n}$ , an invertible matrix  $B_0$ , an  $n$ -dimensional vector of time series  $y_t = [y_{1,t}, \dots, y_{n,t}]'$  and an  $n$ -dimensional vector of i.i.d. structural shocks  $\varepsilon_t = [\varepsilon_{1,t}, \dots, \varepsilon_{n,t}]'$  with mean zero and unit variance.

W.l.o.g., we simplify and omit the lag terms in Equation (1). That is, we focus on the simultaneous interaction of the SVAR given by

$$u_t = B_0 \varepsilon_t, \quad (2)$$

with the reduced form shocks  $u_t = y_t - A_1 y_{t-1} - \dots - A_p y_{t-p}$ , which can be estimated consistently by OLS. The reduced form shocks are equal to an unknown mixture  $B_0$  of the unknown structural shocks  $\varepsilon_t$ . So far, neither the mixing matrix  $B_0$  nor the structural shocks  $\varepsilon_t$  are identified. To see this, define the unmixed innovations  $e_t(B)$  as the innovations obtained by unmixing the reduced form shocks with some matrix  $B$

$$e_t(B) := B^{-1} u_t. \quad (3)$$

Note that for  $B = B_0$ , the unmixed innovations are equal to the structural shocks  $\varepsilon_t$ , i.e.,  $e_t(B_0) = \varepsilon_t$ . However, the structural shocks  $\varepsilon_t$  and the mixing matrix  $B_0$  are unknown and without imposing further structure, there is no criterion to verify, whether our mixing matrix  $B$  and our unmixed innovations  $e_t(B)$  are equal to the true mixing matrix  $B_0$  and the structural shocks  $\varepsilon_t$ .

To identify  $B_0$  and the shocks  $\varepsilon_t$ , the researcher has to impose structure on the SVAR. The structure can be specified in two dimensions: We may

- (i) impose more structure on the interaction of the shocks (see, Sims (1980) for short-run restrictions or Blanchard (1989) for long-run restrictions), or
- (ii) impose more structure on the stochastic properties of the structural shock (see, Lanne et al. (2010) for time-varying volatility or Gouriéroux et al. (2017), Lanne et al. (2017), Lanne and Luoto (2021) Keweloh (2021b), or Guay (2021) for non-Gaussian shocks).

Imposing structure on the stochastic properties of the shocks can be used to derive conditions for the unmixed innovations, while imposing structure on the interaction narrows the space of possible mixing matrices used to unmix the reduced form shocks.

In applied work, the probably most frequently imposed structure are uncorrelated structural shocks (meaning  $\varepsilon_{i,t}$  is restricted to be uncorrelated with  $\varepsilon_{j,t}$  for  $i \neq j$ ) and a recursive interaction (meaning restricting on  $B_0$  such that  $b_{ij} = 0$  for  $i < j$  where  $b_{ij}$  denotes the element at row  $i$  and column  $j$  of  $B_0$ ). Uncorrelated shocks with unit variance can be used to derive  $(n + 1)n/2$  moment conditions from

$$I = E[\varepsilon_t \varepsilon_t'] \stackrel{!}{=} E[e_t(B) e_t(B)'], \quad (4)$$

while a recursive order implies that  $n(n - 1)/2$  parameters of  $B_0$  are known a priori, leaving only  $(n + 1)n/2$  unknown parameters in the mixing matrix  $B$ . It is then straightforward to show that, if the remaining  $(n + 1)n/2$  parameters of the restricted  $B$  matrix generate unmixed innovations  $e(B)$ , which satisfy the  $(n + 1)n/2$  moment conditions in Equation (4), the matrix  $B$  has to be equal to  $B_0$  and hence, the unmixed innovations are equal to the structural shocks, meaning the SVAR is identified.<sup>1</sup>

However, economic theory rarely allows to derive the required  $n(n - 1)/2$  parameter restrictions to ensure identification. More recently, identification methods based on non-Gaussian and independent shocks have been put forward in the literature (see, Gouriéroux et al. (2017), Lanne

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<sup>1</sup>Note that this GMM approach is equivalent to the frequently used estimator based on the Cholesky decomposition.

et al. (2017), Lanne and Luoto (2021) Keweloh (2021b), or Guay (2021)). These identification schemes do not require to impose any restrictions on the impact of the shocks, in particular on the matrix  $B_0$ . Instead, the researcher has to impose structure on the stochastic properties of the shocks. If the structural shocks are not only uncorrelated but independent, this property can be used to derive additional moment conditions. For example, consider the coskewness matrices of the structural shocks  $S_i = E[\varepsilon_t \varepsilon_t' \varepsilon_{i,t}]$  for  $i = 1, \dots, n$ . Then, independent and mean zero shocks imply that all entries of  $S_i$  are zero except for the  $i$ th diagonal element, which contains the (unknown) skewness of the shock  $\varepsilon_{i,t}$ . Hence, the unmixing matrix  $B$  has to generate unmixed innovations, which satisfy the third-order moment conditions derived from

$$S_i = E[\varepsilon_t \varepsilon_t' \varepsilon_{i,t}] \stackrel{!}{=} E[e_t(B) e_t(B)' e_{i,t}(B)], \quad (5)$$

for  $i = 1, \dots, n$  and similarly,  $B$  has to generate unmixed innovations, which satisfy the fourth-order moment conditions derived from

$$K_{ij} = E[\varepsilon_t \varepsilon_t' \varepsilon_{i,t} \varepsilon_{j,t}] \stackrel{!}{=} E[e_t(B) e_t(B)' e_{i,t}(B) e_{j,t}(B)], \quad (6)$$

for  $i, j = 1, \dots, n$ . If at most one component of  $\varepsilon_t$  is unskewed (has an excess kurtosis), second- and third-order moment conditions (second- and fourth-order moment conditions) identify the SVAR without any further restrictions up to labeling of the shocks, see, e.g., Keweloh (2021b).

### 3 Block-recursive SVAR

In this section, we propose a generalization between identification based on recursiveness restrictions and identification based on non-Gaussian shocks. The proposed GMM estimator allows the researcher to impose an arbitrary block-recursive structure and for a given block-recursive structure we derive a conservative set of assumptions on the independence and non-Gaussianity of the shocks to ensure identification. In particular, we prove that it is sufficient to assume (mean) independence of the structural shocks in a given block, while shocks of different blocks only need to be uncorrelated.

Additionally, we use a Monte Carlo study to show that data driven estimators based on non-

Gaussianity suffer from a curse of dimensionality, i.e. the bias and variance increases quickly with an increasing model size and a decreasing sample size. However, we show that exploiting the block-recursive structure can stop the curse of dimensionality. Therefore, we conclude that if in a given application well justified restrictions are available, these restrictions should be used and we propose to use higher-order moment conditions only to supplement the restrictions to ensure identification.

### 3.1 Imposing structure on the interaction of shocks

Traditionally, identification of the SVAR is based on a structure imposed on the interaction of the shocks, e.g. short-run or long-run restrictions. These restriction based approaches require a fixed number of restrictions on the interaction of the shocks to ensure identification. A frequently imposed structure on the interaction is a recursive structure, meaning that each structural shock is restricted to have no simultaneous impact on variables ordered above the shock. The reasoning behind a recursive structure is oftentimes the prejudice that some variables, e.g., some macroeconomic variables like inflation, tend to move slowly, while other variables, e.g. financial variables like stock prices, react faster. However, in practice this intuitive reasoning oftentimes allow to order only some, but not all variables recursively. This motivates us to consider the block-recursive SVAR, meaning that the structural shocks are ordered in blocks of consecutive shocks and each structural shock can simultaneously affect all variables in the same block and in blocks ordered below but explicitly not variables in blocks ordered above.<sup>2</sup> Figure 1 shows different block-recursive structures in a SVAR with four variables. The examples show that a block-recursive structure generalizes the non-recursive SVAR and the fully-recursive SVAR and includes both as extreme cases.

We now introduce the notation for the block-recursive SVAR. Suppose that the structural shocks can be ordered into  $m \leq n$  blocks of consecutive shocks. Let the indices  $p_1 = 1 < p_2 < \dots < p_m \leq n$  denote the beginning of a new block and for  $p_i$  let  $\tilde{\varepsilon}_{p_i,t}$  denote the vector of all structural

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<sup>2</sup>Zha (1999) derives identifying restrictions for the block-recursive SVAR. The author restricts not only the simultaneous interaction, but also the lagged interaction. Our proposed block-recursive structure affects only the simultaneous interaction, while the lagged interaction remains unrestricted.

Figure 1: Examples of Different Block-Recursive SVAR Models.

$$\tilde{u}_{p_1} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{Bmatrix} \Bigg\} \tilde{\varepsilon}_{p_1}$$

(a) One Block

$$\tilde{u}_{p_1} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} b_{11} & b_{12} & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{Bmatrix} \Bigg\} \tilde{\varepsilon}_{p_1}$$

(b) Two Blocks

$$\tilde{u}_{p_1} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{Bmatrix} \Bigg\} \tilde{\varepsilon}_{p_1}$$

(c) Three Blocks

$$\tilde{u}_{p_1} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \end{Bmatrix} \Bigg\} \tilde{\varepsilon}_{p_1}$$

(d) Four Blocks

Note: The figure illustrate how the the block structure can be defined by the structural shocks and our definition of  $\tilde{\varepsilon}_{p_i}$  and  $\tilde{u}_{p_i}$ ,  $i = 1, \dots, m$ .

form shocks in the  $i$ th block. Furthermore,  $\tilde{u}_{p_i,t}$  denotes reduced form shocks in block  $i$  such that

$$\tilde{\varepsilon}_{p_i,t} := [\varepsilon_{p_i,t}, \varepsilon_{p_i+1,t}, \dots, \varepsilon_{p_{i+1}-1,t}]', \quad (7)$$

$$\tilde{u}_{p_i,t} := [u_{p_i,t}, u_{p_i+1,t}, \dots, u_{p_{i+1}-1,t}]', \quad (8)$$

for  $i = 1, \dots, m$  and  $p_{m+1} := n + 1$  for ease of notation. Moreover, let  $l_i$  denote the number of shocks in block  $i$  for  $i = 1, \dots, m$ . The vector of all structural shocks  $\varepsilon_t$  can then be decomposed into the  $m$  blocks  $\varepsilon_t = [\tilde{\varepsilon}'_{p_1,t}, \dots, \tilde{\varepsilon}'_{p_m,t}]'$  and the reduced form shocks can be decomposed analogously  $u_t = [\tilde{u}'_{p_1,t}, \dots, \tilde{u}'_{p_m,t}]'$ . The SVAR is block-recursive with  $m \leq n$  blocks with  $p_1 = 1 < p_2 < \dots < p_m \leq n$ , if shocks in the  $i$ th block have no simultaneous impact on reduced form shocks in blocks  $j$  with  $j < i$  such that for  $i = 1, \dots, m$

$$b_{ql} = 0, \text{ for } l \geq p_i \text{ and } q < p_i. \quad (9)$$

More generally, any block-recursive structure can be described by the following assumption.

**Assumption 1.** *Block-recursive interaction:*

For  $m \leq n$  blocks with  $p_1 = 1 < p_2 < \dots < p_m \leq n$  and  $q, l = 1, \dots, n$  let  $B_0 \in \mathbb{B}_{\text{br ec}} := \{B \in \mathbb{B} \mid b_{ql} = 0 \text{ if } \exists p_i \in \{p_1, \dots, p_m\} \text{ with } l \geq p_i \text{ and } q < p_i\}$ .

### 3.2 Imposing structure on the stochastic properties of shocks

Imposing structure according to Assumption 1 on the interaction is not sufficient to ensure identification and further assumptions on the dependence and potential non-Gaussianity of the shocks are required. Almost all identification approaches at least impose the assumption of mutually uncorrelated structural shocks such that  $E[\varepsilon_{i,t}\varepsilon_{j,t}] = E[\varepsilon_{i,t}]E[\varepsilon_{j,t}]$  for  $i \neq j$ .<sup>3</sup> Mutually uncorrelated shocks are justified by the idea that different structural shocks are orthogonal, e.g., a structural monetary policy shock should not depend on other structural shocks. In general, imposing uncorrelated structural shocks does not rule out that the structural shocks are dependent. In this case, the interpretation of the estimated SVAR via impulse response functions can be misleading. For example consider the two random variables  $\varepsilon_1 \sim \mathcal{N}(0, 1)$  and  $\varepsilon_2 = \varepsilon_1^2 - 1$  such that both random variables are uncorrelated, but dependent. Policy analysis based on impulse response functions typically uses the ceteris paribus assumption that only a single shock varies, while the other shocks remain unchanged. In the example above, both shocks are uncorrelated, but nevertheless always move simultaneously. Therefore, uncorrelated structural shocks are not sufficient to guarantee that the ceteris paribus assumption holds.

A more rigorous implementation of the idea of orthogonal shocks is to impose mutually independent shocks such that  $E[h(\varepsilon_{i,t})g(\varepsilon_{j,t})] = E[h(\varepsilon_{i,t})]E[g(\varepsilon_{j,t})]$  for  $i \neq j$  with bounded, measurable functions  $g(\cdot)$  and  $h(\cdot)$ . If shocks are independent, a structural shock cannot contain any information on any other structural shock. Therefore, independent structural shocks justify the ceteris paribus interpretation used in policy analysis based on impulse response functions. However, several authors argue that the assumption of independent structural shocks is too strong (cf. Kilian and Lütkepohl (2017, Chapter 14), Lanne and Luoto (2021), or Lanne et al. (2021)). In particular, independence also implies that the volatility processes of the shocks are independent, which may be too restrictive for some macroeconomic applications. For example suppose that  $\tilde{\varepsilon}_{1,t}$  and  $\tilde{\varepsilon}_{2,t}$  are drawn independently of each other and represent unscaled structural shocks.

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<sup>3</sup>Proxy-variable identification approaches are different and instead assume that structural shocks are uncorrelated with an external proxy variable.

Additionally, in each period an additional volatility shock  $v_t$  is drawn independently of the other shocks and the structural shocks are given by  $\varepsilon_{1,t} = \tilde{\varepsilon}_{1,t}v_t$  and  $\varepsilon_{2,t} = \tilde{\varepsilon}_{2,t}v_t$ . These structural shocks are uncorrelated but dependent since the variance of one shock contains information on the variance of the other shock.

A compromise between the two extreme cases of mutually uncorrelated and mutually independent shocks is the assumption of mutually mean independent shocks, such that  $E[\varepsilon_{i,t}g(\varepsilon_{j,t})] = E[\varepsilon_{i,t}]E[g(\varepsilon_{j,t})]$  for  $i \neq j$  with a bounded, measurable function  $g(\cdot)$ . If shocks are mutually mean independent, a structural shock cannot contain any information about the mean of other structural shock. Mutually mean independent shocks can justify the ceteris paribus assumption used in impulse response analysis and at the same time allow for depended volatility processes. In particular, the two shocks  $\varepsilon_{1,t} = \tilde{\varepsilon}_{1,t}v_t$  and  $\varepsilon_{2,t} = \tilde{\varepsilon}_{2,t}v_t$  defined above are mean independent, since a given shock contains no information on the mean of the other shock.

Imposing structure on the dependence of the structural shocks allows to derive moment conditions, see, e.g. Lanne and Luoto (2021), Keweloh (2021b), or Guay (2021). In particular, uncorrelated structural shocks with mean zero and unit variance imply  $n$  variance and  $n(n-1)$  covariance conditions

$$E[e(B)_{i,t}^2] = 1 \quad \text{and} \quad E[e(B)_{i,t}e(B)_{j,t}] = 0, \text{ for } i, j = 1, \dots, n \text{ and } i \neq j. \quad (10)$$

Additionally, mean independent structural shocks implies asymmetric cokurtosis conditions

$$E[e(B)_{i,t}^3e(B)_{j,t}] = 0, \text{ for } i, j = 1, \dots, n \text{ and } i \neq j. \quad (11)$$

Moreover, mean independent structural shocks imply additional cokurtosis conditions

$$E[e(B)_{i,t}^2e(B)_{j,t}e(B)_{k,t}] = 0, \text{ for } i, j, k = 1, \dots, n \text{ and } i \neq j \neq k, \quad (12)$$

$$E[e(B)_{i,t}e(B)_{j,t}e(B)_{k,t}e(B)_{l,t}] = 0, \text{ for } i, j, k, l = 1, \dots, n \text{ and } i \neq j \neq k \neq l \quad (13)$$

and independent structural shocks would imply further symmetric cokurtosis conditions

$$E[e(B)_{i,t}^2e(B)_{j,t}^2] = 1, \text{ for } i, j = 1, \dots, n \text{ and } i \neq j. \quad (14)$$

Furthermore, all coskewness conditions

$$E[e(B)_{i,t}^2 e(B)_{j,t}] = 0, \text{ for } i, j = 1, \dots, n \text{ and } i \neq j \quad (15)$$

$$E[e(B)_{i,t} e(B)_{j,t} e(B)_{k,t}] = 0, \text{ for } i, j, k = 1, \dots, n \text{ and } i \neq j \neq k \quad (16)$$

can be derived from mutually mean independent structural shocks.

### 3.3 Identification and estimation

In this section, we show that identification in a block-recursive SVAR can be achieved by including all variance-covariance conditions and additionally the asymmetric cokurtosis conditions  $E[e(B)_{i,t}^3 e(B)_{j,t}] = 0$  where the innovations  $e(B)_{i,t}$  and  $e(B)_{j,t}$  belong to the same block. Therefore, let  $E[f_2(B, u_t)] = 0$  contain all variance-covariance conditions from Equation (10) and  $E[f_{4_{p_i}}(B, u_t)] = 0$  contains all asymmetric cokurtosis conditions from Equation (11) corresponding to shocks in block  $k$ , e.g.,  $E[e(B)_{i,t}^3 e(B)_{j,t}] = 0$  for  $i, j = p_k, \dots, p_{k+1} - 1$  and  $i \neq j$ . We define the conservative set of moment conditions as

$$E[f_{\mathbf{N}}(B, u_t)] := E \begin{bmatrix} f_2(B, u_t) \\ f_{4_{p_1}}(B, u_t) \\ \vdots \\ f_{4_{p_m}}(B, u_t) \end{bmatrix}. \quad (17)$$

Note that the set  $E[f_{\mathbf{N}}(B, u_t)]$  does not contain asymmetric cokurtosis conditions of shocks in different blocks, e.g. the condition  $E[e(B)_{i,t}^3 e(B)_{j,t}] = 0$  for shocks  $e(B)_{i,t}$  and  $e(B)_{j,t}$  in different blocks is not contained in  $E[f_{\mathbf{N}}(B, u_t)]$ . The conditions  $E[f_{\mathbf{N}}(B, u_t)]$  can be justified by the following assumption.

**Assumption 2.** *Block-recursive mean independence:*

For  $m \leq n$  blocks with  $p_1 = 1 < p_2 < \dots < p_m \leq n$ ,

- (i) all shocks are mutually uncorrelated, i.e.,  $E[\varepsilon_{i,t} \varepsilon_{j,t}] = 0$  for  $i \neq j$ .
- (ii) all shocks within the same block are mutually mean independent, i.e.,  $E[\varepsilon_{i,t} | \varepsilon_{-i,t}] = 0$  for  $i \in \{p_k, p_k + 1, \dots, p_{k+1} - 1\}$  and  $-i = \{p_k, p_k + 1, \dots, p_{k+1} - 1\} \setminus i$  for  $k = 1, \dots, m$ .

The conservative set of moment conditions contains  $n$  variance conditions,  $n(n-1)/2$  covariance conditions and  $\sum_{k=1}^m l_k(l_k-1)/2$  asymmetric cokurtosis conditions, where  $l_k := p_{k+1} - p_k$  denotes the number of shocks in block  $k$ . Therefore, each additional specified block refines the conservative set  $f_{\mathbf{N}}(B, u_t)$  such that it contains less higher-order moment conditions. In the extreme case, when the SVAR is specified recursively, meaning each block contains only one variable, the conservative set of moment conditions contains no higher-order moment conditions. In the other extreme case of a single block containing all variables, the conservative set of moment conditions contains all  $n(n-1)$  asymmetric cokurtosis conditions and it is similar to the conditions proposed in Lanne and Luoto (2021).<sup>4</sup>

In the following proposition, we show that the conservative set of moment conditions is sufficient to locally identify the block-recursive SVAR.

**Proposition 1.** *Identification in the block-recursive SVAR:*

Let  $u_t = B_0 \varepsilon_t$  with  $m \leq n$  blocks and  $p_1 = 1 < p_2 < \dots < p_m \leq n$ . Suppose Assumption 1 and Assumption 2 hold. If that at most one structural shock in each block has zero excess kurtosis, the conservative set of moment conditions  $E[f_{\mathbf{N}}(X, u_t)] = 0$  is locally identified at  $X = B_0$  with  $X \in \mathbb{B}_{brec}$ .

*Proof.* For ease of notation, we omit the time index  $t$  and w.l.o.g. consider an example with two blocks

$$\begin{bmatrix} u_{p_1} \\ u_{p_2} \end{bmatrix} = \begin{bmatrix} B_{11} & 0 \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{p_1} \\ \varepsilon_{p_2} \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix}, \quad (18)$$

where  $u_{p_1}$  and  $u_{p_2}$  contain the reduced form shocks of the first and second block,  $\varepsilon_{p_1}$  and  $\varepsilon_{p_2}$  contain the structural shocks of the first and second block, and  $B_{11}$ ,  $B_{21}$ ,  $B_{22}$ ,  $X_{11}$ ,  $X_{21}$ , and  $X_{22}$  are the corresponding blocks of the matrices  $B_0$  and  $X$ .

<sup>4</sup>Lanne and Luoto (2021) propose to select  $n(n-1)/2$  asymmetric cokurtosis conditions, which is sufficient for local identification if none of the asymmetric conditions does include the third power of a Gaussian shock. They advocate to rely on a moment selection criterion to avoid including redundant conditions or conditions of Gaussian shocks. Additionally, Lanne and Luoto (2021) note that including all  $n(n-1)$  asymmetric cokurtosis conditions ensures local identification even if conditions related to Gaussian shocks are included. We argue that the degree of overidentification remains reasonably small even if we include all asymmetric cokurtosis conditions and therefore, including redundant conditions can be expected to be rather harmless. For example, in a SVAR with four variables and no restrictions the conservative set has 22 conditions to identify 16 parameters. Thus, we suggest to use all asymmetric cokurtosis conditions in order to avoid the cumbersome process of selecting a subset of the conditions.

First, let  $E[f_{\mathbf{2}_{p_1}}(X, u)] = 0$  contain all (co-)variance conditions of shocks in the first block. The block-recursive structure implies that  $u_{p_1} = B_{11}\varepsilon_{p_1}$ . It follows from Lanne and Luoto (2021) that the conditions containing only shocks in the first block

$$E \begin{bmatrix} f_{\mathbf{2}_{p_1}}(X, u) \\ f_{\mathbf{4}_{p_1}}(X, u) \end{bmatrix} = 0 \quad (19)$$

locally identify  $X_{11} = B_{11}$ , the impact of the shocks in the first block on the variables in the first block.

Second, let  $E[f_{\mathbf{2}_{p_1 p_2}}(X, u)] = 0$  contain all covariance conditions belonging to shocks in both blocks. At the local solution  $X_{11} = B_{11}$ , the covariance conditions containing shocks of both blocks only hold if  $X_{21} = B_{21}$ . To see this, rewrite the covariance conditions as  $E[e_{p_2}(X)e_{p_1}(X)'] = 0$ . With the partitioned inverse of  $X$  and the block-recursive structure it holds that  $e_{p_2}(X) = -X_{22}^{-1}X_{21}X_{11}^{-1}B_{11}\varepsilon_{p_1} + X_{22}^{-1}(B_{21}\varepsilon_{p_1} + B_{22}\varepsilon_{p_2})$ . Therefore, with  $X_{11} = B_{11}$  it holds that

$$E[e_{p_2}(X)e_{p_1}(X)'] = -X_{22}^{-1}X_{21}E[\varepsilon_{p_1}\varepsilon_{p_1}'] + X_{22}^{-1}B_{21}E[\varepsilon_{p_1}\varepsilon_{p_1}'] + B_{22}E[\varepsilon_{p_2}\varepsilon_{p_2}']. \quad (20)$$

With  $E[\varepsilon_{p_1}\varepsilon_{p_1}'] = I$  and  $E[\varepsilon_{p_2}\varepsilon_{p_2}'] = 0$ , the condition  $E[e_{p_2}(X)e_{p_1}(X)'] = 0$  implies  $0 = -X_{22}^{-1}(X_{21} - B_{21})$  at  $X_{11} = B_{11}$ . Therefore, at the local solution  $X_{11} = B_{11}$  the covariance conditions  $E[f_{\mathbf{2}_{p_1 p_2}}(X, u)]$ , globally identify  $X_{21} = B_{21}$  the impact of shocks in the first block on variables in the second block.

Finally, let  $E[f_{\mathbf{2}_{p_2}}(X, u)] = 0$  contain all (co-)variance conditions of shocks in the second block. At the solution  $X_{11} = B_{11}$  and  $X_{21} = B_{21}$  the unmixed innovations of the second block  $e_{p_2}(X)$  are mixtures of the structural shocks in the second block and are not influenced by shocks from the first block. This follows from the partitioned inverse of  $X$  and the block-recursive structure such that  $e_{p_2}(X) = X_{22}^{-1}B_{22}\varepsilon_{p_2}$ . It then again follows from Lanne and Luoto (2021) that at the solution  $X_{11} = B_{11}$  and  $X_{21} = B_{21}$  the remaining conditions containing only shocks in the second

block

$$E \begin{bmatrix} f_{2_{\mathbf{p}_2}}(X, u) \\ f_{4_{\mathbf{p}_2}}(X, u) \end{bmatrix} = 0 \quad (21)$$

locally identify  $X_{22} = B_{22}$ , meaning the impact of shocks in the second block on variables in the second block.

□

**Remark 1.** In Proposition 1 the impact of shocks on variables in different blocks is identified based on covariance conditions. The interaction of shocks on variables within the same block is identified based on asymmetric cokurtosis conditions and the local identification result of Lanne and Luoto (2021). Importantly, the proposition also holds for different higher-order moment conditions ensuring identification within the blocks. For example, the set  $E[f_{4_{\mathbf{p}_i}}(B, u_t)]$  could contain all coskewness and cokurtosis conditions implied by independent structural shocks in block  $i$ . In this case, global identification up to sign and permutation within a block follows from Keweloh (2021b). Alternatively, if the set contains all symmetric cokurtosis conditions implied by independent structural shocks in block  $i$ , global identification follows under conditions from Lanne et al. (2021). However, these sets may contain invalid higher-order moment conditions, which would destroy the identification result. In particular, symmetric cokurtosis conditions may not be fulfilled if the variance of the shocks is driven by the same process. Additionally, the number of coskewness and cokurtosis conditions implied by (mean) independent shocks increases quickly with the variables in the SVAR, or in our case variables in a block, and lead to problems associated with many moment conditions.

**Remark 2.** Without further restrictions, data driven approaches relying on non-Gaussian and independent shocks can only ensure identification up to sign and permutation. This means that the order and sign of the shocks in the impulse response functions is not identified. In practice, the researcher has to manually assign labels to the shocks. Restricting the solution to a given block-recursive order simplifies the permutation or labeling problem. In particular, shocks can only be permuted inside blocks. For instance, in example (b) in Figure 1 shocks from the second block cannot be permuted into the first block since this violates the block-recursive structure.

Therefore, specifying a finer block-recursive structure simplifies the labeling of the shocks.

Define the block-recursive SVAR GMM estimator which minimizes the variance, covariance and the asymmetric cokurtosis conditions over the set of block-recursive matrices as

$$\hat{B}_{\mathbf{N}} := \arg \min_{B \in \mathbb{B}_{\text{block-recursive}}} g_{\mathbf{N}}(B)' W g_{\mathbf{N}}(B), \quad (22)$$

with a suitable weighting matrix  $W$  and  $g_{\mathbf{N}}(B) := 1/T \sum_{t=1}^T f_{\mathbf{N}}(B, u_t)$ . Consistency, asymptotic normality, and asymptotic optimal weighting of the block-recursive SVAR GMM estimator follow from the identification result obtained in Proposition 1 and standard assumptions, see Hall (2005). In a recursive SVAR, the proposed block-recursive SVAR GMM estimator uses no information contained in moments beyond the variance and collapses to the frequently used estimator obtained by applying the Cholesky decomposition to the variance-covariance matrix of the reduced form shocks. In an unrestricted SVAR corresponding to a single block containing all shocks, the block-recursive SVAR GMM estimator estimates all interactions based on higher-order moment conditions and is equal to the estimator proposed by Lanne and Luoto (2021) containing all asymmetric cokurtosis conditions.

In practice, efficiently estimating the block-recursive SVAR GMM estimator requires to estimate the asymptotically efficient weighting matrix. Most commonly, this is done by a two-step estimator, starting with an initial suboptimal but consistent weighting matrix, e.g.  $W = I$ , leading to an inefficient estimate of  $B_0$  which can then be used to estimate the efficient weighting matrix and thus to efficiently estimate  $B_0$ . Alternatively, the continuously updating estimator (CUE) proposed by Hansen et al. (1996) updates the weighting matrix continuously and optimizes

$$\hat{B}_{\mathbf{N}} := \arg \min_{B \in \mathbb{B}_{\text{block-recursive}}} g_{\mathbf{N}}(B)' W^*(B) g_{\mathbf{N}}(B), \quad (23)$$

where  $W^*(B)$  depends on  $B$  and is an estimator for the asymptotically efficient weighting matrix. The asymptotically efficient weighting matrix is equal to

$$W^* := S^{-1} \quad \text{and} \quad S := \lim_{T \rightarrow \infty} E [T g_T(B_0) g_T(B_0)'], \quad (24)$$

see, e.g., Hall (2005). Keweloh (2021a) shows that in a SVAR with fourth-order moment con-

ditions, the long-run covariance matrix  $S$  contains moments up to order eight and is, therefore, difficult to estimate in small samples. He demonstrates that the inability to precisely estimate  $S$  and in consequence the efficient weighting matrix  $W^*$  leads to a poor finite sample performance of two-step GMM and CUE estimators. Recognizing this downside, Keweloh (2021a) proposes a novel estimator for  $S$  exploiting serially and mutually independent shocks. Keweloh (2021a) illustrates that the estimator for  $S$  substantially increases the small sample performance of the two-step GMM and CUE estimator. For the remainder of the study, we will apply the block-recursive CUE estimator from Equation (23) together with the estimator for  $S$  proposed in Keweloh (2021a).

**Remark 3.** In many applications, the researcher is only interested in some structural shocks. In this case, our proposed block-recursive identification approach is robust to various misspecifications. For simplicity, consider the following SVAR with two blocks

$$\begin{bmatrix} u_{p_1} \\ u_{p_2} \end{bmatrix} = \begin{bmatrix} B_{11} & 0 \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_{p_1} \\ \varepsilon_{p_2} \end{bmatrix}, \quad e(X) = X^{-1}u, \quad \text{and} \quad X = \begin{bmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{bmatrix}, \quad (25)$$

where  $u_{p_1}$  and  $u_{p_2}$  contain the reduced form shocks of the first and second block,  $\varepsilon_{p_1}$  and  $\varepsilon_{p_2}$  contain the structural shocks of the first and second block, and  $B_{11}$ ,  $B_{21}$ ,  $B_{22}$ ,  $X_{11}$ ,  $X_{21}$ , and  $X_{22}$  are the corresponding blocks of the matrices  $B_0$  and  $X$ . For any invertible  $X$  satisfying the block-recursive structure and the covariance conditions  $0 = E[e_{p_2}(X)e_{p_1}(X)']$ , which ensure that the unmixed innovations of block one and two are uncorrelated, it holds that the unmixed innovations of the second block satisfy

$$e_{p_2}(X) = X_{22}^{-1}B_{22}\varepsilon_{p_2}, \quad (26)$$

meaning they are only a mixture of structural shocks from the second block.<sup>5</sup> Therefore, if the covariance conditions  $0 = E[e_{p_2}(X)e_{p_1}(X)']$  are satisfied, the moment conditions containing only unmixed innovations of the second block identify the shocks in the second block and their impact

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<sup>5</sup>To verify this note that with the block-recursive structure and the partitioned inverse it holds that  $e_{p_1}(X) = X_{11}^{-1}B_{11}\varepsilon_{p_1}$  and  $e_{p_2}(X) = -X_{22}^{-1}X_{21}X_{11}^{-1}B_{11}\varepsilon_{p_1} + X_{22}^{-1}(B_{21}\varepsilon_{p_1} + B_{22}\varepsilon_{p_2})$ . With  $E[\varepsilon_{p_1}\varepsilon_{p_1}'] = I$  and  $E[\varepsilon_{p_2}\varepsilon_{p_1}'] = 0$ , the condition  $0 = E[e_{p_2}(X)e_{p_1}(X)']$  implies  $0 = -X_{22}^{-1}(B_{21} - X_{21}X_{11}^{-1}B_{11})B_{11}'(X_{11}^{-1})'$ , which only holds if  $X_{21} = B_{21}B_{11}^{-1}X_{11}$ . Plugging in the condition for  $X_{21}$  yields  $e_{p_2}(X) = X_{22}^{-1}B_{22}\varepsilon_{p_2}$ .

$X_{22} = B_{22}$ . Consequently, identification of the shocks in a given block does not depend on whether the shocks in the previous block are identified as long as the the shocks are uncorrelated with the shocks in the previous block. In particular, this implies that the conditions  $E[f_{\mathbf{N}}(X, u_t)] = 0$  locally identify the shocks in a given block even if all shocks in the previous blocks are Gaussian.

### 3.4 Finite sample performance

We simulate a block-recursive SVAR with  $n = 4$  and  $n = 2$  variables. The mixing matrices  $B_0$  are given by

$$B_0 = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix} \quad \text{and} \quad B_0 = \begin{bmatrix} 1 & 0.5 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 0.5 & 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 0.5 & 1 \end{bmatrix}. \quad (27)$$

The structural shocks are drawn from a mixture of Gaussian distributions with mean zero, unit variance, skewness equal to 0.89 and an excess kurtosis of 2.35. In particular, the shocks satisfy

$$\varepsilon_i = z\phi_1 + (1 - z)\phi_2 \quad \text{with} \quad \phi_1 \sim \mathcal{N}(-0.2, 0.7), \quad \phi_2 \sim \mathcal{N}(0.75, 1.5), \quad z \sim \mathcal{B}(0.79), \quad (28)$$

where  $\mathcal{B}(p)$  indicates a Bernoulli distribution and  $\mathcal{N}(\mu, \sigma^2)$  indicates a normal distribution.

The Monte Carlo study analyzes the impact of imposing a block-recursive order on the SVAR CUE using the conservative moments from Equation (17) and the estimator for  $S$  proposed in Keweloh (2021a). In the small SVAR with  $n = 2$ , we impose no restrictions. In the large SVAR with  $n = 4$ , one estimator (CUE) is calculated without restrictions and a second estimator (block-recursive CUE) is estimated, which uses the block-recursive order in Equation (27).

Table 1 reports the mean and standard deviation of each estimated element of  $B_0$ . The simulation shows, how the performance of estimates based entirely on non-Gaussianity decreases with an increasing model size. This curse of dimensionality is more pronounced in smaller samples. The Monte Carlo study illustrates how in a typical macroeconomic application, which rarely or if at all contains more than a few hundred observations, data driven estimates based on non-Gaussianity

Table 1: Finite sample performance of the CUE and block-recursive CUE

	$n = 2$ CUE	$n = 4$ CUE				$n = 4$ block-recursive CUE			
$T = 150$	$\begin{bmatrix} 0.97 & 0.47 \\ (1.52) & (4.65) \end{bmatrix}$	$\begin{bmatrix} 0.91 & 0.43 & 0.02 & 0.01 \\ (2.08) & (5.02) & (6.02) & (8.1) \\ 0.52 & 0.85 & 0.01 & -0.0 \\ (5.84) & (6.36) & (5.43) & (6.14) \end{bmatrix}$	$\begin{bmatrix} 0.97 & 0.46 & . & . \\ (1.63) & (5.36) & . & . \\ 0.52 & 0.94 & . & . \\ (4.62) & (5.49) & . & . \end{bmatrix}$						
$T = 500$	$\begin{bmatrix} 0.99 & 0.49 \\ (1.83) & (4.45) \\ 0.5 & 0.99 \\ (3.81) & (2.48) \end{bmatrix}$	$\begin{bmatrix} 0.97 & 0.47 & 0.0 & 0.0 \\ (2.59) & (5.59) & (6.66) & (7.77) \\ 0.49 & 0.95 & -0.0 & 0.0 \\ (5.57) & (4.83) & (5.9) & (7.5) \\ 0.48 & 0.48 & 0.96 & 0.47 \\ (8.34) & (9.26) & (4.85) & (10.44) \\ 0.48 & 0.48 & 0.49 & 0.95 \\ (8.04) & (10.01) & (9.06) & (10.76) \end{bmatrix}$	$\begin{bmatrix} 0.99 & 0.49 & . & . \\ (1.82) & (3.74) & . & . \\ 0.5 & 0.98 & . & . \\ (3.69) & (2.11) & . & . \\ 0.5 & 0.49 & 0.98 & 0.5 \\ (2.27) & (2.44) & (1.84) & (4.12) \\ 0.49 & 0.49 & 0.49 & 0.99 \\ (2.25) & (2.33) & (3.99) & (2.78) \end{bmatrix}$						
$T = 5000$	$\begin{bmatrix} 1.0 & 0.5 \\ (1.82) & (3.36) \\ 0.5 & 1.0 \\ (3.35) & (1.82) \end{bmatrix}$	$\begin{bmatrix} 1.0 & 0.5 & 0.0 & -0.0 \\ (2.31) & (3.74) & (4.56) & (4.56) \\ 0.5 & 1.0 & 0.0 & 0.0 \\ (4.09) & (1.91) & (4.51) & (4.33) \\ 0.5 & 0.5 & 1.0 & 0.5 \\ (5.19) & (5.67) & (3.5) & (5.18) \\ 0.5 & 0.5 & 0.5 & 1.0 \\ (5.11) & (6.22) & (5.24) & (3.66) \end{bmatrix}$	$\begin{bmatrix} 1.0 & 0.5 & . & . \\ (1.93) & (3.9) & . & . \\ 0.5 & 1.0 & . & . \\ (3.74) & (2.16) & . & . \\ 0.5 & 0.5 & 1.0 & 0.5 \\ (2.4) & (2.47) & (1.77) & (3.78) \\ 0.5 & 0.5 & 0.5 & 1.0 \\ (2.34) & (2.52) & (3.47) & (2.52) \end{bmatrix}$						

The table reports the mean of  $\hat{b}_{ij}$  and in parentheses the standard deviation of  $\sqrt{T}(\hat{b}_{ij} - b_{ij})$  of all estimated elements  $\hat{b}_{ij}$  over 4000 Monte Carlo replicates for the CUE estimator and the block-recursive CUE estimator. The former estimator does not use any zero restrictions. The latter estimator uses zero restrictions which are highlighted by the dots. The structural shocks are drawn from t-distribution with seven degrees of freedom. The simulated SVAR has  $n = 2$  and  $n = 4$  variables with sample sizes 150, 250, 500, and 5000, respectively.

become less reliable the more variables and shocks the SVAR contains. However, the simulation also stresses that exploiting the block-recursive structure annihilates the deterioration of the performance induced by a larger model.

## 4 The interdependence of monetary policy and the stock market

In this section, we apply the proposed block-recursive SVAR CUE estimator to analyze the interaction of monetary policy and the stock market. Using our estimator, we can leave the

short- and long-run interaction of the stock market and monetary policy unrestricted. However, we are able to include a larger set of control variables compared to fully moment-based approaches without losing too much precision, as we can exploit the information from well established theoretical implications like, for instance, price rigidity that imply no immediate reaction to changes in the macroeconomic environment for a subset of variables. In particular, our SVAR contains two blocks: The first block contains macroeconomic control variables like output growth, investment growth and inflation, while the second block contains stock returns and the federal funds rate. We assume that the first block of variables does not contemporaneously react to the second block of structural shocks, while the second block remains unrestricted.

## 4.1 Specification and Identification

We consider a SVAR in five variables with quarterly U.S. data from 1983Q1 to 2019Q1 of the form

$$\begin{bmatrix} y_t \\ I_t \\ \pi_t \\ s_t \\ i_t \end{bmatrix} = \alpha + \gamma t + \sum_{i=1}^p A_i \begin{bmatrix} y_{t-1} \\ I_{t-1} \\ \pi_{t-1} \\ s_{t-1} \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} u_t^y \\ u_t^I \\ u_t^\pi \\ u_t^s \\ u_t^i \end{bmatrix}, \quad (29)$$

where  $y$  denotes real output growth,  $I$  real investment growth,  $\pi$  the inflation rate,  $i$  the federal funds rate and  $s$  real stock returns<sup>6</sup>. The consideration of control variables here follows from the simple model exercise we perform in the appendix, where we construct a model that either features long-run monetary neutrality or non-neutrality based on the role of physical capital concerning productivity. The standard neoclassical model with exogenous growth predicts a mean reversal of output, investment and stock prices after a monetary policy shock, while an Ak-type endogenous

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<sup>6</sup>The inflation rate is defined as the quarter to quarter growth rate in the quarterly chain-type GDP price index retrieved from the FRED. The GDP growth rate is given by the quarterly log-difference of real GDP retrieved from the FRED. Real investment growth is given by the quarterly growth rate of real gross private domestic investment obtained from the FRED. The nominal interest rate is defined as the Federal Funds Rate (FFR), where the effective FFR (retrieved from FRED) is replaced by the shadow FFR provided by Wu and Xia (2016) for the Zero Lower Bound observations during the Great Recession. Stock returns are defined as the quarterly log-difference in real stock prices, where real stock prices are given by the S&P 500 index (retrieved from macrotrends.net) divided by the chain-type GDP price index.

growth model (see for instance Annicchiarico and Rossi (2013)) implies that output, investment and stock prices should remain below their pre-shock level. The inclusion of investment allows us to check on and discriminate between the two model worlds. Furthermore, we choose to include output, investment and stock prices in growth rates in order to check on the validity of potential long-run restrictions.

Moreover, we set  $p = 2$  as suggested by the AIC. The linear time trend  $t$  is added to account for an eventual long-term decline in the interest rate as discussed by for instance Carvalho et al. (2016). In Appendix 5.2 we conduct robustness checks regarding the observation period, the exclusion of the linear time trend, the lag structure, the estimation method within the blocks, different specifications and the inclusion of further control variables. Our main results are found to be robust and remain qualitatively unchanged: Stock prices and the nominal interest rate both react immediately to monetary policy and stock market shocks and the long-run effect is associated with high uncertainty.

We assume that real investment growth, real output growth and inflation behave sluggishly, meaning they cannot react to monetary policy or stock market shocks within the same quarter. These restrictions can be justified by price rigidities and capital adjustment costs as oftentimes used in standard DSGE models, see for example Smets and Wouters (2007). However, interest rates and stock returns are unrestricted and can contemporaneously respond to all shocks. Therefore, the block-recursive structure is given by

$$\begin{bmatrix} u_t^y \\ u_t^I \\ u_t^\pi \\ u_t^s \\ u_t^i \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 & 0 \\ b_{21} & b_{22} & b_{23} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^I \\ \varepsilon_t^\pi \\ \varepsilon_t^s \\ \varepsilon_t^i \end{bmatrix}. \quad (30)$$

Proposition 1 allows to identify the impact of the monetary policy shock  $\varepsilon_t^i$  and the stock market shock  $\varepsilon_t^s$  without committing to any further restrictions if the monetary policy and stock market shocks are mean independent and at least one of the two shocks is non-Gaussian. In general, the structural shocks are not observable, however, Remark 3 allows to verify whether the two structural shocks are non-Gaussian. In particular, Remark 3 implies that the unmixed innovations

in the second block  $e_t^s(\hat{B})$  and  $e_t^i(\hat{B})$  are equal to a linear combination of the structural stock market and monetary policy. Note that this also holds if the structural stock market and monetary policy were Gaussian. Table 2 shows the skewness, kurtosis and the Jarque-Bera test for normality of the unmixed innovations in the second block. We find evidence that the innovations in the second block are non-Gaussian, which implies that the structural monetary policy and/or structural stock market shock are non-Gaussian.

Table 2: Non-Gaussianity of the unmixed innovations in the second block

	$e_t^s(\hat{B})$	$e_t^i(\hat{B})$
Skewness	-0.596	-0.548
Kurtosis	4.124	14.736
JB-Test	0.00	0.00

Skewness, kurtosis and the p-value of the Jarque-Bera test for normality of the unmixed innovations in the second block.

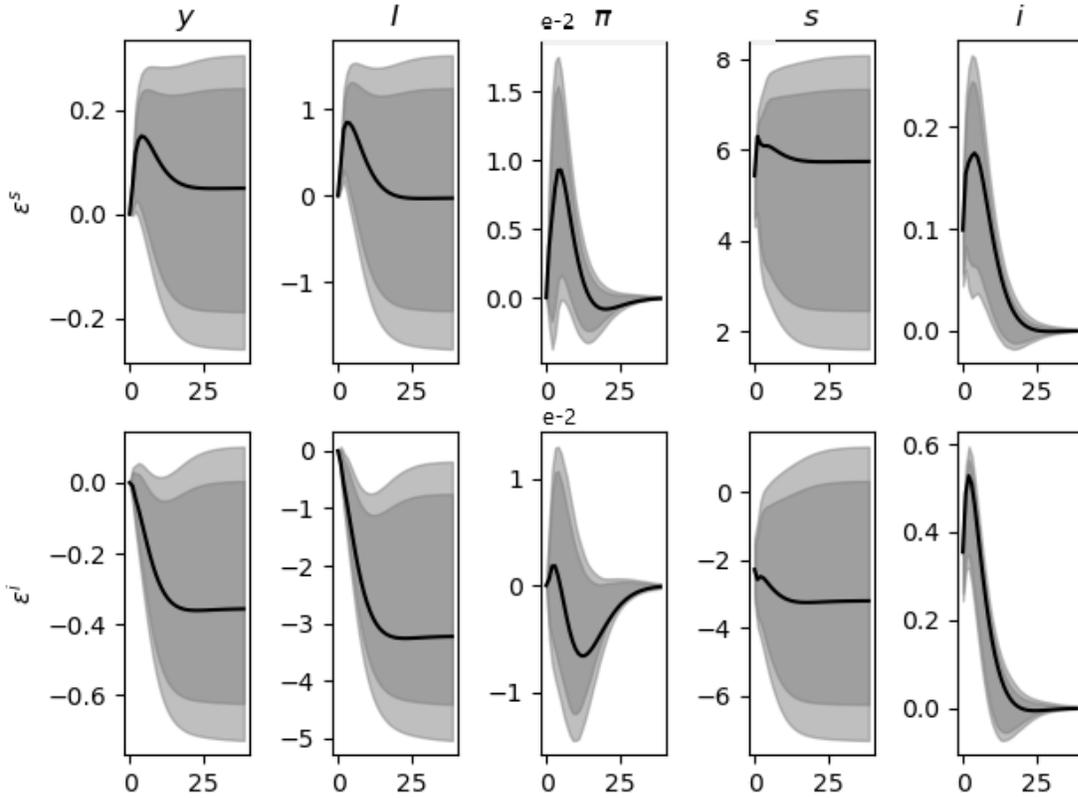
## 4.2 Results for the block-recursive SVAR

The SVAR is estimated by the block-recursive SVAR CUE with the restrictions imposed in Equation (30) and we apply the estimator for  $S$  proposed in Keweloh (2021a).

Figure 2 shows the corresponding impulse response functions (IRFs) to the estimated stock market and monetary policy shocks. The responses of stock returns, real investment growth and real GDP growth are integrated to show the associated level effects, which makes it possible to visually check the validity of long-run restrictions. Exploiting the block-recursive order makes labeling straightforward. There is only one shock, which leads to an increase of the interest rate together with a decrease of output and a medium-run decrease of inflation in the second block, which is what we would expect from a monetary policy shock. The remaining shock is then labeled as the stock market shock.

The estimated lower right block of the B-matrix reads (asymptotic variance, Wald test statistic

Figure 2: Impulse responses to stock market and monetary policy shocks in the block-recursive SVAR.



The figure shows the responses to one standard deviation shocks. The columns  $y$ ,  $I$ , and  $s$  show the cumulative responses of investment growth, output growth, and stock returns. Confidence bands are 68% and 80% bootstrap bands with 5000 replications in the bootstrap algorithm.

with  $H_0 : B_{ij} = 0$  and p-value for the elements in parentheses)<sup>7</sup>

$$\hat{B}_{lowerright} = \begin{pmatrix} 5.44 & -2.29 \\ (30.92/136.79/0.0) & (80.86/9.30/0.0) \\ 0.1 & 0.35 \\ (0.19/7.48/0.01) & (0.45/39.95/0.0) \end{pmatrix}. \quad (31)$$

A one standard deviation stock market shock leads to an immediate increase of stock prices

<sup>7</sup>Based on Keweloh (2021a) we use the assumption of serially and mutually independent shocks to estimate the asymptotic variance.

about 5.44% and to an immediate interest rate increase about 0.1 percentage points. A one standard deviation monetary policy shock leads to a decrease in stock prices of about 2.3%, while the nominal interest rate increases about 0.35 percentage points. The estimated simultaneous interaction is qualitatively comparable to the results in Bjørnland and Leitemo (2009). The Wald tests suggest, that the simultaneous response of stock returns and interest rates to monetary policy and stock market shocks is significant at the 5% level. Additionally, also the confidence bands in Figure 2 suggest that both variables interact simultaneously and cannot be ordered recursively.

Consistent with the news literature around Beaudry and Portier (2006), we find that a positive stock market shock is followed by a business cycle expansion with an increase in the real output growth rate, the real investment growth rate and a positive inflation rate. Therefore, even if the central bank is not interested in stock prices in the first place, a stock market shock implies a business cycle boom with increasing inflation. Thus, the central bank will increase the FFR. Additionally, we find that a contractionary monetary policy shock induces a recession with a decrease in output, investment and prices. The future recession and an efficient stock market, which immediately incorporates all available information, then explains the initial negative response of stock prices to the monetary policy shock.

Unlike Bjørnland and Leitemo (2009), we do not impose long-run neutrality of monetary policy w.r.t. stock prices. Based on the point estimate we find that a contractionary monetary policy shock leads to permanently lower output, investment and stock prices and therefore, we find evidence that monetary policy is not neutral w.r.t stock prices in the long-run. However, based on the broad confidence bands, long-run neutrality of monetary policy w.r.t. stock prices cannot be rejected. In contrast to that, the long-run negative effect on the output level is significant when considering the 68% band, while the investment level effect is persistently negative even at the 80% band.

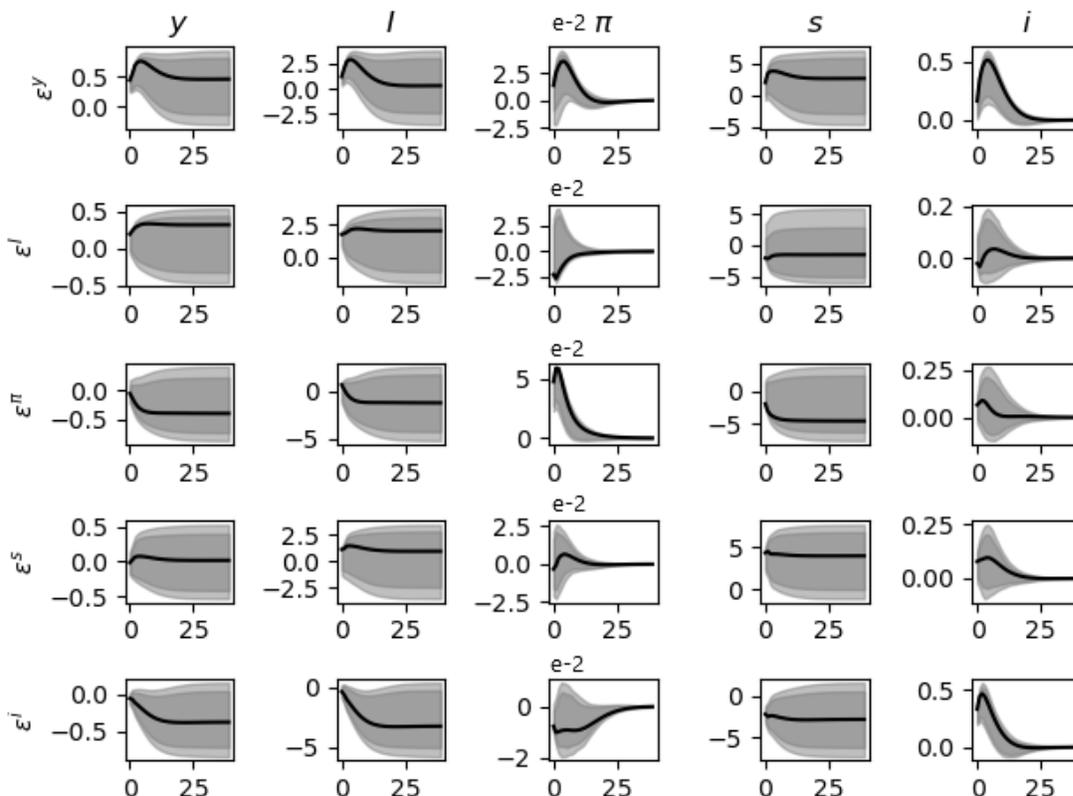
### 4.3 Results for the unrestricted SVAR

We now estimate the SVAR without block-recursive restrictions. In particular, we use the block-recursive SVAR CUE with only one block containing all shocks and we again use the estimator for  $S$  proposed in Keweloh (2021a). We find no strong evidence against the block-recursive structure

imposed in the previous section. Regarding the effects of monetary policy and stock market shocks, we find similar point estimates, but larger confidence bands. The results illustrate how combining a data-driven approach with zero restrictions on the short-run interaction between variables allows to decrease the variance of the estimator and to gain deeper insights into the interrelationship between monetary policy and the stock market.

Figure 3 shows the respective IRFs. The monetary policy and stock market shocks are labeled

Figure 3: Impulse responses in the unrestricted SVAR.



The figure shows the responses to one standard deviation shocks. The columns  $y$ ,  $I$ , and  $s$  show the cumulative responses of investment growth, output growth, and stock returns. Confidence bands are 68% and 80% bootstrap bands with 1000 replications in the bootstrap algorithm.

such that  $\epsilon_i$  is the shock featuring the highest correlation with the reduced form shock  $u_i$  and the stock market shock  $\epsilon_s$  is the shock featuring the highest correlation with the reduced form shock  $u_s$ . Moreover, the IRF shows that the shock labeled as the monetary policy shock is the

only shock, which leads to an increase in the interest rate accompanied by a decrease in GDP, investment and a medium-run decrease in inflation, which reinforces our labeling. The labeling of a demand shock  $\varepsilon_d$ , an investment specific shock  $\varepsilon_I$  and an inflation shock  $\varepsilon_\pi$  is also based on the highest correlation between reduced form errors and structural shocks (the correlation matrix between reduced form errors and structural shocks can be found in the appendix). However, this labeling is to be taken with a grain of salt, as the conclusiveness of a fully-moment based approach with five variables visibly deteriorates compared to the last section. For instance the first and fourth row of the impulse responses show similar qualitative results, which are associated with high uncertainty. So convincingly labeling a stock market shock becomes difficult here, which is the same problem other solely data-driven approaches face (see, e.g., Lanne et al. (2017)).

The last two columns of the estimated B-matrix corresponding to the stock market and monetary policy shock read (asymptotic variance, Wald test statistic with  $H_0 : B_{ij} = 0$  and p-value for the elements in parentheses)

$$\hat{B}_{columns\ 4/5} = \begin{pmatrix} -0.02 & -0.06 \\ (0.87/0.05/0.83) & (0.43/1.16/0.28) \\ 1.12 & -0.35 \\ (27.12/6.61/0.01) & (12.97/1.36/0.24) \\ -0.33 & -0.78 \\ (218.65/0.07/0.79) & (72.81/1.19/0.28) \\ 4.32 & -2.17 \\ (92.59/28.76/0.0) & (78.11/8.66/0.0) \\ 0.08 & 0.33 \\ (0.27/3.3/0.07) & (0.45/34.97/0.0) \end{pmatrix}. \quad (32)$$

The unrestricted estimation confirms our finding on the interaction of monetary policy and the stock market: A tightening of monetary policy induces a recession with a decrease in output, investment, inflation and stock prices, and a positive stock market shock is accompanied by an immediate increase in interest rates. Additionally, we again find evidence that no recursive ordering of both variables is viable. However, consistent with the finding in the Monte Carlo simulation in Section 3, confidence intervals are larger compared to the block-recursive estimation

in the previous section. In particular, the stock market shock appears to have no significant impact on investment, GDP or inflation, thus making it difficult to explain the response of the interest rate.

Turning to the validity of the block-recursiveness assumption used in the previous section, we find mixed results. In particular, we perform a joint Wald test with the null hypothesis that the stock market and monetary policy shock have no simultaneous impact on output growth, investment growth and inflation. The Wald statistic of this test is 19.58 with a p-value  $< 0.01$  and therefore, we would reject the hypothesis of the block-recursive order used in the previous section at any conventional significance level. However, Keweloh (2021a) shows that in larger SVARs Wald tests reject the null hypothesis too often, especially when multiple hypotheses are tested jointly. Based on the element wise Wald tests shown above, we can only reject the null hypothesis that stock market shocks have a simultaneous impact on investment at the 5% level. However, based on Figure 3 we find that the 80% confidence band of the simultaneous response of investment to a stock market shock contains the null, so compared to the element wise Wald test we cannot reject an initial zero at a much higher confidence level here. Given that the Wald test rejects the null hypothesis too often, we conclude, that there is no strong evidence against a block-recursive structure.

## 5 Conclusion

Identifying SVARs is challenging as for identification some structure has to be imposed either on the interaction between variables or the statistical properties of the shocks. Most approaches either fully rely on restrictions concerning the interaction or use no restrictions and completely rely on statistical properties. We argue that a combination of both approaches is best suited for many macroeconomic applications.

We propose a framework to combine block-recursive restrictions with non-Gaussian shocks, such that identification depends as little on information from higher moments as possible. In particular, identification of our proposed block-recursive SVAR GMM estimator relies on asymmetric cointegration conditions to ensure identification of the shocks within a given block, however, the impact of shocks on variables in different blocks is identified only by covariance conditions. Therefore,

with each additionally specified restriction the estimator uses less higher-order moment condition. From an asymptotic point of view, using restrictions is unnecessarily restrictive in a non-Gaussian SVAR with independent shocks. We use a Monte Carlo simulation to illustrate that purely data-driven estimators perform good in small SVARs with just a few variables, however, the finite sample performance decreases substantially when more variables are included. We show that exploiting a block-recursive order can stop the performance decrease in large SVARs and small samples.

We apply the proposed block-recursive SVAR estimator to analyze the interaction of monetary policy and the stock market. We argue that on the one hand there are not enough credible short- or long-run restrictions available to identify the SVAR based on restrictions on the interaction of the variables. At the other hand, purely data-driven approaches struggle to find conclusive answers on the interaction of both variables or even fail to reliably label monetary policy and stock market shocks. Employing our new estimator, we find that stock returns and interest rates both react simultaneously to stock market and monetary policy shocks. In particular, a tightening of monetary policy leads to a recession and a decrease of stock returns and a positive stock market shock indicates a future business cycle expansion and an immediate increase in interest rates. We find mixed results regarding the long-run neutrality of monetary policy. The point estimates show that a tightening of monetary policy leads to permanently lower output, investments, and stock prices. However, the long-run responses are associated with large confidence bands and for some of the variables we cannot reject the null hypothesis that the long-run effect is zero.

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# Appendix

## 5.1 The uncertainty about theoretical restrictions on the interaction between monetary policy and stock markets

In the main body of the paper we argued that there is a lack of indisputable short- or long-run restrictions on the effect of monetary policy or stock market shocks. In this section we use a simple asset pricing model to show that standard theory predicts an immediate reaction of monetary policy and stock markets to shocks in the respective other variable and that the validity of a long-run restriction crucially depends on the assumptions one makes concerning economic growth.

Consider that households can save by buying firm stocks of firm  $i$  at price  $v_{i,t}$ , yielding dividend  $d_{i,t+1}$  in the next period or by a non-contingent bond  $b_t^f$  yielding a guaranteed real interest at rate  $r_t$ . The no-arbitrage condition then is

$$1 + r_t = E_t \frac{v_{i,t+1} + d_{i,t+1}}{v_{i,t}}. \quad (33)$$

From this, one can acquire the central asset pricing equation of the form

$$v_{i,t} = E_t \sum_{s=1}^{\infty} \frac{d_{i,t+s}}{\prod_{j=1}^s (1 + r_{t+j-1})}, \quad (34)$$

so the current stock price is the expected discounted sum of future dividends. On the firm side assume a continuum of infinitely small firms with mass 1 and dividends of firm  $i$  are given by

$$d_{i,t+s} = y_{i,t+s} - j_{i,t+s} + b_{i,t+1+s}^f - (1 + r_{t+s-1})b_{i,t+s}^f - \bar{w}\bar{n}, \quad (35)$$

where  $y_{i,t}$  is output,  $j_{i,t}$  investment in the physical capital stock,  $b_{i,t}^f$  are debt sales (where  $\int_0^1 b_{i,t}^f di = b_t^f$ ),  $\bar{w}$  the constant real wage and  $\bar{n}$  labor input, also assumed constant for simplicity. We assume further an accumulation of physical capital  $k_{i,t}$  of the form

$$k_{i,t+1} = (1 - \delta)k_{i,t} + j_{i,t}, \quad \delta \in (0, 1). \quad (36)$$

The production function reads

$$y_{i,t} = Ak_{i,t}^\alpha (Z_t \bar{n})^{1-\alpha}, \quad \alpha \in (0,1), \quad (37)$$

with  $A$  an exogenous scaling factor and  $Z_t$  an aggregate productivity factor exogenous to the individual firm. Consequently, the firm maximization problem reads

$$\max_{\{k_{i,t+1+s}, b_{i,t+s}^f\}} \sum_{s=0}^{\infty} E_t \Lambda_{t+s} d_{i,t+s}, \quad (38)$$

with  $\Lambda_t$  the firm's discount factor and subject to (36)-(37). The optimality conditions yield the common interest rate parity condition of the form

$$E_t A \alpha k_{i,t+1}^{\alpha-1} (Z_{t+1} \bar{n})^{1-\alpha} + (1 - \delta) = 1 + r_t, \quad (39)$$

which says that in equilibrium the interest rate on foreign capital and the return on capital investment will coincide. Now inserting (35)-(37) into (34) yields

$$v_{i,t} = E_t \sum_{s=1}^{\infty} \frac{Ak_{i,t+s}^\alpha (Z_{t+s} \bar{n})^{1-\alpha} - k_{i,t+s+1} + (1 - \delta)k_{i,t+s} + b_{i,t+1+s}^f - (1 + r_{t+s-1})b_{i,t+s}^f - \bar{w}\bar{n}}{\prod_{j=1}^s (1 + r_{t+j-1})}. \quad (40)$$

Imposing the limiting condition  $\lim_{T \rightarrow \infty} b_T = 0$  then leads to future debt sales dropping out from the asset pricing equation, as dividends cannot be debt-financed indefinitely. As becomes evident, the dynamics of the numerator are then entirely driven by the evolution of capital. Using Equation (39) then allows to find the evolution of capital as

$$k_{i,t+1} = E_t \left[ \left( \frac{\alpha A}{r_t + \delta} \right)^{\frac{1}{1-\alpha}} \bar{n} Z_{t+1} \right]. \quad (41)$$

Now consider that the real interest rate increases once such that  $r'_t > r_t^*$  and for the rest of the time  $r'_{t+s} = r_{t+s}^*$ ,  $\forall s > 0$  (primes denote variables after the shock, asterisks variables without the shock). The resulting response of dividends and stock prices now crucially depends on what we assume about the productivity factor  $Z_t$ :

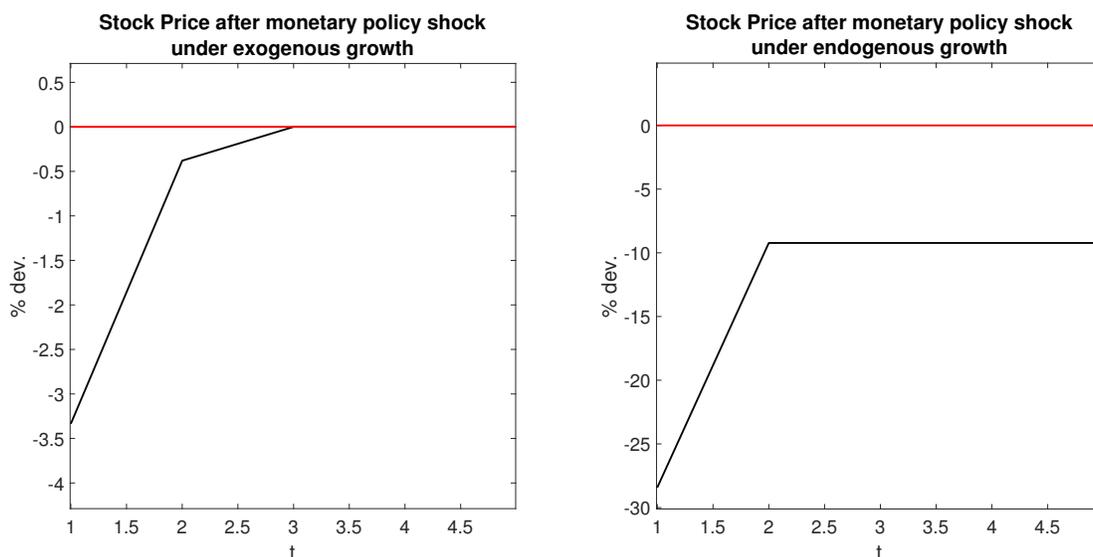
1.) Exogenous growth: Assume a neoclassical growth model with decreasing marginal returns to

capital, so  $Z_t$  is some exogenously growing variable.

2.) Endogenous growth: Assume an endogenous growth model, for instance a standard learning-by-doing technology with  $Z_t = \int_0^1 k_{i,t-1} di = K_{t-1}$ .

Figure 4 shows the effect of an interest rate shock on stock prices for an exogenous and endogenous growth model<sup>8</sup>. Assuming sticky prices, thus nominal and real variables move in the same

Figure 4: Simulated response of real stock prices to a one-time exogenous interest rate increase of about one percentage point as implied by the exogenous and endogenous growth model.



direction in the short-run, we can interpret the exogenous real interest rate increase as equivalent to a monetary policy shock. Both models imply an immediate reaction of stock prices to the monetary policy shock. However, in the exogenous growth model with decreasing returns to capital, stock prices revert back to their long-run level, while under endogenous growth with the learning-by-doing technology, the decrease in stock prices is permanent. This is because in the first case the lower capital stock implies a higher marginal return of capital in the future, which drives back capital to its old steady state, while in the second case it does not, because

<sup>8</sup>For simplicity we assume  $\bar{n} = 1$ , the initial debt  $b_t^f = 0$ ,  $\bar{w} = 0$  and use a standard calibration of  $\alpha = \frac{1}{3}$ ,  $\delta = 0.1$  and setting  $A = 0.46$  to ensure a long-run output growth rate of about 3%.

the lower aggregate capital stock implies lower capital investment return for the individual firm. Analogously, one sees from equations (41), (36) and (37) that the decrease in productivity  $Z$  will lead to a lower future capital stock and thus in the short- and long-run lower investment and output, so one would expect stock prices to comove with output and investment in the short- and long-run.

Furthermore, interpret a stock price shock as news about higher future productivity that is not realized today like in Beaudry and Portier (2006). For instance assume  $A$  is no longer a constant, but time dependent. Assume now that in the next period  $A'_{t+1} > A^*_{t+1}$ . From Equation (40) it becomes evident that an increase in future dividends leads to an increase in stock prices now. Because of  $A'_{t+1} > A^*_{t+1}$ , we also know that  $y'_{t+1} > y^*_{t+1}$ , so households expect a business cycle boom in the future, feel richer and will increase their demand even today. A central bank aiming at flattening business cycle fluctuations would immediately adjust its policy rate, which becomes evident from Equation (39). Consequently, stock prices will contemporaneously react to monetary policy shocks, as will monetary policy to stock market shocks.

To sum up, the validity of theoretical restrictions here depends on only a few modeling assumptions for which, to the best of our knowledge, there exists up to now no consensus in the literature. Exogenous and endogenous growth models are found left and right in the theoretical literature and we are not willing to discard any of these models from the set of potentially "true" models. But with our new estimator we do not have to and can remain agnostic here without giving up too much precision due to the availability of less disputable assumptions like for instance price rigidity that can be exploited to ease the burden on the identification by higher moment conditions.

## 5.2 Application appendix

This section contains supplementary material for the application presented in Section 4. Table 3 shows some descriptive statistics of the variables used in the SVAR.

Table 4 shows the standard deviation, skewness, kurtosis and p-value of the Jarque-Bera test of all estimated reduced form errors of the SVAR in Section 4.1.

Table 5 shows the correlation between the estimated structural shocks from the non-recursive

Table 3: Descriptive statistics

	Mean	Median	Mode	Std. deviation	Variance	Skewness	Kurtosis
$y$	0.71	0.74	-2.19	0.61	0.37	-0.83	3.46
$I$	1.1	0.96	-11.56	3.16	9.97	-0.28	2.3
$\pi$	2.28	2.09	0.27	0.87	0.76	0.36	-0.28
$i$	1.56	2.11	-26.45	6.5	42.25	-1.08	2.88
$s$	3.69	4.02	5.25	3.44	11.84	-0.03	-0.93

Table 4: Moments of estimated reduced form errors

	$u^y$	$u^I$	$u^\pi$	$u^s$	$u^i$
Standard deviation	0.488	2.521	0.055	6.062	0.39
Skewness	-0.729	0.103	-0.026	-0.581	-1.127
Kurtosis	5.128	3.754	2.838	4.326	11.088
JB-Test	0.00	0.162	0.918	0.00	0.00

Standard deviation, Skewness, Kurtosis and p-value of the Jarque-Bera test of all estimated reduced form errors in the SVAR of Section 4.

SVAR in section 4.3 and the reduced form shocks that we use to label the structural shocks retrieved from the fully moment-based approach. As it becomes evident, the strongest correlation

Table 5: Correlation of reduced form and estimated structural shocks

	$u^y$	$u^I$	$u^\pi$	$u^s$	$u^i$
$\varepsilon^y$	0.9	0.49	0.25	0.35	0.45
$\varepsilon^I$	0.4	0.69	-0.41	-0.34	-0.07
$\varepsilon^\pi$	-0.11	0.26	0.86	-0.34	0.19
$\varepsilon^s$	-0.0	0.46	-0.05	0.74	0.21
$\varepsilon^i$	-0.1	-0.13	-0.11	-0.36	0.87

Correlation of estimated structural shocks and reduced form shocks from Section 4.3.

between reduced form errors and structural shocks is found on the main diagonal, thus we label the fourth structural shock as the stock market shock and the fifth one as the monetary policy shock. We stress however that the labeling task becomes increasingly difficult the more variables are considered in one block, so the labeling here should always be taken with a grain of salt.

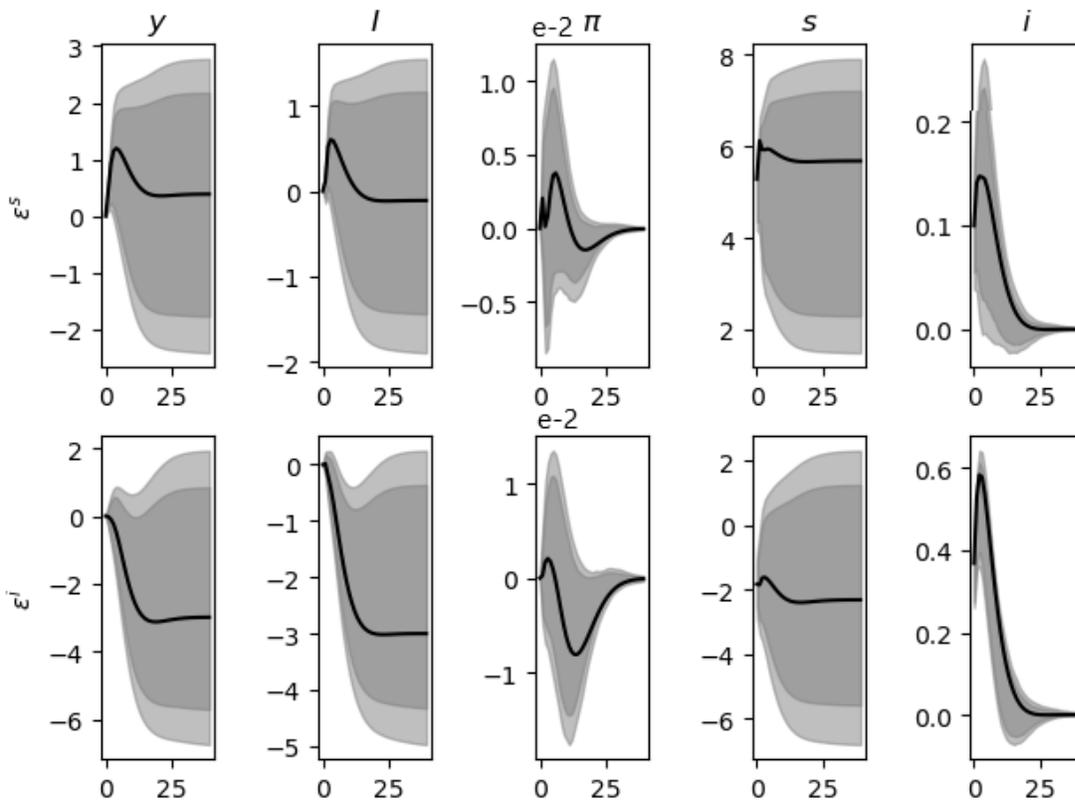
We proceed by checking on the robustness of the results presented in Section 4. All robustness checks exploit the block-recursive order described in the main body of the paper. First we replace output growth by the growth rate in the industrial production index, which is used by Bjørnland and Leitimo (2009). Table 6 shows the skewness, kurtosis and Jarque-Bera test results for the estimated structural shocks. Figure 5 shows the IRFs for the stock market and monetary policy shocks. The qualitative results from our main paper are robust to this change of specification.

Table 6: Moments of estimated structural shocks in the block-recursive SVAR with the industrial production index instead of GDP.

	$e^s(\hat{B})$	$e^i(\hat{B})$
Skewness	-0.646	-1.081
Kurtosis	4.08	12.915
JB-Test	0.00	0.00

Skewness, Kurtosis and p-value of the Jarque-Bera test of the estimated structural shocks in the second block for the specification.

Figure 5: Impulse responses to stock market and monetary policy shocks in the block-recursive SVAR with the industrial production index instead of GDP.



The figure shows the responses to one standard deviation shocks. The columns  $y$ ,  $I$ , and  $s$  show the cumulative responses of investment growth, output growth, and stock returns. Confidence bands are 68% and 80% bootstrap bands with 1000 replications in the bootstrap algorithm.

We now check if our results are dependent on the estimation technique for the data-driven identification within the blocks. Therefore, we employ the PML estimator proposed by Gouriéroux et al. (2017) to estimate both blocks. In particular, we assume that the structural shocks are t-

distributed with seven degrees of freedom. Additionally, the PML estimator is whitened, meaning we optimize the likelihood over the space of uncorrelated innovations with unit variance. Table 7 shows the skewness, kurtosis and Jarque-Bera test results for the estimated structural shocks. Figure 6 shows the impulse responses. As it becomes evident, the change of the estimation

Table 7: Moments of estimated structural shocks in the block-recursive SVAR with the PML estimator (see Gouriéroux et al. (2017)).

	$e^s(\hat{B})$	$e^i(\hat{B})$
Skewness	-0.626	-0.494
Kurtosis	4.116	14.448
JB-Test	0.00	0.00

Skewness, Kurtosis and p-value of the Jarque-Bera test of the estimated structural shocks from the second block.

technique does not change our main results.

Third, we increase the number of lags to  $p = 4$ . Table 8 shows the skewness, kurtosis and Jarque-Bera test results for the estimated structural shocks. As becomes evident from Figure 7, the

Table 8: Moments of estimated structural shocks in the block-recursive SVAR with a lag order of  $p = 4$ .

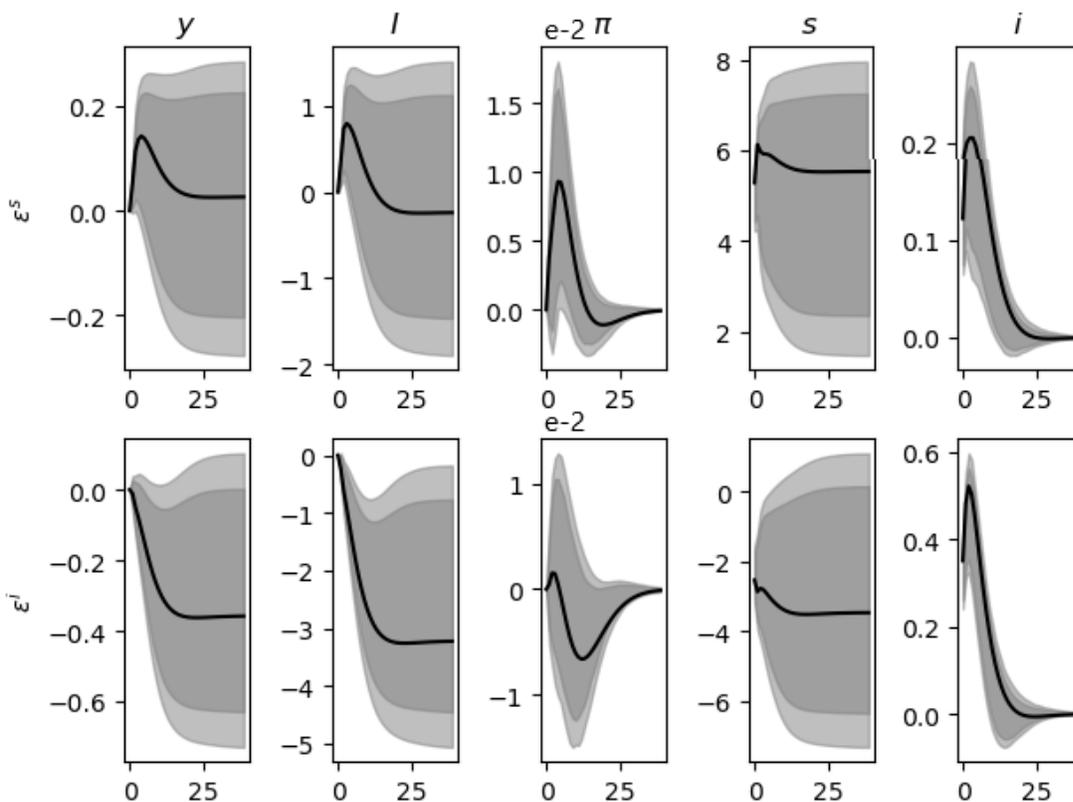
	$e^s(\hat{B})$	$e^i(\hat{B})$
Skewness	-0.832	-0.386
Kurtosis	4.755	11.982
JB-Test	0.00	0.00

Skewness, Kurtosis and p-value of the Jarque-Bera test of all estimated structural shocks from the second block.

estimated simultaneous interaction is again similar to our baseline specification. However, the confidence bands are large and we cannot reject the long-run neutrality of monetary policy with respect to stock prices, but on the other side there is not much evidence for it either, as due to the broad confidence bands many other long-run outcomes are possible.

Fourth, we consider the inclusion of commodity price inflation (named  $\pi_c$ ), defined as the logarithmic difference in the producer price index (also taken from the FRED). For instance, Bjørnland and Leitemo (2009) argue that the inclusion of commodity price inflation helps to get more reliable results regarding the reaction of the overall inflation rate with respect to the structural shocks and thus should be included into the SVAR specification. We assume that commodity price inflation shocks are ordered in the first block, so commodity price inflation cannot react immediately to stock market and monetary policy shocks, but to all other shocks in the first block.

Figure 6: Impulse responses to stock market and monetary policy shocks in the block-recursive SVAR with the PML estimator (see Gouriéroux et al. (2017)).



The figure shows the responses to one standard deviation shocks. The columns  $y$ ,  $I$ , and  $s$  show the cumulative responses of investment growth, output growth, and stock returns. Confidence bands are 68% and 80% bootstrap bands with 1000 replications in the bootstrap algorithm.

Table 9 shows the skewness, kurtosis and Jarque-Bera test results for the estimated structural shocks in this specification. Figure 8 shows the resulting IRFs. As it can be seen, the inclusion

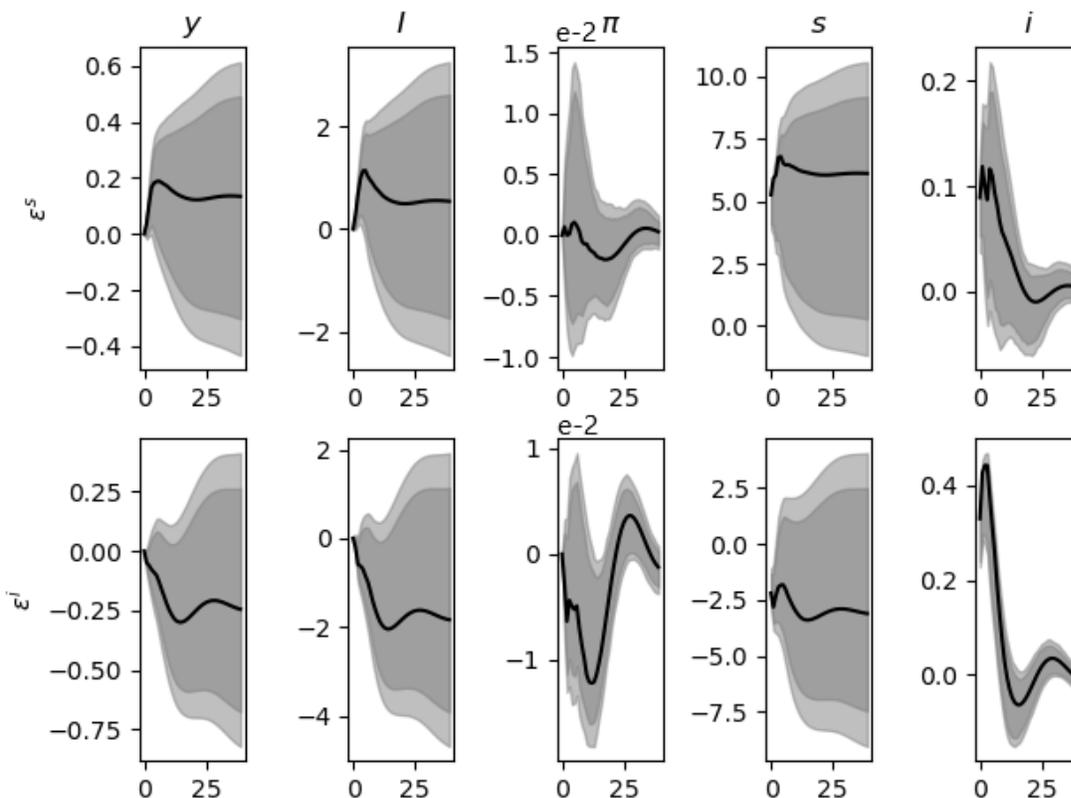
Table 9: Moments of estimated structural shocks in the block-recursive SVAR with commodity price inflation included in the first block.

	$e^s(\hat{B})$	$e^i(\hat{B})$
Skewness	-0.615	-0.7
Kurtosis	4.071	13.757
JB-Test	0.00	0.00

Skewness, Kurtosis and p-value of the Jarque-Bera test of the estimated structural shocks from the second block.

of commodity price inflation has no impact on the estimated interaction of monetary policy and

Figure 7: Impulse responses to stock market and monetary policy shocks in the block-recursive SVAR with a lag order of  $p = 4$ .

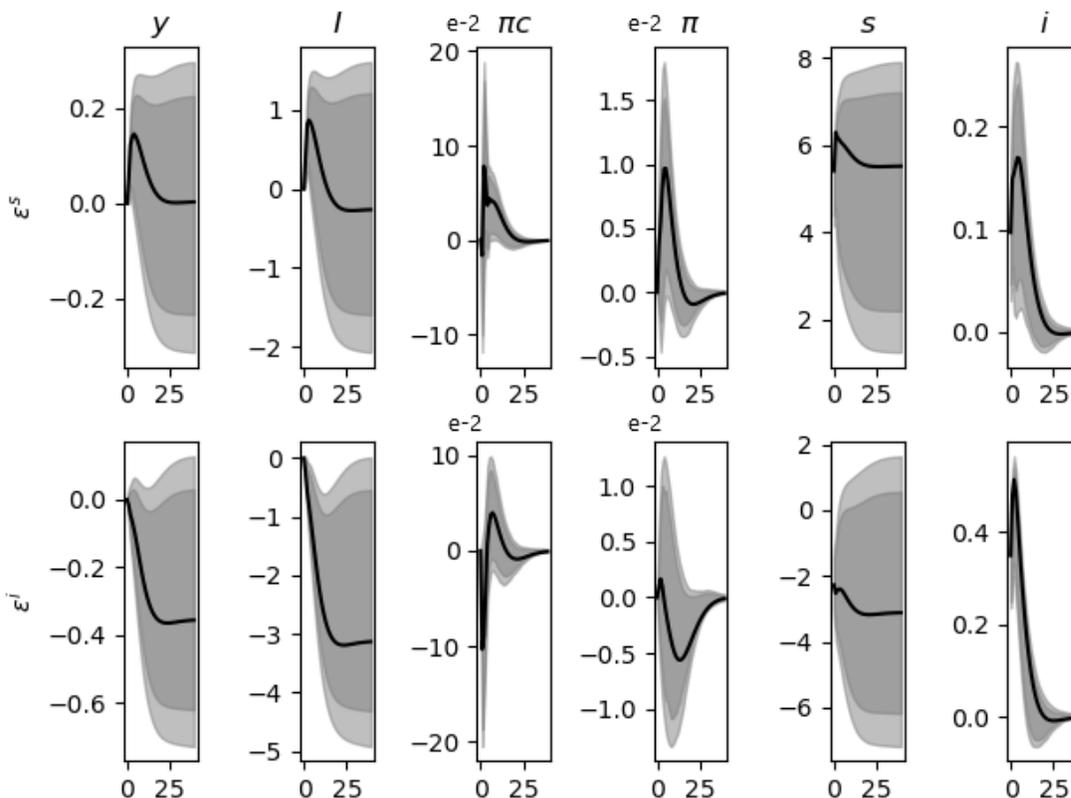


The figure shows the responses to one standard deviation shocks. The columns  $y$ ,  $I$ , and  $s$  show the cumulative responses of investment growth, output growth, and stock returns. Confidence bands are 68% and 80% bootstrap bands with 1000 replications in the bootstrap algorithm.

stock markets compared to Section 4.2.

Fifth, we exclude all observation from 2007Q4 onward from the sample to have a similar observation period as Bjørnland and Leitemo (2009). Table 10 shows the skewness, kurtosis and Jarque-Bera test results for the second block shocks. Figure 9 shows the resulting IRFs. Our main results remain unchanged. We again find mixed evidence regarding long-run neutrality of monetary policy. In particular, the long-run response of investments is negative. However, the long-run response of stock prices reverts back to zero, but is again associated with a large confidence band.

Figure 8: Impulse responses to stock market and monetary policy shocks in the block-recursive SVAR with commodity price inflation included in the first block.



The figure shows the responses to one standard deviation shocks. The columns  $y$ ,  $I$ , and  $s$  show the cumulative responses of investment growth, output growth, and stock returns. Confidence bands are 68% and 80% bootstrap bands with 1000 replications in the bootstrap algorithm.

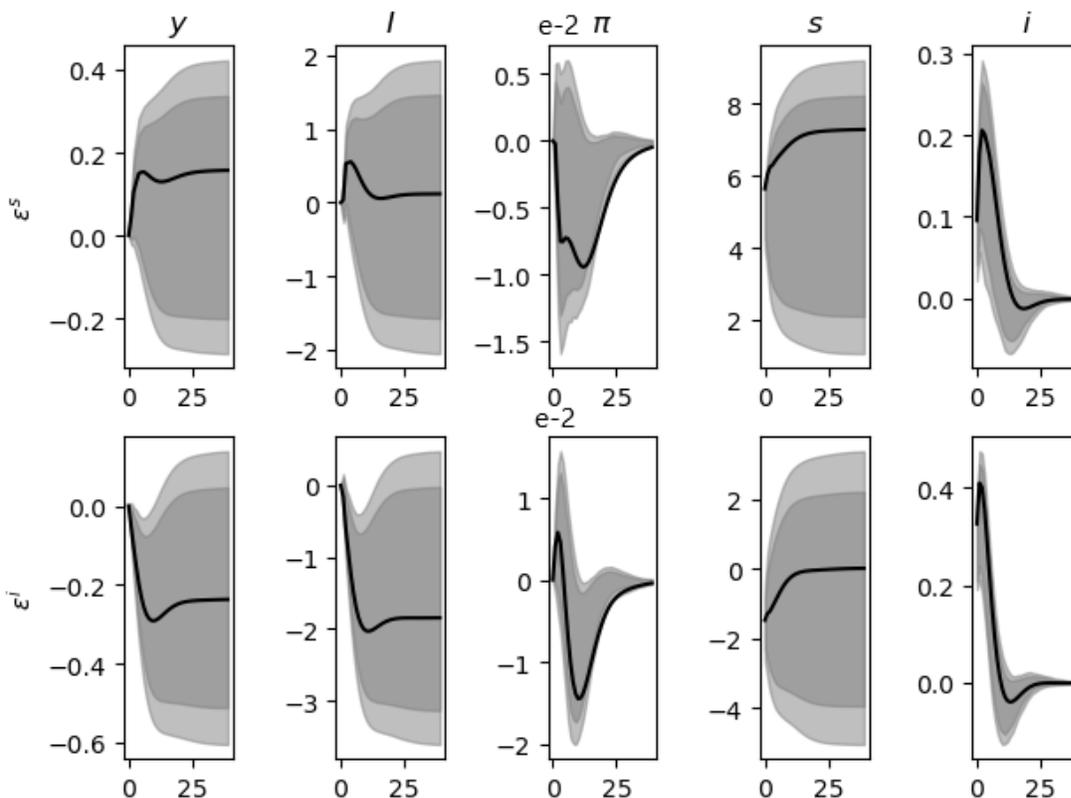
Table 10: Moments of estimated structural shocks in the block-recursive SVAR with the observation period 1983Q1-2007Q3.

	$e^s(\hat{B})$	$e^i(\hat{B})$
Skewness	-0.719	-2.032
Kurtosis	4.584	12.819
JB-Test	0.00	0.00

Skewness, Kurtosis and p-value of the Jarque-Bera test of the estimated structural shocks from the second block.

At last, we check on the relevance of the time trend included in our specification. Table 11 shows the skewness, kurtosis and Jarque-Bera test results for this specification. Figure 10 shows the impulse responses of the stock price and FFR to a stock market and monetary policy shock

Figure 9: Impulse responses to stock market and monetary policy shocks in the block-recursive SVAR with the observation period 1983Q1-2007Q3.



The figure shows the responses to one standard deviation shocks. The columns  $y$ ,  $I$ , and  $s$  show the cumulative responses of investment growth, output growth, and stock returns. Confidence bands are 68% and 80% bootstrap bands with 1000 replications in the bootstrap algorithm.

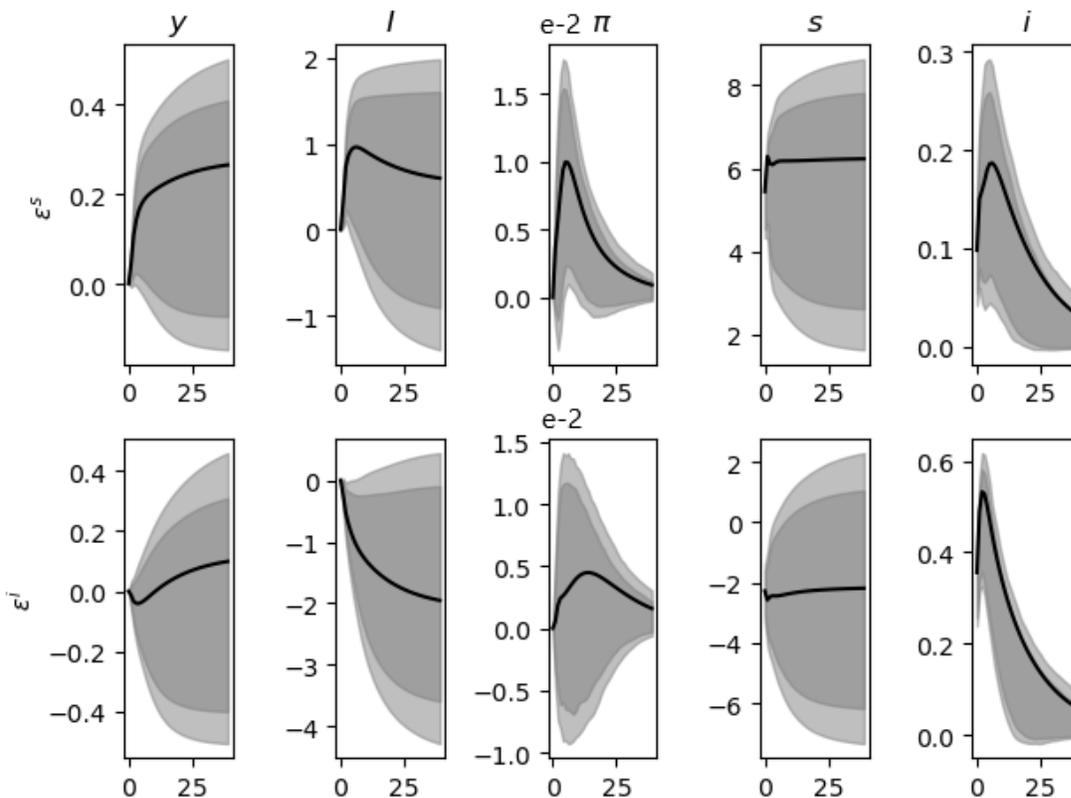
Table 11: Moments of estimated structural shocks in the block-recursive SVAR without the linear time trend.

	$e^s(\hat{B})$	$e^i(\hat{B})$
Skewness	-0.593	-0.528
Kurtosis	4.117	14.888
JB-Test	0.00	0.00

Skewness, Kurtosis and p-value of the Jarque-Bera test of the estimated structural shocks from the second block.

under the specification without a linear time trend. The main qualitative and quantitative insights remain unchanged. However, the confidence band of the stock price response to a monetary policy shock is a broader, thus the response becomes insignificant earlier and there is no conclusive

Figure 10: Impulse responses to stock market and monetary policy shocks in the block-recursive SVAR without the linear time trend.



The figure shows the responses to one standard deviation shocks. The columns  $y$ ,  $I$ , and  $s$  show the cumulative responses of investment growth, output growth, and stock returns. Confidence bands are 68% and 80% bootstrap bands with 1000 replications in the bootstrap algorithm.

answer about the long-run behavior. Furthermore, the output answer becomes positive in the long-run, which is something one would not expect after a contractionary monetary policy shock.

To summarize the results from our robustness checks, we find that

- i) the on impact effect of a monetary policy shock on stock returns is negative,
- ii) the on impact effect of a stock market shock on the FFR is positive and,
- iii) the long-run impact of monetary policy shocks on stock prices is highly uncertain.



