

# Numerical simulation of low cycle fatigue behavior, combining the phase-field method and the Armstrong-Frederick model

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The present work couples the phase field method of fracture to the Armstrong-Frederick model of plasticity with the kinematic hardening. The chosen approach inherits the advantages of both techniques and is aimed at the study of low cycle fatigue effects in ductile materials. However, the numerical implementation of this promising concept brings with it several challenges, such as the definition of a unique framework for both setups, the derivation of coupled evolution equations, the distinction between tension and compression mode and the development of a computationally efficient algorithm. In the approach developed, the derivation of evolution equations uses the minimum principle of the dissipation potential. This step requires the expression of the dissipation potential of the classic Armstrong-Frederick model in terms of the internal variable rates by using the Legendre transformation. The model is eventually implemented in the FE-program and applied in order to investigate the life-time of the cold-formed carbon steel and the cold-formed stainless steel.

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## 1 Introduction

Fatigue failure significantly influences the safety measures during the service life of engineering structures. It particularly holds for structures subjected to alternate loading with high amplitudes, which frequently occurs for transportation vehicles. The investigation of fatigue effect by using the phase field method has gained attention in the recent literature, however, it is mainly related to the high cycle fatigue (HCF) and very high cycle fatigue (VHCF). Here, the Wöhler curves and the Paris' law are taken as basis. In contrast to HCF and VHCF, extensions of the phase field method to capture the main features of low cycle fatigue (LCF) are still an open issue. A suitable representation of low-cycle fatigue must include cyclic plasticity, which is the main motivation to use the Armstrong-Frederick approach for this purpose [1].

## 2 Coupling of the Armstrong-Frederick model to the phase-field approach

The present model uses the phase field framework proposed by Miehe et al. [2] as a basis. The first extension is related to the definition of the coupled free energy  $\Psi^c$  including elastic contributions and a term due to the hardening ( $\Psi^{\text{hard}}(\xi)$ )

$$\Psi^c(\epsilon^e, d, \xi) = \omega \Psi_+^{\text{el}}(\epsilon^e) + \Psi_-^{\text{el}}(\epsilon^e) + \bar{\omega} \Psi^{\text{hard}}(\xi), \quad \Psi^{\text{hard}}(\xi) = \frac{1}{2} c \xi : \xi. \quad (1)$$

Here,  $\xi$  is a strain-like internal variable,  $\epsilon^e$  is the elastic part of the strain tensor,  $d$  is the damage variable and  $c$  represents the kinematic hardening modulus. The elastic energy is split into a tension part  $\Psi_+^{\text{el}}$  and a compression part  $\Psi_-^{\text{el}}$  and the reduction of the tension energy due to damage is achieved by introducing function  $\omega = [(1-d)^2 + k_d]$  with the numerical constant  $k_d$ . Function  $\bar{\omega}$  is introduced to distinguish the pure compression mode from the pure tension and from the mixed mode [1].

In a further step, the coupled dissipation potential is defined as follows:  $\Phi^c = \Phi^d + \bar{\omega} \Phi^{\text{AF}}$ . It consists of two contributions: the damage potential  $\Phi^d$  and the dissipation potential of the Armstrong-Frederick model  $\Phi^{\text{AF}}$ . The damage influence on the dissipation functional related to the plastic deformations follows the same argumentation as in the case of the hardening energy such that  $\Phi^{\text{AF}}$  is weighted by the function  $\bar{\omega}$ . The dissipation potential related to damage  $\Phi^d$  is expressed in terms of the damage variable  $d$ , its gradient  $\nabla d$  and their respective rates [2]

$$\Phi^d = \Phi + P(\dot{d}) = g_c \dot{\gamma}(\dot{d}, \nabla \dot{d}, d, \nabla d) + \frac{k_p}{2} \langle \dot{d} \rangle_-^2, \quad \gamma(d, \nabla d) = \frac{1}{2l} d^2 + \frac{l}{2} |\nabla d|^2. \quad (2)$$

In this definition, potential  $\Phi$  depends on the critical Griffith-type fracture energy  $g_c$  and on the rate of the crack surface density function  $\dot{\gamma}$ . Condition  $\dot{d} \geq 0$  leading to an increasing damage evolution is implemented by introducing penalty term  $P(\dot{d})$ , where symbol  $\langle \bullet \rangle_- = (\bullet - |\bullet|)/2$  represents negative Macaulay brackets and where  $k_p$  is the penalty constant.

The dissipation potential of the Armstrong-Frederick model depends on the thermodynamical forces in its original form [3]. However, the coupling of two models requires this potential to be expressed in the space of internal variables and their rates, which is achieved by applying the Legendre transformation [1]. The current contribution only presents the final expression

$$\Phi^{\text{AF}}(\dot{\epsilon}^p, \dot{\xi}) = \bar{\sigma}_Y \|\dot{\epsilon}^p\| + \frac{b}{2} \|\dot{\epsilon}^p\|^2 + \frac{b}{2} \|\dot{\xi}\|^2 + b \dot{\epsilon}^p : \dot{\xi}, \quad \bar{\sigma}_Y = \sqrt{2/3} \sigma_Y, \quad (3)$$

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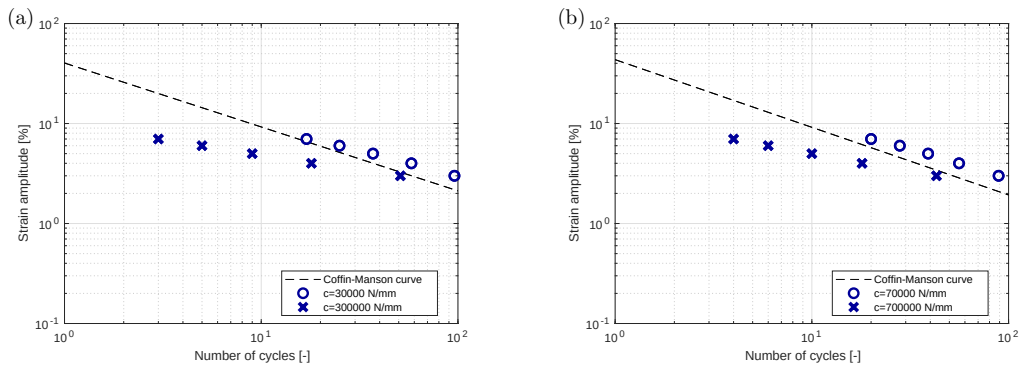
where  $\sigma_Y$  is the yield limit and where  $b$  denotes the pseudo-viscoelasticity. Equations (1)-(3) serve as basis for the application of the minimum dissipation potential approach (MDP) where the minimization of the Lagrange function  $\mathcal{L}^{\text{MDP}} = \Psi^c + \Phi^d + \Phi^{\text{AF}}$  yields driving forces

$$\mathbf{q}_{\epsilon^p} = \bar{\sigma} = \frac{\partial \mathcal{L}^{\text{MDP}}}{\partial \dot{\epsilon}^p} = \bar{\omega} \left[ \bar{\sigma}_Y \frac{\dot{\epsilon}^p}{\|\dot{\epsilon}^p\|} + b \dot{\epsilon}^p + b \dot{\xi} \right], \quad \mathbf{q}_\xi = \bar{\chi} = \frac{\partial \mathcal{L}^{\text{MDP}}}{\partial \dot{\xi}} = \bar{\omega} \left[ b \dot{\epsilon}^p + b \dot{\xi} \right]. \quad (4)$$

The latter expressions enable recovering of evolution equations for internal variables  $\epsilon^p$  and  $\xi$  which are numerically implemented in a predictor-corrector scheme at the Gauss point level.

### 3 Numerical results

The capabilities of the model are demonstrated on examples studying the life-time of the carbon and stainless steels. The simulations are performed for constant strain amplitudes until complete failure of the sample at a time increment  $\Delta t = 4.28 \times 10^{-4}$  s and 2000 time steps per load cycle. The load amplitudes amount to 3%, 4%, 5%, 6% and 7% in different tests also studying the influence of the value of kinematic hardening parameter  $c$ . The dogbone sample geometry typical of tensile tests is assumed. Moreover, a material defect is generated in the center of the sample in order to stipulate the crack initiation. This effect is simulated by reducing the Young modulus and the yield limit to 75% of their actual values. The defect is placed in the material bulk, since it is assumed that post-processing techniques can significantly reduce the surface roughness. The obtained numerical results are compared to experimental data as shown in Fig. 1. Experimental values stem from the work by Nip et al. [4] where Coffin-Manson curves are used to present the relationship between the applied strains and the number of cycles up to the total failure. In all cases, an excellent agreement between the experimental data and the simulations is observed, particularly for higher strains. The linear dependence between the applied strains and the number of cycles is clearly reflected in the numerical results, as well as the observation that a higher hardening parameter corresponds to a faster damage evolution.



**Fig. 1:** Comparison of the numerical results to the experimental Coffin-Manson curve for (a) carbon steels with  $c = 30000$  N/mm and  $c = 300000$  N/mm, (a) stainless steels with  $c = 70000$  N/mm and  $c = 700000$  N/mm.

### 4 Conclusion

This contribution presents a combination of the phase-field model for damage and the Armstrong-Frederick model for plasticity. The derivation of evolution equations for inelastic deformations is based on a Legendre transformation, which enables the capturing of both models and their interaction in one framework. With the formulation at hand, accurate life time predictions for materials under cyclic load are possible, which is demonstrated by simulating several kinds of steels. Here, an excellent qualitative and quantitative agreement of experimental and numerical data has been achieved.

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