Inheritance Taxation, Unemployment, and Asset Pricing in Frictional Labour Markets

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Veröffentlichung als Dissertation in der Wirtschaftswissenschaftlichen
Fakultät der Technischen Universität Dortmund

Dortmund, 13. März 2023

Acknowledgment

I wrote this thesis as a doctoral researcher at TU Dortmund and the Ruhr Graduate School in Economics (RGS Econ). Throughout my research, I have benefited from the help of kind and generous people, without whom this dissertation would not have been possible.

First and foremost, I owe thanks to my doctoral advisor Prof. Dr. Philip Jung for his invaluable advice, support and guidance. Further, I thank my dissertation committee, Prof. Dr. Ludger Linnemann and Prof. Galina Zudenkova, Ph.D., for their time dedicated to reviewing this dissertation.

For financial support, I want to thank TU Dortmund, the Ruhr Graduate School in Economics (RGS Econ), and the Deutsche Forschungsgemeinschaft (DFG). I am grateful for the opportunity to partake in courses and conferences organized by RGS Econ.

I want to thank my colleagues at TU Dortmund and RGS Econ for their mental support, helpful comments and fruitful discussions. Special thanks go to my friend and colleague, Philipp Korfmann, with whom I had the honor to share an office and the long and difficult road towards becoming PhDs.

Most of all, I am indebted to my friends and family for their love and patience. I want to thank my parents Petra and Otmar, my brother Nicolas, and my partner Linda.

Contents

List of Figures

List of Tables

Introduction

This thesis examines the impact of labour market frictions on (i) the optimal inheritance taxation of family-owned firms and (ii) investment and asset prices and their co-movement with labour market flows. The equilibrium unemployment model attributed to Peter A. Diamond, Dale T. Mortensen and Christopher A. Pissarides (DMP) has become the paradigm of the macroeconomic study of the labour market.[1] The DMP model explains the coexistence of vacant jobs and unemployed workers, how institutions shape the labour market, and how adverse shocks propagate in the macroeconomy, raising unemployment instead of lowering wages. This thesis shows that the framework is also a fruitful avenue to study optimal taxation (Chapter 1), and asset pricing (Chapters 2 and 3).

In the standard version of the neoclassical labour market, large numbers of firms and households trade in a frictionless market with complete information. Hence, a match between a worker and an employer does not yield a match-specific rent. Workers earn their labour's product and should a match separate, they will find an identical job instantaneously. There is no discrete effect of a job loss on earnings.[2] On the demand side, firms rent labour on a spot market and earn no profit from it, i.e. the firm value does not reflect human capital embedded in the firm. Both implications take issue with practical experience. Firstly, workers fear job loss and, at the same time, firms strive to retain experienced workers because they are hard to

[1] The authors produced a series of papers, earning the Nobel Memorial Prize in Economic Studies. See Pissarides (2000) for the standard treatment of the DMP model and Rogerson et al. (2005) for a comprehensive survey of search-theoretic labour market models.

[2] Earnings differentials following from human capital have been examined in neoclassical labour markets, as well (e.g. Becker, 1994). Robinson (1969) explains earnings differentials in a monopsonistic labour market.

substitute. We observe considerable wage dispersion across firms (Goux and Maurin, 1999). Secondly, the market value of a firm very much depends on its workforce.

Labour market frictions include, among others, the non-transferability of human capital across jobs, imperfect information, and the costs and risks of hiring. In the DMP model, match-specific rents arise naturally from costly stochastic search. Firms and workers bargain over these rents, implying that the wage will typically exceed the worker's outside option (the worker's expected earnings outside of this match). A wage that exceeds the outside option becomes worth protecting; for example, via policies that promote the preservation of long-standing matches. On the demand side, when a firm derives profits from labour input, the stock of employment becomes a determinant of the firm's value and equity price. From an investment perspective, the costs of search are employment adjustment costs. Like capital adjustment costs in Tobin's q theory, equity prices reflect the costly procurement of workers (Tobin, 1969; Merz and Yashiv, 2007).

This thesis presents three self-contained chapters, building on frictions as major improvements of our understanding of the labour market. Chapter 1 studies optimal inheritance taxation of family-owned businesses, focussing on match-specific earnings losses that may be averted by granting tax deductions for business assets. Chapters 2 and 3 study the co-movement of asset prices and labour market flows, following from costly labour adjustment. The chapters seek to establish a framework that consistently solves the equity premium puzzle and the unemployment volatility puzzle.

Chapter 1 studies the optimal inheritance taxation of family businesses in an analytical model, providing a rationale for generous deductions found in Germany's legal framework. Using the German inheritance and gift tax statistic, the Chapter first establishes several empirical observations and connects those to the German legal framework. I find that inheritances of business assets are unequally distributed, even compared to the dispersion of inheritances in general. A more valuable company has a higher probability of an intra-family succession. Effective inheritance tax rates are not progressive because testators of large estates can apply a few deductions. The most prominent is the §13a tax deduction for business assets: until 2009, the law allowed to deduct 40% of business assets if the heir held onto the company for at least five years. In 2009, §13a was expanded to 85% and 100% if the company is kept for five or seven years, respectively. Lastly, I find that testators accompany businesses with a sizeable stock of liquid assets. Those liquid assets slightly increase the probability of an intra-family succession.

Chapter 1 presents a tractable model that motivates the favourable tax treatment, offering two rationales. The first rationale states that, in incomplete capital markets, paying the tax may demand divestment and lay-offs. Laid off workers suffer sizable earnings losses as they loose job- and industry-specific human capital. Quantitatively, this argument is weak: testators accompany business inheritances with a large stock of liquid assets ready to pay the tax and inheritance taxes can be deferred and paid in instalments. The government steps in as a lender when the private financial sector does not suffice. These findings are consistent with Holtz-Eakin et al. (2001) who study U.S. data.

The second rationale is that deductions incentivize intra-family firm succession, which has a positive externality on the earnings of workers employed in the firm. It rests on two major incomplete market assumptions: first, the firm must not be marketable to outside buyers; second, it must not be possible or profitable to hire an external manager. Under these assumptions, the family-owned firm is dissolved if there is no intra-family succession. An externality arises from the career choice of the company heir. If she decides to pursue a career outside of the parent's firm, the firm is dissolved and workers suffer earnings losses as they loose job- and industry-specific human capital. The social planner can set a deduction for business assets, such that the heir internalizes the cost to workers in her career choice. I analytically derive an optimal tax formula with only a small number of function arguments and quantitatively apply the formula to German data. The optimal tax rate for small businesses is close to zero. However, it is confiscatory for large firms that are marketable: if the heir's career choice is irrelevant for the continuation of the business, there is no externality and no reason for deductions.

Chapter 1 contributes to the literature on inheritance and wealth taxation, and to the taxation of family-owned firms in particular. It is most closely related to Grossmann and Strulik (2010) who study optimal inheritance taxation in an overlapping-generation variant of the span-of-control framework (Lucas, 1978). They stress the trade-off, faced by lawmakers, between capital destruction and the Carnegie conjecture (Holtz-Eakin et al., 1993): taxation raises inter-generational capital destruction but taxation decreases the fraction of untalented entrepreneurs who manage large companies. This idea is closely related to Guvenen et al. (2017) who propose a wealth tax in place of the capital income tax to raise the efficiency of capital allocation. In the neoclassical labour market model, workers benefit from wealth and inheritance taxation via a general equilibrium effect: taxation diverts capital from untalented to talented entrepreneurs and the aggregate marginal product of labour (the wage) rises.

In contrast, frictions introduce a detrimental partial equilibrium effect of capital reallocation on earnings because match separations eradicate match-specific rents. A displaced worker looses job-specific human capital, faces search costs, might become long-term unemployed, might settle for a job that pays a lower wage, or has to relocate. In an influential contribution, Jacobson et al. (1993) show that displaced workers suffer sizeable long-term costs.[3] Wage bargaining, standard in the search theoretical literature, allows to model these discrete earnings losses in a partial equilibrium framework.

Chapters 2 and 3 study the co-movement of output, consumption, asset prices, and labour market flows. In the simplest neoclassical economy, a competitive firm rents labour and capital on spot markets, bundles inputs, makes no profits and has a firm value of zero. With frictions, a firm is not just a collection of rental contracts: physical and human capital embedded in a firm are reflected in the firm's value. In the q theory of investment, equity prices reflect the replacement costs of capital when capital adjustment is costly (Tobin, 1969; Hayashi, 1982; Cochrane, 1991). Correspondingly, Merz and Yashiv (2007) show that equity prices reflect the replacement costs of employment, when employment adjustment is costly. Merz and Yashiv present a competitive equilibrium model that does not allow for the coexistence of involuntary unemployment and open vacancies, and the model's performance of matching labour market data or wages is questionable: in a competitive equilibrium, productivity shocks cause fluctuations in wages rather than employment because labour supply is - in line with empirical estimates - quite inelastic (Cahuc et al., 2014, p.582). This contradicts empiri-

[3]See Bertheau et al. (2022) for a recent review of estimates.

cal evidence that productivity shocks have a stronger impact on employment than on wages (Rogerson and Shimer, 2011).

The DMP framework avoids these shortcomings, dropping the assumption of competitive wages and introducing costly search and stochastic matching. A firm faces hiring costs, subsuming the cost of search, screening, interviewing, formal and informal training, red tape, and a productivity gap between new workers and tenured workers.[4] In the framework, the cost of filling a vacancy is time-varying: in a boom, many firms search for scarce unemployed workers and the cost of procuring a worker is high; in a recession, few firms search even though unemployed workers are abundant and the procurement costs are low. The firm value, which equals employment and capital instalment costs, inherits this procyclicality. This cyclical co-movement of unemployment and equity prices is mirrored in data, where we observe a strong correlation of stock price indices and labour market transition rates.

When we want to numerically analyse this link between unemployment and stock prices, we run into two major issues, or puzzles: the equity premium puzzle and the unemployment volatility puzzle (Shimer puzzle). At the latest since Mehra and Prescott (1985), it has been well known that the workhorse neoclassical growth model struggles to provide a theoretical basis for the equity premium observed in data.[5] Since Shimer (2005), if not before, it it has been well known that the workhorse DMP model fails to replicate the fact that unemployment is about seven times more volatile than GDP. Chapters 2 and 3 seek to simultaneously solve the two puzzles.

These chapters contribute to the growing literature on the connection of asset prices and labour markets in the DMP framework. Hall (2017) and Kehoe et al. (2019) study how fluctuations in the stochastic discount factor can raise unemployment volatility and drive the correlation of employment and asset prices. Other authors have introduced disaster risk in the DMP model (Wachter and Kilic, 2018; Petrosky-Nadeau et al., 2018; Bai and Zhang, 2021).

Chapter 2 examines and ultimately falsifies the application of two popular theories in the DMP framework: the disaster risk theory motivates the risk

[4]Barron et al. (1997, 1999); Mühlemann and Pfeifer (2016); Mühlemann and Strupler Leiser (2018); Silva and Toledo (2009) provide estimates of the cost of search.

[5]For an overview of macro-finance theory, see Cochrane (2011, 2017).

premium with rare deep drops of consumption and equity prices, and the long-run risk theory, which assumes that growth rates have a small, highly persistent component.

In the disaster risk literature, investors fear rare disasters, such as a collapse of the financial system, the Great Depression, wars, pandemics or natural disasters because they reduce both equity prices and consumption at the same time. To carry the risk of excessively low returns during periods of low consumption, investors demand a premium. Disaster risk was popularized in macro-finance models by Rietz (1988), Barro (2006, 2009), Gourio (2012), and Ghosh and Anisha (2012) and applied to the DMP framework by Wachter and Kilic (2018) and Petrosky-Nadeau et al. (2018). The latter is most closely related to the model studied in Chapter 2: disasters are endogenously generated in a globally-solved DMP framework with cyclical fluctuations.

I derive the global solution of a DMP model with cyclical fluctuations, recursive preferences, endogenous separations and wage rigidity and use the simulated method of moments to estimate the model to match key moments of output and labour market variables. Via a small surplus calibration (Hagedorn and Manovskii, 2008; Ljungqvist and Sargent, 2017), the model offers a robust solution of the Shimer puzzle, but it produces no equity premium. While the model can generate disasters endogenously, these are too rare and too small to be a sufficient motivation for a risk premium. Why does the related model by Petrosky-Nadeau et al. (2018) solve the equity premium puzzle? In Section 2.3.4, I show that their parametrization to historical international panel data generates disasters far too frequently.

The second model studied in Chapter 2 assumes a productivity growth process with a small, stochastic, highly persistent component. It builds on the long-run consumption risk theory (LRR), conceptualized by Kandel and Stambaugh (1991) and popularized by Hansen et al. (2008), Bansal and Yaron (2004) and Bansal et al. (2012). These LRR models are consumption-based asset pricing models, that assume consumption and returns to be stochastic endowments. In contrast, Croce (2014) and Chapter 2 apply long-run risk to a general equilibrium framework, substituting exogenous consumption and dividend risk with productivity risk. To my knowledge, this chapter offers the first attempt to globally solve and carefully estimate long-run risk in the

DMP framework.[6]

Compared to the DMP model with cyclical fluctuations, I find that the long-run risk model is a slight improvement in terms of asset prices but inferior in the labour market dimension. In simulations, the equity premium is somewhat higher but still insufficient compared to data. The long-run risk model solves the Shimer puzzle only if we assume very strong wage rigidity that leads to counterfactual separation rates.

Chapter 2 concludes with an examination of the transmission of shocks and explains why the two models fail to solve the equity premium puzzle. In both models, equity returns and marginal utility are negatively correlated, i.e. equity is a risk not an insurance. Yet, investors do not demand a sizeable premium to hold equity because i) equity returns are not sufficiently volatile and ii) the conditional volatility of marginal utility is too small. Intuitively, investors perceive holding equity as a risk, but neither equity nor the economy are perceived as very risky. Similar problems are known since Rouwenhorst (1995), at the latest. In summary, the models studied in Chapter 2 must be improved in two dimensions. First, we want to raise the conditional volatility of marginal utility without raising its unconditional volatility. Second, we want to raise the volatility of equity.

Chapter 3 exploits these insights, introducing slow-moving habits and capital adjustment costs into the DMP framework. Habits, in style of Campbell and Cochrane (1999), address the insufficient conditional volatility of marginal utility, as households now derive utility from excess consumption relative to habit. Risk aversion becomes time-varying: investors, who consume close to their habit, become more risk-averse even for a low parameter of risk aversion. The time-varying risk aversion raises the conditional volatility of marginal utility in response to small perturbations of consumption. Investors demand a premium for holding risky equity. Still, the unconditional volatility of consumption and the volatility of the risk-free rate remain at empirically plausible levels. Capital adjustment costs address the insufficient volatility of equity prices. In this framework, the equity price has two components: the replacement costs of the capital stock (Tobin's q) and the

[6]In a working paper version, Kehoe et al. (2019) argue that the long-run risk model, solved by perturbation, does not generate sufficiently strong discount factor volatility to raise the unemployment volatility significantly.

replacement costs of employment (vacancy-posting costs). Both costs are time-varying and together they generate a volatility of equity returns in line with empirical estimates.

In simulations, the model outlined in Chapter 3 reaches key targets: it yields a large risk premium, while maintaining a low consumption and interest rate volatility. It robustly solves the unemployment volatility puzzle.

Similar to a filter, I match the parametrized model to empirical time series. The model is effective in matching the unemployment series, equity prices, and the equity premium. The model replicates the high correlation of employment and equity prices, that motivated Chapters 2 and 3. However, the exercise overestimates the volatility of consumption and the risk-free rate. I conclude that assuming a monetary friction, together with a Taylor rule are likely to raise the model's goodness-of-fit.

CHAPTER 1

Inheritance Taxation of Mature Family-Owned Firms

Chapter Abstract

Using administrative tax data, this paper shows that generous deductions for family-owned businesses reduce effective inheritance tax rates in Germany. The tax code does not achieve horizontal equity. A tractable model rationalizes the favourable tax treatment under incomplete capital markets. First, taxation may demand divestment and lay-offs, but empirically this is rarely necessary. Secondly, a firm that is not marketable and cannot hire an external manager needs an intra-family succession to survive. Liquidation causes considerable earnings losses for employees with match-specific human capital. Inheritance tax deductions for business assets let firm heirs internalize these earnings losses and incentivize succession to the parents' business. I analytically derive an optimal tax formula with only a small number of function arguments; I then quantitatively apply the model to German data. In the baseline calibration, the optimal tax rate for small businesses is close to zero, while it is confiscatory for large firms.

1.1 Introduction

In light of a surge in wealth inequality, arguments for the introduction and expansion of capital and inheritance taxes have been gaining ground, both in academia and politics. Beginning in the second half of the 21st century, Piketty and Zucman (2015) observe rising income and wealth inequality in developed nations. While some degree of wealth inequality is a manifestation of meritocracy, wealth inequality is deemed unjust when rooted in luck instead of merit and talent. To combat persistent and increasing wealth inequality, economists propose capital gains taxes (Straub and Werning, 2014; Piketty and Saez, 2012) and inheritance taxes (Piketty and Saez, 2013; Farhi and Werning, 2013; De Nardi and Yang, 2016).

In those countries that levy inheritance or wealth taxes, we observe great possibilities for deductions and deferments for taxpayers. This holds especially true for business owners. Germany, for example, grants sizeable deductions for business heirs if they retain ownership of the inherited company and its workers: until 2009, the law allowed for a 40% deduction in business assets if heirs held onto the company for at least five years. In 2009, the deduction was expanded to 85% and 100% if the company is kept for five or seven years, respectively. Why do lawmakers grant large deductions for the inheritance and gift tax, which is in principle a progressive tax? In practice, bequest and wealth taxes are met with scepticism from politicians, both on the left and right of the political spectrum, who fear that asset taxation will affect the self-employed, mom-and-pop shops in the U.S. and Germany's *Mittelstand*. Taxation makes firm succession less appealing and can tighten borrowing constraints of entrepreneurs and force business owners to sell (parts of) the company, ultimately leading to job destruction and losses of human capital.

To the best of my knowledge, I am the first to provide a positive theory which justifies deductions for business capital based on firm-specific human capital of entrepreneurs and match-specific human capital of workers embedded in old firms. The managers of small, mature firms hold firm-specific human capital: they know their customers, workforce, local politicians, and the product. Asymmetric information prevents outsiders from taking over family-controlled firms. Acquiring this firm-specific human capital is very costly to outsiders, but less so for the owner's children. In a traditional family-owned firm, children are expected to accumulate this capital, while its acquisition can be unprofitable or too risky for outsiders. My theory focuses on the employees of mature companies. Tenured workers have accumulated match-specific human capital, which is not easily carried over to a new employer. A tenured worker suffers from considerable and permanent wage penalties in the event of a lay-off. Prominently, Jacobson et al. (1993) estimate that, five years on, a laid off high-tenure worker has suffered a 25% wage penalty. I consider a partial equilibrium model which accounts for workers' earnings losses and the human capital costs of becoming the company's manager. In this scenario, an entrepreneur owns an established firm and employs high-tenure workers. She bequeaths the firm and some liquid assets to her offspring. The child has to make a discrete career choice: either learn to become the firm's manager or "break free" and choose another career. The former allows her to earn the firm's profits, but may come at large opportunity costs. If she decides to relinquish control, however, the firm will be dissolved and matched workers will incur earnings losses. In this decision, the company heir does not take into account the fact that workers will be subjected to wage penalties if she closes the company. If she follows the entrepreneurial career path, she uses equity, inherited cash and debt subject to a borrowing constraint to choose a firm size. My theory provides two rationales for deductions. First, bequest taxation reduces equity: if capital and employment are complements, a firm heir will have to lay off workers to finance her tax liabilities if no perfect capital market is available and her assets are not sufficient. Secondly, inheritance taxation of business capital decreases the value of a firm, reducing the successor's propensity to follow in the parent's footsteps: deductions for business assets incentivize firm continuation and let the company heir internalize workers' earnings losses if the firm is dissolved.

My theory provides a rationale for deductions of business assets that is limited to small firms and is predicated on two important assumptions: one, that the firm cannot be managed by an external manager and, two, that the firm is not marketable. First, I assume that the firm's owner cannot hire an external manager. Unlike external managers, the company heir has been educated in the firm and knows its product, employees and workers. For an external manager acquiring this firm-specific human capital is too costly given the small profits this company generates.[1] Secondly, I assume that the firm is too small to be marketable. Asymmetric information prevents external buyers from observing unrealized capital gains and the human capital embedded in the firm. To the outside buyer, true productivity is private information. Again, costs of acquiring human capital necessary to manage this firm are high compared to the firm's limited profits. Contrary to outside buyers, a company heir selected from the owner family does not face an asymmetric information problem and can easily accumulate the firm-specific human capital to manage this firm. As a consequence, if the firm is not owned by a family member, it is dissolved, not sold. These two core assumptions limit the analysis to small firms, but small firms make up the vast majority of businesses: 80.9% of all German companies have at most 10 employees, but, in sum, these companies account for 18.9% of total employment and 11.2% of gross value added.[2] In the U.S., 76% of all companies employ less than 10 workers, accounting for 10.6% of total employment.[3]

Importantly, my theory pertains to small firms and only small firms. First, firms with large turnovers can easily hire managers. Owning and managing a firm are two separate decisions, so the career choice problem of an heir becomes insignificant. Secondly, the true value of a large or medium-sized firm is not subject to the same asymmetric information problem. External buyers can proxy productivity and unrealized capital gains from published information and the costly acquisition of further information is profitable for buyers of larger firms with higher absolute profits. If a company heir is forced to sell a part of a large company, she will find buyers for the company stocks. Hence, when lawmakers grant deductions for business owners regardless of the company size, they allow beneficiaries of very large inheritances to

[1] Another motivation is a principal-agent problem: External managers can extract profits extensively and effective screening is too costly to be profitable given the small firm's profits. No external managers will be hired and the owner must be the manager (see Song et al., 2011)

[2] Federal statistical office. Retrieved Oct 2017. https://www.destatis.de/DE/ZahlenFakten/GesamtwirtschaftUmwelt/UnternehmenHandwerk/KleineMittlereUnternehmenMittelstand/Tabellen/Insgesamt.html.

[3] US Census: Firm Characteristics Data Tables (2014). Retrieved Oct 2017. https://www.census.gov/ces/dataproducts/bds/data_firm2015.html.

free-ride on a solution to problems pertaining only small firms.

The empirical section of this paper uses the German inheritance and gift tax statistic of 2002, and makes four major observations: i) inherited business assets are more unequally distributes than other inherited assets; ii) deductions significantly reduce the progressive pattern of the tax code; iii) together with business assets, company owners bequeath large stocks of liquid assets; iv) the probability of firm continuation rises with the size of the company; and v) also rises slightly with liquid assets inherited together with the company. These findings contribute to the empirical research on inheritances in Germany and provide the basis for my theory.

Finally, I use the statistic to parametrize the optimal inheritance tax rate for business assets conditional on business continuation. First, the argument that taxation might force divestment and lay-offs via borrowing constraints is weak: testators accompany firm assets with sizeable stocks of liquid assets which often suffice to pay the tax liability, especially as the government grants generous deferment of this tax liability. Second, setting the tax on business assets is a trade-off between tax revenue and potential earnings losses. A high tax rate generates revenue. A low tax on business assets incentivizes heirs to continue the parent's firm at the cost of foregone tax revenue. Crucially, the optimal tax rate for business assets depends on the level of the inheritance tax applicable if the heir decides not to continue the parent's business. If the non-business inheritance tax rate is low, the government imposes only a minor penalty on scrapping the firm. To incentivize an heir to continue the parent's firm, business continuation must be subsidized and the optimal business inheritance tax rate is negative. If the non-business tax rate is high, the tax rate on businesses can be positive. Given the current tax rate for business inheritances of between € 1m and € 5m, the optimal business inheritance tax rate is -1%, i.e. business assets should not be taxed and instead the government ought to grant a small subsidy. Importantly, the theory pertains to small firms, which cannot be sold as is at the capital market. If the firm is marketable, the optimal inheritance tax rate is confiscatory: the heir's "wrong" career choice does not cause earnings losses for workers and the planner confiscates all of the firm's excess returns using inheritance tax.

This paper is organized as follows: The next section reviews the related literature. Section 1.4 outlines the model and the decentralized equilibrium.

Section 1.5 describes the constrained efficient allocation and compares it to the German tax system. Section 1.6 calibrates the model. Section 1.7 concludes.

1.2 Related literature

The normative literature on general inheritance taxation is extensive and results depend heavily on the assumed motive for bequests[4] and the type of government expenditures financed by the tax. There seems to be some consensus that accidental bequests should be taxed at a 100%, even though Kopczuk (2003) establishes that accidental bequests are the results of a lack of annuity markets. If intergenerational links are studied in the fashion of a Barro-Becker-type infinite horizon dynastic framework and markets are complete, zero optimal tax results of Chamley (1986) and Judd (1985, 1999) can be applied to inheritance taxation. Given the availability of labour taxes, capital should remain untaxed. Externalities of bequests can give rise to an inheritance subsidy (Kaplow, 1995, 1998), e.g. when the motive is primarily emotional ("warm glow"), the parent derives utility from bequeathal and the child derives utility from the transfer; this is an external effect of the bequest. An inheritance subsidy lets the donor internalize the externality. Similarly, in an infinite-horizon framework, the parent's inter-generational discount factor need not align to the social discount factor in the social welfare function.[5]

Even abstracting from redistribution, the case for an optimal positive inheritance tax rate can be made: Grossmann and Poutvaara (2009) assume two types of inter-generational transfers; bequests and inter-vivo funding for schooling. An estate tax leads parents to substitute from bequests to school funding, which, in general equilibrium, increases wages and interest rates and can be Pareto improving. This result is akin to Jones et al. (1997) and Stiglitz (2018), who break zero optimal capital tax results by assuming that labour is a composite of supplied hours and endogenous human capital. Michel and Pestieau (2004) derive positive optimal inheritance taxes in an OLG growth framework featuring a government that finances exogenous

[4]See Kopczuk (2013) for a review.

[5]See Piketty and Saez (2012)'s discussion of Atkinson and Stiglitz (1976).

government expenses with linear taxes on labour, savings and bequests.

Turning to redistribution, Piketty and Saez (2013) and Farhi and Werning (2013) break zero tax results in Mirrlesian frameworks. Farhi and Werning identify positive optimal tax rates if parents differ by their degree of altruism and the social welfare function is Rawlesian. Piketty and Saez find that estates should be taxed if the social welfare function weighs heavily on the less fortunate, bequests are highly concentrated, and the elasticity of intergenerational transfers with respect to taxes is low. In a heterogeneous agent framework, De Nardi and Yang (2016), accounting for inter-generational linkages of ability, find that an increase in the estate tax depresses output, wealth and inequality and increases welfare. They argue that the estate tax has little effect on the aggregate capital stock, its distribution, and dependency of outcomes on parental background, while the welfare gains from redistribution are substantial.

Given the great availability of rationales for positive inheritance taxation, I do not pose the same question. Many industrialized countries tax intergenerational transfers. I start from this factual position and ask whether family business assets should be treated favourably. Starting with the firm's founders, inheritance taxes disturb decisions on a number of margins: i) the decision to found a firm, ii) considerations regarding its size, and iii) the intertemporal consume-bequest decision. Building on the heterogeneous agent framework with entrepreneurship by Quadrini (2000), Cagetti and de Nardi (2009) find that the progressive estate tax distorts the investment decisions of large businesses, with a negligible effect on the decision of small companies. They find that abolishing the estate tax is not welfare-improving if fiscal balance must be achieved via an increase in other taxes. Following Quadrini, a number of authors have studied optimal capital taxation in heterogeneous agent models in the presence of entrepreneurship (Boháček and Zubrický, 2012; Kitao, 2008; Meh, 2005). I contribute to this literature by putting emphasis on the labour market outcomes of workers employed by family-owned firms. A neoclassical labour market explains the adverse effect of taxation on wages indirectly: if taxation reduces the entrepreneurs' optimal capital choice, the tax depresses wages in general equilibrium. In my model, a reduction of capital is followed by a reduction of its complementary labour input. This is realized by laying off workers, who lose their firm-specific human capital and suffer from substantial earnings losses.

In general, favourable taxation of family businesses is not compatible with horizontal equity and promotes tax evasion. Using Norwegian data, Alstadsaeter et al. (2014) show that closely-held family firms respond to favourable tax treatments and can serve as tax shelters. Though my data does not allow for the study of tax evasion, I show that in terms of establishing horizontal equity, German inheritance tax laws fail miserably.

Turning to efficiency of management, intergenerational transfers of managerial control can have positive effects on productivity by alleviating the principle-agent problem and adverse effects because entrepreneurial ability might decline between generations. Dubbed the "Carnegie conjecture", large expected or realized fortunes reduce the incentive to work and accumulate human capital. Across generations entrepreneurial ability can diminish endogenously and preferential tax treatment then hinders the substitution of bad heirs with good external managers (Holtz-Eakin et al., 1993). Additionally, just because their parents were good managers, heirs need not be good managers *ex ante*: this mean reversion of ability, or simply lack of talent, is an exogenous source of inferior managerial skills. Keeping business control within the family limits the amount of eligible CEOs to a group of people who might not be best matches: for estimates on the adverse effects of intra-family transmission of leadership, see Bloom and Van Reenen (2007), Bennedsen et al. (2007) and Pérez-González (2006). On the other hand, selecting a CEO among the firm owners helps to circumvent the principal-agent problem between firm owner and manager. In companies held by a family and a number of independent shareholders, Villalonga and Amit (2006) find that this effect is great as long as the company founder is the CEO. Once the chair of CEO is transferred to a family member, conflicts arise between the family and other shareholders. Villalonga and Amit estimate that these family-shareholder conflicts become quantitatively more detrimental than the alleviation of the principal-agent problem.

The article most closely related to this chapter is Grossmann and Strulik (2010), who study optimal inheritance taxation in an OLG variant of Lucas (1978)'s span-of-control framework. Inheritance taxation faces a trade-off between capital destruction and the Carnegie conjecture: taxation causes elevated capital destruction due to firms being sold, but decreases the fraction of untalented entrepreneurs managing large inherited companies beyond the scope of their limited talent. Exempting company heirs from the tax can motivate untalented heirs to manage excessively large companies, thereby reducing total output. In contrast to my paper, Grossmann and Strulik assume a perfect labour market which downplays the role of income losses to workers - ultimately the public's main concern about inheritance taxation. In a similar spirit, Guvenen et al. (2017) simulate a revenue-neutral tax reform replacing capital income taxes by a flat tax on wealth. Substituting a capital income tax with a wealth tax increases the burden on non-profitable entrepreneurs while decreasing the burden on profitable, non-wealthy entrepreneurs. The tax reform increases welfare by enhancing efficiency ("use it or loose it") by almost 8%, but redistributes towards more profitable entrepreneurs which increases inequality. Again, when Guvenen et al. promote a wealth tax, stating that it is "like pruning: it eliminates weak branches, strengthens stronger ones"[6], they overlook the fact that not only entrepreneurs are subject to pruning, but also their employees.

Inheritance taxes have to be financed via company cash flow, family savings, or debt. Whichever, under collateralized borrowing, taxes tighten the heirs' budget constraint, which in turn limits room for new debt and investments. Potrafke et al. (2014) ask leading managers of German SMBs about their reaction to the hypothetical abolishment of favourable treatment of business bequests: 52% of their respondents say that they would have to reduce the number of vacancies, 65.9% of managers would reduce investment and 43% claim that they would have had to sell business property. Using the same survey data, Hines et al. (2019) find that inheritance taxation influences the timing and composition of bequests and conclude that the effects of inheritance taxes on business successions are stronger when companies find themselves in adverse economic situations. Note that this survey was financed by a family business lobby organization and the researchers asked managers of family-owned firms for their opinion on the taxation of family-owned firms. The results should be taken with, at least, a grain of salt. Gale and Slemrod (2001) cast doubt on the validity of similar studies in the U.S., pointing to inconsistencies in survey responses and the fact that the

[6] Presentation slides by the authors: `http://piketty.pse.ens.fr/files/Guveneneta12016.pdf`

accumulation of assets to finance an expected tax is good business practice: Holtz-Eakin et al. (2001) find that up to 58% of business owners can pay their estate taxes using liquid assets only. Business owners refrain from taking out life insurance policies as an insurance against estate taxes, referring to low taxes or other assets which have already been transferred to the heir. The empirical section of this paper affirms this finding for the German case.

German lawmakers justify the generous exemptions by the spatial dimension of labour demand, the role of entrepreneurs as charitable donors, the role of SMBs as economic stabilizers and by a distrust of capital markets.[7] A traditional family business is often the sole major employer in a rural area and whole small towns depend on their labour demand. If firms are relocated or closed due to taxation, workers will incur earnings losses if they cannot easily relocate. Spatial frictions are an underlying cause of earnings differentials studied in this paper. Secondly, lawmakers view entrepreneurs as charitable donors who show "commitment in the social and cultural field, furthering social cohesion in their region". However, welfare gains by charitable donations must be contrasted with losses in tax revenue. Diamond (2006) explores the role of tax free donations financing public goods. Thirdly, lawmakers attribute a part of "Germany's jobs miracle" to the composition of German firms, claiming that small and medium sized firms act as stabilizers in turbulent economic times, while large businesses are held to be more vulnerable. Yet given that large public companies have a diversified product portfolio while small firms are highly specialized, it is highly questionable whether smaller firms are less prone to economic turbulence. Finally, lawmakers support tax exemptions for privately-owned firms, arguing that is is only so that production can be guaranteed to take place in Germany, a hypothesis predicated on home bias among entrepreneurs. Empirical evidence points in the opposite direction: selecting CEOs from a limited number of family members reduces productivity (Villalonga and Amit, 2006) and increases the chances of firm destruction. The traditional distrust of capital markets belongs to history books and should not form the basis for a tax code.

Note that many arguments favouring large tax exemptions equally favour

[7] See, for example, the German ministry of finance's 2015 draft for a new inheritance taxation bill [in German]: `http://dip21.bundestag.de/dip21/btd/18/059/1805923.pdf`

possibilities for deferment. Gale and Slemrod (2001) argue that deferments reduce the present value of tax liabilities, thus reducing the tax liability for business heirs. The German government realizes this, granting a 10 year window over which bequest taxes can be deferred free of interest for business heirs. Additional interest-bearing deferment is possible if paying taxes is classified an undue hardship for the beneficiary. In the U.S. liabilities can be paid over a 14-year window at a 2% interest rate administered only in the first two years.

The next section gives an introduction to the German tax code and reports descriptive statistics, predicating the theory in Section 1.4 and quantifying the optimal tax derived in Section 1.5.

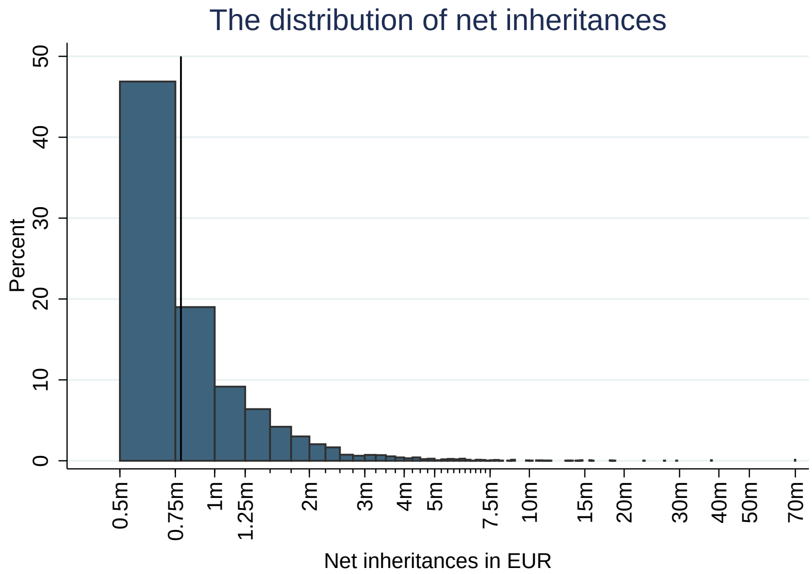


Figure 1.1: The distribution of inheritances. For comparability, I focus on inheritances between close relatives, whose complete portfolio distribution is known and which exceed € 500,000. The vertical line marks the median.

1.3 Data and legal framework

This section gives a brief introduction to the German inheritance tax code and reports five empirical observations from the inheritance and gift tax statistic of 2002. The dataset includes all inheritances and gifts for which tax authorities have set a tax. As outlined below, inheritances and gifts are tax free as long as they do not exceed thresholds of up to € 500,000. Inheritances below these thresholds (which make up the bulk of all inheritances) are not scrutinized and will not be recorded in the data. Hence, this dataset is informative about the right-hand tail of the inheritance distribution. For comparability, I focus on inheritances between close relatives whose complete portfolio distribution is known and which exceeds € 500,000.

First, I compare the subset of data used in this section to estimates of the asset distribution's right-hand tail. Figure 1.1 shows the distributions of inheritances above € 500,000. This right-hand tail of the inheritance distribution resembles a Pareto distribution. Following Vermeulen (2017), a Pareto distribution has the complementary cumulative distribution function ($ccdf$)

$$P(W > w) = \left(\frac{w_{min}}{w}\right)^{\alpha},$$

defined on the interval $[w_{min}, \infty[$ for $\alpha > 0$. Parameter w_{min} is the $ccdf$'s lower bound. The parameter of interest is the Pareto-Lorenz coefficient α: the lower α, the fatter the tail of the Pareto distribution. When estimating α given some lower bound w_{min}, the researcher assumes that the right-hand tail of the distribution is well approximated by a Pareto distribution. I use maximum likelihood estimation to derive estimates of α for my sample; see Appendix 1.A.2 for all estimates of the Pareto-Lorenz coefficient and the related inverted coefficient. If I set the lower bound to €500,000, the Pareto coefficient is 1.61. When I raise the threshold to €1m, I find an estimate of 1.62. This is close to other estimates of the German Pareto coefficient: Vermeulen (2017) uses the European central bank's Household Finance and Consumption Survey (HFCS) and supplements it with the Forbes World's billionaires list. He estimates the tail index of asset holdings, α, to be between 1.37 and 1.61. Atkinson et al. (2011), citing Dell (2007), estimate the coefficient to be 1.67.

Figure 1.2 shows the distributions of business assets, liquid assets, and real estate in the sample defined above. The data reveals that many business inheritances are very small, reflecting that the majority of businesses is very small. Still, the diminutive sizes of companies in the dataset are surprising: the median value of business assets is only € 128,600 which is considerably lower than the median amount of inherited real estate (€ 432,000) and median of liquid assets (€ 371,300). Almost half of business inheritances are worth less € 100,000 and about 72% are worth less than half a million Euros. These small companies can be owned by the self-employed which have no tangible value besides the owner's human capital. These can also be heavily leveraged companies or companies whose assets are tax depreciated. Unfortunately, the statistic only reports a Euro amount of transferred business assets. Turning to the dispersion, the range of business assets exceeds the range of real estate and liquid assets in the sample. To the right-hand side of the distributions, 2.4% of business inheritances exceed € 5m compared to 2.5% of real estate and 0.9% of liquid asset inheritances. To the far right of the distribution, 99% quartile of business assets is € 9.72m compared to € 7.2m for real estate and € 4.8m for liquid asset inheritances. This points to:

Empirical fact 1 *Inheritances of business assets are more unequally distributed than other asset classes.*

Compared to liquid assets and real estate, business assets are often quite small, but the distribution includes the transfer of vast companies which, overall, results in a very unequal distribution of business assets. This is the starting point of this paper: the majority of businesses are small companies with few employees and low net worth whose stocks are hardly marketable at a financial market. However, there are transfers of vast business assets and stocks. When lawmakers allow company heirs to deduct business assets from their inheritance tax bill, lawmakers target the majority of all inheritances, but the extremely wealthy can exploit the deductions as well.

Unlike the Anglo-Saxon estate tax, levied on the testator, Germany collects an inheritance and gift tax, levied on the beneficiary. As most countries, German law treats inheritances and gifts almost equally, leaving them free of tax as long as they do not exceed general thresholds. Table 1.1 summarizes these

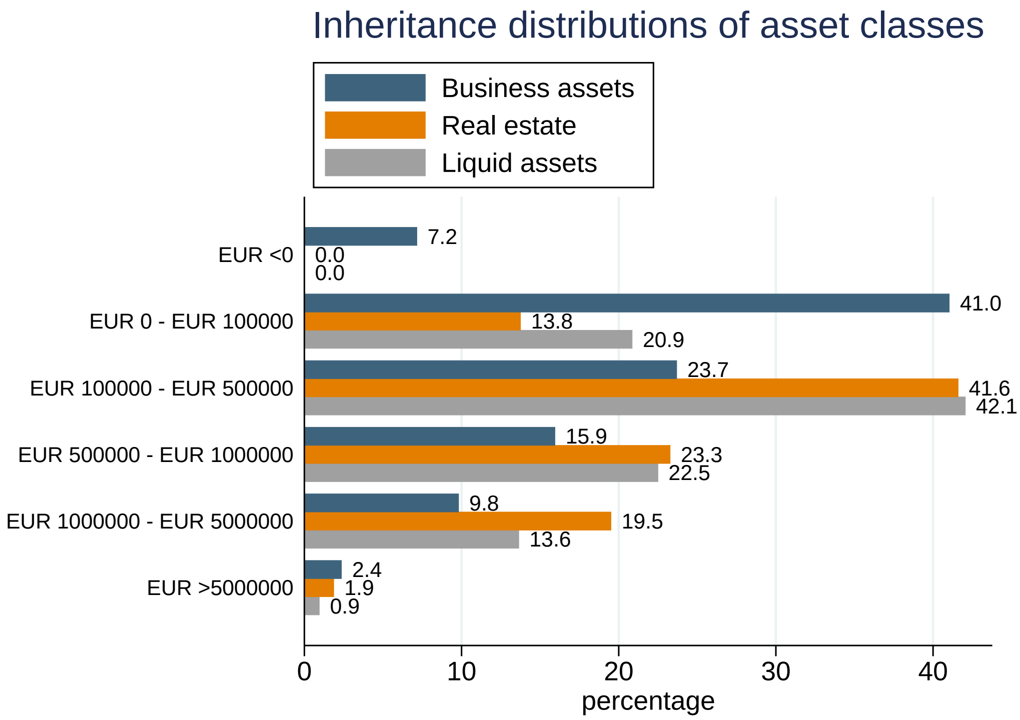


Figure 1.2: Distribution of asset classes across inheritances. If an inheritance included multiple asset classes, this observation will be part of more than one distribution. For comparability, I focus on inheritances between close relatives, whose complete portfolio distribution is known and which exceed €500,000.

	General tax deduction	Tax class
Spouse	€ 500,000	I
Children	€ 400,000	I
Grandchildren	€ 200,000	I
(Grand-)parents	€ 100,000	II
Siblings, their children, parents-in-law, children-in-law	€ 20,000	II
Non-relatives	€ 20,000	III
Business property		I

Table 1.1: Tax deduction (§16 ErbStG) and inheritance tax classes by degree of kinship between donor and donee.

	Inheritance tax class		
Inheritance/gift after deduction	I	II	II
< € 75,000	7 %	15 %	30 %
< € 300,000	11 %	20 %	30 %
< € 600,000	15 %	25 %	30 %
< € 6,000,000	19 %	30 %	30 %
< € 13,000,000	23 %	35 %	50 %
< € 26,000,000	27 %	40 %	50 %
> € 26,000,000	30 %	43 %	50 %

Table 1.2: Marginal tax rates by tax class and inheritance/gift received. Business assets are always subject to tax rates of class I (§19 ErbStG).

deductions, which increase with the proximity of blood between testator and beneficiary. The degree of kinship also determines the applicable inheritance tax class I-III. Business property is always taxed at tax class I. Marginal tax rates are a function of this tax class and the total amount received (Table 1.2). The tax is calculated in ten year windows: every ten years, families can exploit the deduction levels and lower marginal tax rates, creating a straightforward avenue for tax evasion via premortal gifts. The deductions of Table 1.1 and tax rates of Table 1.2 are calculated per testator-beneficiary pair. Bequests from different testators are all taxed individually. In the U.S., the general deduction level varies year by year. In 2002, $ 1m could be transferred free of taxes; in 2018, the general deduction will rise to $11.18m. Note that U.S. law applies deductions to the estate, not the testator-beneficiary pair. In addition to the standard deductions of Table 1.1, heirs can draw on numerous additional discounts, e.g. §17 ErbStG determines "sustenance

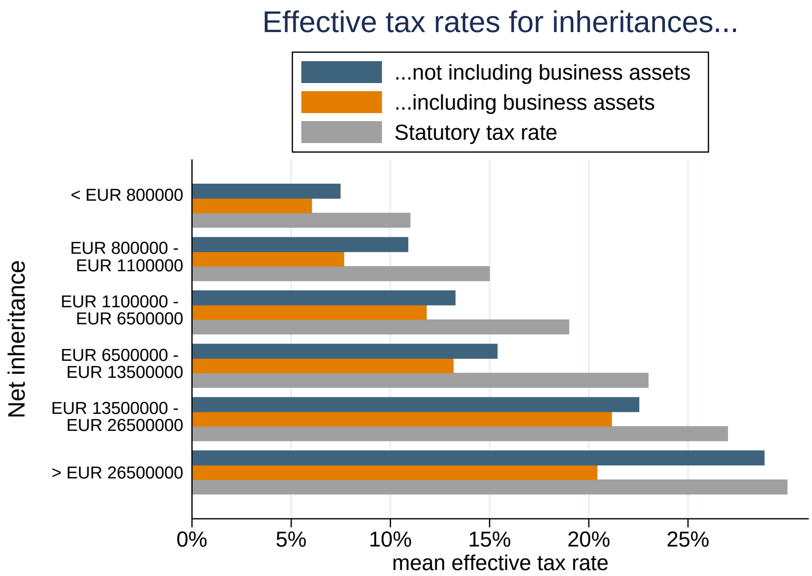


Figure 1.3: Mean effective inheritance tax rates computed as the ratio of tax liabilities to the size of transfers, including earlier transfers from the same donor and excluding transferred debt. Transfers lower than EUR 500,000 have been excluded. The figure only shows transfers subject to inheritance class I. The depicted statutory tax rates are computed for the net inheritance after a deduction of EUR 500,000.

deductions" for children and the spouse of a deceased.[8] This sustenance deduction starts at €52,000 for toddlers and decreases with the age of the child. Spouses can always deduct €256,000. Other deductions reduce the tax for a number of intangible assets like real estate by a fixed percentage (§13 Abs.1 and §13c). Among the most extensively used discounts are generous deductions for business and stock owners, defined in §13a. Prior to 2009, the legal framework for §13a was as follows: if a testator who holds more than 25% of a company's equity bequeaths or gifts her stake, the beneficiary can deduct €225,000 plus 40% of the firm's value if she pledges to keep the company for five years. This is also the code of law used by the individuals in this sample.[9]

Figure 1.3 compares the statutory tax rate in each inheritance bracket to the

[8] All paragraphs refer to the German inheritance taxation code of law.

[9] After 2009, lawmakers expanded the §13a deductions: In 2014, §13a reduced the tax base by more than €66 billion (Source: Inheritance and gift tax statistic 2014), compared to total intergenerational annual inheritances and gifts of approximately €220 billion estimated by Schinke (2012). The extreme deductions in 2014 and 2015 were the result of extensive tax evasion in anticipation of a reform as the German Constitutional Court declared the law's current state to be unconstitutional because it contradicted the concept of horizontal equity; wealthy families anticipated reforms and gifted business assets to younger children - even turning toddlers into billionaires (Bach and Mertz, 2016).

mean effective tax rate, defined as the ratio of tax liability to net inheritance. Effective tax rates are lower than the statutory tax rate throughout the sample. This is not worrying *per se*: general deductions (Table 1.1) decrease the mean rates and the tax code imposes some concavity in average tax rates to reduce the tax for those who receive an inheritance which is just high enough to qualify for a higher tax bracket. Yet, the effective tax rates for inheritances including business assets are considerably lower than the tax rates levied on inheritances without business assets throughout the sample. For example, business assets reduce the mean effective tax rate in the € 6.5m - € 13.5m bracket from a statutory 23% to 13%. Inheritances of half this size which do not contain business assets are subject to the same effective tax rate; hardly an element of a progressive tax. The role of deductions is most pronounced for recipients of extraordinarily high net inheritances. Recipients of inheritances exceeding € 26.5m are subject to a statutory tax rate of 30%. In this bracket, the mean effective tax rate for those not receiving business assets is in fact 28.8%. But those who receive business assets pay only 20.4% on average, with effective tax rates ranging from 16.5% to 26.5%. Note that the mean effective tax rate in this bracket is even lower than in the adjacent bracket. Though the deductions do not fully convert the progressive statutory tax code into an effective regressive tax, it is safe to state that:

Empirical fact 2 *Effective inheritance tax rates do not abide by a purely progressive pattern because of deductions for business assets. Horizontal equity is not given.*

This paper provides a positive theory for lower effective tax rates for small, mature firms. I stress again that my theory does not rationalize lower tax rates for huge fortunes as observed in the highest brackets in Figure 1.3. In fact, I claim that wealthy families free-ride on deductions designed for intergenerational transfers of small firms and that this exploitation is not compatible with either horizontal or vertical equity.

One rationale for low effective tax rates for business assets is that borrowing constraints might lead to divestment and lay-offs. Yet, testators accompany business inheritances and gifts with liquid assets, and this raises the question of whether beneficiaries cannot pay taxes exclusively with inherited liquid assets. Table 1.3 reports ratios of liquid assets to company

Firm net worth in €	<0.1m	<0.5m	<1m	<5m	<10m	>10m	all
Mean cash to firm value ratio	216.38	0.70	0.37	0.27	0.06	0.03	0.95
Liquid assets > tax liabilities	0.60	0.56	0.40	0.26	0.19	0	0.53
N	743	578	217	137	16	13	1704

Table 1.3: Inherited liquid assets relative to firm values. The second row shows the share of heirs who receive sufficient liquid assets to finance their complete tax liability. Cases where a complete portfolio composition of an inheritance is not reported have been excluded. Only transfers to close relatives who are eligible to tax class I are included in the sample.

assets. First, the average ratio decreases with the inherited company's size: while small firms are accompanied by enormous cash transfers (21638%), this ratio decreases sharply with the firm's net worth and drops to 3% for inheritances whose firm share exceeds € 10m. Companies with a net worth of € 1m to € 5m are accompanied by a cash transfer of 27% of the firm's value on average. I also compute the fraction of firm heirs who can pay the total tax liability exclusively with the inheritance's liquid assets. This ratio is approximately 60% for very small firms and drops to zero for firms whose value exceeds € 10m. Still, 26% of the beneficiaries who receive firms with a value between € 1m and € 5m can pay the taxes out of their parents' pockets. In the complete sample, the ratio of business heirs who can pay the tax in this manner is 53%. This result is consistent with findings by Holtz-Eakin et al. (2001) for the U.S.: they find that up to 58% of business owners can pay their estate taxes using only liquid assets. Their main finding is that business owners do not use sufficient life insurance as an insurance against estate taxes, referring to low tax liability or other assets which have already been transferred to the heir. In summary,

Empirical fact 3 *Business inheritances are accompanied by large inheritances of liquid assets.*

While the inheritance statistic does not explicitly report whether an heir continues the inherited business, it reports whether she made use of the §13a deductions for family-owned firms. §13a allows for a deduction of € 225,000 plus 40% of the firm's value if the recipient pledges to keep the company for five years. I assume that those recipients who used §13a deductions decided to continue the inherited company while the others sold or dissolved the inherited business. To study how the size of the business inheritance and cash affect the probability of §13a usage, I collapse the dataset to the

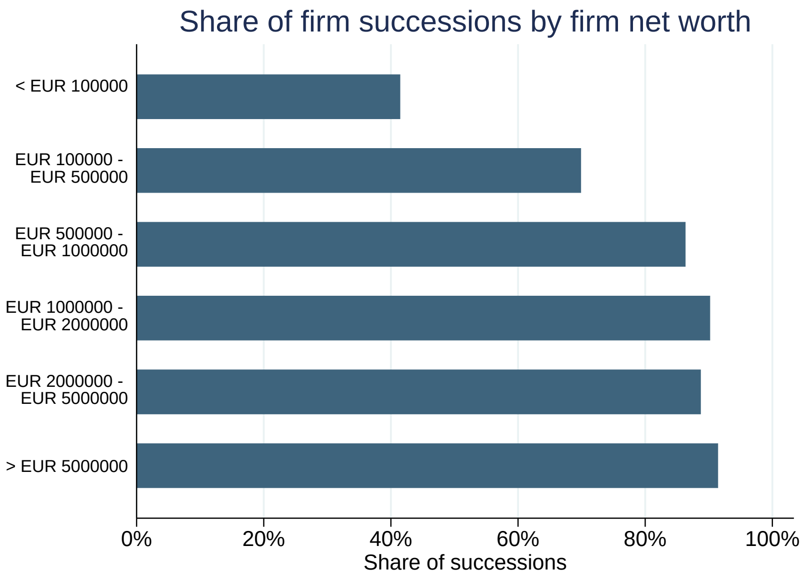


Figure 1.4: Probability of succession by firm net worth.

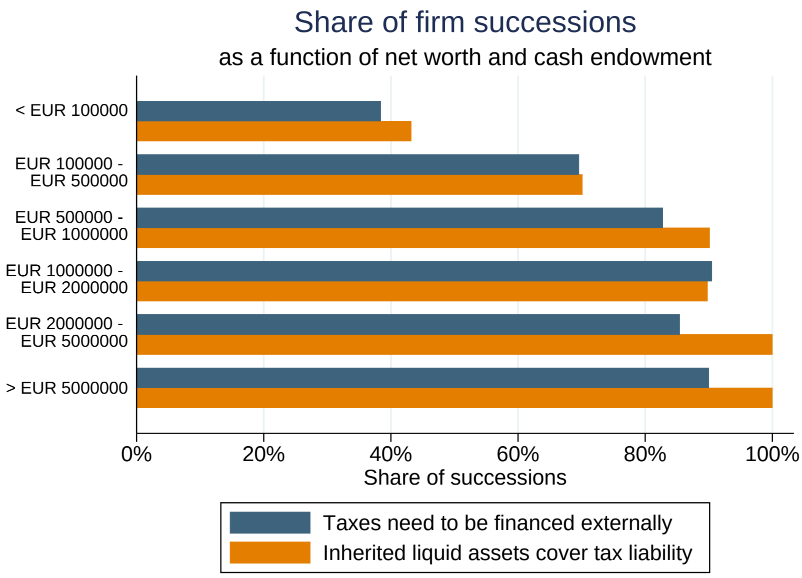


Figure 1.5: Probability of succession conditional on liquid assets. The figure shows the unconditional vs the conditional probability of succession given that liquid assets inherited are sufficient to finance all inheritance tax liabilities. Liquid assets are defined as the sum of cash, "Bausparguthaben" and other stocks in companies, which are not subject to§13a.

testator level. All beneficiaries who received assets from the same testator are treated as a family who jointly decides to use §13a or not. In the 2002 data, 63% of families who inherited a non-agricultural business worth at least €1000 made use of the deductions.[10] Figure 1.4 reports the probability of business succession by firm net worth. Only around 40% of heirs of very small companies worth less than €100,000 will follow in their parents' footsteps. If the firm net worth exceeds €0.5m, the share of continued firms rises to almost 90%. Appendix 1.E provides estimates of a regression analysis: a 1% increase in firm value correlates with a 0.1% increase in the succession probability. It is evident that,

Empirical fact 4 *The size of a company increases the probability of an intra-family succession.*

Empirical fact 3 states that business inheritances are accompanied by large intergenerational transfers of liquid assets. Figure 1.5 shows the role of liquid assets in succession probabilities. For inheritances in the lowest and the highest two net worth brackets, the ability to pay taxes with inherited liquid assets raises the probability of intra-family succession. For inheritances between €100,000 to €20m the influence of cash on firm succession is ambiguous. Regression analysis in Appendix 1.E indicates that there is a small, albeit insignificant, positive correlation between inherited cash and the succession of firms.

Empirical fact 5 *Liquid assets inherited together with the firm can increase the probability of an intra-family succession slightly.*

The following section outlines the main model that builds on the empirical facts: the theory differentiates between small and large firms, addresses liquid assets as a means to finance tax liabilities, and models the endogenous firm succession probability of facts 4 and 5. The main question is therefore whether we can rationalize the lack of horizontal equity and progressiveness in the tax code without resorting to arguments of crony capitalism.

[10] Heirs can only use the §13a deductions if they do not sell (parts of) the company in the upcoming five years. Although they can revoke their decision, the relatively low pick up rate of 63% allows us to posit that most decisions are terminal. The European commission expects that around 30% of businesses will be closed for lack of a successor, corroborating my estimate (Commission, 1998).

1.4 Model

This section outlines a partial equilibrium model that rationalizes inheritance tax deductions for business assets via borrowing constraints and the externality of the heir's career choice. The analysis focuses on one firm and its employees. The timing is as follows. First, the parent of the family, who is the firm's founder, uses equity and debt to choose a firm size and employment. Second, the heir makes a career choice: either she liquidates firm capital and works outside of the firm or follows in her parent's footsteps. In the former case, the firm's workers lose their jobs and incur earnings losses. This is the core externality in this theory. Third, in case of a firm succession, the heir makes an investment decision using equity, inherited cash, and debt to determine a new firm size. This investment decision is subject to a borrowing constraint tightened by taxation. As outlined above, liquid assets inherited with firms and deferment of inheritance tax liabilities cast doubt on the borrowing constraint argument.

1.4.1 Technology, bargaining and profits

First, all agents are risk-neutral. Workers do not have access to a savings technology; entrepreneurs can save in a deposit and invest in the firm. An entrepreneur j operates with a Leontief production function,

$$f(A_j, n_j, K_j) = A_j \min\left(n_j, \frac{K_j}{\phi}\right), \tag{1.1}$$

where A_j denotes productivity or the entrepreneur's talent, K_j denotes total capital input and ϕ is a parameter. Given capital input, the optimal employment choice is $n(K_j) = K_j\phi^{-1}$, where ϕ denotes the firm's capital labour ratio.[11] Per matched worker, the entrepreneur earns the match productivity net of wages. Operating profits read

$$\Pi(A_j, n_j) = n_j[A_j - w(A_j)]. \tag{1.2}$$

The entrepreneur with ability A_j and each worker bargain over the match's

[11] Specifically, I rule out that parents allocate more assets into the firm than $n_{j-1}\phi$, which rules out tax avoidance by parents.

period surplus $(A_j - \phi s R^d - \underline{w})$. Workers earn wage $w(A_j)$ if the bargaining succeeds and $\underline{w}$ otherwise. The entrepreneur earns operating profit $A_j - w(A_j)$ per match if the bargaining succeeds. Otherwise, she liquidates the associated physical capital, which depreciates by factor $0 < s < 1$, and earns the deposit interest rate, R^d, on the scrap value. Nash bargaining with worker's bargaining power $\eta \in (0,1)$ yields the wage

$$w(A_j) = \eta(A_j - s\phi R^d) + (1-\eta)\underline{w}. \tag{1.3}$$

The negotiated wage exceeds the worker's outside option if $(A_j - s\phi R^d) > \underline{w}$.

Using wage (1.3) and the number of employed workers, operating profits read

$$\Pi(A_j, n_j) = n_j[(1-\eta)(A_j - \underline{w}) + \eta s \phi R^d], \tag{1.4}$$

are linear in firm size and using (1.1) rewrite

$$\Pi(A_j, K_j) = \rho(A_j) \cdot K_j, \text{ with } \rho(A_j) = \frac{1}{\phi}[(1-\eta)(A_j - \underline{w}) + \eta s \phi R^d]. \tag{1.5}$$

1.4.2 Financial frictions

Entrepreneurs can borrow d_j units of capital at net interest rate R^l, subject to collateralized borrowing as in Kiyotaki and Moore (1997),

$$R^l d_j \leq \lambda e_j, \tag{1.6}$$

where e_j denotes collateral and parameter λ summarizes the quality of financial institutions, e.g. its ability to seize profits. Only funds invested in the firm serve as collateral. Assume A_j is large enough to always ensure that the return on capital exceeds the borrowing rate, $R^l < \rho(A_j)$.

1.4.3 Investment problem

An entrepreneur's state vector consists of her entrepreneurial ability, A_j, cash, x_j, and the inherited firm's current equity, k_j. The government levies tax rate τ_e on k_j and τ_s on x_j. The entrepreneur allocates cash and equity into a save deposit and/or invests in the firm. She can use funds invested in the firm as collateral to borrow d_j. Debt must be repaid at net interest rate $R^l > R^d$. Appendix 1.B proves that it is optimal to invest the complete inheritance in the firm rather than deposit at the lower interest rate R^d. It follows for funds invested by the heir: $e_j = (1-\tau_e)k_j + (1-\tau_s)x_j$.[12] Here, I anticipate this result for brevity. Denote the total capital input $K_j = e_j + d_j$.

The debt choice problem reads:

$$\begin{aligned} V_e(x_j, k_j, A_j) \quad \max_{d_j} \quad & \rho(A_j)[(1-\tau_s)x_j + (1-\tau_e)k_j + d_j] - R^l d_j \\ \text{s.t.} \quad R^l d_j \quad & \leq \lambda[(1-\tau_s)x_j + (1-\tau_e)k_j] \\ d_j \quad & \geq 0. \end{aligned} \tag{1.7}$$

In the optimum, entrepreneurs exhaust the borrowing constraint, choosing maximum leverage,

$$d_j = \frac{\lambda}{R^l}[(1-\tau_s)x_j + (1-\tau_e)k_j], \tag{1.8}$$

which implies a total capital input of

$$K_j = \left(1 + \frac{\lambda}{R^l}\right)[(1-\tau_s)x_j + (1-\tau_e)k_j]. \tag{1.9}$$

Linearity of operating profit and (1.8), are used to identify the value of a firm,

$$V_e(x_j, k_j, A_j) = \widetilde{\rho(A_j)}[(1-\tau_s)x_j + (1-\tau_e)k_j], \tag{1.10}$$

where $\widetilde{\rho(A_j)} = \rho(A_j)\left(1 + \frac{\lambda}{R^l}\right) - \lambda$ denotes the (leveraged) return on equity-financed capital, which includes the exhaustion of the borrowing constraint and subtraction of the costs of debt.

Given the Leontieff production function, the optimal number of employed workers is

$$n_j = \frac{K_j}{\phi} = \frac{1}{\phi}\left(1 + \frac{\lambda}{R^l}\right)[(1-\tau_s)x_j + (1-\tau_e)k_j]. \tag{1.11}$$

All n_j workers earn and consume wage income $w(A_j)$.

Children who have chosen to follow in their parents' footsteps and manage the firm solve problem (1.7). They choose debt (1.8) and new employment

[12] The proof relies on the assumption $\rho(A_j) > R^l > R^d$: as long as the firm is more profitable than saving, a risk-neutral agent will invest exclusively in the firm. As long as leverage has a positive net return, $\rho(A_j) > R^l$, the risk-neutral agent will also exhaust the borrowing constraint (see Appendix 1.B).

(1.11). Denote their indirect utility from problem (1.7) by $V_e(x_j, k_j, A_j)$. Assume that parents, who are the firm founders, invest equity k_{j-1} and have entrepreneurial ability A_{j-1}. They solve the related problem and choose debt $d_{j-1} = \frac{\lambda}{R^l} k_{j-1}$, implying total capital input of $K_{j-1} = (1 + \frac{\lambda}{R^l})k_{j-1}$ and firm size $n_{j-1} = \phi^{-1}(1 + \frac{\lambda}{R^l})k_{j-1}$ in terms of employment.

1.4.4 Career choice

Between the heir's and parent's investment problem, the heir makes a discrete career choice: to either manage the family firm as an entrepreneur or sell the firm and pursue her talents elsewhere, i.e. in a job outside the family company.[13]

The entrepreneurial path yields indirect utility $V_e(x_j, k_j, A_j)$, but the heir incurs idiosyncratic utility cost ϵ_j drawn from a distribution with cumulative density function $F(\epsilon)$. ϵ_j subsumes the costs of acquiring human capital to become a manager, working with parents in their firm, and talent for jobs outside of the family business. ϵ_j can also be interpreted as impatience: an impatient heir might want to sell the company and consume now rather than manage and consume profits over time.

The talent path yields an outside wage w_{out} and non-stochastic interest R^d on the cash inheritance, x_j, plus the scrap value of equity, sk_j. Both, cash and sold capital, are then subject to the non-corporate tax rate, τ_s. The children's career choice has external effects on the workers' human capital: if the firm is scrapped, the n_{j-1} workers formerly employed by the parents lose their high-tenure jobs and will earn and consume the outside wage. The difference $w(A_j) - \underline{w} > 0$ reflects the substantial earnings losses after lay-off. The children's career choice is the main motivation for optimal tax deductions: an heir may abstain from pursuing her own talents if the tax penalty on this path is high enough.

Formally, the heir chooses the entrepreneurial path if the value of en-

[13]The heir's outside wage need not equal the workers' outside wage. Assuming that an entrepreneur's child has benefited from excellent education, she can be a "Jack of all trades" (Lazear, 2005) with the human capital to earn a higher wage than the attached workers outside of the company ($w_{out} \geq \underline{w}$).

trepreneurship net of utility costs exceeds the utility of firm liquidation,

$$V_e(x_j, k_j, A_j) - \epsilon \geq w_{out} + (1 - \tau_s)R^d[x_j + sk_j]. \tag{1.12}$$

The solution to the discrete choice problem is a reservation talent,

$$\bar{\epsilon}(x_j, k_j, A_j) \equiv V_e(x_j, k_j, A_j) - w_{out} - (1 - \tau_s)R^d[x_j + sk_j], \tag{1.13}$$

and the firm is liquidated if $\epsilon_j > \bar{\epsilon}(x_j, k_j, A_j)$. Denote the indirect utility of the career choice problem (1.12) by $V(x_j, k_j, A_j)$.

1.4.5 Bequest choice

Assume a bequest policy function:[14] the parent will always bequeath the company's equity $k_j = k_{j-1}$ and a cash bequest, x_j, which is an increasing function of k_{j-1} as a proxy of wealth,

$$x_j = \tilde{x}_j(k_{j-1}, \tau_s), \text{ with } \tilde{e}_{x_j,k_{j-1}} \geq 0, \text{ and } \tilde{e}_{x_j,\tau_s} \leq 0.$$

$\tilde{e}_{x_j,k_{j-1}}$ denotes the elasticity of cash bequests to capital bequests and $\tilde{e}_{x_j,\tau_s}$ denotes the tax elasticity of cash bequests. For tractability and to rule out tax sheltering, assume $\tilde{e}_{x_j,\tau_e} = 0$, i.e. parents who are entrepreneurs do not adjust the cash bequest in response to changes in the tax rate of the continued firm.

Excess profitability of equity (Appendix 1.C) yields the choice $k_j = k_{j-1}$, which is credible in light of low empirical estimates of the tax bequest elasticity and special role of a family business for donors: Slemrod and Kopczuk (2001) identify a long-run tax elasticity of between -0.1 and -0.16. The tax elasticity of business assets is probably smaller: first, entrepreneurs, especially firm founders, tend to take pride in their work and their business and feel some responsibility for their workers (Kammerlander, 2016). Secondly, a company is not easily divided into small chunks which are consumed or bequeathed without friction.

In my framework, tax sheltering is ruled out by assumption, but as shown by Alstadsaeter et al. (2014), favourable taxation of business assets leads

[14]See Appendix 1.C for a microfoundation of policy functions. The main text deviates from the microfoundation by assuming $\tilde{e}_{x_j,\tau_e} = 0$.

to a shift in portfolio choices. In Germany, §13a spawned *Cash-GmbHs*, pseudo-companies designed to hoard liquid assets for the next generation.

1.4.6 Taxation and divestment

This subsection provides the first motivation for a deduction of business assets. Taxation might lead to divestment and lay-offs in a continued firm. When employment and capital are complements, employment rises with capital input. Taxation can reduce employment via four channels: (i) if the elasticity of bequests with respect to taxes $\tilde{e}_{x_j,\tau_s}$ is negative, taxes reduce bequests; (ii) taxation reduces capital by reducing the net value of cash or capital the heir receives; (iii) by reducing an heir's asset holdings, the borrowing constraint tightens; (iv) taxation affects career choice. The influence of taxation on career choice (iv) is discussed in the presentation of the central planner's problem below. For now, focus on the bequest decision of parents and the investment decisions of children. The effects of inheritance taxation on employment are

$$\frac{\partial n_j}{\partial \tau_s} = -x_j \phi^{-1}\left(1+\frac{\lambda}{R^l}\right)\left[1+\frac{1-\tau_s}{\tau_s}\tilde{e}_{x_j,\tau_s}\right] < -x_j \phi^{-1}\left[1+\frac{1-\tau_s}{\tau_s}\tilde{e}_{x_j,\tau_s}\right] \text{ and}$$
$$\frac{\partial n_j}{\partial \tau_e} = -k_{j-1}\phi^{-1}\left(1+\frac{\lambda}{R^l}\right) < -k_{j-1}\phi^{-1}.$$

The term $-x_j \phi^{-1}\left[1+\frac{1-\tau_s}{\tau_s}\tilde{e}_{x_j,\tau_s}\right]$ is the direct effect of taxation on employment if entrepreneurs cannot borrow ($\lambda = 0$). It is the sum of channels (i) and (ii). Each unit of net cash bequest invested into the company allows for the employment of ϕ^{-1} additional workers. Similarly, $-k_{j-1}\phi^{-1}$ is the direct effect of taxation of corporate assets absent debt ($\lambda = 0$).

Additionally, a lower net value of cash or equity tightens the borrowing constraint (iii). With each unit of cash or equity the heir can increase debt, $\left(1+\frac{\lambda}{R^l}\right) > 1$. Leverage increases total capital input and amplifies the detrimental effect of taxation on employment. When we study optimal taxation with entrepreneurship but ignore borrowing constraints, we overlook the amplifying effect leverage can have on labour demand.

Importantly, a reduction of employment may translate into earnings losses for workers because the wage paid in the firm exceeds the workers' outside option. This channel is widely overlooked in the optimal taxation literature based on neoclassical labour markets. In a neoclassical labour market, taxation reduces aggregate capital, which in turn depresses wages via general equilibrium effects. In this model, taxation can reduce the wages for some agents considerably, if company heirs are forced to lay off workers.

If an inheritance is not accompanied with sufficient liquid assets to pay the tax liability, heirs must reduce the firm size and lay off workers to finance the taxes, i.e. heirs operate firms with smaller total capital and employment if

$$\tau_s x_j + \tau_e k_{j-1} < x_j.$$

Empirically, Germany's estate and gift tax statistic lets us compute the ratio of liquid assets to company assets. As an example, consider an heir who inherits some cash and a company worth € 4m (after deduction of € 400,000, see Table 1.1). The applicable tax rates for this inheritance are $\tau_s = 19\%$ and $\tau_e = (1 - 0.4) \times 19\% = 11.4\%$ for business assets in the year 2002 (see Table 1.2). Table 1.3 shows that the average cash bequest this heir can expect is worth 27% of the firm's value. It follows $19\% \cdot 27\% + 11.4\% < 27\%$ and the average heir can pay the inheritance tax liabilities using only the transferred cash. In fact, even if there were no favourable treatment of company assets ($\tau_e = \tau_s$), the average company heir could pay the tax liabilities out of the parent's pocket.

Note that recipients of company assets have access to deferments. The German government grants a 10 year window over which bequest taxes can be deferred free of interest for business heirs. Additional interest-bearing deferment is possible if paying taxes is classified an undue hardship for the beneficiary. This has two consequences: as Gale and Slemrod (2001) note, the present value of the tax is lower because of intertemporal discounting. Secondly, not all taxes need to be paid out of cash or by selling the company. Instead the government acts as an extended financial sector, providing loans free of collateral constraints. Those findings are in line with Gale and Slemrod (2001) who point out that the accumulation of assets to finance an expected tax is good business practice. Holtz-Eakin et al. (2001) find that up to 58% of business owners can pay their estate taxes using only liquid assets. Deferments, *inter vivo* gifts, life insurance, cash transfers, and financial markets cast doubt on the idea that taxation leads to divestment as outlined in this subsection. How, then, can we justify the deductions granted by lawmakers? The main rationale in this paper is the externality of an heir's career choice discussed in the next section.

1.5 Central planner

The prior section casts doubt on the idea that inheritance taxation causes divestment in continued firms. It is a weak rationale for generous deductions. This paper's main rationale builds on the externality of an heir's career choice, who in the decentralized solution does not take earnings losses of workers into account. In the centralized solution of the career choice problem, the planner accounts for the earnings losses. This section solves the problem and decentralizes the planner's career choice.

Consider a *constrained-efficient* allocation: a social planner chooses quantities and the reservation talent which determine whether an heir continues the parent's firm. The planner obeys the borrowing constraint and the inheritance tax rate, τ_s, levied on cash bequests and business assets if the company is not continued by the heir. The purpose of this paper is to examine to which degree family business assets should be treated favourably. The purpose of this paper is not to identify the optimal inheritance tax rate for all assets. Hence, the planner takes τ_s as a given parameter.

1.5.1 Social welfare function

The planner chooses debt $\{\widetilde{d}_j, \widetilde{d}_{j-1}\}$, employment $\{\widetilde{n}_j, \widetilde{n}_{j-1}\}$, and a reservation talent $\bar{\omega}$. If it is optimal to continue a firm and $A_j > R^l + \underline{w}$ holds, it is optimal to exhaust the borrowing constraint. Anticipating the optimal choices for debt and employment, the social planner's objective function reads

$$\begin{aligned}\Omega = \max_{\bar{\omega}} F(\bar{\omega}) &\overbrace{\left\{((1-\tau_s)x_j + k_j)\phi^{-1}(1+\frac{\lambda}{R^l})\left[A_j - \underline{w} - \phi\frac{R^l\lambda}{R^l+\lambda}\right]\right\}}^{\equiv\Omega_e(k_j,x_j)} - \int_{-\infty}^{\bar{\omega}} \epsilon\,\mathrm{d}F_\epsilon(\epsilon) \\ &+ [1-F(\bar{\omega})]\left\{w_{out} + ((1-\tau_s)x_j + k_j s)R^d\right\}. \qquad (1.14)\end{aligned}$$

Consult Appendix 1.D for the derivation. In short, the firm is continued with probability $F(\bar{\omega})$ and sold otherwise. In the latter case, the heir earns the outside wage, taxes are levied on the scrap value of capital, and the sum of the inheritance is deposited and earns interest R^d. The idiosyncratic utility costs are not realized. In the former case, the planner uses equity and the after tax value of cash as collateral to expand capital input by factor $(1+\frac{\lambda}{R^l})$. Per unit of capital, the planner employs ϕ^{-1} workers who produce output A_j but forego the outside option $\underline{w}$. Denote by $\Omega_e(k_j, x_j)$ the social value of the firm, which is the indirect utility derived from the firm if the planner employs socially optimal quantities.

1.5.2 Career choice

For comparison, I repeat the reservation utility costs of the private career choice,

$$\bar{\epsilon}(x_j,k_j,A_j) = \underbrace{V_e(x_j,k_j,A_j)}_{\widetilde{\rho}[(1-\tau_s)x_j+(1-\tau_e)k_j]} - w_{out} - [(1-\tau_s)x_j + (1-\tau_s)sk_j]R^d. \tag{1.15}$$

When the heir makes her career choice, she compares the leveraged return on equity to its scrap value at the financial market plus her personal outside option. Her return on equity is a function of the share of production she can reap given her Nash bargaining power $(1-\eta)$. In contrast, the planner chooses to continue the firm if $\epsilon \leq \bar{\omega}(x_j,k_j,A_j)$, with

$$\bar{\omega}(x_j,k_j,A_j) = \Omega_e(x_j,k_j,A_j) - w_{out} - [(1-\tau_s)x_j + sk_j]R^d. \tag{1.16}$$

The social planner compares the complete value of production to the heir's utility costs and the outside option. The latter includes tax income if the firm is scrapped. The most relevant difference between the social value of the firm and its private counterpart is the worker's share of production, η. This is the core externality in the model: in the decentralized solution, heirs do not take into account that high-tenure workers earn a higher wage in the firm and incur earnings losses if the heir makes the "wrong" career choice. A private solution that internalizes the worker's losses demands that career choice enters the bargaining between workers and the heir, but career choice takes place many years before the first bargaining. Now, if the government grants deductions for firm continuation, the company heir internalizes the earnings losses of workers when making her career choice. This rationale is robust to any changes of the tax rate τ_s. Even if the government levies no inheritance taxes, it is still optimal to subsidize firm continuation.

The planner can decentralize the constrained-efficient solution by imposing a tax rate $\tilde{\tau}_e$ which equalizes the private and the planner's reservation talents,

$$\Omega_e(x_j, k_j, A_j) - V_e(x_j, k_j, A_j) = \tau_s s k_j R^d.$$

The planner sets a tax rate $\widetilde{\tau_e}$ such that the difference between the social and the private value of the firm equals tax revenues of scrapping the firm. The optimal tax reads

$$\tilde{\tau}_e(k_j, x_j) = \frac{k_j \tau_s s R^d - \left[(1-\tau_s)x_j + k_j\right]\left[\left(1+\frac{\lambda}{R^l}\right)\phi^{-1}\overbrace{\eta(A_j - \underline{w} - s\phi R^d)}^{\equiv \Delta w}\right]}{k_j \widetilde{\rho(A_j)}},$$

which can be simplified to (1.17).

Theorem 1 (Optimal corporate inheritance tax rate) *The corporate inheritance tax rate $\tilde{\tau}_e$ that maximizes social welfare and is levied only if the heir chooses to continue the firm is given by*

$$\tilde{\tau}_e(k_j, x_j) = \frac{T - \Delta w n_j|_{\tau_e=0}}{k_j \widetilde{\rho}}, \tag{1.17}$$

where $T \equiv k_j \tau_s s R^d$ denotes the tax revenue earning the deposit interest rate if the firm is scrapped, $n_j|_{\tau_e=0}$ is the employment level absent corporate inheritances taxes, $\Delta w = w(A_j) - \underline{w}$ denotes the earnings losses and $k_j \widetilde{\rho}$ is the gross return on equity.

The optimal inheritance tax on equity can be positive or negative depending on the level of the standard inheritance tax, τ_s, and the size of the wage losses if the firm is scrapped. If earnings losses are large $\Delta w >> 0$, a higher firm succession rate is socially efficient. To encourage more heirs to choose the entrepreneurial path, the planner grants a larger tax discount for firm succession, $\tau_e << \tau_s$. Turning to the level of τ_s: a high non-corporate inheritance tax rate has two implications: first, the higher the tax rate, the larger the tax revenue if the firm is scrapped. The same holds for the opportunity costs of capital, i.e. the deposit interest rate and the reversibility of capital investments s. The higher τ_s, the higher the optimal fraction of scrapped firms and the higher τ_e. Second, a larger tax rate τ_s creates a heftier penalty on the heir's outside option. If τ_s is low, firm continuation must be subsidized to let the heir internalize the workers' earnings losses and the optimal $\widetilde{\tau_e}$ is negative. If tax rate τ_s is very high, the outside option is costly in terms of tax liabilities and a smaller yet positive $\widetilde{\tau_e}$ can implement the socially-optimal reservation talent. The private leveraged return on equity $\widetilde{\rho}$ affects the size, not the sign, of $\tilde{\tau}_e$. Suppose it is optimal to subsidize firm continuation, $\tilde{\tau}_e < 0$: a larger $\widetilde{\rho}$ reduces the size of this subsidy because the firm is already more profitable for the heir. Suppose $\tilde{\tau}_e$ is positive: a high return on equity reduces the optimal tax rate, because the private return is also part of the social welfare function.

The assumptions which drive the main result (1.17) are obviously restrictive. Firms with a small numbers of employees face difficulties trying to find a suitable, yet affordable manager. Asymmetric information and the non-transferability of human capital prevent potential external buyers from buying and managing the firm. However, these assumptions cease to be credible once a company has reached a certain size. As shown in Section 1.3, favourable treatment of business assets is used extensively for very large companies. A large company can be sold at the capital market. Assume that the value of a large firm is not private information ($s = 1$) and that external buyers will inject the same amount of cash into the firm as the heir would. It follows that the socially-optimal probability of continuing the firm is $F(-w_{out})$, i.e. the firm should not play any role in the private career choice of the heir, who should instead choose the career which bests matches her personal tastes and talents. In the case of large firms, the planner confiscates the whole excess return of firm heirs:

Corollary 1.1 (Large firm inheritance tax rate) *The socially optimal inheritance tax rate $\tilde{\tau}_e$ given that the firm can be bought and continued by external* *buyers who choose capital input* $x_{j-1}(1-\tau_s)+sk_{j-1}$ *is given by*

$$\tilde{\tau}_e^{large}(k_j,x_j)=\frac{(\widetilde{\rho}-R^d)(1-\tau_s)x_j+(\widetilde{\rho}-(1-\tau_s)R^d s)k_j}{\widetilde{\rho}k_j}, \tag{1.18}$$

where the numerator summarizes the excess return on capital in the firm compared to the return on scrapped capital at the market.[15]

1.6 Parametrization

To determine the optimal tax rate numerically, I parametrize a small dynamic perturbation of the model presented in the main text. I calibrate the model to annual data and add an exogenous rate of firm destruction. The latter is motivated by high rates of return on capital reported by managers of SMBs, which partly reflect risk premia. Assume an annual survival probability q. With probability $(1-q)$, the firm is destroyed entirely, the bank seizes remaining assets, and workers lose their jobs. For simplicity, assume no productivity differences between children and parents $A_j = A_{j-1}$ (consult Grossmann and Strulik (2010) for a Carnegie conjecture model). The legal framework dictates that a company heir who wants to use the §13a deductions must keep the company for five years if she chooses the basic and seven years if she chooses the extended deduction plans. I use six years as the time horizon. After inheriting the company, the heir chooses debt and employment and pays taxes. Throughout the following six years, she abstains from divestment and investment. Profits are consumed. Assume that the same limitations on investment hold for the planner. Capital depreciates completely after six years.

The optimal tax in the dynamic model reads

$$\tilde{\tau}_e(k_j,x_j)=\frac{\tau_s s k_j-\sum_{t=0}^{5}\Delta w n_j|_{\tau_e=0}\left(\frac{q}{R^d}\right)^t}{\sum_{t=0}^{5}k_j\widetilde{\rho}\left(\frac{q}{R^d}\right)^t}.$$

[15]The planner eliminates the firm from the heir's career choice, $\bar{\omega}=-w_{out}$. Solving $\bar{\omega}\overset{!}{=}\bar{\epsilon}$ for τ_e yields (1.18).

1.6.1 Baseline calibration

I need to assign numerical values to the capital labour-ratio ϕ, the scrap value of capital s, the worker's wage differential Δw, the borrowing and deposit interest rates $\{R^d, R^l\}$, the financial sector's quality λ, and the tax rate on non-business assets τ_s.

Starting with the driving force of the results, Jacobson et al. (1993) document that displacements reduce earnings permanently: wages decline sharply at the time of the lay-off and quickly recover for six quarters. Thereafter, relative earnings grow slowly. Five years after the separation, high-tenure workers still suffer a 25% wage penalty. More recently, Jung and Kuhn (2019) and Couch and Placzek (2010) find earnings differentials of 11% and 13% six years after the displacement. Choosing a midpoint of the recent estimates, I assume that displaced workers suffer from a 12% wage differential. Conservatively, this 12% underestimates the direct losses after displacement, which are quite sizeable - in the range of 25% (Couch and Placzek, 2010) to 37% (Jung and Kuhn, 2019) one year after displacement. Normalizing the outside option $\underline{w}$ to unity, I set the annual wage in the company to $w(A_j) = 1.12$.

I set the annual deposit interest rate to 2% and following Kitao (2008), the annual loan-deposit interest rate spread to 5%, yielding $R^d = 1.02$ and $R^l = 1.07$. Following Kitao (2008), who in turn cites Evans and Jovanovic (1989), set the quality of the financial sector such that total capital invested must not exceed 0.5 times own assets, implying $\lambda = 0.5 \times R^l$.

To estimate the return on capital ρ, I use results of a survey among managers of German SMBs.[16] I use values of 2013 and I focus on SMBs with an annual turnover lower than € 0.5m and between € 0.5m and € 1m. The average return on total capital of is 14.8% in the former and 15% in the latter bracket. The report points to a relatively constant return on equity over time for firms with a turnover not exceeding € 1m.

Compared to the riskless return on savings, the return on capital is high, reflecting a risk premium to some degree. Fackler et al. (2013) report firm exit rates from the German Establishment History Panel (BHP). They find

[16] Mittelstand im Mittelpunk 2016: `https://www.dzbank.de/content/dzbank_de/de/home/unsere_kunden/firmenkunden/publikationen/mittelstandsstudie.html`

that the average hazard rate of a German businesses decreases with the number of employees and increases over time from 1985 to 2006. The hazard rate is U-shaped in business maturity, lending support to both a liability of newness as well as a liability of adolescence. I use their reported linear probability regression results and descriptive statistics and infer that the average annual exit rate of a manufacturing firm, which is at least five years old and has between 10 and 19 employees, is 2.1% (in 2006).

To parametrize the capital-labour ratio, ϕ, I set the capital share of production to $\frac{1}{3}$, implying $\phi = 1.5$.[17].

Parameters A_j, s and η are collinear. I set the liquidation factor, s, because it is one of the driving parameters of the model. Officer (2007) estimates the acquisition discount of non-publicly traded firms to 15% to 30% compared to publicly listed firms. In this framework, the firm is liquidated rather than sold. The liquidation value might well be below 70% of the company's value of capital. On the other hand, setting a low liquidation factor biases the model in direction of a hefty tax discount because a low liquidation factor corresponds to low tax revenue when the firm is scrapped. Following Officer's estimate, the baseline case is $s = 0.7$. A sensitivity analysis covers values from 0 (capital is firm-specific and its value is zero outside the company) to 1 (the frictionless case). Once the liquidation factor is set, equations (1.3) and (1.5) pin down the productivity of a match, $A_j = 2.84$, and the Nash bargaining power of workers, $\eta = 0.16$. In the baseline case, assume away the cash bequest $x_j = 0$. I examine the case $x_j > 0$ in extension 1.6.3.

1.6.2 Baseline results

Table 1.4 shows the optimal corporate tax rate $\widetilde{\tau_e}$ for the benchmark parametrization and robustness checks. The results apply the more beneficial small firm optimal tax formula (1.17). The large firm optimal tax rate is always confiscatory. The table reports $\widetilde{\tau_e}$ as a function of the statutory tax rates τ_s, which depend on the size of the inheritance. The table also includes the baseline optimal tax by Piketty and Saez (2013) and a confiscatory tax rate, $\tau_s = 100\%$. $\widetilde{\tau_e}$ increases in τ_s which is the penalty on firm destruction. Quantitatively,

[17]The capital share of production, $\alpha A_j = (\rho - 1)K_j$ and labour share of production $(1 - \alpha)A_j = w(A_j)n_j$ imply $\phi = \frac{K}{N} = \frac{\alpha w(A_j)}{(1-\alpha)(\rho-1)}$

the optimal $\widetilde{\tau}_e$ is around zero for tax rates applied in Germany, i.e. tax rates between 15 and 30%. Unless the statutory tax rate is 30%, the planner subsidizes firm continuation slightly. To internalize earnings losses of workers, firm heirs demand a small subsidy on top of a complete inheritance tax deduction if the firm is continued.

Columns denoted Δw show $\widetilde{\tau}_e$ for different levels of earnings losses. First, the planner grants a much larger subsidy when earnings losses are 31% (midpoint of estimates of the wage differential one year after a lay-off). Secondly, if there are no wage differentials, the planner still grants deductions. This is a result of the lack of a positive role of taxation. This model purposefully excludes redistribution or public investment into human capital. Including redistribution or schooling would raise $\widetilde{\tau}_e$, but the main mechanism, i.e. promoting firm continuation via a tax discount, would remain intact.

Next, turn to the quality of the financial sector. When entrepreneurs can raise debt up to a debt-equity ratio of 100%, $\widetilde{\tau}_e$ becomes strongly negative because, with limited equity, a lot of debt can be raised and many workers employed. If entrepreneurs face a borrowing constraint of $\lambda = 0$, $\widetilde{\tau}_e$ hardly changes. As shown in Section 1.4.3, the financial sector can impact the investment decision, but is not equally important for career choice.

Lastly, how does the frictional market for a firm's capital affect the optimal tax? The liquidation factor s raises the reselling value of the firm's capital. $\widetilde{\tau}_e$ rises with s. When $s = 0$, the tax revenue of a scrapped firm is zero; the planner favours a lower firm liquidation rate and sets a lower $\widetilde{\tau}_e$. When capital can be used elsewhere without friction, in setting a larger tax, the planner allows more firms to be liquidated. A low liquidation value also affects the career choice of the heir because it penalizes firm destruction and itself promotes firm continuation.

In this section, bequests of liquid assets are neglected. The reason for this is that liquid assets allow firm heirs to expand employment above the level that their parents managed, i.e. they create new jobs which, by assumption, have the same productivity and human capital as the old matches. New matches are not the jobs this policy seeks to protect. The next section rules out employment above the parent's level.

Firm value	τ_s	$\widetilde{\tau}_e$	Δw		$\frac{\lambda}{R^l}$		ρ	s	
			0%	31%	0%	100%	1.08	0	1
<€ 600,000	15	-1.43	1.64	-5.13	-0.41	-2.39	0	-3.06	-0.72
<€ 6,000,000	19	-0.99	2.08	-4.69	0.03	-1.96	0	-3.06	-0.1
<€ 13,000,000	23	-0.55	2.51	-4.26	0.48	-1.53	0.01	-3.06	0.53
<€ 26,000,000	27	-0.11	2.95	-3.82	0.93	-1.11	0.01	-3.06	1.15
>€ 26,000,000	30	0.21	3.28	-3.49	1.27	-0.79	0.02	-3.06	1.62
Piketty/Saez	50	2.4	5.46	-1.3	3.51	1.34	0.04	-3.06	4.74
Confiscatory	100	7.86	10.93	4.16	9.12	6.67	0.1	-3.06	12.55

Table 1.4: Optimal company tax, $\widetilde{\tau}_e$, as a function of firm size and parameters. All rates in percent.

1.6.3 Mature human capital

This section extends the baseline model by an inequality constraint that rules out an heir's firm size beyond the parent's firm size. In the baseline model, if an heir receives a sufficiently large cash gift, she will increase investment beyond the level needed to replenish the amount of capital lost by taxation. She will create new matches and in the baseline model, every match produces the same amount of output A_j and new workers enjoy the same wage differential as old workers. Investment and divestment have symmetrical effects. However, human capital that policymakers seek to protect is not newly acquired. Assume that high-tenure matches n_{j-1} which are active before the inheritance each produce A_j units. For simplicity, assume that the heir does not invest further into the firm: she exhausts the borrowing constraint and uses cash and inherited equity to replenish taxed capital but any additional cash is only invested at the capital market and earns R^d.[18] Additional cash is not pledged as collateral because $R^l > R^d$. Introduce the constraint $K_j \leq (1 + \frac{\lambda}{R^l})k_{j-1}$, where k_{j-1} denotes the parent's equity and K_j the heir's total capital choice. This constraint binds if the parent's cash gift exceeds the tax liability, $x_j \geq \tau_s x_j + \tau_e k_{j-1}$ and is potentially confusing: the heir who receives a cash gift that exceeds tax payments is constrained in her choice, while the heir with little cash is unconstrained.

[18] Alternatively, I could assume that the heir can create new matches with no embedded human capital. I would need to make arbitrary assumptions about new matches' productivity, which must be more productive than the capital market's R^d and wages would therefore lie somewhere above the worker's outside option.

The heir chooses to exhaust the borrowing constraint up to the same point her parent did or as far as her funds allow

$$d_j = \min\left\{\frac{\lambda}{R^l}k_{j-1},\ \frac{\lambda}{R^l}[(1-\tau_s)x_j + (1-\tau_e)k_{j-1}]\right\},$$

giving total capital input

$$K_j = \min\left\{\left(1+\frac{\lambda}{R^l}\right)k_{j-1},\ \left(1+\frac{\lambda}{R^l}\right)[(1-\tau_s)x_j + (1-\tau_e)k_{j-1}]\right\}$$

and employment $n_j = \frac{K_j}{\phi}$. Liquid assets, that are not used to replenish taxed capital, are invested at the capital market and earn $R^d[(1-\tau_s)x_j - \tau_e k_{j-1}]$. The value of entrepreneurship is

$$V_e(x_j, k_{j-1}, A_j) = \begin{cases} \rho(\widetilde{A_{j-1}})k_{j-1} + R^d[(1-\tau_s)x_j - \tau_e k_{j-1}] & \text{if } x_j \geq \tau_e k_{j-1} + \tau_s x_j \\ \rho(\widetilde{A_{j-1}})[(1-\tau_s)x_j + (1-\tau_e)k_j] & \text{else.} \end{cases}$$

The social value of the firm is

$$\Omega_e(x_j, k_{j-1}, A_j) = \begin{cases} \frac{k_{j-1}}{\phi}(1+\frac{\lambda}{R^l})\Big[A_j - \underline{w} - \phi\frac{R^l\lambda}{R^l+\lambda}\Big] + (1-\tau_s)x_j R^d & \text{if } x_j \geq \tau_e k_{j-1} + \tau_s x_j \\ ((1-\tau_s)x_j + k_{j-1})\phi^{-1}(1+\frac{\lambda}{R^l})\Big[A_j - \underline{w} - \phi\frac{R^l\lambda}{R^l+\lambda}\Big] & \text{else.} \end{cases}$$

As before, the planner decentralizes the socially optimal career choice by setting a τ_e that satisfies

$$\Omega_e(x_j, k_{j-1}, A_j) - V_e(x_j, k_{j-1}, A_j) = \tau_s s k_{j-1} R^d,$$

which yields:

Corollary 1.2 (Inheritance tax rate with constrained investment) *Under the constraint that the heir's firm size choice cannot exceed the parent's and additional liquid assets are invested at interest rate R^d, the socially optimal inheritance tax rate $\tilde{\tau}_e$ is*

$$\tilde{\tau}_e^{bound}(k_{j-1}, x_j) = \begin{cases} \frac{T - \Delta w n_{j-1}}{R^d k_j} & \text{if } x_j \geq \tau_e k_{j-1} + \tau_s x_j \\ \frac{T - \Delta w n_j|_{\tau_e=0}}{k_{j-1}\widetilde{\rho}} & \text{else.} \end{cases} \tag{1.19}$$

where $T \equiv k_{j-1}\tau_s s R^d$ denotes the tax revenue earning the deposit interest rate *if the firm is scrapped, n is the employment level the parent chose,* $n_j|_{\tau_e=0}$ *is the heir's employment level absent corporate inheritances taxes,* $\Delta w = w(A_j) - \underline{w}$ *denotes the earnings losses and* $k_{j-1}\widetilde{\rho}$ *and* $k_{j-1}R^d$ *are the gross returns on equity and savings.*

The difference between the two cases is the rate of return in the denominator. In the majority of parametrizations, the optimal equity tax rate is negative, i.e. the inheritance is subsidized if the firm is continued. The subsidy is larger ($\widetilde{\tau}_e$ is lower) when the heir receives a sufficiently large cash gift to cover tax payments. This result is counter-intuitive at first glance: why should the wealthier, constrained heir receive a larger subsidy? The unconstrained heir earns $\widetilde{\rho}$ on cash invested in the firm relative to R^d in the outside option. This incentivizes the unconstrained heir to continue the firm. In contrast, the constrained heir earns R^d on additional cash in both the firm and the outside option. The planner must grant a larger subsidy to the constrained, wealthy heir to incentivize more firm continuation. Table

				$\widetilde{\tau}_e$	
Firm value	τ_s	$\frac{x_j}{k_j}$	% constrained heirs	constrained	unconstrained
€0.5m - €1m	15	37	40	-15.3	-2.4
€1m - €5m	19	27	26	-10.6	-1.7
€5m - €10m	23	6	19	-4.4	-0.7
>€10m	27	3	0	-1.2	-0.2

Table 1.5: Optimal company tax accounting for mature human capital. Optimal $\widetilde{\tau}_e$ using (1.19). Estimates of the cash-to-capital share and the percentage of constrained heirs from Table 1.3.

1.5 reports optimal inheritance taxes in the investment-constrained model. The firm size brackets, cash-to-capital ratios $\frac{x_j}{k_j}$ and the percentage of constrained heirs are taken from Table 1.3. Recall that heirs are 'constrained' when they inherit sufficient liquid assets to pay their tax liability. As an example, for 26% of heirs who receive a company worth between €1m and €5m this is the case. On average, an heir in this bracket receives 27cent per euro of firm value as liquid assets. For these 26%, the planner chooses a subsidy of 10.6%. For the remaining 74%, heirs in this firm value bracket the planner barely provides any subsidy. The favourable taxation of wealthy heirs is counter-intuitive and appears unfair. The focus of this paper is on efficiency and purposefully ignores equity, but the result indicates that the model would benefit from an introduction of a positive role for taxation, e.g. redistribution.

1.7 Conclusion

This paper derives an optimal inheritance tax rate for heirs of family-owned businesses conditional on succession. It rationalizes the generous tax discounts for small business owners that are observed in German bequest taxation laws, among others. The paper highlights earnings losses of workers who lose match-specific human capital as an important determinant of optimal capital taxes. To the best of my knowledge, this channel is widely overlooked in the optimal taxation literature. Importantly, in my framework, welfare gains are only achieved if firms are not marketable and cannot hire external managers. The German inheritance and gift tax statistic shows that tax deductions are extensively used by heirs of very large firms, but my theory calls for confiscatory taxation of large fortunes. Policymakers should improve on half-hearted rules to limit access to the generous deductions for heirs of large fortunes; such deductions are incompatible with horizontal equity and come at exorbitant cost to tax-payers.

This analysis could be extended in a number of directions. First, the career choice externality could be embedded in a general equilibrium model which assigns a different, welfare-improving purpose to inheritance taxation, e.g. redistribution (Piketty and Saez, 2013; De Nardi and Yang, 2016; Farhi and Werning, 2013), human capital acquisition (Grossmann and Poutvaara, 2009), financing of schooling (Jones et al., 1997; Stiglitz, 2018), or financing displaced workers' training. This could yield a higher optimal tax rate for business heirs. Second, the mechanism could be embedded in a general equilibrium analysis with heterogeneity in asset holdings in the spirit of Quadrini (2000); Cagetti and de Nardi (2009). A promising starting point is given by Buera et al. (2015), who introduce labour frictions in a Quadrini (2000)-type model. This could be useful to quantitatively weigh earnings losses highlighted in this paper against efficiency gains stressed by Grossmann and Strulik (2010) and Guvenen et al. (2017) and redistributive motives.

Appendix

1.A Data

1.A.1 Data description and editing

The German inheritance and gift tax statistic of 2002 records all inheritances and gifts for which tax authorities have set a tax. As outlined in the main text, recipients of inheritances and gifts can apply general exemptions of up to € 500,000 (Table 1.1) and sustenance exemptions of up to € 265,000. Only if the inheritance or gift exceeds these thresholds, the recipient of the inheritance or gift becomes an observation in the dataset. For comparability, I only use inheritances and transfers which exceed € 500,000. The observational unit is the recipient of an inheritance, but observations are linked by a testator ID, such that it is possible to collapse the data to the testator level. The 2002 data contains 164,289 observations from 96,600 testators of which 79% are inheritances and 21% are gifts. 20% of the observations are so-called *"Vermächtnisse"*, which are directed inheritances, e.g. the eldest son gets the company, the second son the house. For these observations the portfolio distribution is often unknown. In the analysis I drop all inheritances and gifts whose complete portfolio distribution is unknown.

I perform a number of data editing steps: I infer the applicable tax classes from reported family relationships where missing and drop 61 cases with inconsistent family relationships and tax classes. In many cases there is no information whether inherited firms are located in Germany or abroad. The reported cases are overwhelmingly (99.3%) German firms, which I assume for the non-reported locations as well. To compute the effective tax rates, I divide the levied tax by the gross inheritance, defined as the inheritance plus *inter vivo* gifts in the last 10 years minus debt and claims by the testator's spouse in case he/she did not inherit. In 18 cases I infer the age of the recipient from the age-dependent sustenance assumption. To estimate the succession probability, I use the variable which determines whether §13a deductions have been used. Firm assets are defined as the sum of shares of business assets, corporations and agricultural and forestry assets. I define liquid assets as the sum of stocks, securities, deposits, building society

threshold x_{min}	>500,000	>1m	>2m	>5m	>10m
		observational unit: beneficiary			
Pareto-Lorenz α	1.6132	1.6204	1.5907	1.6307	1.4450
Inverted Pareto-Lorenz β	2.6309	2.6117	2.6928	2.5855	3.247
N	4711	1543	488	114	35
		observational unit: testator			
Pareto-Lorenz α	1.4762	1.5347	1.4872	1.4563	1.6493
Inverted Pareto-Lorenz β	3.0999	2.8703	3.0526	3.1916	2.5401
N	6112	2229	753	189	75

Table 1.6: Pareto-Lorenz coefficients for all inheritances. Inheritances smaller than € 500,000 have been excluded for better comparability.

savings and cash.

1.A.2 Inheritance distribution

The inverted coefficient relates to the standard Pareto-Lorenz coefficient α via $\beta = \frac{\alpha}{\alpha-1}$, where α is the Pareto distribution's shape parameter (Atkinson et al., 2011). A larger inverted Pareto coefficients implies a fatter right-hand tail of the distribution. My estimates for both coefficients are reported in Table 1.6.

1.B Portfolio choice problem

This section proves the policy functions:it is optimal to invest available cash and equity in the firm, to exhaust the borrowing constraint and not to invest in the risk-less deposit.

The heir faces the following portfolio choice problem: she inherits equity and cash denoted $I_j = (1-\tau_s)x_j + (1-\tau_e)k_j$ and chooses investment in the firm e_j, savings a_j, and debt d_j. The financial sector allows borrowing up to $R^l d_j \leq \lambda e_j$. Only assets invested in the firm serve as collateral. Capital earns rate ρ, deposits earn R^d and borrowing costs are R^l. The core assumption used throughout this section is $\rho > R^l > R^d$. A risk-neutral agent always chooses at least some investment in the firm, which allows to neglect the non-negativity constraint of e_j. The portfolio choice problem reads

$$\begin{aligned} V_e = \max_{e_j,a_j,d_j} & \rho(e_j + d_j) - R^l d_j + R^d a_j \\ s.t. I_j &= a_j - e_j \\ -a_j &\leq 0 \\ (R^l d_j - \lambda e_j) &\leq 0 \\ -d_j &\leq 0. \end{aligned}$$

Denote by γ the Lagrange multiplier of the budget constraint, μ_1 and μ_3 are the multipliers of the non-negativity constraints of savings and debt. Non-negativity constraints are important here to rule out arbitrage. The Lagrange function reads

$$\begin{aligned} V_e = \max_{e_j,a_j,d_j} & \rho(e_j + d_j) - R^l d_j + R^d a_j \\ & + \gamma(I_j - a_j - e_j) \\ & + \mu_1 a_j \\ & - \mu_2(R^l d_j - \lambda e_j) \\ & + \mu_3 d_j, \end{aligned}$$

and the first-order conditions for a maximum are

$$\frac{\partial V_e}{\partial e_j} = \rho - \gamma + \mu_2 \lambda \qquad = 0 \tag{1.20}$$

$$\frac{\partial V_e}{\partial a_j} = R^d - \gamma + \mu_1 \qquad = 0 \tag{1.21}$$

$$\frac{\partial V_e}{\partial d_j} = \rho - R^l - \mu_2 R^l + \mu_3 = 0 \tag{1.22}$$

$$(I_j - a_j - e_j) \qquad = 0 \tag{1.23}$$

$$\mu_1 a_j \qquad = 0 \tag{1.24}$$

$$\mu_2(R^l d_j - \lambda e_j) \qquad = 0 \tag{1.25}$$

$$\mu_3 d_j \qquad = 0 \tag{1.26}$$

$$\{\mu_1, \mu_2, \mu_3\} \qquad \geq 0. \tag{1.27}$$

The proof proceeds as follows: First, by contradiction show that $d_j > 0$ and $a_j > 0$ cannot hold simultaneously. On the way, show that if $d_j > 0$, then it is optimal to exhaust the borrowing constraint. Second, with $\{d_j > 0, a_j > 0\}$ ruled out, there are only three remaining cases to check. Only one fulfils (1.20)-(1.27).

Start with (1.22) and multiply with d_j for $d_j > 0$:

$$
\begin{aligned}
(\rho - R^l)d_j - \mu_2 R^l d_j + \mu_3 d_j &= 0 \\
&\text{add and subtract } \mu_2 \lambda e_j \\
(\rho - R^l)d_j - \mu_2 R^l d_j + \mu_2 \lambda e_j + \mu_3 d_j &= \mu_2 \lambda e_j \\
&\text{by (1.25) and (1.26)} \\
(\rho - R^l)d_j &= \mu_2 \lambda e_j.
\end{aligned}
$$

Since $\rho - R^l$ and λe_j are positive, $\mu_2 > 0$ if $d_j > 0$, i.e. if the heir decides to use leverage, she will exhaust the borrowing constraint, $d_j = \frac{\lambda e_j}{R^l}$.

It follows

$$
\mu_2 = \frac{\rho - R^l}{R^l}. \tag{1.28}
$$

Now, multiply (1.21) with a_j for $a_j > 0$:

$$
\begin{aligned}
R^d a_j - \gamma a_j + \mu_1 a_j &= 0 \\
&\text{using (1.24) and } a_j \geq 0, \\
R^d &= \gamma
\end{aligned} \tag{1.29}
$$

Substitute (1.28) and (1.29) into (1.20),

$$
\rho - R^d + \frac{\lambda(\rho - R^l)}{R^l} = 0.
$$

Per assumption $\rho - R^d > 0$, $\rho - R^l > 0$, and $\{\lambda, R^l\} > 0$ which contradicts this equality. Hence, $\{d_j > 0, a_j > 0\}$ cannot be optimal.

It follows that either of the three must be the optimum

Case 1: $d_j = 0$ and $a_j > 0$

Case 2: $d_j = 0$ and $a_j = 0$

Case 3: $d_j > 0$ and $a_j = 0$.

Case 1: $a_j > 0$ implies $R^d = \gamma$ and $d_j = 0$ implies $R^l d_j < \lambda e_j$, which gives $\mu_2 = 0$. For (1.20) follows $\rho - \gamma + 0 \cdot \lambda = 0$ and equivalently $\rho = \gamma$,which contradicts $R^d = \gamma$.

Case 2: $d_j = 0$ implies $\mu_2 = 0$ which yields $\rho = \gamma$. This implies for (1.22):

$$
\begin{aligned}
(\rho - R^l) - 0 + \mu_3 &= 0 \\
\mu_3 &= -(\rho - R^l) < 0.
\end{aligned}
$$

The necessary condition for a maximum demands that $\mu_3 > 0$, which rules out Case 2 (and Case 1). If the shadow price μ_3 was smaller zero, implying $\rho < R^l$, the investor would like to short debt. The non-negativity rules this out.

Case 3: $d_j > 0$ implies $\mu_2 = \frac{\rho - R^l}{R^l} > 0$ and $\mu_3 = 0$.

By (1.20),

$$
\begin{aligned}
\rho - \gamma + \frac{\rho - R^l}{R^l}\lambda &= 0 \\
\rho + \frac{\rho - R^l}{R^l}\lambda &= \gamma,
\end{aligned}
$$

i.e. the shadow price of the constraint increases in the return on equity and the quality of the financial sector λ. Leverage raises the shadow price above ρ. No savings in the deposit, $a_j = 0$, implies

$$
\begin{aligned}
\mu_1 &= \gamma - R^d \\
&= \frac{\rho(R^l - 1) + R^l(1 - R^d)}{R^l} > 0,
\end{aligned}
$$

which is consistent with $\rho > R^l > R^d$. This proves that (1.20)-(1.27) hold in the case $d_j > 0, a_j = 0$.

The policy functions read

$$d_j = \frac{\lambda e_j}{R^l}$$
$$a_j = 0$$
$$e_j = I_j.$$

1.C Bequest choice

This section provides an exemplary microfoundation of the bequest policy functions in the main text. For tractability, in the main text, I deviate from the model outlined in this Appendix and assume $\tilde{e}_{x_j,\tau_e} = 0$.

Following Jung and Kuester (2015), assume that ϵ is *iid* logistically distributed with mean 0 and variance $\psi_\epsilon^2\pi^2/3$. Denote by $f(\epsilon)$ the pdf and denote by $F(\epsilon)$ the cdf of random variable ϵ. The *ex ante* probability to choose the entrepreneurial path is

$$\Pr(\epsilon \leq \bar{\epsilon}(x_j, k_j, A_j)) = F(\bar{\epsilon}(x_j, k_j, A_j)) = \left[1 + \exp\left(-\frac{\bar{\epsilon}(x_j, k_j, A_j) - \mu_\epsilon}{\psi_\epsilon}\right)\right]^{-1}.$$

Exploiting properties of the logistic distribution, the indirect utility reads

$$\begin{aligned} V(x_j, k_j, A_j) =& F(\bar{\epsilon})V_e(x_j, k_j, A_j) + \int_{-\infty}^{\bar{\epsilon}} \epsilon df(\epsilon) + (1 - F(\bar{\epsilon}))\left[w_{out} + (1 - \tau_s)R^d(x_j + sk_j)\right] \\ =& F(\bar{\epsilon})V_e(x_j, k_j, A_j) + \Psi + (1 - F(\bar{\epsilon}))\left[w_{out} + (1 - \tau_s)R^d(x_j + sk_j)\right], \\ \Psi =& -\psi(1 - F(\bar{\epsilon}))\log(1 - F(\bar{\epsilon})) + F(\bar{\epsilon})\log F(\bar{\epsilon}), \end{aligned}$$

where Ψ denotes the option value of the career choice. $V(x_j, k_j, A_j)$ enters the objective function of a a dynastic bequest problem.

The parent faces a classic consumption-bequest problem. Her state vector reads $\{k_{j-1}, A_{j-1}\}$, and she chooses consumption, bequests in business assets and bequests in liquid assets,

$$\begin{aligned}
\max_{c_{j-1}, x_j, k_j} \; & u(c_{j-1}) + \beta V(x_j, k_j, A_j) \\
\text{s.t. } & k_j + x_j + c_{j-1} = \rho\widetilde{(A_{j-1})}k_{j-1} \\
& k_j \leq k_{j-1}.
\end{aligned}$$

The parent is not allowed to bequeath more equity than she uses in production, $k_j \leq k_{j-1}$, ruling out tax sheltering.

First-order conditions read

$$\begin{aligned}
u'(c_{j-1}) &\leq \beta \frac{\partial V}{\partial k_j} \quad \begin{cases} < \text{when } k_j = k_{j-1} \text{ (i)} \\ = \text{when } k_j \leq k_{j-1} \text{ (ii)} \end{cases} \\
u'(c_{j-1}) &= \beta \frac{\partial V}{\partial x_j}.
\end{aligned}$$

For reasonable values of τ_e, τ_s, s and $F(\bar{\epsilon})$, it follows $\frac{\partial V}{\partial x_j} < \frac{\partial V}{\partial k_j}$.[19] Hence,

$$\beta \frac{\partial V}{\partial x_j} = u'(c_{j-1}) < \beta \frac{\partial V}{\partial k_j}.$$

It is optimal to always bequeath the firm, even if that means $x_j < 0$, which in legal terms turns the bequest into an "onerous gift".

The first-order condition with respect to x_j indicates that the cash bequest rises with parent's wealth, $\{k_{j-1}, A_{j-1}\}$. Bequests are a normal good. Consumption and bequests are substitutes: when the price of a bequest, τ_s, rises, $\frac{\partial V}{\partial x_j}$ falls. $u'(c_{j-1})$ must fall which implies higher consumption by the parent.

Since cash is more profitable in the firm than in the outside option, $\partial\left(\frac{\partial V}{\partial x_j}\right)/\partial \tau_e < 0$. When τ_e is large, fewer heirs choose the entrepreneurial path. A lower fraction of heirs has access to the more productive entrepreneurial technology, which reduces the average return of bequeathed cash and conse-

[19]Numerical assumptions are necessary because an heir who chooses the talent path prefers cash rather than equity: Equity is subject to a scrap factor while cash goes unscathed. If most or all capital is destroyed when the firm is scrapped and the probability of this event is large, the heir *ex ante* prefers cash. Empirically, governments grant generous deductions, s should be somewhere around 80% and $F(\bar{\epsilon})$ somewhere around 70%, more than enough to ensure $\frac{\partial V}{\partial x_j} < \frac{\partial V}{\partial k_j}$.

quently the marginal value of the cash bequest. This resembles the "strategic component" of firm bequests. The firm founder might have a taste for wealth preferences or might genuinely care about the firm's employees. If the heir is more likely to choose the entrepreneurial path, the firm founder endows the heir with more cash.

In summary, for the policy functions follows

$$\begin{aligned} k_j &= k_{j-1} \\ x_j &= \hat{x}(A_{j-1}, k_{j-1}, \tau_s, \tau_e) \\ \tilde{e}_{x_j,k_{j-1}} &\geq 0 \\ \tilde{e}_{x_j,\tau_s} &\leq 0 \\ \tilde{e}_{x_j,\tau_e} &\leq 0. \end{aligned}$$

To ensure tractability of the model in the main text, I assume $\tilde{e}_{x_j,\tau_e} = 0$.

1.D Social welfare function

Utility is linear and the planner can redistribute output intratemporally. Hence, maximizing social welfare in terms of utility is equal to maximizing output minus utility and borrowing costs. The planner chooses employment and debt, $\{\widetilde{n_{j-1}}, \widetilde{n_j}, \widetilde{d_{j-1}}, \widetilde{d_j}\}$, and the reservation talent for the company heir, $\bar{\omega}(x_j, k_j, A_j)$. The total number of workers in this economy is denoted $\bar{n}_{j-1}$ in the first and $\bar{n}_j$ in the second period. Define the social welfare function,

$$\begin{aligned} \max_{\widetilde{d_{j-1}}, \widetilde{n_{j-1}}, \widetilde{d_j}, \widetilde{n_j}, \bar{\omega}} \quad & \widetilde{n_{j-1}} A_{j-1} - R^l \widetilde{d_{j-1}} + (\bar{n}_{j-1} - \widetilde{n_{j-1}}) \underline{w} \\ & + \beta^s \left\{ F(\bar{\omega}) \left[\widetilde{n_j} A_j - R^l \widetilde{d_j} \right] - \int_{-\infty}^{\bar{\omega}} \epsilon \, dF_\epsilon(\epsilon) \right. \\ & + (1 - F(\bar{\omega})) \left[\widetilde{n_j} \underline{w} + w_{out} + ((1-\tau_s) x_j + (1-\tau_s) k_j s) R^d + \tau_s k_j s R^d \right] \\ & \left. + \tau_s x_j R^d + (\bar{n}_j - \widetilde{n_j}) \underline{w} \right\} \end{aligned}$$

which equals the sum of output minus borrowing costs, expected utility costs and foregone labour income outside the firm in both periods with social discount factor β^s. In the first period, output consists of the output produced by the $\widetilde{n_{j-1}}$ workers minus debt service. The $(\bar{n}_{j-1} - \widetilde{n_{j-1}})$ workers, who are not employed in the firm in the first period, earn the outside wage in any state of production. Before the second period of production takes place, the planner commands a reservation talent, $\bar{\omega}(x_j, k_j, A_j)$. For any $\epsilon \leq \bar{\omega}$ the firm will be continued and utility costs are realized. Given the *cdf* of ϵ, the firm is continued with probability $F(\bar{\omega}(x_j, k_j, A_j))$ and the planner chooses quantities $\widetilde{n_j}$ and $\widetilde{d_j}$. Production and debt service proceed as before. With probability $(1 - F(\bar{\omega}(x_j, k_j, A_j)))$ the heir draws a utility cost realization exceeding $\bar{\omega}(x_j, k_j, A_j)$ and the firm's exit is socially optimal: the $\widetilde{n_j}$ workers and the heir earn their outside wages and after-tax equity and cash are deposited, earning interest R^d. The exogenously given tax rate τ_s will be applied to the remaining equity, and tax revenue will be deposited at rate R^d. The last line represents the government's income from taxing the cash bequest and the labour income of workers, who are in this economy but cannot be employed by the firm due to financial constraints.

I simplify the social welfare function to highlight those channels that demand the social planner's attention. First, it is both privately and socially optimal to exploit the borrowing constraint, as long as $A_j > R^l + \underline{w}$, which I assume henceforth. Both the planner and the entrepreneurs will exploit the constraint and *in the first period* will choose the same capital input and employment levels, $\widetilde{d_{j-1}} = d_{j-1} = \frac{\lambda}{R^l} k_{j-1}$ and $\widetilde{n_{j-1}} = n_{j-1} = \phi^{-1} \frac{R^l + \lambda}{R^l} k_{j-1}$. This allows to drop output in the first period. In the second period, though both will exploit the borrowing constraint, the private decision may be subject to a tax on equity, τ_e, leading to different levels of debt and employment in the private and centralized solution. Second, I drop exogenously given state values which neither the planner nor the agents can alter. Those are: $\bar{n}_{j-1}\underline{w}$ and $\bar{n}_j\underline{w}$ and the tax income on cash bequests $\tau_s x_j$. The social welfare function reduces to

$$\max_{\widetilde{d_j}, \widetilde{n_j}, \bar{\omega}} F(\bar{\omega}(x_j, k_j, A_j)) \Big[\widetilde{n_j} A_j - R^l \widetilde{d_j}\Big] - \int_{-\infty}^{\bar{\omega}} \epsilon \, dF_\epsilon(\epsilon)$$
$$+ (1 - F(\bar{\omega}(x_j, k_j, A_j))) \Big[\widetilde{n_j}\underline{w} + w_{out} + [(1 - \tau_s)x_j + k_j s] R^d\Big]$$

Substituting socially optimal choices for debt and employment

$$\tilde{d}_j = \frac{\lambda}{R^l}\Big[(1-\tau_s)x_j + k_j\Big] \text{ and } \tilde{n_j} = \phi^{-1}\Big(1+\frac{\lambda}{R^l}\Big)\Big[(1-\tau_s)x_j + k_j\Big]$$

and subtracting $\widetilde{n'\underline{w}}$ for clarity,[20] we arrive at the social welfare function, with only one remaining choice variable, the reservation talent which determines whether to continue or destroy a firm:

$$\begin{aligned}\Omega = \max_{\bar{\omega}} F(\bar{\omega})&\overbrace{\Big\{((1-\tau_s)x_j + k_j)\phi^{-1}(1+\frac{\lambda}{R^l})\Big[A_j - \underline{w} - \phi\frac{R^l\lambda}{R^l+\lambda}\Big]\Big\}}^{\equiv\Omega_e(k_j,x_j)} - \int_{-\infty}^{\bar{\omega}} \epsilon dF_\epsilon(\epsilon)\\ &+ [1-F(\bar{\omega})]\Big\{w_{out} + ((1-\tau_s)x_j + k_j s)R^d\Big\}, \end{aligned} \tag{1.14}$$

where I denote by $\Omega_e(k_j, x_j)$ the social value of the firm with equity k_j and cash holdings x_j.

1.E Probability of firm continuation

What is the correlation of an intra-family succession and the size of inherited business assets? What is the correlation of a succession and the stock of inherited liquid assets (cash)? Table 1.7 reports average marginal effects obtained from a Logit model,

$$\begin{aligned}&\mathbf{x_i}'\beta = \log(business\ assets_i)\beta_k + \log(cash_i)\beta_x + \ldots + \epsilon_i,\\ &P(succession_i|\mathbf{x_i}') = \Big[1+\exp\Big(-\mathbf{x_i}'\beta\Big)\Big]^{-1}\end{aligned} \tag{1.30}$$

and Table 1.8 reports the regression coefficients. In specification (1) cash inheritances are omitted. Specifications (2) and (3) include cash inheritances, with an interaction term in (3). Specifications (4) and (5) include heirs of agricultural and forestry firms with a corresponding dummy variable, more than doubling the sample size.

To begin with, I interpret marginal effects and coefficients as correlations, not causal effects (see below). Starting with the main finding, throughout the

[20] Subtracting $\widetilde{n'\underline{w}}$ from the SWF does not affect the socially optimal career choice, as it reduces welfare for either career choice.

	(1)	(2)	(3)	(4)	(5)	(6)
log(business assets)	0.0948***	0.0953***	0.0968***	0.107***	0.108***	0.131***
	(19.79)	(15.87)	(16.30)	(34.81)	(35.60)	(21.31)
log(cash)		0.00997	0.0141	0.00484	0.00872	
		(1.26)	(1.73)	(1.11)	(1.86)	
Includes agriculture& forestry				0.0187	0.0210	0.0552
				(1.04)	(1.17)	(1.50)
Age						0.00127
						(1.56)
sample includes agricultural & forestry	-	-	-	✓	✓	✓
interaction term log(business) × log(cash)	-	-	✓	-	✓	-
Pr[firm succession]	0.627	0.607	0.607	0.328	0.328	0.384
N	1468	947	947	2846	2846	897

z statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1.7: Marginal effects on the firm succession probability. Dependent variable is the usage of §13a deductions, assumed to equal the succession of an inherited business. Beneficiaries who did not receive a firm have been excluded. I reduced the sample to inheritances for which whole decomposition is known. Data: German inheritance and gift tax statistic 2002.

	(1)	(2)	(3)	(4)	(5)	(6)
log(business assets)	0.486***	0.478***	1.440***	0.742***	1.480***	0.825***
	(14.23)	(11.40)	(4.67)	(22.62)	(7.27)	(13.04)
log(cash)		0.0501	0.970***	0.0336	0.707***	
		(1.26)	(3.33)	(1.11)	(3.84)	
log(business assets) ×log(cash)			-0.0788**		-0.0635***	
			(-3.20)		(-3.73)	
Includes				0.130	0.147	0.347
agriculture& forestry				(1.04)	(1.17)	(1.49)
Age						0.0463
						(1.85)
Age^2 receipient						-0.000355
						(-1.48)
sample includes agricultural & forestry	-	-	-	✓	✓	✓
Pr[firm succession]	0.627	0.607	0.607	0.328	0.328	0.384
N	1468	947	947	2846	2846	897

z statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1.8: Coefficients of the Logit model. Dependent variable is the usage of §13a deductions, assumed to equal the succession of an inherited business. Transfers not including a firm or firms worth less than € 1000 have been excluded. I reduced the sample to inheritances for which the whole decomposition is known. Data: German inheritance and gift tax statistic 2002.

specifications, the company size is positively correlated with the propensity to use §13a deductions. Robustly, a 1% larger inherited business raises the propensity to use §13a by approximately 0.1 percentage points.

Liquid assets The correlation of inherited liquid assets and the succession probability is positive but small. The marginal effect is insignificant across all specifications. However, for some beneficiaries cash inheritances can be more relevant in the decision to continue the firm than for others: specifications (3) and (5) include interaction terms of liquid and business assets. With the interaction term, the coefficient of cash inheritances becomes positive and significant, while the interaction term's coefficient is negative (Table 1.8). Liquid assets inherited together with a company have a positive but diminishing correlation with firm succession. This can be explained by prior *inter vivo* gifts, life insurance policies or borrowing constraints that permit heirs of larger companies to borrow to pay their taxes.

Agricultural and forestry firms Including agricultural and forestry firms in the sample increases the marginal effects slightly, but at the same time the probability that §13a is used drops significantly from around 60% to 32%. Many agricultural and forestry firms in the sample are very small with a mean net worth of only €5600. Hence, for many beneficiaries it is impractical to go through the hassle of applying for §13a deductions and keeping the company for five years. Yet, controlling for the firm size, farm and forestry firms have a 0.02 percentage point higher succession probability. This estimate is insignificant and very small compared to Laband and Lentz (1983) who find that succession rates of farms are higher than non-farms by a multiple of five. Laband and Lentz argue that heirs of family-owned farms are born into their occupation. They help out on the parents' farm and accumulate occupational human capital and farm-specific human capital, such as knowing the weather and soil, which cannot be transferred across farms. Using the inheritance and gift tax statistic, I cannot confirm this claim as the difference in succession rates measured by usage of firm deductions is not statistically different from zero.

Recipient's age In specification (6), I add the main recipient's age (and age squared) to the regression model. The age effect is positive and diminishing, but not significant (Table 1.8). Using the U.S. ARMS data and respectively Israelian data, Mishra et al. (2004) and Kimhi and Nachlieli (2001) find that the age of the eldest son has a positive and diminishing effect on the succession probability of family farms. In contrast, when I reduce my sample to agricultural and forestry firms, the effect of age on firm succession vanishes altogether (not in table). My results concerning age should be interpreted with caution: First, the dataset only contains the main beneficiary's age instead of the age of the beneficiary who will manage the company. Second, there is a selection problem: the age of a recipient is not relevant for taxation in many cases. It is relevant for "sustenance exemptions", only the spouse of the deceased or beneficiaries younger than 28 can apply for. Consequently, the variable age is missing in many cases, explaining the lower sample size of specification (6). Redeeming yet surprising, the mean reported age is 53 with a standard error of 17.

Causal effects? Why should the findings not be interpreted as causal effects? Firstly, many firm successions are "smooth" to facilitate the transfer of managerial control and save inheritance taxes. Smooth transitions drive up the succession probability and reduce the firm's value at the testator's death. This biases the estimates downward if the gifts are not reported in the statistic. Secondly, family firms can be used as tax shelters (Alstadsaeter et al., 2014). Testators will try to minimize the portfolio share of liquid assets they bequeath in order to save taxes. In Germany, this has spawned so-called "cash GmbHs", pseudo companies whose sole purpose it is to transfer assets intergenerationally at a reduced tax rate. Thirdly, many business owners take pride in their work, see responsibilities for their employees and want to build an estate that lasts. This is interpreted as a taste-for-wealth motive and can bias the results: i) a taste-for-wealth entrepreneur, who does not find a successor in the family, might sell (parts of) the company before his death to avoid liquidation. The firm will either not appear in the statistic at all or only a fraction of the firm's worth is reported; ii) the combination of a taste-for-wealth and a strategic bequest motive (Bernheim et al., 1985) can bias the estimates via reverse causality: suppose an entrepreneur bar-gains with her child over the firm succession. The company owner might make firm succession a prerequisite for the inheritance. Consequently, a planed firm succession by the child will drive up business and liquid assets. In summary, I abstain from calling my estimates causal effects. Research in this field could immensely benefit from better data or the possibility to connect this dataset with others, e.g. income tax data. In its current state, the inheritance and gift tax statistic lacks information about testators and beneficiaries and linking the dataset to others is impossible.

CHAPTER 2

The Equity Premium and Unemployment: Endogenous Disasters or Long-Run Risk?

Chapter Abstract

This paper studies two extensions of the Diamond-Mortensen-Pissarides framework to jointly generate i) a high volatility of unemployment and stock prices, ii) the striking correlation of unemployment and stock prices and iii) a large equity premium. First, a globally solved DMP model with endogenous separations and wage rigidity is unable to generate a large risk premium but succeeds in matching key macroeconomic moments. The introduction of endogenous separations improves the model's goodness of fit and helps to match the volatile 1950s U.S. economy. Second, a DMP model driven by a small, autoregressive component of productivity growth can solve the Shimer puzzle if wage rigidity is assumed to be excessively strong. Facing long-run risk, investors demand a slightly larger equity premium which still falls short of empirical estimates.

2.1 Introduction

This paper studies the co-movement of stock prices and unemployment. Risky assets command a substantial premium, and their annual returns are pro-cyclical and four times more volatile than GDP. Since Mehra and Prescott (1985), if not before, it has been well known that the workhorse neoclassical growth model struggles to provide a theoretical basis for these facts. Furthermore, labour market transitions are volatile and move with the cycle. The unemployment rate is seven times more volatile than GDP. Since Shimer (2005), at the latest, it has been known that the workhorse model of the frictional labour market struggles to explain this volatility. Can a deviation of the canonical Diamond, Mortensen and Pissarides model (DMP) explain both the risk premium and high volatilities in stock prices and labour market flows? This paper's main findings are as follows. Firstly, a globally solved DMP model with cyclical fluctuations, endogenous separations, and rigid wages matches U.S. labour market data but cannot generate the risk premium. Secondly, a small, stochastic, persistent component of productivity growth, following Bansal and Yaron (2004) and Croce (2014), increases the equity premium marginally; yet the premium is insufficient and the model struggles to offer a robust solution to the Shimer puzzle.

This paper augments the canonical DMP model with Epstein and Zin (1989)-preferences, endogenous separations, wage rigidity and two different productivity processes: i) standard, cyclical TFP-shocks (Mehra and Prescott, 1985) and ii) shocks to the persistent component of productivity growth (Bansal and Yaron, 2004). Since large deviations from steady state (disasters) may be responsible for the risk premium, the models are solved globally (Fernández-Villaverde and Levintal, 2018; Petrosky-Nadeau and Zhang, 2017). The models are parametrized to moments of post-war U.S. time series. I specifically target means and volatilities of the employment-to-unemployment and unemployment-to-employment transition rates.

This paper is motivated by the striking correlation between labour market data and stock prices, shown in Figure 2.1. The first panel documents the negative correlation between stock prices (S&P 500 index) and unemployment. Figure 2.1's second panel shows that job-finding rates and stock prices are positively correlated: when news about better productivity surfaces, the expected profit derived from a new worker rises. Firms increase their hiring; stock prices surge as employment and the expected firm surplus rise. The third panel documents the negative correlation of separations and asset prices. Adverse news about productivity raises the number of terminated jobs and depresses stock prices at the same time.

Why do labour market flows and stock prices co-move in the DMP framework? Just like equity, a new hire is a risky asset that is priced with the "fundamental asset pricing equation": in the case of equity, the equilibrium stock price equals the present value of future dividends; in the case of employment, the equilibrium price of a hire (cost of hiring and training) equals the present value of the firm's share of the product of labour.

Solving the equity premium puzzle remains a challenge for consumption-based asset pricing models. Going back to Rietz (1988), Barro (2006), Barro (2009), Gourio (2012), and Ghosh and Anisha (2012), a small probability of deep and persistent drops in output and consumption (=disasters) can generate sizeable risk premia. Disasters can be interpreted as a Peso problem: *ex post* the researcher observes a low volatility of fundamentals, but *ex ante* investors assigned a positive probability to a disaster. The Sharpe ratio observed by the researcher appears large *ex post*, but merely reflects the positive probability of a rare event. In a DMP framework, Wachter and Kilic (2018) assume a time-varying probability of exogenous disasters. Relatedly, Petrosky-Nadeau et al. (2018) solve the DMP model globally with cyclical fluctuations, calibrate to long-term data and show that the model can generate disasters endogenously. In their standard framework, labour market frictions amplify the effect of cyclical shocks on employment and production.[1] They claim that endogenous disasters and wage inertia generate a large equity premium. In contrast, I show that the DMP model with wage rigidity and cyclical TFP solves the Shimer puzzle, but fails to generate a large equity premium, even though the model is capable of generating endogenous disasters. I show that the parametrization used by Petrosky-Nadeau et al. (2018) does not match labour market transition rates and predicts disasters far too frequently. A careful parametrization falsifies the endogenous disaster story.

As emphasized by Bansal and Yaron (2004), Bansal et al. (2012), and

[1]See Hairault et al. (2010), Jung and Kuester (2011) and Den Haan et al. (2020).

Schorfheide et al. (2018), long-run consumption risk and recursive preferences can generate large risk premia. Croce (2014) shows that a production-based asset pricing model with capital-adjustment costs and a small predictable component of productivity growth can generate sufficiently large risk premia. Convex capital-adjustment costs reduce the volatility of investment and give rise to a time-varying Tobin's q. The volatile Tobin's q raises the volatility of equity prices, which leads investors to demand a risk premium. In contrast to Croce (2014), my long-run risk model (LRR) assumes a frictional labour market in an economy without capital. The time-varying cost of filling a vacancy takes the place of convex capital-adjustment costs: the time-varying vacancy-filling rate translates into time-varying vacancy-posting costs, which generate a volatile Tobin's q. This point has been made by Merz and Yashiv (2007), who establish that equity prices reflect employment and labour-adjustment costs in a frictional labour market. To my knowledge, the long-run risk model with a frictional labour market has not yet been studied extensively.[2] This paper shows that a small persistent component of productivity growth raises the equity premium marginally, but simulated unemployment volatility and the equity premium fall short of their empirical counterparts. In the LRR model, wage rigidity must be very strong to solve the Shimer puzzle. This makes the LRR model less robust: very rigid wages affect separations and may turn them pro-cyclical.

This paper makes two additional contributions. Firstly, I use the models' policy functions together with employment and output time series to estimate processes for cyclical fluctuations and the long-run growth component. Using the estimated productivity series, I simulate the U.S. economy from 1929 to 2018 and study the goodness-of-fit of the generated series to the data. Endogenous separations improve the model's goodness of fit, especially in turbulent times. The canonical model with a constant separation rate will fail to replicate large hikes in unemployment, for example the Great Depression. Endogenous separations allow the model to match such hikes. Comparing time series reveals that the long-run risk model is better suited for the moderate post-1950s U.S. economy than the volatile years prior to the

[2] A working paper version of Kehoe et al. (2019) argues that the long-run risk model with recursive preferences, solved by perturbation, does not generate sufficiently strong volatility in the discount factor to drive a volatility of unemployment as large as observed empirically. After solving the model globally, I agree.

1950s. This model puts the burden of matching fluctuations completely on a component that is assumed to be *small and highly-persistent*. Deep recessions and quick recoveries, e.g. the Great Depression, are difficult to match with this component alone.

Secondly, I study why my models fail to generate a risk premium: Although investors dislike the pro-cyclical nature of equity returns, they do not demand a large premium because the conditional variance of marginal utility is too small. I show that the variance is larger in the long-run risk model (LRR) than the RBC model because small shocks can have long-lasting effects. But, neither cyclical nor long-run risk shocks suffice to introduce enough risk in the model, calibrated to post-war U.S. data. In the calibration by Petrosky-Nadeau et al. (2018), the prospect of (too) frequent disasters raises the volatility of marginal utility and a premium follows. I conclude that a habit model might be a step towards solving the equity premium puzzle in this framework: with habits, the variance of marginal utility rises even though consumption and output volatility remain at empirically plausible levels.

The paper proceeds as follows: The next section outlines the model. The subsequent sections Section 2.3 and Section 2.4 analyze two different versions of this model. First, Section 2.3 parametrizes the DMP model with endogenous separations and wage rigidity (RBC), simulates and matches the 1928-2018 time series. Second, Section 2.4 parametrizes the model with a small persistent component of productivity growth (LRR), simulates and matches time series. Section 2.5 studies the transmission of shocks in the models and their failure to replicate the equity premium.

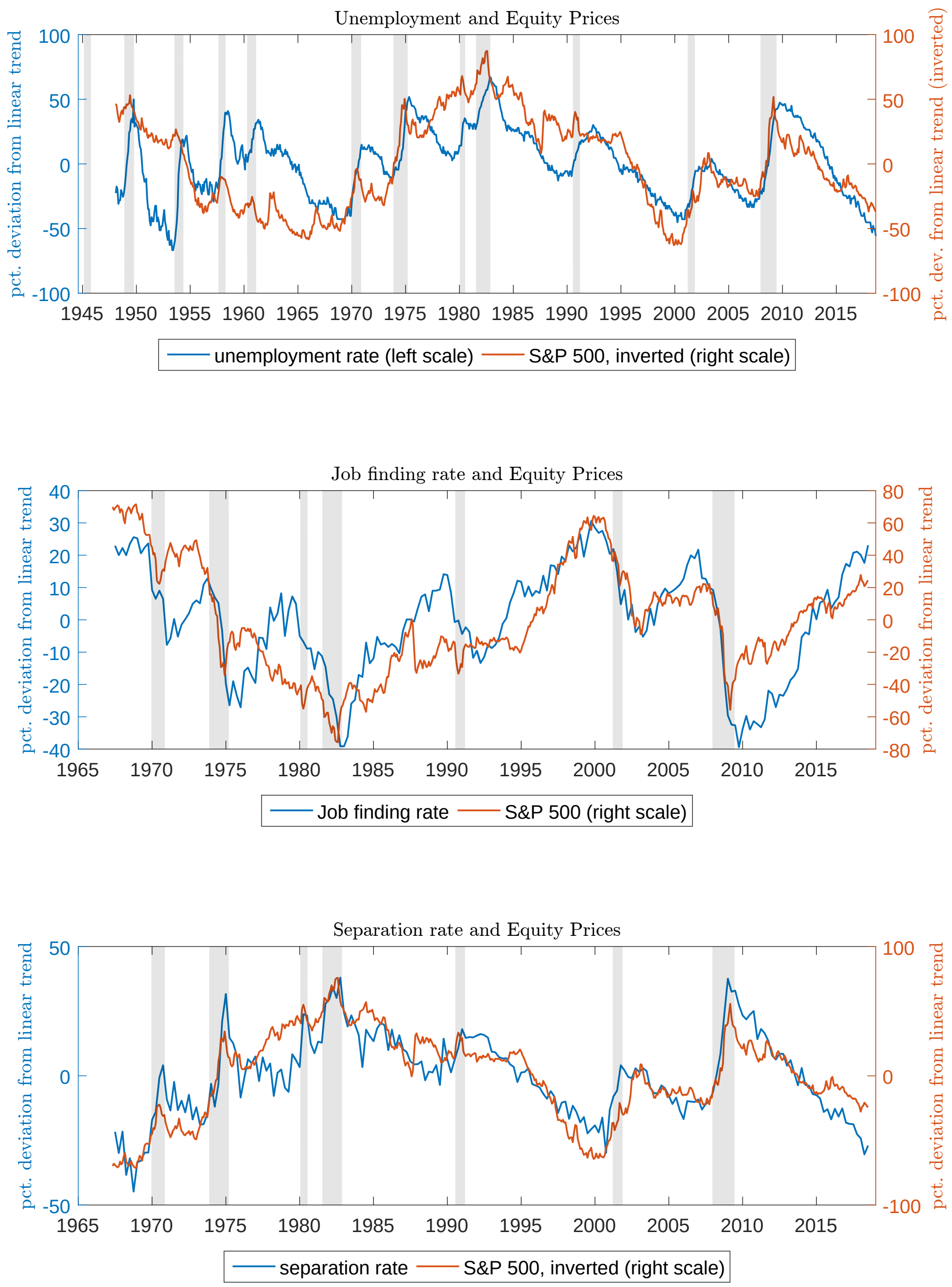


Figure 2.1: Equity prices and labour market data. S&P 500 vs unemployment (inverted), the job-finding rate and the separation rate (inverted) in percentage deviations from a linear trend. U.S. quarterly data. Grey bands denote NBER recessions. These correlations are remarkable compared to other time series: as shown by Hall (2017), the correlation between TFP and unemployment is low. Appendix 2.A shows the comparatively low correlations of wages and dividends to equity prices and unemployment.

2.2 Model

This section describes the model, which is an extension of the canonical Diamond-Mortensen-Pissarides framework. A time-varying separation rate increases the model's goodness-of-fit of labour data. Wage rigidity raises the volatility of unemployment. Recursive preferences disentangle risk aversion and the intertemporal elasticity of substitution and give meaning to news shocks. We compare two different productivity processes: a standard RBC shock (Mehra and Prescott, 1985) and a small, predictable component of growth (Bansal and Yaron, 2004). Disaster risk, endogenously generated by large RBC shocks, and the Bansal-Yaron model are prominent solutions of the equity premium puzzle. The model is solved globally.

Preferences Time is discrete. The representative consumer has Epstein and Zin (1989) recursive preferences given by

$$V_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}} \tag{2.1}$$

with the intertemporal elasticity of substitution ψ and risk aversion γ. In the special case of $\gamma = \frac{1}{\psi}$, (2.1) reduces to a power utility function. We make the standard assumptions that $\gamma > 1/\psi < 1$: when $\gamma > 1/\psi$, investors prefer an early resolution of risk and demand a risk premium for assets that are positively correlated with expected consumption growth; when $1/\psi < 1$, news about consumption growth raise returns.

The stochastic discount factor M_{t+1} is given by

$$M_{t+1} = \frac{\frac{\partial V_t}{\partial C_{t+1}}}{\frac{\partial V_t}{\partial C_t}} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{V_{t+1}}{\mathbb{E}_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}. \tag{2.2}$$

Production At the beginning of a period, there exist l_t firm-worker matches in the economy. Let u_t denote the unemployed and normalize the mass of all workers to unity, $u_t + l_t = 1$. Following Jung and Kuhn (2014), match j's production function is linear,

$$y_{j,t}|_{\epsilon_{j,t}>\underline{\epsilon_t}} = e^{z_t}A_t + \epsilon_{j,t}, \tag{2.3}$$

where $e^{z_t}A_t$ denotes aggregate productivity and $\epsilon_{j,t}$ is a match's idiosyncratic productivity drawn from a logistic distribution with mean $-\mu_{\epsilon,t}$ and variance $\frac{\psi_{\epsilon,t}^2\pi^2}{3}$. A firm and a worker jointly decide whether to produce or to separate the match using a cut-off rule $\underline{\epsilon_t}$. If the idiosyncratic shock is lower than the cut-off point, the match separates. The cut-off rule is an outcome of the bargaining discussed below. In case of a separation, the worker becomes unemployed and the firm incurs wasteful separation costs $\tau_{eu,t}$. Using properties of the logistic distribution we obtain the expected output of a match,

$$\begin{aligned}
\widetilde{Y}_t &= \int_{\underline{\epsilon}}^{\infty}(e^{z_t}A_t + \epsilon_{j,t})df(\epsilon_{j,t}) + \int_{-\infty}^{\underline{\epsilon}} -\tau_{eu,t}df(\epsilon_{j,t}) \\
&= (1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t &(2.4)\\
\Psi_t &= -\psi_{\epsilon,t}[(1-\pi_{eu,t})\log(1-\pi_{eu,t}) + \pi_{eu,t}\log\pi_{eu,t}] - \pi_{eu,t}\tau_{eu,t}, &(2.5)
\end{aligned}$$

where Ψ_t denotes the option value of separation. The probability of separation before the revelation of $\epsilon_{j,t}$ reads

$$\pi_{eu,t} = P(\epsilon_{j,t} < \underline{\epsilon}) = \left[1 + \exp\left(\frac{-\underline{\epsilon_t} - \mu_{\epsilon,t}}{\psi_{\epsilon,t}}\right)\right]^{-1}.$$

The economy's total output, available for consumption and investment is given by

$$Y_t = l_t\widetilde{Y}_t. \tag{2.6}$$

Assume the following process for exogenous productivity with $a_t \equiv \log(A_t)$:

$$\begin{aligned}
a_{t+1} - a_t &= g_a + x_t + \sigma_a\epsilon_{a,t+1} \\
x_{t+1} &= \rho_x x_t + \sigma_x\sigma_a\epsilon_{x,t+1} \\
z_{t+1} &= \rho_z z_t + \sigma_z\epsilon_{z,t+1} &(2.7)
\end{aligned}$$

where $\epsilon_{a,t+1}$, $\epsilon_{x,t+1}$ and $\epsilon_{z,t+1}$ are iid standard normal variables. Productivity growth has a deterministic mean component g_a and two stochastic components: $\epsilon_{a,t+1}$ is a transitory shock to the technology growth rate, $\epsilon_{x,t+1}$ is a shock to the persistent component of the growth rate.

Vacancy posting and matching The timing in a period is as follows. At the beginning of a period, l_t matches exist. The aggregate states of productivity and the idiosyncratic productivities are revealed. Firms and workers bargain over wages or jointly decide to separate a match. Next, the $l_t(1-\pi_{eu,t})$ non-separated matches produce output which is used for consumption and investment. Finally, vacancies and unemployed workers are matched in a frictional market. The sum of old and new matches determines next period's l_{t+1}.

The representative firm, which is the mutual fund that owns all matches, discounts dividends with the representative agent's stochastic discount factor (2.2). It takes the probability of a match q_t as given and chooses vacancies and employment to maximize its cum-dividend stock price,

$$\begin{aligned}
P_t^c &= \max_{\{v_{t+\tau},l_{t+\tau}\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau} \\
\text{s.t. } l_{t+1} &= l_t(1-\pi_{eu,t}) + q_t v_t \\
D_t &= l_t\left[(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t} - W_t) + \Psi_t\right] - \kappa_{1,t} v_t - q_t v_t \kappa_{2,t} \\
v_t q_t &\geq 0.
\end{aligned}$$

The firm's value of a match at the beginning of a period is the derivative of P_t^c with respect to l_t,

$$J_t \equiv (1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t - (1-\pi_{eu,t})W_t + (1-\pi_{eu,t})\mathbb{E}_t M_{t+1} J_{t+1}, \quad (2.8)$$

where W_t denotes the worker's wage. Firms post vacancies at a cost $\kappa_{1,t}$ and pay training costs, $\kappa_{2,t}$, if the vacancy is filled. Via the first-order conditions of the problem, we derive the free-entry condition

$$\kappa_{1,t} - \Theta_t = q_t(\mathbb{E}_t M_{t+1} J_{t+1} - \kappa_{2,t}). \quad (2.9)$$

Investment equals the resources spent on the matching process,

$$\kappa_{1,t} v_t + q_t v_t \kappa_{2,t}.$$

I follow Petrosky-Nadeau and Zhang (2017) and impose a non-negativity constraint on posted vacancies, $v_t \geq 0$. The constraint's Lagrange multiplier is Θ_t. See Appendix 2.B.2 for a detailed solution of the firm's dynamic problem.

Assume that firms post vacancies v_t and find workers according to the matching function

$$\Xi_m(u_t, v_t) = \frac{u_t v_t}{(u_t^\iota + v_t^\iota)^{\frac{1}{\iota}}} \quad \iota > 0 \tag{2.10}$$

borrowed from den Haan et al. (2000). This function has the advantage that transition rates stay within $[0, 1]$. This improves the computational stability compared to a standard Cobb-Douglas matching function.[3] Denoting labour market tightness $\theta_t \equiv \frac{v_t}{u_t}$, the job-finding rate $\pi_{ue,t}$ and vacancy-filling rate q_t are given by

$$\pi_{ue,t} = (1 + \theta_t^{-\iota})^{-\frac{1}{\iota}} \tag{2.11}$$

$$q_t = (1 + \theta_t^{\iota})^{-\frac{1}{\iota}}. \tag{2.12}$$

Finally, labour's law of motion reads

$$l_{t+1} = l_t(1 - \pi_{eu,t}) + \pi_{ue,t}(1 - l_t).$$

Bargaining and separation The representative family earns wages for each productive match and replacement income b_t per unemployed worker. Replacement income is financed with lump-sum taxes, denoted T_t. The

[3]When the discounted firm surplus is low, firms post few (or even no) vacancies. With a Cobb-Douglas matching function, a reduction of vacancies increases job-finding without bound, jeopardizing stability. Instead of the new matching function, one could restrict the vacancy-filling rate, but this introduces a kink which again jeopardizes stability. The den Haan et al. (2000) function restricts the elasticity with respect to job-seekers, $\frac{1}{1+\theta^{-\iota}} \in [0, \frac{1}{2}]$: i) Unless the job-finding rate exceeds the vacancy-filling rate, labour market tightness $\theta = \frac{\pi_{ue}}{q}$ is smaller than one. Hence, the upper bound for the elasticity of the matching function with respect to unemployment is $\frac{1}{2}$ achieved at $\iota \to 0$. Sedláček (2016) and others estimate this elasticity to exceed $\frac{1}{2}$ (see their Table 1). ii) If one assumes Nash bargaining and the Hosios condition at steady state values, the bargaining power of job-seekers cannot exceed $\frac{1}{2}$ for the same reasons.

mutual fund's shares s_t trade at price P_t. Bonds $\widetilde{B}_t$ return a risk-free interest rate R_t^f. The family's consumption reads

$$C_t = W_t l_t (1-\pi_{eu,t}) - T_t + b_t(1 - l_t + \pi_{eu,t} l_t) + s_t(D_t + P_t) - s_{t+1} P_t + \widetilde{B}_t - \frac{1}{R_{t+1}^f}\widetilde{B}_{t+1}.$$

The family's value of an additional worker in terms of consumption goods reads

$$\Delta_t \equiv \frac{\frac{\partial V_t}{\partial l_t}}{(1-\frac{1}{\psi})C_t^{-\frac{1}{\psi}}} = [(W_t - b_t)(1-\pi_{eu,t})] + \mathbb{E}_t M_{t+1}\Delta_{t+1}(1-\pi_{eu,t}-\pi_{ue,t}).$$

Define the joint surplus of a match $\Sigma_t \equiv \Delta_t + J_t$ and denote the worker's bargaining power with ρ_t. Nash bargaining determines the wage and the separation rate,

$$(\pi_{eu,t}, W_t) = \arg\max_{\pi_{eu,t}, W_t} \Delta_t^{\rho_t} J_t^{1-\rho_t}. \tag{2.13}$$

The first-order condition of (2.13) yields the efficient reservation productivity $\underline{\epsilon}_t = b_t - e^{z_t}A_t - \tau_{eu,t} - \mathbb{E}_t M_{t+1}\Sigma_{t+1}$. If the idiosyncratic productivity of a match falls below this cut-off value, a match is separated. By property of the logistic distribution the separation rate reads

$$\pi_{eu,t} = \left[1 + \exp\left(\frac{\mathbb{E}_t M_{t+1}\Sigma_{t+1} + e^{z_t}A_t - \mu_{\epsilon,t} + \tau_{eu,t} - b_t}{\psi_{\epsilon,t}}\right)\right]^{-1} \tag{2.14}$$

Via the free-entry condition (2.9) and the first-order conditions of (2.13), obtain the Nash wage (see Appendix 2.B.3)

$$\begin{aligned} W_t =& \Big[(1-\pi_{eu,t})\Big]^{-1} \\ & \Big\{\rho_t \left[(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t + (1-\pi_{eu,t})\mathbb{E}_t M_{t+1} J_{t+1}\right] \\ & + (1-\rho_t)\left[b_t(1-\pi_{eu,t}) - \mathbb{E}_t M_{t+1}\Delta_{t+1}(1-\pi_{eu,t}-\pi_{ue,t})\right]\Big\}. \end{aligned} \tag{2.15}$$

To introduce wage rigidity, I follow Jung and Kuester (2015) and assume that the worker's bargaining power decreases in the persistent component of the technology growth rate x_t and TFP's transitory component z_t,

$$\rho_t = \bar{\rho} e^{-x_t \alpha_x - z_t \alpha_z}. \tag{2.16}$$

This turns the shocks ϵ_z and ϵ_x into joint shocks to wages and productivity, or surplus shocks. In Section 2.4, I describe why this assumption is useful in the context of a long-run risk model. In short: when the worker's outside option and vacancy-posting costs are proportional to productivity, the fundamental surplus is not volatile enough to solve the Shimer puzzle, even if the fundamental surplus is calibrated to be small.

Aggregation and productivity adjustment Bonds are in zero net supply, $\widetilde{B}_t = 0 \; \forall t$, and the representative family owns the mutual fund, $s_t = 1 \; \forall t$. Taxes are used to pay unemployment benefits, so $T_t = b_t(1 - l_t + \pi_{eu,t} l_t)$. Together, aggregate consumption reduces to output minus investment,

$$C_t = W_t l_t (1 - \pi_{eu,t}) + D_t = Y_t - \kappa_{1,t} v_t - q_t v_t \kappa_{2,t}.$$

Finally, the return of the risk-free bond reads

$$\frac{1}{R^f_{t+1}} = \mathbb{E}_t M_{t+1}. \tag{2.17}$$

See Appendix 2.B.2 for a derivation of stock returns and the stock price,

$$P_t = \underbrace{\left(\frac{\kappa_{1,t} + q_t \kappa_{2,t}}{q_t} - \Theta_t \right)}_{\mathbb{E}_t M_{t+1} J_{t+1}} l_{t+1}. \tag{2.18}$$

The term in parentheses is Tobin's marginal q, the shadow price of employment in the firm's optimization problem.

I solve the stationary version of the model (Appendix 2.B.1). In short, A_t scales the constant parameters $\{\mu_\epsilon, \psi_\epsilon, \kappa_1, \kappa_2, \tau_{eu}, b\}$, e.g. $b_t = bA_t$. Define the productivity-adjusted variables $c_t = \frac{C_t}{A_t}$, $w_t = \frac{W_t}{A_t}$ $d_t = \frac{D_t}{A_t}$, $p_t = \frac{P_t}{A_t}$, $\widetilde{\Psi}_t = \frac{\Psi_t}{A_t}$, $\widetilde{\Sigma}_t = \frac{\Sigma_t}{A_t}$. The model is solved globally with the projection method outlined in Appendix 2.F.1 and estimated with the method of simulated moments outlined in Appendix 2.F.2. The next two sections parametrize the models:

First, Section 2.3 parametrizes the RBC model with cyclical fluctuation z_t. Next, Section 2.4 parametrizes the LRR model with a small, persistent component of growth x_t. In each section, I simulate the model and match the parametrized model to unemployment and output time series to examine the models' predictive power for other variables.

2.3 Real business cycle fluctuations

I estimate the model for two different specifications of productivity: a model driven by cyclical volatility (RBC) and a long-run risk model (LRR) with a small persistent component of the productivity growth rate. In the RBC model, the sources of volatility are standard cyclical fluctuations and, for comparability, an iid innovation to the growth rate. The parametrized RBC model generates sufficient volatility of labour market variables to solve the Shimer puzzle. It can generate rare disasters endogenously, but does not yield a large risk premium. I show that the risk premium found by Petrosky-Nadeau et al. (2018) is a result of their extreme parametrization, which generates disasters too frequently.

2.3.1 Parametrization

In this section productivity of a match $A_t e^{z_t}$ follows

$$\begin{aligned} \log A_{t+1} - \log A_t &= g_a + \sigma_a \epsilon_{a,t+1} \\ z_{t+1} &= \rho_z z_t + \sigma_z \epsilon_{z,t+1}, \end{aligned}$$

with $\epsilon_{a,t+1} \overset{iid}{\sim} N(0,1)$ and $\epsilon_{z,t+1} \overset{iid}{\sim} N(0,1)$.

I parametrize all models to monthly frequency and use post-war U.S. data to estimate the models. Table 2.1 summarizes the parametrization of the RBC model. Following Bansal and Yaron (2004) and Croce (2014), I set risk aversion γ to 10 and the intertemporal elasticity of substitution ψ to 1.5. For $\psi > \frac{1}{\gamma}$ agents prefer an early resolution of risk and dislike assets which are correlated with consumption growth, a necessary condition for the existence of an equity premium (Campbell, 2017). For $\psi > 1$, an increase in expected future consumption raises the return of wealth and an increase in consumption volatility drives down wealth and prices. If $\psi < 1$, more volatile consumption would raise prices, e.g. a looming disaster would cause a stock boom. In a production-based model, Croce (2014) illustrates that an intertemporal substitution smaller one induces households to raise consumption in light of positive news about growth, leading to a reduction of investment, a large and volatile risk-free rate and a negative equity premium.

Parameters $\{\beta, g_a, \mu_\epsilon, \iota, \kappa_1, \kappa_2, b, \tau_{eu}\}$, are set in a stochastic steady state.[4] As the annual risk-free rate I use the post-war average of the bond interest rate, 2.3%, which pins down β. The exogenous growth rate g_a sets annual output growth to 1.79%. The mean idiosyncratic productivity of a match, μ_ϵ, normalizes the expected productivity of a match before the separation decision to unity.

Vacancy-posting costs, κ_1, subsume search, screening, and interviewing. The training costs, κ_2, subsume formal and informal training, administration costs associated with a new worker, the productivity gap to workers with tenure, and moving cost that the firm might cover. The full extent of hiring and training cost is difficult to assess and estimates vary substantially: Silva and Toledo (2009) posit hiring costs of 4.3% of the quarterly wage of a new hire, excluding the productivity gap to workers with tenure. Barron et al. (1999) estimate that a new worker in sum spends only one month in training.[5] For Germany, Mühlemann and Pfeifer (2016) find that hiring costs of skilled labour amount to two months of wages. For Switzerland, Mühlemann and Strupler Leiser (2018) estimate hiring costs of four months of wages. Given the range of estimates, I assume that total hiring costs are equivalent to 6% of total monthly wages and training costs equal one-and-a-half monthly wages. For one match, this amounts to expected hiring and training costs, $\kappa_1/\bar{q} + \kappa_2$, of around three monthly wages.

Davis et al. (2013) use establishment-level data and find a daily vacancy-filling rate of 5%. At 20 business days per month, the daily estimate aggregates to a 64.15% monthly filling rate. I set the steady state vacancy-filling rate, $\overline{q}$, to 64.15% to estimate the curvature of the matching function ι. In my 1967-2018 dataset, the mean probability of a transition from employment to unemployment is 1.92% and the mean probability of transitioning from unemployment to employment is 26.05%. I target these steady state transition probabilities $\overline{\pi}_{eu}$ and $\overline{\pi}_{ue}$ to pin down the separation cost τ_{eu} and the outside option b.

[4]This stochastic steady state uses corrections for Jensen's inequality to evaluate expected values. The stochastic steady state is not equivalent to the ergodic distribution or the model solution as in Farhi and Gourio (2018).

[5]Barron et al. (1999) estimate that a new worker spends 29.5% (1982 EOPP training measure) to 36.2% (1992 SBA training measure) of his first three month work time in training. Standard deviations for the estimates are 38.7% (EOPP) and 51.2% (SBA).

Finally, I set parameters $\{\sigma_z, \sigma_a, \rho_z, \psi_\epsilon, \alpha_z, \bar{\rho},\}$ via Simulated Method of Moments Estimation (SMM, see Appendix 2.F.2). The real business cycle literature typically targets moments of HP-filtered data. In the LRR model (Section 2.4), movement of trend growth is the essential driver of the model and might be eliminated by this filter. Hence, I only target moments of quarterly or annual growth rates. I target the volatility and autocorrelation of annual output growth, the volatilities of quarterly unemployment, the quarterly separation rate and annual wage growth, the correlation between unemployment and output, and, finally, the correlation between unemployment and separations.

The idea behind this choice of moments is as follows: The standard deviation of separations estimates the elasticity ψ_ϵ. The correlation of unemployment and separations rules out pro-cyclical separations that may arise with strong wage rigidity. The autocorrelation of output growth estimates ρ_z. I target the volatility of unemployment to estimate the wage rigidity parameter α_z.

The volatility of wage growth estimates a low bargaining power which serves two purposes: first, empirically, wages do not strongly co-move with productivity; second, rigid wages seek to avoid counter-cyclical dividends which can arise if wages equal the marginal product of labour. Broadly speaking, dividends equal production minus wages minus investment. In a flexible wage model, following an adverse productivity shock, production, wages, and investment drop simultaneously which can raise dividends. Investors do not demand a positive risk premium for stocks that pay countercyclical dividends and a version of the equity premium puzzle arises. When wages are rigid, they do not fall strongly with productivity. Dividends do not rise in a recession and investors demand a premium for holding the stock. Hagedorn and Manovskii (2008) set a low worker's bargaining power to match their estimate of the elasticity of wages with respect to productivity. Though this is possible in this RBC model, the same strategy fails in the estimation the LRR model because wages scale with long-run productivity and the low target elasticity of Hagedorn and Manovskii (2008) cannot be reached.

I introduce *iid* innovations to the growth rate ϵ_a for two reasons: first, the parametrization ought to be compatible with the LRR model's; second, ϵ_a helps to match data as it can be interpreted as measurement error (see

Section 2.3.3). With this structural model, it is possible to discriminate the *iid* shocks from the RBC shocks by exploiting the implications of the shocks for the unemployment rate. ϵ_a randomizes productivity growth, but once materialized the shock carries no information for the following periods' TFP growth rate. The economy instantaneously converges to the new balanced growth path and, in productivity-adjusted terms, households and firms do not change their behaviour.[6] In contrast, TFP shocks (and in the next section, LRR shocks) carry information about productivity in the next periods. Households and firms react to ϵ_z by raising or reducing investment in employment. The correlation of unemployment and output discriminates between the shocks and together with the volatility of output growth identify σ_z and σ_a.

[6]The existence of the shock affects policy functions via expectations, but its realizations do not. Farhi and Gourio (2018) exploit this property to derive straightforward closed-form solutions of their model.

Parametrization				
Parameter		Value	Target	Source
γ	risk aversion	10.000	-	Bansal and
ψ	EIS	1.5000	-	Yaron (2004)
β	time discount	0.9992	$\overline{r^f} = 2.3\%$	U.S. Data
g_a	constant growth rate	0.0015	$\overline{\Delta^a Y} = 1.79\%$	U.S. Data
μ_ϵ	mean idiosyncratic shock	0.0805	$\frac{v}{l} = 1$	-
ι	matching function	0.8560	$\bar{q} = 64.15\%$	Davis, Faberman, Haltiwanger (2013)
κ_2	training costs	1.4409	$1.5 \times \bar{w}$	-
κ_1	vacancy-posting costs	0.9695	$\frac{\kappa_1 \bar{v} + \kappa_2 \bar{v} \bar{q}}{\bar{w} \bar{l}(1-\bar{\pi}_{eu})} = 6\%$	-
b	unemployment benefits	0.8160	$\bar{\pi}_{ue} = 26.05\%$	U.S. Data
τ_{eu}	separation costs/tax	2.5222	$\bar{\pi}_{eu} = 1.92\%$	U.S. Data
σ_z	volatility RBC shock	0.0057		
σ_a	std *iid* growth shock	0.0066		
ρ_z	autocorrelation TFP	0.9593		estimated with SMM
ψ_ϵ	Elasticity of separations	1.5457		
α_z	wage rigidity	1.4739		
$\bar{\rho}$	bargaining power	0.1464		

SMM estimation				
Moment		Simulation	Target	Source
$\rho(\Delta^q Y, \Delta^q u)$	corr. GDP & unemployment	-0.54%	-0.58%	
$\sigma(\Delta^a Y)$	volatility GDP	2.62%	2.32%	
$\sigma(\Delta^q \pi_{eu})$	volatility separations	7.03%	6.67%	
$\sigma(\Delta^a W)$	volatility wages	2.22%	2.70%	all SMM targets: US data 1948-2017
$\sigma(\Delta^q u)$	volatility unemployment	7.51%	7.68%	
$\rho(\Delta^a Y)$	autocorr. GDP	0.20	0.19	
$\rho(\Delta^q \pi_{eu}, \Delta^q Y)$	corr. separations & GDP	-0.45	-0.44	

Table 2.1: Parametrization of the RBC model. Parameters missing in this table are zero in this specification. All SMM targets are U.S. Data (see Appendix 2.A).

		Mean	Std	$\rho(X)$	$\rho(X,\Delta^q Y)$			Mean	Std	$\rho(X)$	$\rho(X,\Delta^a Y)$
	Data	5.770					Data	1.792	2.316	0.186	
u	RBC	7.170				$\Delta^a Y$	RBC	1.815	2.642	0.200	
	LRR	7.558					LRR	1.810	2.366	0.212	
	Data		7.683	0.442	-0.578		Data	1.889	1.632	0.267	0.849
$\Delta^q u$	RBC		7.508	0.616	-0.538	$\Delta^a C$	RBC	1.827	3.087	0.191	0.984
	LRR		7.363	0.612	-0.610		LRR	1.830	3.134	0.128	0.970
	Data	1.915					Data	1.603	2.701	0.351	0.893
π_{eu}	RBC	1.934				$\Delta^a W$	RBC	1.804	2.160	0.203	0.956
	LRR	2.044					LRR	1.820	2.707	-0.028	0.618
	Data		6.673	-0.204	-0.447		Data	2.326	2.436	0.752	-0.056
$\Delta^q \pi_{eu}$	RBC		7.030	0.203	-0.453	r^f	RBC	2.289	2.970	0.653	-0.025
	LRR		6.446	0.614	-0.378		LRR	2.327	2.982	0.050	0.441
	Data	26.048					Data	6.461	11.206	0.013	-0.030
π_{ue}	RBC	26.034				r^s	RBC	2.736	4.673	0.127	0.386
	LRR	26.048					LRR	3.289	7.943	0.057	0.450
	Data		5.103	-0.013	0.291		Data	4.801	11.318	0.005	-0.022
$\Delta^q \pi_{ue}$	RBC		4.574	0.347	0.517	$r^s - r^f$	RBC	0.447	3.313	-0.011	0.565
	LRR		7.060	-0.013	-0.045		LRR	0.961	5.262	0.031	0.430

Table 2.2: Simulation results. $\Delta^q X$ denotes the quarterly and $\Delta^a X$ the annual growth rate of X. $\rho(X)$ is X's autocorrelation. $\rho(X, \Delta^i Y)$ is the correlation of X and output growth at the matching frequency. Mean and standard deviations in percent. Simulation moments of 1000 economies over 60 years with a burn-in phase of equal length.

2.3.2 Simulation

This section discusses simulation results of the parametrized RBC model, summarized in Table 2.2. Simulations match the volatilities of unemployment and transition rates, π_{ue} and π_{eu}. Unemployment and separations are counter-cyclical and job-finding is pro-cyclical. The model matches the targeted mean transition rates, but the mean unemployment rate exceeds that in the data by about 1.5 percentage points. The mean unemployment rate can exceed the steady state approximation $\frac{\bar{\pi}_{eu}}{\bar{\pi}_{eu}+\bar{\pi}_{ue}}$ if the covariance of unemployment and the flows are non-zero, which arises easily with a non-constant discount factor. Assume unemployment is high and productivity is at its steady state level. In this state, low consumption increases marginal utility today, which reduces the discount factor. Agents prefer to consume sooner than later, firms post fewer vacancies and the job-finding rate remains low. In the canonical DMP model with linear utility, the discount factor is constant and the economy returns to its steady state quickly because the vacancy-filling rate does not react to high marginal utility. But, even in the canonical model, the steady state relationship need not hold because a high job-finding rate occurs in economic expansions when the unemployment rate is already low.[7]

Turn to the right-hand side of Table 2.2. The mean, volatility, and autocorrelation of output growth are estimation targets. The model closely links consumption to output; it predicts an excess volatility of consumption and a lack of autocorrelation inherited from the output targets. Still, consumption volatility is not large enough to generate a risk premium on its own, which would be a convenient but faulty way to "solve" the premium puzzle.

The risk-free rate's mean, volatility, autocorrelation, and cyclicality match their empirical counterparts. Interestingly, in the following Section 2.3.3, matching time series shows a poor data fit in the model's implied interest rate. This shows the limit of focussing on simulation results. Equity return, r^s, is neither as large nor as volatile as its empirical counterpart. The RBC model fails to produce a noteworthy equity risk premium. My parametrization generates almost no disasters endogenously (see Table 2.4). Hence, the stochastic discount factor's small variance is insufficient to generate a premium. Section 2.5 explores this argument further. My finding contradicts Petrosky-Nadeau et al. (2018). They parametrize the DMP model to generate disasters endogenously and investors demand a premium to accommodate disaster risk. In Section 2.3.4, I show that their parametrization generates disasters too frequently.

Overall, simulation results show that this Diamond-Mortensen-Pissarides model with cyclical fluctuations and a small surplus calibration is a good description of the labour market. Yet, it fails to solve the equity premium puzzle. Can the model generate the striking correlation of unemployment and stock prices regardless? And how does the endogenous disaster model perform in the Great Depression? The next section answers these questions.

2.3.3 Matching time series

Simulation results have shown that the RBC model solves the Shimer puzzle but does not solve the equity premium puzzle in simulations. This section studies whether the model can match the time series of key macroeconomic variables. This section matches the unemployment time series to estimate a process for TFP from the model's policy function; using this TFP series as an

[7] See Hairault et al. (2010) and Den Haan et al. (2020).

input, I simulate the model and compare key macroeconomic variables to their empirical counterparts.

In applications like the canonical New-Keynesian model, researchers apply a Kalman filter to estimate parameters of a linear model and match time series. However, this model is solved globally, which renders the Kalman filter inapplicable.[8] Schorfheide et al. (2018) demonstrate how to use a non-linear filter to estimate a higher order approximation of the standard Bansal-Yaron model. Compared to the models outlined in this paper, the standard Bansal-Yaron model is much simpler and can be solved with perturbation. A global solution is too time intensive and thus a non-linear filter is not an economical choice.

Still, it is possible to match unemployment and a second time series with the parametrized model: I feed a starting point for employment and stock prices in April 1929 into the model. The monthly unemployment series allows to estimate a time series for productivity, z_t, such that the model's policy function maps into the unemployment time series. Then, the output time series (linearly interpolated to monthly frequency) allows us to estimate the iid growth innovations $\epsilon_{a,t}$. The orthogonality of $\epsilon_{a,t}$ to unemployment is the key to estimate the two series separately: once $\epsilon_{a,t}$ is revealed it contains no information about future changes of productivity. The investment decision of the forward-looking firm is not affected; so investment, and therefore unemployment, must be completely driven by z_t. See Appendix 2.C for details. By construction, the model-generated output and unemployment series should match the empirical time series. At times, the policy function does not allow us to reach a high employment rate, leading to minor discrepancies between model-generated and empirical series.

Figures 2.2 - 2.4 compare matched time series of the RBC model against data. The model is parametrized to post-war data, but the longer time frame used here allows us to check how a endogenous disaster model performs in a true economic disaster - the Great Depression. In the title of each panel, ρ denotes the correlation coefficient of the annualized model-generated and

[8] Petrosky-Nadeau and Zhang (2017) and Fernández-Villaverde and Levintal (2018) show that the perturbation solution is a bad approximation of the global policy function in a rare disaster model.

empirical time series.

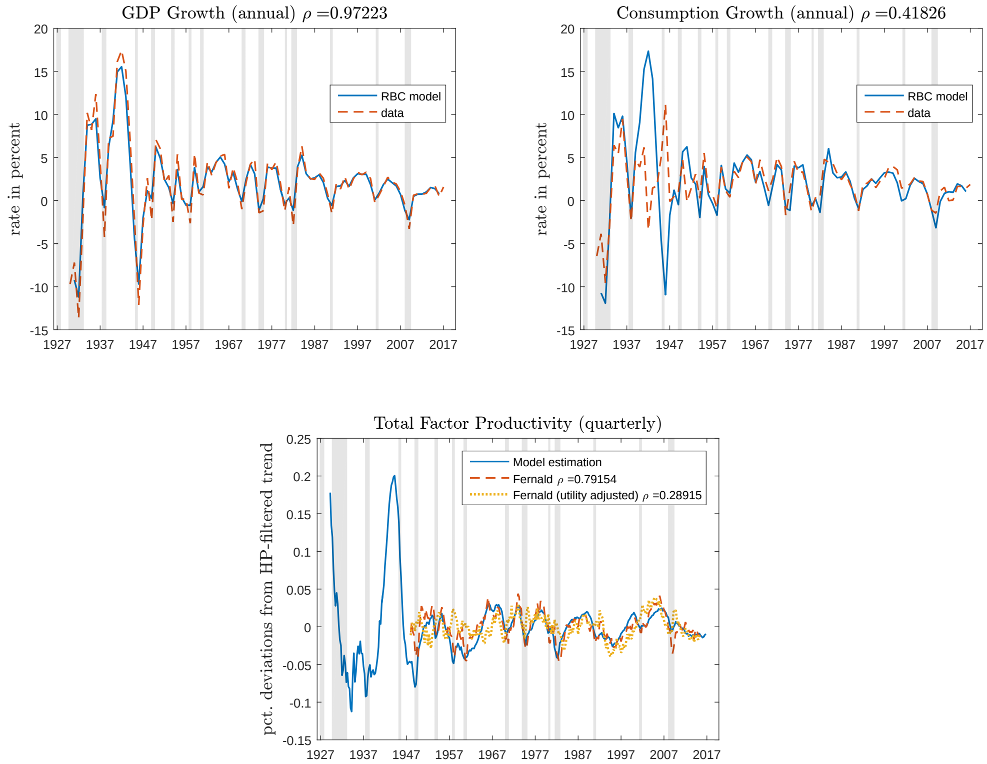


Figure 2.2: Matched time series of the RBC model: output, consumption and productivity. Grey bands denote NBER recessions. ρ denotes the correlation coefficient of annualized simulated and empirical data. HP-filter smoothing parameters: $\lambda = 129,600$ for monthly and $\lambda = 100,000$ for quarterly data.

Consistent with simulation results, the RBC model matches post-war consumption, but fails to match the consumption series before the war. Unsurprisingly, the model fails to reproduce the low-consumption, high-output, and low-unemployment war years 1941-1945. The last panel of Figure 2.2 shows that my estimated TFP series is more volatile than estimates by Fernald (2014). The panel also illustrates that, to match the pre-1947 unemployment series, we need to assume large fluctuations of TFP.

Figure 2.3 shows the matched labour market series. Model-generated separations are slightly too smooth, but job-finding rates are matched well. Figure 2.3 reveals the appeal of endogenous separations when firms face a non-negativity constraint on vacancies. Endogenous separations allow us to match wide fluctuations of the unemployment rate, as exemplified during depressions or the war years. The minimum of employment growth

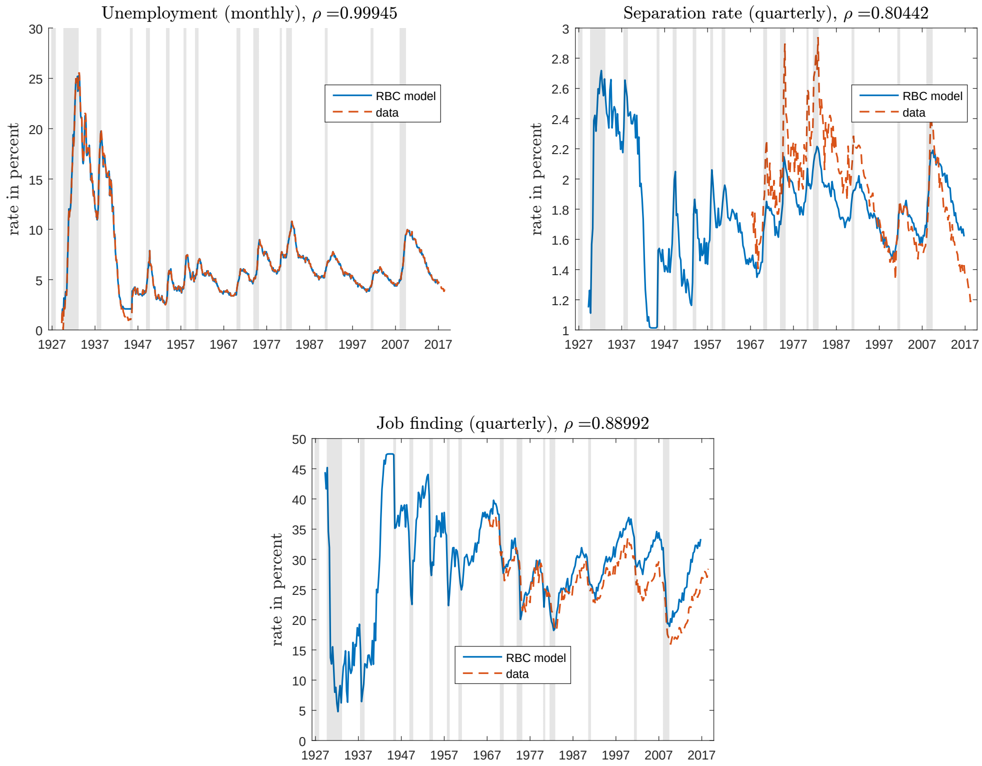


Figure 2.3: Matched time series of the RBC model: labour market. Grey bands denote NBER recessions. ρ denotes the correlation coefficient of annualized simulated and empirical data.

is $\min[\log l_{t+1} - \log l_t] = -\pi_{eu,t}$. If separations are fixed, employment cannot fall as quickly as it does during the Great Depression. In the opposite direction, employment can, at most, grow at rate

$$\max[\log l_{t+1} - \log l_t] = \frac{1-l_t}{l_t}\pi_{ue,t} - \pi_{eu,t}.$$

For a low unemployment rate, the term $\frac{1-l}{l}$ is very small. Under fixed separations, employment can only grow rapidly if the job-finding rate is excessive. This problem can be observed in the war years. The employment rate is already at 97% in late 1942 and rises to 99% in 1945. In an exogenous separation model, positive employment growth at a 97% employment rate demands an excessively high job-finding rate. Additionally, under exogenous separations, the burden of creating fluctuations in unemployment lies completely on the job-finding rate, which must fluctuate too strongly.

Turning to Figure 2.4, dividends in the first panel are almost constant, whereas empirical dividends fluctuate strongly pro-cyclically. Model dividends are the residual of output after wages and investment costs. In a boom, firms generate higher profits but households increase savings. In a recession, the reverse holds. These net transfers from firms to households are very stable compared to empirical dividends, which are true profit shares. Model predicted wages are less volatile than their empirical counterpart, but the two series display a high degree of correlation.

Turning to asset prices, the model does not match the time series of the risk-free rate. Expected monthly consumption growth (and the value function) determines the risk-free rate entirely; a link that is clearly too strong. Adding New-Keynesian frictions and a Taylor rule would probably improve the model's fit here. The model simulation neither matches the volatility of stock prices nor the mean, volatility or direction of equity returns. The RBC model has problems turning modest fluctuations of productivity, which are sufficient to match the unemployment rate and output growth, into strong fluctuations of equity returns and prices.

Finally, Figure 2.5 depicts the return predictability in the post-war U.S. economy. Empirically, a below-average price-to-earnings ratio is a good predictor of higher future returns. To erase the effect of cyclical volatility, the figure plots the ratio of price to earnings over the past ten years (PE10) and the average future return in the following ten years. In the panel titles, ρ denotes the correlation of PE10 and future returns in the data and the model respectively. Empirically, the correlation coefficient is -0.65; the model-generated coefficient is about null, i.e. the model generated PE10 does not predict future equity returns. Quantitatively, the model does not reproduce returns or price-to-earnings ratios as volatile as in the data (see y-axes). Concerning levels, the price-to-earnings ratio is much larger in the model than in the data, while the stock returns in the model are low compared to data averages. The former follows from the definition of model dividends as net transfers from firms to households. The latter follows from the model's difficulties solving the equity premium puzzle. Interestingly, the model underestimates returns in the 2010s. Recall that I match output growth and the unemployment rate. Output growth has been modest compared to returns. In the 2010s, the unemployment rate fell to record lows, but at 3-4 percent, has almost no room to decrease further. Hence, the model fails to match large returns in 2010s. A labour-leisure decision could account for employment growth beyond the unemployment rate and improve data fit.

In summary, the RBC model is effective in matching post-war output, consumption and labour market data. Endogenous separations improve data fit, especially in volatile periods like the Great depression. However, the model's ability to match asset pricing data is miserable. Petrosky-Nadeau et al. (2018) claim that a very similar model solves the equity premium puzzle via endogenously generated disasters. The next section examines their model.

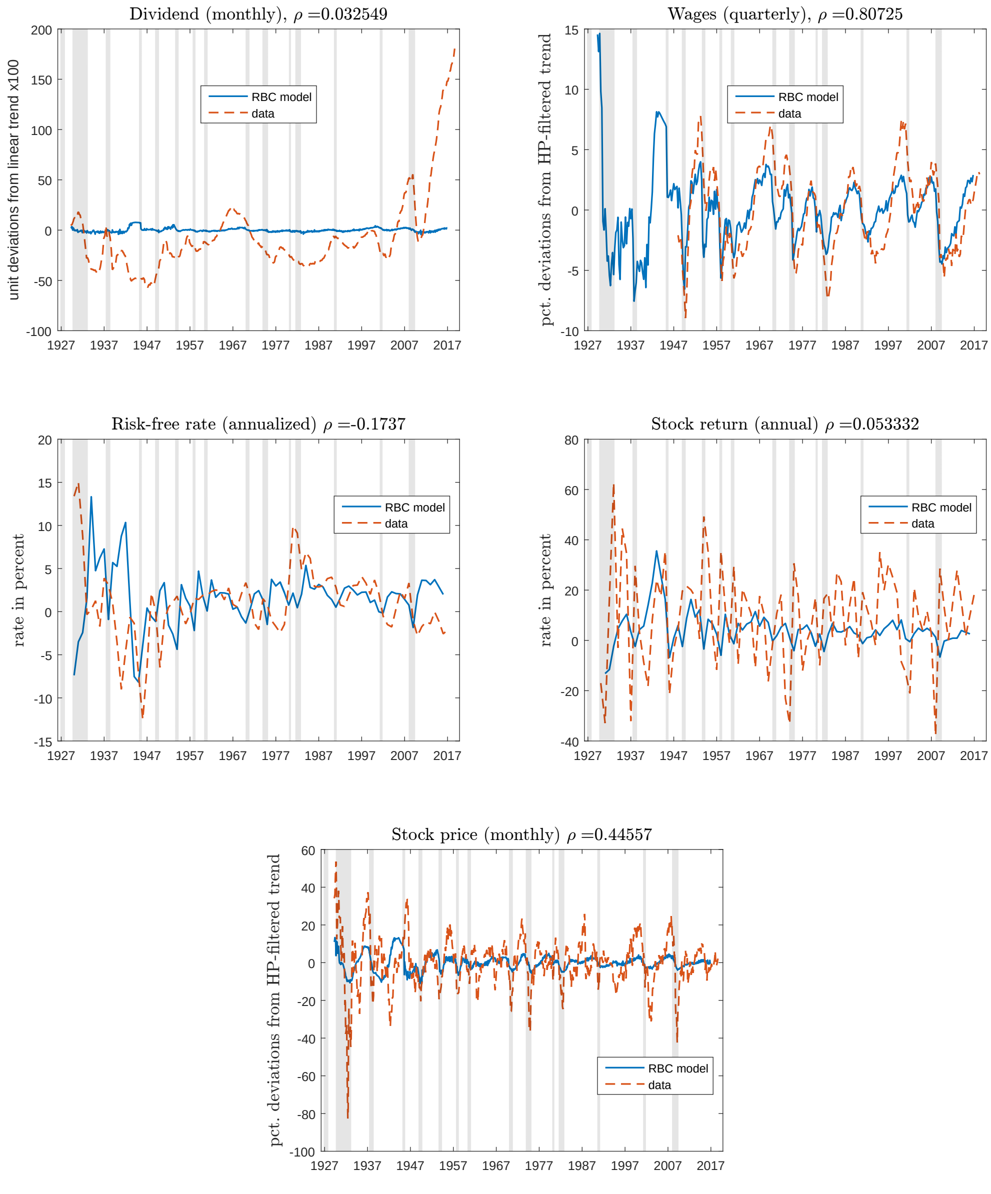


Figure 2.4: Matched time series of the RBC model: asset prices. Grey bands denote NBER recessions. ρ denotes the correlation coefficient of annualized simulated and empirical data. HP-filter smoothing parameters: $\lambda = 129,600$ for monthly and $\lambda = 100,000$ for quarterly data.

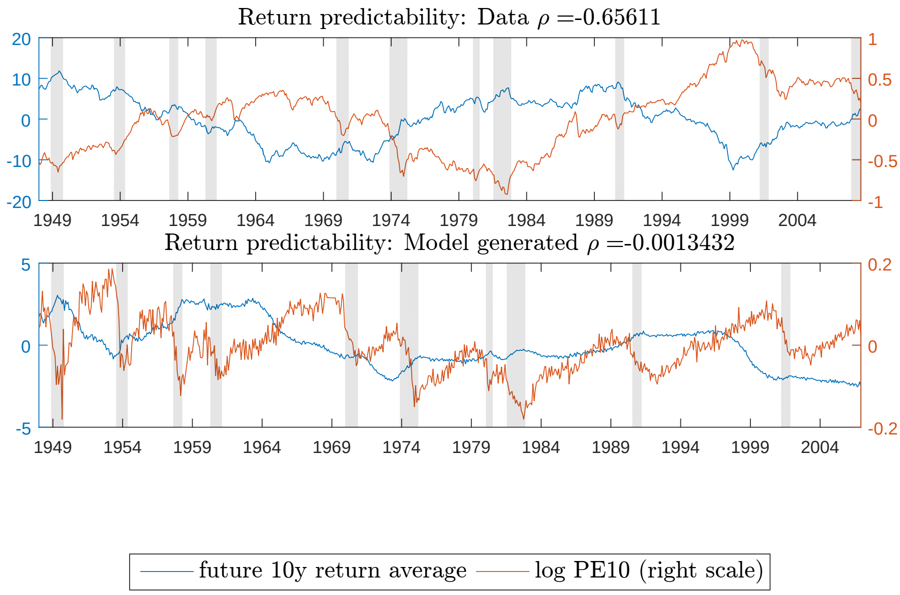


Figure 2.5: Matched time series of the RBC model: return predictability. The price earnings ratio (PE10) is the ratio of the current price to average earnings over the past ten years (also called Cyclically Adjusted PE Ratio (CAPE) or Shiller PE Ratio). I compute log PE10 as $\log \frac{P}{D+1}$ to avoid explosive PE10 for close to zero dividends. Future 10 year returns are the geometrical average of annual returns over the next ten years. The figure shows absolute deviations from mean. Here, ρ denotes the correlation coefficient of PE10 and CAPE in both panels.

2.3.4 Endogenous disasters

Why do Petrosky-Nadeau et al. (2018) (henceforth PNZK) find that a basic DMP model can generate a large equity premium while I find the opposite? This section uses their calibration, simulates the model, and repeats the time series matching.

PNZK parametrize the DMP model to moments of an extensive dataset. They target the mean and volatility of U.S. unemployment from 1929 to 2013. Their target output volatility is the average volatility of international GDP data compiled by Barro and Ursúa (2008). This dataset covers output and consumption growth rates for a large number of countries and spans from 1790 to present. With this calibration, the DMP model can generate disasters endogenously. Investors demand an equity premium to accommodate the endogenous disaster risk. I demonstrate that this calibration implies excess volatility of the vacancy-filling rate, consumption, output, stock prices, and asset returns compared to data. It also predicts several disasters in the post-war U.S. economy. Dupraz et al. (2019) and Kehoe et al. (2019) state that the linear productivity function is responsible for excessively frequent disasters. The RBC model discussed in the previous section assumes linear productivity as well, but the parametrization does not generate frequent disasters.

PNZK assume cyclical productivity without growth,

$$\begin{aligned} z_{t+1} &= \rho_z z_t + \sigma_z \epsilon_{z,t+1} \\ A_{t+1} - A_t &= 0. \end{aligned}$$

The calibration of replacement rate and bargaining power follows Hagedorn and Manovskii (2008). In contrast to my model, separations are constant.

Table 2.4 compares key moments of data to simulations from PNZK and my RBC and LRR model. The LRR model is discussed in the following section. First off, moments from cross-country historic data make rather extreme targets: PNZK's target volatility of GDP is about 50% above GDP volatility in the 1929-2018 U.S. sample and almost two times larger than my target, the post-war volatility. This translates into a large consumption volatility. PNZK target the volatility of HP-filtered unemployment in the

		value
γ	Risk aversion	10.000
ψ	EIS	1.500
β	time discount	0.998
σ_z	Std RBC shock	0.0089
ρ_z	autocorrelation RBC shock	0.983
$\bar{\rho}$	bargaining power	0.040
ι	matching function parameter	1.250
b	unemployment insurance	0.850
$\overline{\pi_{eu}}$	Constant separation rate	0.040
κ_2	training costs	0.500
κ	vacancy-posting costs	0.500

Table 2.3: Parametrization of Petrosky-Nadeau et al. (2018). Any parameter not listed in the table is zero.

	$\sigma(\Delta^a Y)$	$\sigma(\Delta^a C)$	$\sigma(d^q u)$	$r^s - r^f$	$Prob(\Delta C < 0.1)$	$Size(\Delta C < 0.1)$	$Dur(\Delta C < 0.1)$
RBC	2.64	3.09	15.12	0.45	0.42	12.22	3.61
LRR	2.37	3.13	13.70	0.96	0.37	11.77	3.31
PNZK	6.64	6.02	24.27	3.13	2.68	32.77	4.83
Data 1929-2018	4.32	3.80	22.30	4.39	4.52	19.56	4.00
Post-war Data	2.32	1.63	13.92	4.80	1.75	13.74	12.00

Table 2.4: Disaster risk. Moments of simulated and empirical U.S. data. $Prob()$, $Size()$, $Dur()$ denote the annual probability, mean size and mean duration of a disaster as in Barro and Ursúa (2008). $\Delta^a X$ denotes the annual growth rate of X and Δ^q its quarterly growth rate. $\sigma(d^q u)$ denotes the quarterly standard deviation of unemployment, measured as log deviations from HP-filtered trend ($\lambda = 1600$). All rates and standard deviations in percent. Simulation moments of 1000 economies over 60 years with a burn-in phase of equal length.

1929-2018 sample, repeated in the table. The extreme calibration generates disasters endogenously in simulations: The annual probability of a disaster is 2.68%. The average disaster reduces consumption by 33% and lasts for five years. These numbers are close to the historic U.S. sample and motivate a risk premium. In the post-war RBC and LRR models, disasters are extremely rare and small in size. Here, the negligible disaster risk does not rationalize a risk premium.

In contrast to Section 2.3, cyclical volatility is the only source of uncertainty here, i.e. $\epsilon_a = 0$. I estimate a series for z by matching the historic U.S. employment series. Figures 2.6 - 2.8 show the predicted time series given the estimated z-series.

Figure 2.6 show the model-generated output, consumption and TFP series. Since PNZK only assume one shock, the output series is not matched here

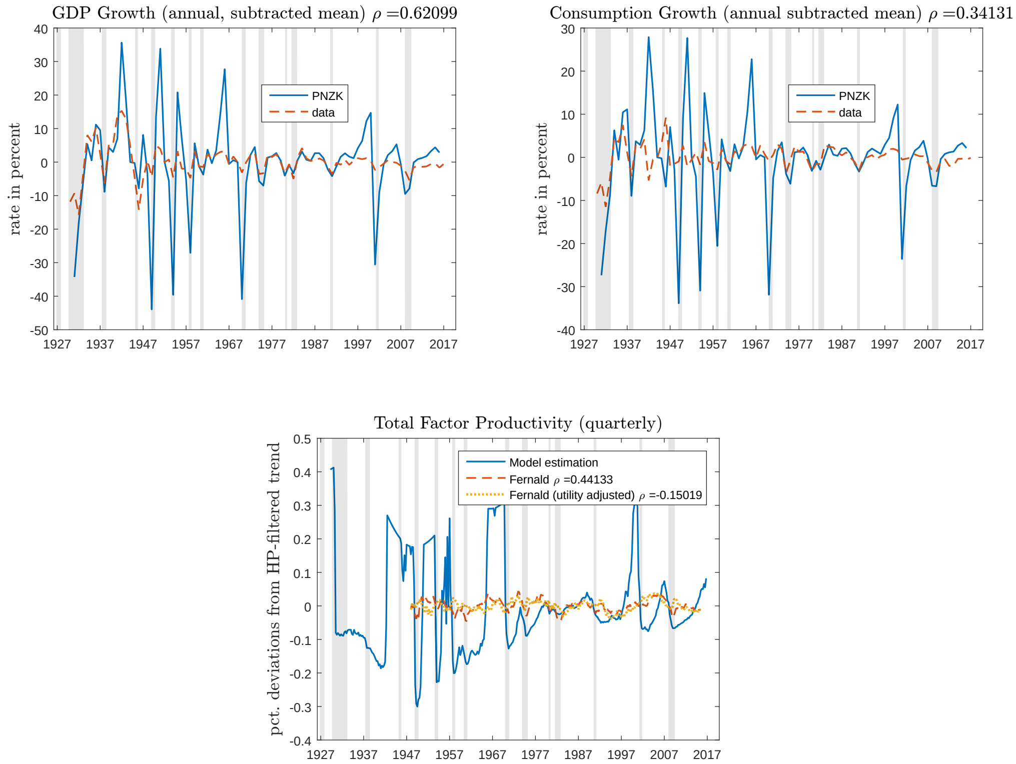


Figure 2.6: Matched time series of the PNZK model: Output, consumption and productivity. Grey bands denote NBER recessions. ρ denotes the correlation coefficient of annualized simulated and empirical data. HP-filter smoothing parameters: $\lambda = 129,600$ for monthly and $\lambda = 100,000$ for quarterly data.

and the predicted output and consumption series are too volatile. Strikingly, the model predicts six consumption and output disasters after 1945. The only disaster in post-war data is the combined effect of two oil crises, excluding Covid-19, which is not part of the sample. The estimated TFP series is too volatile compared to Fernald's estimates.

Figure 2.7 compares model-generated and empirical labour market data. Three observations stand out: (i) The mean separation rate and job-finding rate are about double the empirical rate. (ii) The model is not able to match the employment series. (iii) The job-finding rate fluctuates too strongly.

I view (i) as an erroneous specification of the model at monthly frequency. A mean job-finding rate of 71% is inconsistent with my data and during expansions the rate rises to unrealistically high levels of more than 90%. (i) has consequences for the related (ii) and (iii): the parametrization has problems matching hikes of unemployment and low unemployment rates

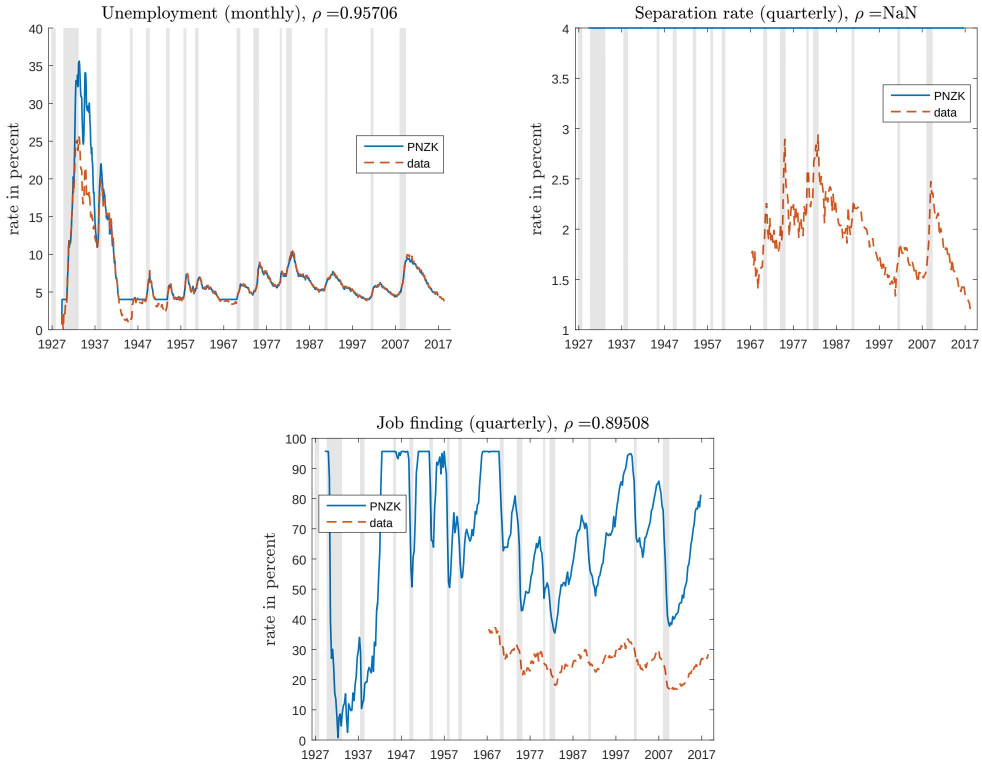


Figure 2.7: Matched time series of the PNZK model: The labour market. Grey bands denote NBER recessions. ρ denotes the correlation coefficient of annualized simulated and empirical data.

because of the large and constant separation rate of 4%.

Recall that unemployment tomorrow is unemployment today minus new matches plus laid off workers,

$$u_{t+1} = u_t - q_t v_t + (1 - u_t)\pi_{eu}. \tag{2.19}$$

Start with favourable times that the model cannot match, e.g. the 1950s. In times of low unemployment, two forces drive the unemployment rate to its higher steady state. Firstly, when the separation rate is constant and unemployment is low, total separations rise because the pool of employed workers is large, i.e. the flow $(1-u_t)\pi_{eu}$ is large because unemployment is low. Secondly, vacancy posting reduces labour market tightness. In equilibrium, the vacancy-filling rate falls, reducing the number of new matches, $q_t v_t$.[9]

[9]Hairault et al. (2010) investigate these non-linear effects prevalent in the DMP framework.

Strong vacancy posting could offset the two effects, but vacancy posting to the necessary extent is inconsistent with the first-order condition of the firm. Hence, the model does not match the most favourable periods in the employment series. A lower constant separation rate could alleviate this problem. Better yet, in an endogenous separation model, the unemployment rate can reach very low levels, because separations become less frequent.

The model predicts excessively high unemployment in the Great Depression, because in these states, the vacancy policy function becomes flat. This is due to the combination of the non-negativity of vacancies, $v_t \geq 0$, and the den Haan et al. (2000) matching function. The latter imposes bounds on the vacancy-filling rate, $q_t \in [0,1]$. In the canonical DMP model, the matching function is Cobb-Douglas with vacancy-filling rate

$$q_t^{CD} = \xi \left(\frac{v_t}{u_t} \right)^{-\alpha},$$

i.e. $v_t \rightarrow 0$ implies $q_t^{CD} \rightarrow \infty$. A firm can fill numerous jobs at the cost of just one vacancy. Unless the expected firm surplus net training costs is negative, firms post some vacancies even in very adverse times. In the model at hand, q_t cannot exceed unity. At the onset of the Great Depression, unemployment rises, and consumption and investment fall. Firms have no incentive to post vacancies. Since q_t is bounded at unity, the constraint $v_t \geq 0$ binds. The vacancy policy is a straight horizontal line at zero. Combined with a constant separation rate of 4%, unemployment rises excessively.

In summary, the codomain of labour's policy function is not sufficient to match the worst times in the historic U.S. sample, which PNZK use to parametrize the model. During the Great Depression, vacancy posting is not sufficient and the unemployment rate counter-factually reaches 35%. This detour shows how endogenous separations can improve data fit in volatile years.

Finally, the excessive volatility of the job-finding rate is mirrored by an excess volatility of the vacancy-filling rate, which regularly falls close to zero (Figure 2.8). This boosts the volatility of equity prices by the relationship

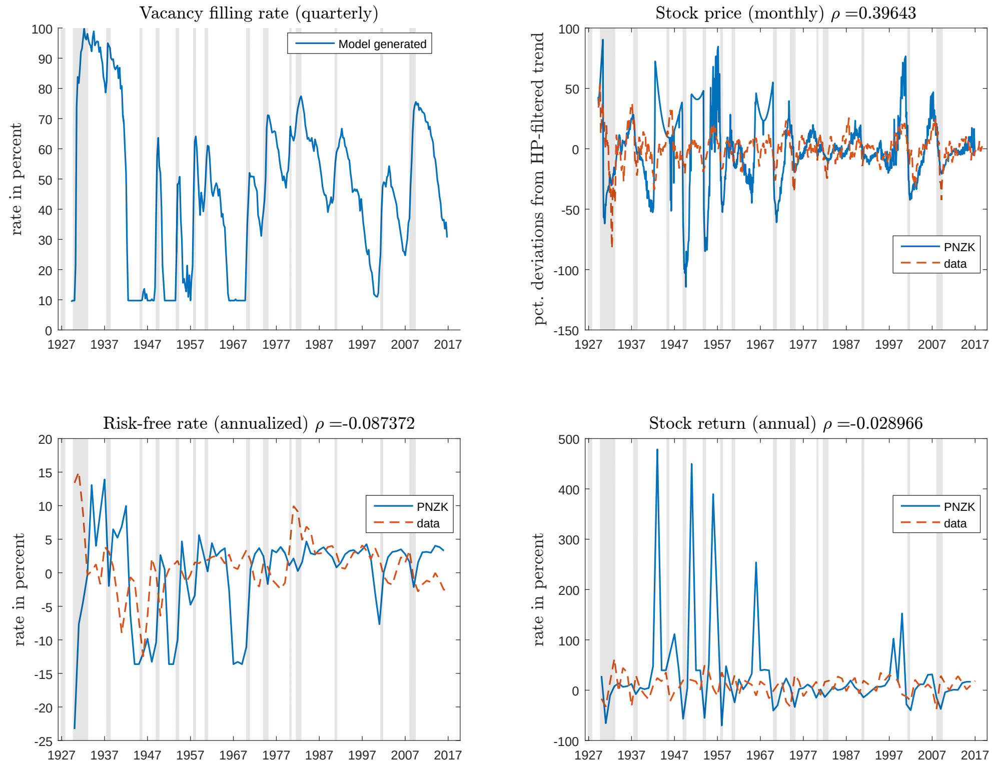


Figure 2.8: Matched time series of the PNZK model: Asset prices. Grey bands denote NBER recessions. ρ denotes the correlation coefficient of annualized simulated and empirical data. HP-filter smoothing parameters: $\lambda = 129,600$ for monthly and $\lambda = 100,000$ for quarterly data.

(ignoring the non-negativity constraint)

$$p_t = \left(\frac{\kappa_1}{q_t} + \kappa_2\right) l_{t+1}.$$

Frequent endogenous disasters are accompanied by large drops of equity prices. These drops are followed by excessive equity returns, which raises the mean equity return in simulations. Figure 2.14 shows that consumption disaster risk yields an expected risk premium in this model. In all fairness, adding a second shock $\epsilon_{a,t}$ to match the output series will eradicate the frequent disasters at the cost of the equity premium puzzle.

In summary, cyclical fluctuations do not robustly solve the equity risk premium. Parametrized to post-war data, the RBC model matches key macro-variables but fails to generate a premium. Using the PNZK parametrization, the fear of frequent disasters motivates a risk premium, but the parametrization has unrealistic implications for output, consumption and labour market transition rates. The next section departs from cyclical fluctuations and endogenous disasters. It introduces a small persistent component of growth - a prominent solution of the equity premium puzzle.

2.4 Long-run risk

Going back to Bansal and Yaron (2004), the long-run risk (LRR) literature has shown how a persistent, time-varying component of consumption and dividend growth can solve the equity premium puzzle. Following Croce (2014), this section assumes that productivity growth has a persistent component while consumption and dividends are equilibrium quantities. Cyclical shocks are disregarded, $\sigma_z = 0$. Productivity grows by a predictable component $g_a + x_t$ and by a completely transitory iid innovation $\epsilon_{a,t}$,[10]

$$\begin{aligned} a_{t+1} - a_t &= g_a + x_t + \sigma_a \epsilon_{a,t+1} \\ x_t &= \rho_x x_{t-1} + \sigma_x \sigma_a \epsilon_{x,t} \\ z_t &= 0 \ \forall t. \end{aligned}$$

In the canonical DMP model, a small fundamental surplus translates volatility of cyclical productivity z_t into high unemployment rate volatility (Ljungqvist and Sargent, 2017). For instance, in the common Hagedorn and Manovskii (2008)-calibration, the outside option is assumed to be large and does not scale with productivity. A reduction of the cyclical z_t vis-à-vis the outside option triggers a reduction of firm surplus, investment and the job-finding rate. Neither do sticky wages (Hall, 2005a), alternating-offer wages (Hall and Milgrom, 2008), or training costs (Pissarides, 2000) scale with productivity z_t. In this LRR model, the outside option and training costs do scale with productivity, A_t. Even if the surplus is parametrized to be small, a reduction of the growth rate does not reduce investment via the surplus channel, i.e. the surplus is small but not volatile. The LRR model will have problems solving the Shimer puzzle.

As an example, consider a Hagedorn and Manovskii (2008)-calibration of a large outside option b and a low bargaining power of workers ϱ. Compare the following detrended surplus functions $\tilde{j}_t$ which deviate from the main text's j_t by a "standard" timing of separations, constant separations and a constant bargaining power ϱ. Here, I present the "standard" DMP surplus function because wages have a simpler closed form. Adjust the firm surplus

[10] In the long-run consumption risk literature, the baseline model is often extended with a time-varying volatility, $\sigma_{x,t}$, to capture time-varying risk premia.

$\tilde{J}_t$ by productivity A_t:

$$\begin{aligned}\tilde{J}_t =& A_t e^{z_t} - [\rho(A_t e^{z_t} + \kappa_t \theta_t) + (1-\rho) b_t] + \mathbb{E}_t M_{t+1} \tilde{J}_{t+1} \\ \tilde{j}_t^{RBC} =& e^{z_t} - [\rho(e^{z_t} + \kappa \theta_t) + (1-\rho) b] + \mathbb{E}_t m_{t+1} \tilde{j}_{t+1}^{RBC} \qquad (2.20) \\ \tilde{j}_t^{LRR} =& 1 - [\rho(1 + \kappa \theta_t] + (1-\rho) b) + \mathbb{E}_t m_{t+1} \tilde{j}_{t+1}^{LRR} \qquad (2.21)\end{aligned}$$

In (2.20) a reduction of productivity e^{z_t} depresses the surplus because productivity falls vis-à-vis the outside option. For a small ρ, the reduction of wages via productivity (ρe^{z_t}) and lower market tightness is small and barely raises the surplus. The small ρ is not the solution of the Shimer puzzle but does help to depress the surplus in bad states. Additionally, the reduction of autoregressive z_t reduces the discounted future surplus $\mathbb{E}_t m_{t+1} \tilde{j}_{t+1}$ which in turn reduces $\tilde{j}_t$. Key in the Hagedorn and Manovskii (2008) model is that the large outside option does not scale with productivity. This is what happens in the LRR model's surplus function (2.21), where b_t scales with productivity A_t. The negative LRR component can still depress the expected discounted future surplus, but compared to the direct productivity channel, this channel is weak. Effects of the SDF on vacancy posting are small (see also Kehoe et al., 2019).

To generate sufficient labour market fluctuations in the LRR model, the fundamental surplus must be linked more tightly to the persistent component of growth. This could be done by negatively linking wages, vacancy-posting costs, training costs, or the outside option to productivity growth. I resort to prominent wage rigidity because I find it the most convincing alternative. Assume that the worker's bargaining power decreases in the persistent component of the technology growth rate,

$$\rho_t = \bar{\rho} exp[-x_t \alpha_x]. \qquad (2.22)$$

Specification (2.22) implies that the bargaining power can rise more than it can fall during high-growth times. This can be interpreted as a downward rigidity of wages: independent of bargaining power, a low productivity growth rate reduces the trend growth of wages $W_t = A_t w_t$. Additionally, detrended wages fall as labour market tightness falls in response to low growth. Workers do not accept this wage reduction and their bargaining power rises. Following Hall (2005a), wages must be acceptable to both parties which might be violated if the bargaining power rises or falls too much; I impose lower and upper bounds on wages in the model solution (see Appendix 2.B.4).

This section repeats the main steps of Section 2.3: parametrization, simulation, and finally matching the unemployment and output time series in order to observe the model's goodness-of-fit of other variables.

2.4.1 Parametrization

The LRR model's parametrization follows the same strategy and uses the same targets as the parametrization of the RBC model in Section 2.3. Parameters $\{\sigma_x, \rho_x, \alpha_x\}$ replace the RBC model's $\{\sigma_z, \rho_z, \alpha_z\}$. Table 2.5 summarizes the parametrization of the LRR model.

2.4.2 Simulation

Table 2.2 reports simulation results of the parametrized LRR model. Many simulation results of the RBC carry over to the LRR model: average transition rates, π_{ue} and π_{eu}, match their counterparts, but the model overestimates the mean unemployment rate. The model matches the volatility of the separation and unemployment rate.

Importantly, the correlation of separations and output growth is negative. This is an important target because the model is prone to pro-cyclical separations: when the productivity growth rate is low, $x_t < 0$, the rigidity keeps wages afloat. As outlined above, the LRR model has difficulties in solving the Shimer puzzle because vacancy-posting costs scale with productivity. To magnify the effect of growth news on the match surplus, I need to assume strong wage rigidity. Wages may even rise when growth is low. In this event, the worker's surplus rises because existing jobs deliver high payments in a bad labour market. In the bargaining, $x_t < 0$ may trigger a reduction of the separation rate. To avoid counterfactual pro-cyclical separations, I target the correlation of separations and output in the estimation. The strong wage rigidity comes at a cost. The job-finding rate is too volatile and it is not pro-cyclical. Compared to the RBC model, wages are badly specified. In summary, the LRR model does not robustly solve the Shimer puzzle or

	Parametrization			
	Parameter	Value	Target	Source
γ	risk aversion	10.000	-	Bansal and
ψ	EIS	1.5000	-	Yaron (2004)
β	time discount	0.9991	$\overline{r^f} = 2.3\%$	U.S. Data
g_a	constant growth rate	0.0015	$\overline{\Delta^a Y} = 1.79\%$	U.S. Data
μ_ϵ	mean idiosyncratic shock	0.0453	$\frac{v}{l} = 1$	-
ι	matching function	0.8560	$\bar{q} = 64.15\%$	Davis, Faberman, Haltiwanger (2013)
κ_2	training costs	1.4409	$1.5 \times \bar{w}$	-
κ_1	vacancy-posting costs	0.9695	$\frac{\kappa_1 \bar{v} + \kappa_2 \bar{v} \bar{q}}{\bar{w} \bar{l} (1 - \bar{\pi}_{eu})} = 6\%$	-
b	unemployment benefits	0.9041	$\bar{\pi}_{ue} = 26.05\%$	U.S. Data
τ_{eu}	separation costs/tax	-2.7286	$\bar{\pi}_{eu} = 1.92\%$	U.S. Data
σ_x	std LRR shock	0.1584		
σ_a	std *iid* growth shock	0.0051		
ρ_z	autocorrelation LRR	0.9792		estimated with SMM
ψ_ϵ	elasticity of separations	0.1200		
α_z	wage rigidity	218.50		
$\bar{\rho}$	bargaining power	0.0629		

	SMM estimation			
		Simulation	Target	Source
$\rho(\Delta^q Y, \Delta^q u)$	corr. GDP & unemployment	-0.61%	-0.58%	
$\sigma(\Delta^a Y)$	volatility GDP	2.35%	2.32%	
$\sigma(\Delta^q \pi_{eu})$	volatility separations	6.45%	6.67%	all SMM targets:
$\sigma(\Delta^a W)$	volatility wages	2.71%	2.70%	US data 1948-2017
$\sigma(\Delta^q u)$	volatility unemployment	7.36%	7.68%	
$\rho(\Delta^a Y)$	autocorr. GDP	0.21	0.19	
$\rho(\Delta^q \pi_{eu}, \Delta^q Y)$	corr. separations & GDP	-0.38	-0.44	

Table 2.5: Parametrization of the LRR model. Parameters missing in this table are zero in this specification. All SMM targets are U.S. Data (see Appendix 2.A).

generate the correct cyclicality and magnitude of labour market flows. The RBC model is superior in the labour dimension. Appendix 2.D.1 explores a variant of the model with counter-cyclical vacancy-posting costs $\kappa_{1,t}$. In this exercise, designed to overcome the dependency of unemployment volatility on sticky wages, separations prove to be pro-cyclical.

As the RBC model, the LRR links consumption too closely to output; consumption is too volatile and too pro-cyclical. Compared to the RBC model, equity returns are more volatile and equity pays a slightly higher risk premium. Still, mean and volatility of returns are not comparable to empirical estimates and the equity premium puzzle persists. Finally, Table 2.4 shows that endogenous disaster risk plays no role in the model.

2.4.3 Matching time series

Figures 2.9 - 2.11 display matched series of the long-run risk model derived with the same strategy as in Section 2.3.3.

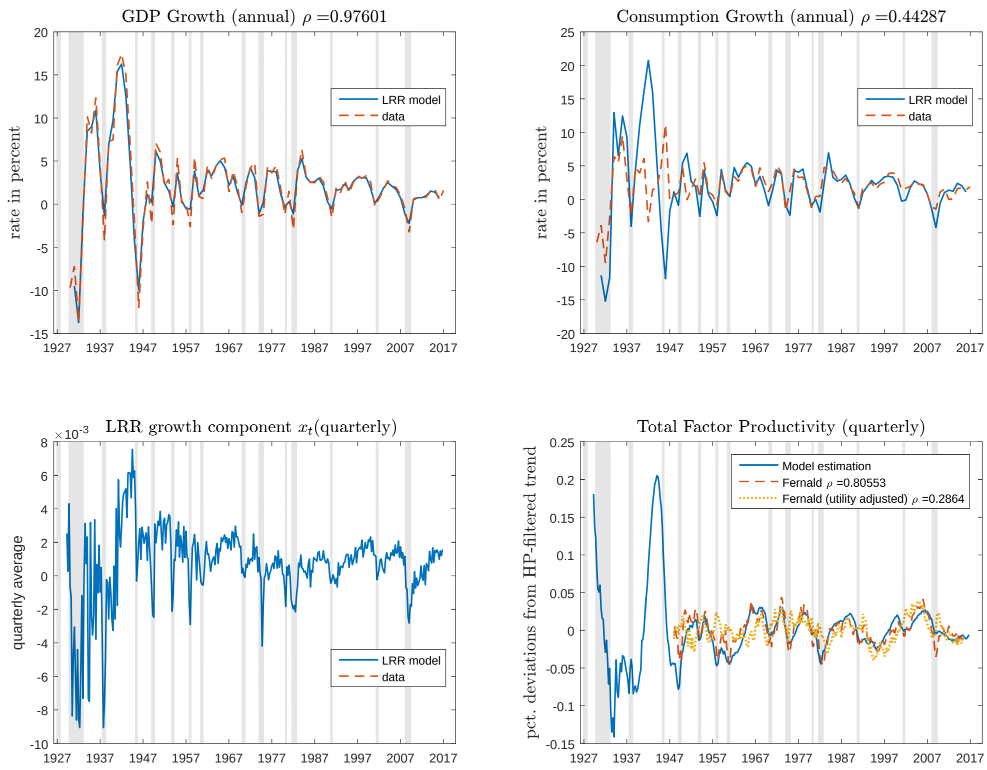


Figure 2.9: Matched time series of the LRR model: output, consumption and productivity. Grey bands denote NBER recessions. ρ denotes the correlation coefficient of annualized simulated and empirical data. HP-filter smoothing parameters: $\lambda = 129,600$ for monthly and $\lambda = 100,000$ for quarterly data.

By construction, the model tracks output growth and unemployment. The model predicts post-war consumption reasonably well. Similarly to the RBC model, the LRR model does not match consumption growth in the high-output, high-employment, low-consumption war years. Prior to 1950, the LRR model fails to match the consumption series. Consistent with Schorfheide et al. (2018), who identify a reduction of consumption volatility between 1940 and 1960, the introduction of stochastic volatility could improve the data fit.

The bottom panels of Figure 2.9 show the estimated series for x_t and TFP. x_t is the only exogenous process that can drive the unemployment rate. Hence, the model estimates a low x_t during the Great Depression, but in the

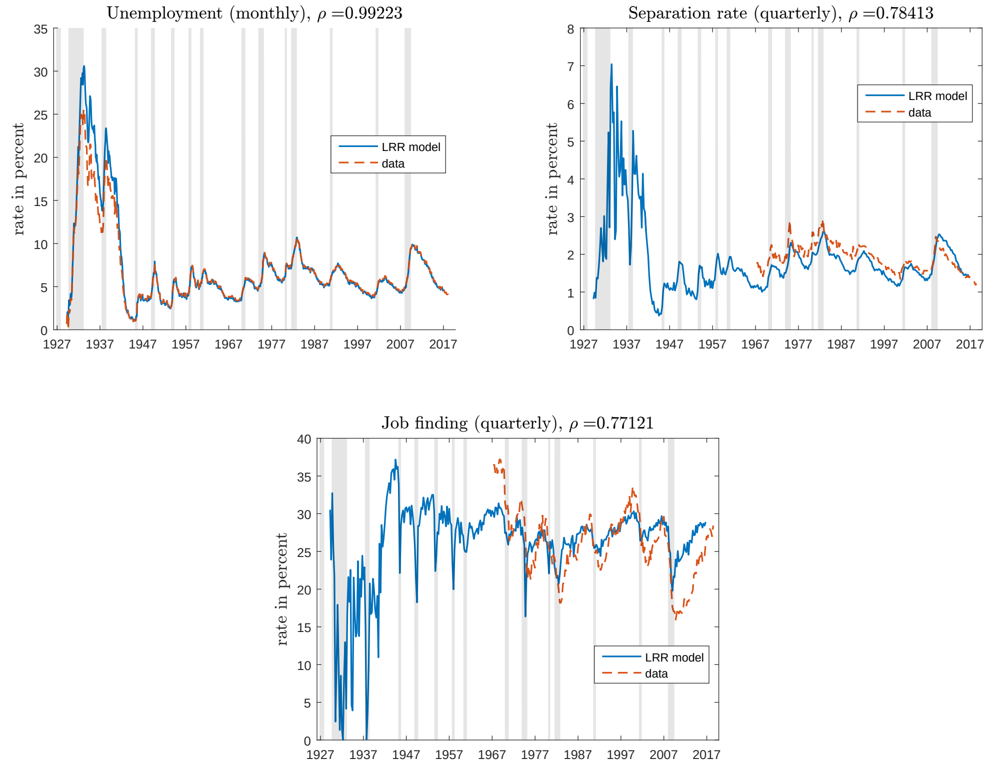


Figure 2.10: Matched time series of the LLR model: labour market. Grey bands denote NBER recessions. ρ denotes the correlation coefficient of annualized simulated and empirical data.

war years, unemployment is almost non-existent which calls for an extreme, sudden jump of x_t. Beginning in the late 50s, the x_t series much more resembles an AR(1) process, consistent with its assumed functional form. Together x_t and $\epsilon_{a,t}$ imply a series for TFP which I compare to estimates by Fernald (2014). After 1950, my estimated TFP series is correlated with the non-utilization adjusted series by Fernald (2014), but it is less volatile. It is the model's success to translate modest x_t-fluctuations into large fluctuations of output and employment.

Turn to Figure 2.11: as in the RBC model, dividends are the residual of output after wages and investment and should be interpreted as net transfers from firms to households rather than the shareholder's profit share. Again, the net transfers are far less volatile than empirical dividends. The LRR model does not track empirical wages in magnitude or direction; the correlation coefficient of data and model wages is almost -50%. The LRR model does not track the risk-free rate (see Section 2.3.3). It generates slightly more volatile stock prices and a slightly larger equity premium than the RBC model. Still, the model fails to generate prices and premia consistent with data.

Finally, return predictability does not hold in the LRR model (Figure 2.12). Compared to the RBC model, returns are somewhat more volatile, but the PE10 is almost constant.

In summary, the RBC model dominates the LRR model in the labour dimension: it is a tedious job to parametrize LRR to solve the Shimer puzzle and with labour market flows moving in the correct direction. At the core of this problem lies the weak effect of the small growth component on the job surplus. The LRR model - just like the RBC model - fails to track dividends, wages and the risk-free rate. The LRR model is slightly superior to the RBC model in the dimension of asset prices. Unless calibrated to generate disasters frequently, neither the RBC nor the LRR model solve the equity premium puzzle. The following section answers why the models fail to do so.

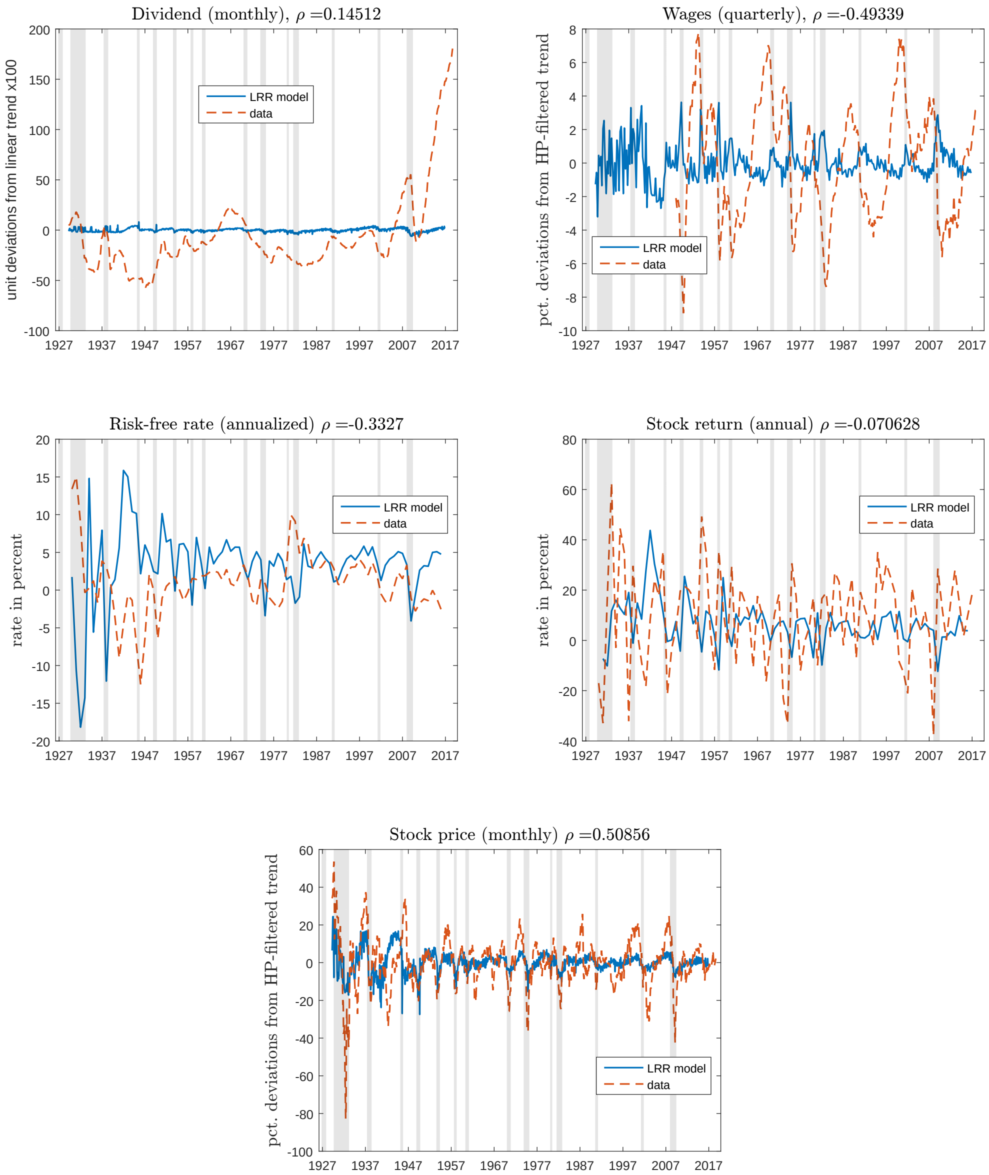


Figure 2.11: Matched time series of the LRR model: asset prices. Grey bands denote NBER recessions. ρ denotes the correlation coefficient of annualized simulated and empirical data. HP-filter smoothing parameters: $\lambda = 129,600$ for monthly and $\lambda = 100,000$ for quarterly data.

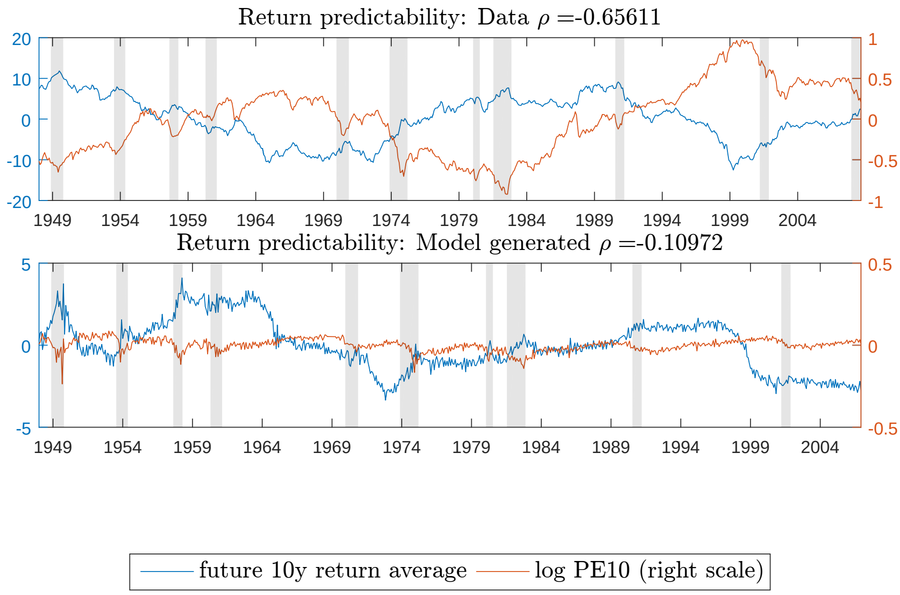


Figure 2.12: Matched time series of the LRR model: return predictability. The price earnings ratio (PE10) is the ratio of the current price to average earnings over the past ten years (also called Cyclically Adjusted PE Ratio (CAPE) or Shiller PE Ratio). I compute log PE10 as $\log \frac{P}{D+1}$ to avoid explosive PE10 for close to zero dividends. Future 10 year returns are the geometrical average of annual returns over the next ten years. The figure shows absolute deviations from mean.

2.5 Transmission mechanism

The last two sections show that neither the RBC model nor the LRR model solve the equity premium puzzle. This section explains this failure. Firstly, I show how news about productivity spreads in the RBC and LRR model. Secondly, I work out the difference between this news-driven model and a habit model (e.g. Hall (2017)). Last, I show that a low conditional standard deviation of marginal utility is responsible for the equity premium puzzle in this model. In order to raise that standard deviation, one could assume an extreme calibration (Petrosky-Nadeau et al., 2018) or introduce habits (Kehoe et al., 2019).

Transmission of news How do shocks spread in the model? At this juncture, it is informative to repeat the stochastic discount factor (SDF),

$$M_{t+1} = \quad = \quad \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{V_{t+1}}{\mathbb{E}\left[V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{-(\gamma - \frac{1}{\psi})} \tag{2.23}$$

with $\gamma > \frac{1}{\psi}$. Figure 2.13 considers adverse shocks $\epsilon_{z,t} < 0$ and $\epsilon_{x,t} < 0$. While the cyclical shock reduces productivity on impact by 1%, the LRR shock reduces productivity in the long-run by 1% (panel b)). Panels c) and d) show the impulse responses of consumption and the expected SDF. In response to the shock, households reduce investment to smooth consumption. Autoregressive productivity and lower investments lead to sustained lower consumption. Negative expected consumption growth raises the expected value of the SDF because, firstly, it raises marginal utility in $t+1$ keeping continuation utility V_{t+1} constant. The first term in parentheses in (2.23) falls. Secondly, lower consumption growth reduces the continuation utility relative to its expected value. In (2.23), this is a reduction of the second term in parentheses. In the Epstein-Zin model, a shock to autoregressive productivity affects the SDF because the shock is adverse news about future consumption growth. Panel c) shows that only the LRR shock causes a small consumption hike before a bust. The RBC shock depresses productivity instantaneously, while the LRR shock depresses productivity gradually and has no immediate effect on production. Since the continuation utility enters the SDF, we do not observe an equal pattern in the impulse response of $\mathbb{E}_t M_{t+1}$ in panel d).

Households discount the future less; so we would expect savings and investment to rise. Savings do rise, which reduces the equilibrium risk-free rate (panel j)). Yet, as panel e) shows, investment falls because the shock reduces the profitability of investment - the firm surplus J_{t+1} - which overcompensates the increase of the SDF. Wage rigidity amplifies the reduction of J_{t+1} because it stops wages from falling one-to-one with productivity. The unemployment rate rises in response to lower investment and higher separations (panel f)). In terms of the free-entry condition,

$$\frac{\kappa_{1,t}}{q(\theta_t)} + \kappa_{2,t} = \mathbb{E}_t M_{t+1} J_{t+1},$$

the right-hand side falls, fewer vacancies are posted, labour market tightness falls and the vacancy-filling rate rises, $q'(\theta_t) > 0$. Less investment translates into lower future employment, l_{t+1}, and a higher vacancy-filling rate. Via Tobin's q,

$$P_t = A_t \left(\frac{\kappa_1}{q(\theta_t)} + \kappa_2 \right) l_{t+1},$$

the lower l_{t+1} and θ_t cause a stock bust (panel g)). Productivity A_t does not fall with ϵ_z and only falls slowly in response to $\epsilon_x < 0$; A_t is not responsible for the stock bust, but news about its growth rate cause the bust in the LRR model.

Finally, panels h) and k) show the impulse responses of the expected equity return $\mathbb{E}_t R^s_{t+1}$ and the expected risk premium $\mathbb{E}_t[R^s_{t+1} - R^f_{t+1}]$. Both models predict low expected returns after the shock, followed by above-average returns during recovery. In the LRR model, the risk-free rate falls more strongly in the recession. Hence, we observe a large equity premium. In a recession, the equity premium falls in the RBC model while it rises in the LRR model. Scientific consensus (e.g. Cochrane, 2011) and Figure 2.4, for instance, show a large equity premium in a recession. When adverse news is announced, it is priced in and the equity return falls. But during the recovery, equity returns are large. The LRR model dominates in this regard.

Epstein-Zin (1989) or habits Hall (2017) describes the co-movement of labour and consumption somewhat differently: his theory is that, in a recession, investment and employment fall because "the value that employers attribute to a new hire declines on account of the higher discount rate." The discount rate is inversely related to the discount factor (Cochrane, 2011). The reduction of the SDF itself (and not a productivity shock) cause a reduction of investment. This is akin to Campbell and Cochrane (1999), where habits raise the present marginal utility in a recession. As a consequence of high present marginal utility, investors abstain from investment.

Figure 2.13 paints a different picture: $\epsilon_{z,t}$ and $\epsilon_{x,t}$ are essentially news about consumption growth and a recession is characterized by two stages: first, an adverse shock lowers expectations about future consumption growth, the continuation utility and expected productivity. The latter overcompensates the effects of the higher discount factor and firms and households reduce investment and separate more jobs. Stock prices fall. Secondly, over the course of the next months, depressed productivity and higher unemployment reduce consumption. In this recession, consumption is low, the discount factor is low and - in terms of Hall - the discount rate is high. Nevertheless, in the recession, investment rebounds because $\mathbb{E}_t M_{t+1} J_{t+1}$ rises and job creation is cheap when a high number of unemployed workers seek jobs. Hall's theory does not differentiate between the anticipation and the recession itself. In reaction to an exogenous shock to the discount factor, the economy simply jumps into the recession with depressed consumption, a low SDF, and depressed investment.

The risk-free rate is pro-cyclical in the RBC and LRR models, while shocks to the SDF yield counter-cyclical interest rates. Figure 2.20 in Appendix 2.D.2 shows the impulse responses of the interest rate to a impatience ($\beta_t \downarrow$) shock.[11] Contrary to a New-Keynesian demand shock, increased impatience causes a recession via low investment. The adverse shock raises the interest rate unambiguously, i.e. in Hall's framework interest rates are counter-cyclical. Habit formation by Campbell and Cochrane (1999) yields a risk-free rate that is less volatile and pro-cyclical and might alleviate this problem of Hall's framework.

Kehoe et al. (2019) experiment with different productivity shocks and

[11] See Appendix 2.D.2 for my parametrization of the Hall model.

habits to solve the Shimer and the equity premium puzzle. They settle on habit formation by Campbell and Cochrane (1999) together with on-the-job human capital accumulation. As outlined above, the news-driven story has to overcome counteracting forces on investment: when bad news arrives, the expected value of the SDF rises and this new household patience must be overcompensated by adverse expectations about TFP. In the habit model, the SDF always moves in the "correct" direction. In a recession, present marginal utility is large and households are impatient, causing lower investment.

Lack of equity premium Investors demand a a premium to hold a risky asset if the asset's return is negatively correlated with the investor's marginal utility. In this framework, the SDF describes marginal utility. Panels d) and h) show that, in expectations, SDF and equity return are indeed negatively correlated. Then why do simulations not yield an equity premium? What matters for the risk premium are not the expected values of equity return and SDF, but the relation of their possible future realizations. Investors demand an equity premium if the covariance of the excess return and the stochastic discount factor is negative across future states,[12]

$$\mathbb{E}_t\left[R^s_{t+1} - R^f_{t+1}\right] = -(R^f_{t+1})Cov_t\left(M_{t+1}, R^s_{t+1}\right).$$

The SDF, $M_{t+1} = \frac{\partial V/\partial C_{t+1}}{\partial V/\partial C_t}$, is the ratio of marginal utilities. Investors discount future consumption more if present marginal utility is large. Specifically, they discount a future state more if marginal utility in that particular state is low. Hence, investors demand a risk premium for assets with low payoffs in low consumption states. The covariance is time-varying, i.e. it describes the investor's expectations about the co-movement of the random variables across states in $t+1$ conditional on the state vector in t.

Following Campbell (2017), denote by tilde innovations to one-period-

[12] Start with the fundamental asset pricing equation and use the definition of a covariance and the fact that the risk-free rate is the reciprocal of the expected value of the SDF:

$$1 = \mathbb{E}_t M_{t+1} R^s_{t+1} = \mathbb{E}_t M_{t+1} \mathbb{E}_t R^s_{t+1} + Cov_t(M_{t+1}, R^s_{t+1})$$
$$\mathbb{E}_t R^s_{t+1} - R^f_{t+1} = -R^f_{t+1} Cov_t(M_{t+1}, R^s_{t+1} - R^f_{t+1}) = -R^f_{t+1} Cov_t(M_{t+1}, R^s_{t+1}).$$

ahead expectations, $\tilde{y}_{t+1} = y_{t+1} - \mathbb{E}_t y_{t+1}$. The log SDF can be expressed as

$$\widetilde{M}_{t+1} = -\frac{1}{\psi}\tilde{C}_{t+1} - \underbrace{\left(\gamma - \frac{1}{\psi}\right)}_{>0}\tilde{V}_{t+1}.$$

As before, news about positive productivity growth reduces the SDF via a direct effect of high consumption growth and an increase of continuation utility.

Figure 2.14 plots the innovations to one-period-ahead expectations, $\widetilde{M}_{t+1}$, against the excess return, $R^s_{t+1} - R^f_{t+1}$, across all states and evaluation nodes in the model. In this figure, a risky asset has realizations in the top-left and bottom-right corners, while an insurance has realizations in the top-right and bottom-left. Two main observations stand out: Firstly, in both models, bad news about consumption growth (high $\widetilde{M}_{t+1}$) is correlated with low returns as indicated by the regression line's slope. Secondly, the variance of $\tilde{M}_{t+1}$ and $R^s_{t+1} - R^f_{t+1}$ are larger in the LRR model than the RBC model (note the x-axes' limits). The covariance can be expressed as

$$Cov_t\,(M_{t+1}, R^s_{t+1}) = \rho_t\,(M_{t+1}, R^s_{t+1})\,\sigma_t(M_{t+1})\sigma_t(R^s_{t+1}).$$

The correlation coefficient ρ_t, approximated by the regression line's slope, is negative for both models, but the $\sigma_t(M_{t+1})$ differ substantially. In a beta representation of the return,

$$\mathbb{E}_t R^s - R^f = \underbrace{\left(\frac{Cov_t\left(M_{t+1}, R^s_{t+1}\right)}{\sigma_t(M_{t+1})}\right)}_{\text{equity's } \beta}\underbrace{\left(-\frac{\sigma_t(M_{t+1})}{\mathbb{E}_t M_{t+1}}\right)}_{\text{price of risk}},$$

we would say that the price of risk is too small. Crucially, the ability to generate an equity premium depends on the state-dependent standard deviation of the SDF, or the price of risk.

How can the model designer achieve a large $\sigma_t(M_{t+1})$? As has been shown, modest RBC shocks do not have persistent effects on consumption growth and utility. The LRR model outperforms the RBC model because of the high persistence of an LRR shock. The shock alters consumption growth for many periods and pushes the economy onto a new balanced growth path. In my parametrization, the volatility and persistence of the long-run growth component are still not sufficient for a significant risk premium. Rare disaster risk offers a solution: Investors perceive a large $\sigma_t(M_{t+1})$ *ex ante* as they fear disasters. *Ex post*, the econometrician observes only a risk premium but the disaster itself did not occur. For example, Figure 2.14 includes the Petrosky-Nadeau et al. (2018) calibration. Compared to my RBC model, the correlation between excess returns and the SDF is weaker but the variance of the SDF is larger as investors fear the frequent consumption disasters in this calibration. Another option is to assume a different functional form of utility, e.g. habits, that can raise the volatility of the SDF.

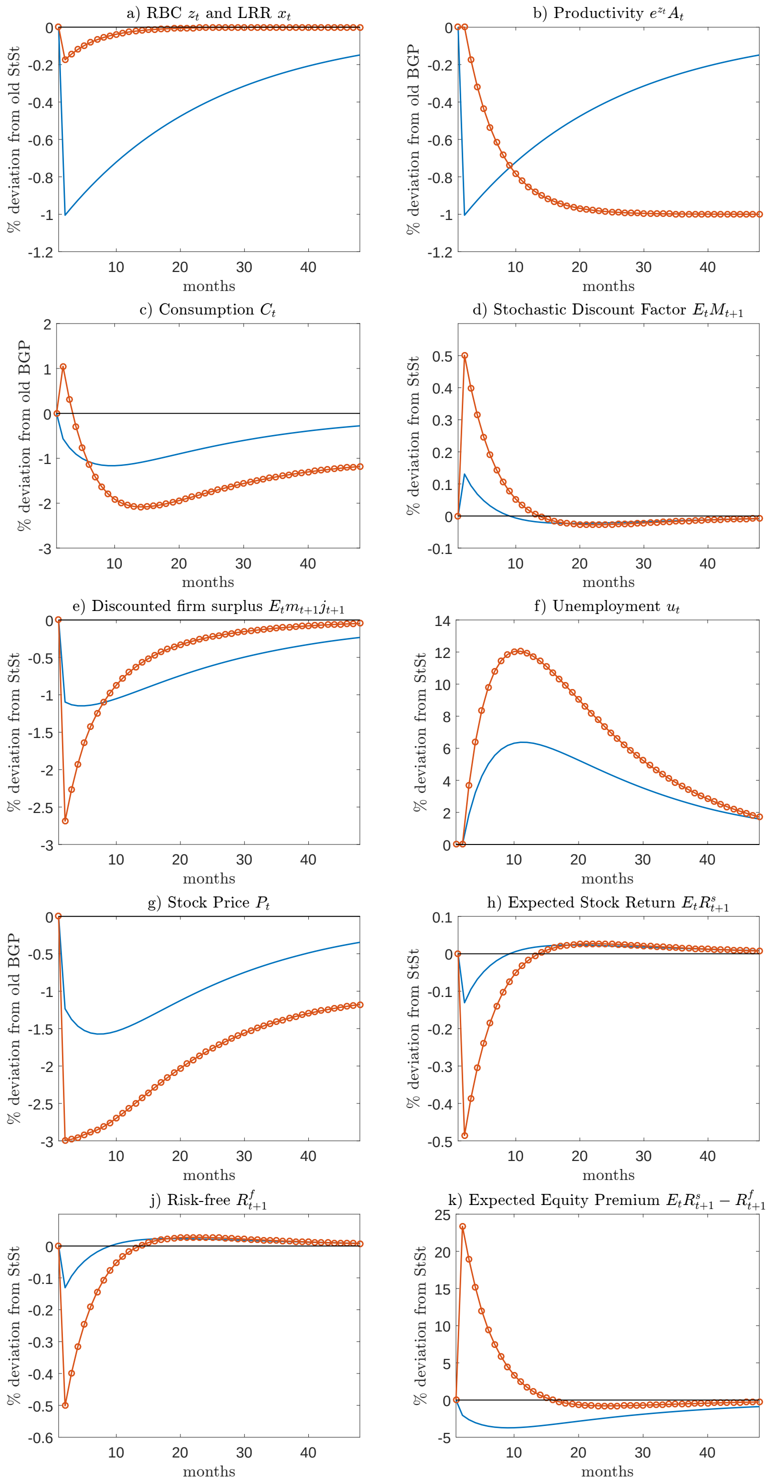
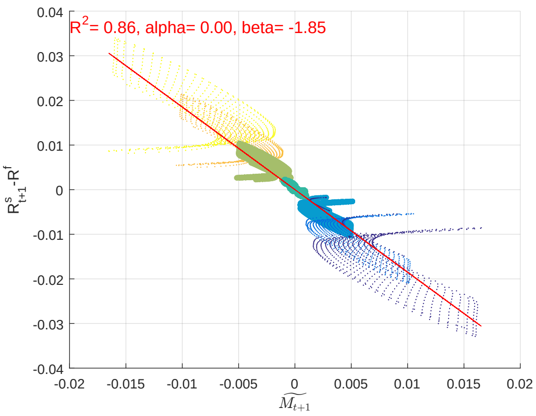
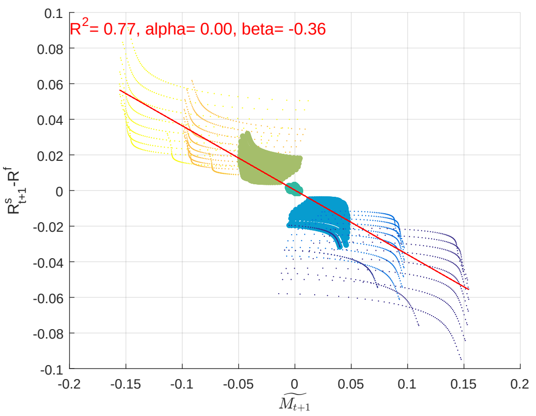


Figure 2.13: IRFs to RBC shock ϵ_z (solid, blue) and LRR shock ϵ_x (dots, red). The RBC shock reduces TFP upon impact by 1%; the LRR reduces long-term TFP by 1%. After the initial shock z_t and x_t follow their laws of motion without further innovations. Variables that grow with trend, e.g. Y, C, P, are plotted in deviations from a balanced growth path, which did not experience the shock ("old BGP"). Variables that do not grow with trend are plotted in percentage deviations from steady state (StSt).

(a) RBC model



(b) LRR model



(c) PNZK

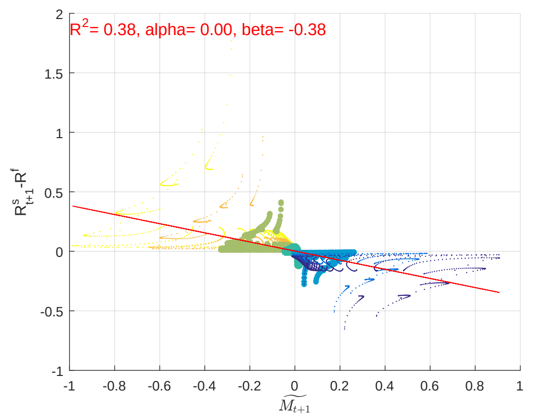


Figure 2.14: Excess returns and the stochastic discount factor in RBC, LRR and PNZK. The figure shows all evaluation nodes across gridpoints used in the three model solutions: The thicker a node in the figure, the larger its probability ω_i in the discrete approximation of the integral. The brighter the colour of the node, the more expansionary the realization of productivity. The red line marks the fitted values of a ω_i-weighted regression; alpha and beta are the regression coefficients. To declutter the figure somewhat, I ignore innovations of ϵ_a which raise the variance of returns and the SDF but do not change the figures qualitatively.

2.6 Conclusion

The paper studies two extensions to the canonical Diamond-Mortensen-Pissarides framework. First, I examine a real business-cycle model with rigid wages and endogenous separations. Second, I introduce a small, stochastic, persistent component of productivity growth. The first model is a good representation of the labour market, production and consumption. However, it fails to generate a risk premium. The long run-risk model offers a minor improvement in the dimension of asset prices, but it provides a distinctly inferior representation of the labour market. At its core, the small growth component alone cannot generate a large volatility of unemployment. Mechanisms to boost the volatility lead to counter-factual employment transition rates. A large, state-dependent variance of consumption growth is necessary to generate an equity premium: neither of the two models succeeds here. The endogenous disaster risk model by Petrosky-Nadeau et al. (2018) features a large variance of consumption growth, but the model creates disasters too frequently.

Building on this paper, I see two avenues for future research: even though two prominent solutions can now be ruled out, the task of finding a framework that robustly explains labour market transitions and asset prices remains. The key challenge is to find a mechanism that increases the volatility of the stochastic discount factor without causing counter-factual movements of transition rates, output and consumption.

Unless they are directly related to productivity growth, this paper does not account for long-term trends in labour market and asset-price data: for example, dividends and share repurchases have risen, but wages have not kept pace. This paper is also silent on the negative trend of labour transition rates or the trends of labour force participation. As in Farhi and Gourio (2018), studying these long-term trends in a DMP framework is an interesting avenue of future research.

Appendix

2.A Data

This Appendix section reports additional post-war data moments and moments of historic data. It discusses higher order moments of unemployment. The second part reports data sources and data composition.

2.A.1 Empirical moments of historic data

In addition to the main text's Table 2.2, Tables 2.6-2.8 display higher moments of the post-war data as well as available moments of the full sample, which is used in the matching series sections, e.g. Figure 2.2. The models are parametrized to post-war data, but the longer sample allows to study the model's out-of-sample predictions and behavior during the great depression. The latter is interesting because we can test whether the endogenous disaster model can replicate the United States' major disasters of the 20th century, which occur before the war.

Table 2.7 reports moments of unemployment and transition rates. Unemployment in the 1929-2018 sample is about twice as volatile as in the narrower post-war sample. The unemployment series is positively skewed: deviations of unemployment from its mean are stronger in an upward direction. This is not news: Milton Friedman (1964, 1993) coins the term "plucking model" to describe that economic activity is often characterized by strong adverse pulls away from a relatively steady trend. The economy then recovers back to trend. The mirror image, a strong positive pull followed by a slow-down does not occur in the same magnitude (see the plucking model by Dupraz et al. (2019)). Hairault et al. (2010) show that a DMP model generates asymmetric unemployment data intrinsically because of unemployment's low mean and properties of labor's law of motion. The $U \rightarrow E$ flow can fall more strongly than it can rise: In a boom, unemployment is low and the job-finding rate, π_{ue} is large. The $U \rightarrow E$ flow, which equals $u\pi_{ue}$, is restricted by low unemployment. In a recession, unemployment is large and the job-finding rate low. The $E \rightarrow U$ flow, which equals $(1-u)\pi_{eu}$, falls significantly because the hike of unemployment magnifies the reduction of

US Data	Historic data	Post-war data
Output	1791 - 2017	1948 - 2017
$\mathbb{E}[\Delta^a Y]$	1.69	1.79
$\sigma(\Delta^a Y)$	4.32	2.32
$Skewness(\Delta^a Y)$	0.27	0.02
$Kurtosis(\Delta^a Y)$	5.24	3.32
$\rho(\Delta^a Y)$	0.25	0.19
$\rho(\Delta^q Y)$		0.36
$Prob(\Delta^a Y < 0.1)$	5.88	1.75
$Size(\Delta^a Y < 0.1)$	-16.28	-14.50
$Dur(\Delta^a Y < 0.1)$	3.64	12.00
Consumption	1835 - 2017	1948 - 2017
$\mathbb{E}[\Delta^a C]$	1.54	1.89
$\sigma(\Delta^a C)$	3.80	1.63
$Skewness(\Delta^a C)$	0.09	-0.30
$Kurtosis(\Delta^a C)$	3.56	2.97
$\rho(\Delta^a C)$	0.03	0.27
$\rho(\Delta^q C)$		-0.13
$Prob(\Delta^a C < 0.1)$	4.52	1.75
$Size(\Delta^a C < 0.1)$	-19.56	-13.74
$Dur(\Delta^a C < 0.1)$	4.00	12.00

Table 2.6: Empirical moments of the U.S. economy: Output and consumption. $\mathbb{E}()$ and $\sigma()$ denote mean and standard deviation. $\Delta^q X$ denotes the quarterly and $\Delta^a X$ the annual growth rate of X. $\rho(X, Y)$ denotes the correlation between X and Y and $\rho(X)$ is X's autocorrelation. $Prob$, $Size$, Dur denote the annual probability, mean size and mean duration (in years) of a disaster. A disaster is defined as a cumulative reduction of ouptut or consumption by at least 10% using the peak-to-trough method by Barro and Ursúa (2008). All rates in percentage.

the transition rate.

The distribution of unemployment has a kurtosis greater than three, i.e. the distribution exhibits fat tails compared to a normal distribution, with kurtosis of three. Post-war, the distribution of unemployment resembles a normal distribution more closely with very low skewness and almost no kurtosis.

Table 2.8 shows moments of wages and asset returns. Asset pricing data are taken from Shiller[13] and Jordà et al. (2019). To some degree the large risk premium reflects leverage of US corporations. Following Petrosky-Nadeau and Zhang (2013) I reduce the risk premium by by 29% , the aggregate

[13]See webpage: `http://www.econ.yale.edu/~shiller/data`.

US Data	Historic data	Post-war data
Unemployment	1929 - 2018	1948-2018
$\mathbb{E}[u]$	6.87	5.77
$\sigma(du)$	41.18	21.33
$Skewness(u)$	2.10	0.63
$Kurtosis(u)$	7.64	3.09
$\sigma(\Delta^q u)$	13.30	7.68
$Skewness(\Delta^q u)$	2.39	1.40
$Kurtosis(\Delta^q u)$	35.36	8.39
$\rho(\Delta^q u)$	0.27	0.44
$\rho(\Delta^q u, \Delta^q Y)$		-0.58
$E \rightarrow U$ Flow		1967 - 2018
$\mathbb{E}[\pi_{eu}]$		1.92
$\sigma(d\pi_{eu})$		10.93
$E[\Delta^q \pi_{eu}]$		-0.18
$\sigma(\Delta^q \pi_{eu})$		6.67
$\rho(\Delta^q \pi_{eu})$		-0.20
$\rho(\Delta^q \pi_{eu}, \Delta^q u)$		0.39
$\rho(\Delta^q \pi_{eu}, \Delta^q Y)$		-0.45
$U \rightarrow E$ Flow		1967 - 2018
$\mathbb{E}[\pi_{ue}]$		26.05
$\sigma(d\pi_{ue})$		12.64
$E[\Delta^q \pi_{ue}]$		-0.12
$\sigma(\Delta^q \pi_{ue})$		5.10
$\rho(\Delta^q \pi_{ue})$		-0.01
$\rho(\Delta^q \pi_{ue}, \Delta^q u)$		-0.49
$\rho(\Delta^q \pi_{ue}, \Delta^q Y)$		0.29

Table 2.7: Empirical moments of the U.S. economy: Labor market. Data are quarterly or quarterly averages. $\mathbb{E}()$ and $\sigma()$ denote mean and standard deviation. $\Delta^q X$ denotes the quarterly and $\Delta^a X$ the annual growth rate of X. $\rho(X,Y)$ denotes the correlation between X and Y and $\rho(X)$ is X's autocorrelation. dX denotes percentage deviations from an HP-filtered trend with smoothing parameter 10^5. All rates in percentage.

leverage of US corporations estimated by Frank and Goyal (2008). Interestingly, average annual returns and their volatility have only changed little in between 1871 and 2018. Most notably, the volatility of the risk-free rate has fallen.

Finally, Figure 2.15 and Figure 2.16 show the time series of wages, dividends, productivity, unemployment and equity prices. Productivity and

US Data	Historic data	Post-war data
Wages		1948 - 2018
$\mathbb{E}[\Delta^a w]$		1.60
$\sigma(dw)$		3.48
$\sigma(\Delta^a w)$		2.70
$\epsilon_{w,exp(z)}$		0.42
Returns	1871 - 2018	1948 - 2018
$\mathbb{E}[r^f]$	2.11	2.33
$\sigma(r^f)$	5.55	2.44
$E[r^s]$	5.86	6.46
$\sigma[r^s]$	12.46	11.21
$E[R^s - R^f]$	4.39	4.80
$\sigma[R^s - R^f]$	12.52	11.32

Table 2.8: Empirical moments of the U.S. economy: Wages and returns. $\mathbb{E}()$ and $\sigma()$ denote mean and standard deviation. $\Delta^a X$ denotes the annual growth rate of X. dX denotes percentage deviations from an HP-filtered trend with smoothing parameter 10^5. $\epsilon_{w,z}$ denotes the elasticity of wages with respect to technology, filtered and in log deviations. r^s denotes the annual equity return and r^{real} the real rate of U.S. treasury bills. All rates in percentage.

wages co-move strongly, while wages and unemployment (or equity prices) are loosely correlated. Compared to wages, dividends are hardly correlated to productivity, unemployment and equity prices at all. The latter is noteworthy given that equity prices reflect discounted future dividends. The years following the Great Recession are especially interesting as dividends rise significantly.

2.A.2 Data sources

The parametrization uses post-war data, starting in January 1948. The historic samples are used in the time series figures.

- Inflation control

  1948-

  - GDPDEF: Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted
    Source: U.S. Bureau of Economic Analysis retrieved from FRED

historic sample

- CPI: CPI-U (Consumer Price Index-All Urban Consumers) published by the U.S. Bureau of Labor Statistics and Warren and Pearson's price index for years before 1913 [14], Index 2012=100, Monthly interpolation of quarterly data
  Source: Robert Shiller: Irrational Exuberance [Princeton University Press 2000, Broadway Books 2001, 2nd ed., 2005] retrieved from: `http://www.econ.yale.edu/~shiller/data`

- Population size control
  - POPINDEX = CNP16OV/CNP16OV(2012)
    CNP16OV: Population Level, Thousands of Persons, Monthly, Not Seasonally Adjusted
    Source: U.S. Bureau of Labor Statistics retrieved from FRED

- Output

1948-

- GDP: Gross Domestic Product, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate
  Source: U.S. Bureau of Economic Analysis retrieved from FRED
- In chained 2012 Dollars (GDPDEF) and at 2012 population level (POPINDEX)
- OUTNFB: Nonfarm Business Sector: Real Output, Index 2012=100, Seasonally Adjusted Annual Rate
  Source: U.S. Bureau of Economic Analysis retrieved from FRED
- At 2012 population level (POPINDEX)

historic sample

- Barro and Ursúa (2008): Macroeconomic Data, Annually, Index 2012=100

[14] Compared to Shiller and this paper, Jordà et al. (2019) use a different historical estimate of the CPI by Lawrence H. Officer and Samuel H. Williamson, "The Annual Consumer Price Index for the United States, 1774-Present," MeasuringWorth, 2020. For this paper, differences between CPI estimates are negligible.

retrieved from: `https://scholar.harvard.edu/barro/publications/barro-ursua-macroeconomic-data`

- Consumption

  1948-

  - PCE: Personal Consumption Expenditures, Billions of Dollars, Monthly, Seasonally Adjusted Annual Rate
    Source: Federal Reserve Bank of St. Louis
  - Before 1959: PCEC Personal Consumption Expenditures, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate
    Source: Federal Reserve Bank of St. Louis
  - PCE and PCEC in chained 2012 Dollars (GDPDEF) and at 2012 population level (POPINDEX)

  historic sample

  - Barro and Ursúa (2008): Macroeconomic Data, Annually, Index 2012=100
    retrieved from: `https://scholar.harvard.edu/barro/publications/barro-ursua-macroeconomic-data`

- S&P500 Equity prices, Dividends, Earnings

  1948-

  - Source: Robert Shiller: Irrational Exuberance [Princeton University Press 2000, Broadway Books 2001, 2nd ed., 2005] retrieved from: `http://www.econ.yale.edu/~shiller/data`
  - In chained 2012 Dollars (GDPDEF)

  historic sample

  - Source: Robert Shiller: Irrational Exuberance [Princeton University Press 2000, Broadway Books 2001, 2nd ed., 2005] retrieved from: Robert Shiller's homepage: `http://www.econ.yale.edu/~shiller/data`

  - In 2012 prices using CPI by Shiller (Warren and Pearson's price index)

- Risk-free interest rate:

  1948-

  - 10-Year Treasury Constant Maturity Rate, Percent, Monthly, Not Seasonally Adjusted
    Source: Board of Governors of the Federal Reserve System (US) retrieved from FRED
  - Before 1953-04-01: TB3MS: 3-Month Treasury Bill: Secondary Market Rate, Percent, Monthly, Not Seasonally Adjusted
    Source: Board of Governors of the Federal Reserve System (US) retrieved from FRED
  - Adjusted for inflation with GDPDEF

  historic sample

  - Bond rate by Jordà et al. (2019)

  - Inflation adjustment with CPI by Shiller (Warren and Pearson's price index) Retrieved from `http://www.macrohistory.net/data/`

- Vacancies

  - Composite Help-Wanted Index by Barnichon (2010)
    Source: `https://sites.google.com/site/regisbarnichon/data`

- Total factor productivity

  - log TFP by Fernald (2014), quarterly
    Available with and without utility adjustment
    I linearly interpolate to monthly frequency
    Source: retrieved from Ramey (2016)'s homepage: `https://econweb.ucsd.edu/~vramey/research/Ramey_HOM_technology.zip`

- Wages

  - Gross domestic income: Compensation of employees, paid wages and salaries (A4102C1Q027SBEA)
    Billions of Dollars, Quarterly, Seasonally Adjusted
    Source: U.S. Bureau of Economic Analysis retrieved from FRED
  - In chained 2012 Dollars (GDPDEF) and at 2012 population level (POPINDEX)

- Profits
  - Corporate Profits After Tax with Inventory Valuation Adjustment (IVA) and Capital Consumption Adjustment (CCAdj), Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate
    Source: U.S. Bureau of Economic Analysis retrieved from FRED
  - In chained 2012 Dollars (GDPDEF) and at 2012 population level (POPINDEX)

- Unemployment rate
  I follow Petrosky-Nadeau and Zhang (2013) and composite four datasets:
  - For 01/1929 - 12/1942, use NBER's macrohist unemployment data m08292a, seasonally adjusted by NICB
  - For 01/1940 - 12/1946, use NBER's macrohist unemployment data m08292b, seasonally adjusted by NBER
  - For 01/1947 - 12/1966, use NBER's macrohist unemployment data m08292c, not seasonally adjusted. I apply a x12-Arima-filter to seasonally adjust this series.
  - For 01/1948 - 09/2018, use U.S. Bureau of Labor Statistics, seasonally adjusted.

  When series overlap, I use the newer series. Figure 2.17 shows how the four series comprise the long unemployment series: The solid line is the composite series.

- Job-finding and separation rates
  The 04/1967 - 09/2018 job-finding rates and separation rates consist of two datasets:

- 04/1967 - 04/2007 unemployment-to-employment and employment-to-unemployment data by Shimer.[15]
- 04/1990 - 09/2018 labor force status flows (UE and EU) from the Current Population Survey and the stock of employed (E) and unemployed (U) provided by the U.S. Bureau of Labor Statistics. The job-finding and separation rates are UE/U and EU/E.

From 04/1990 onward, I use the BLS data. The BLS and Shimer data have different means. I calculate the ratio between mean BLS and mean Shimer transition rates in the years 1990 to 1991.[16] I adjust the Shimer series by these ratios $\pi_{i,t}^{composite} = \pi_{i,t}^{Shimer} \frac{\pi_{i,90-91}^{BLS}}{\pi_{i,90-91}^{Shimer}}$ and use the adjusted Shimer data for periods before 04/1990. Figure 2.18 shows the composite transition rates.

[15] This data was constructed by Robert Shimer. For additional details, please see Shimer (2005) and his webpage `http://home.uchicago.edu/shimer/data/flows/`. The data from June 1967 and December 1975 were tabulated by Joe Ritter and made available by Hoyt Bleakley.

[16] The choice of dates and width of this window has a negligible effect on the composite series.

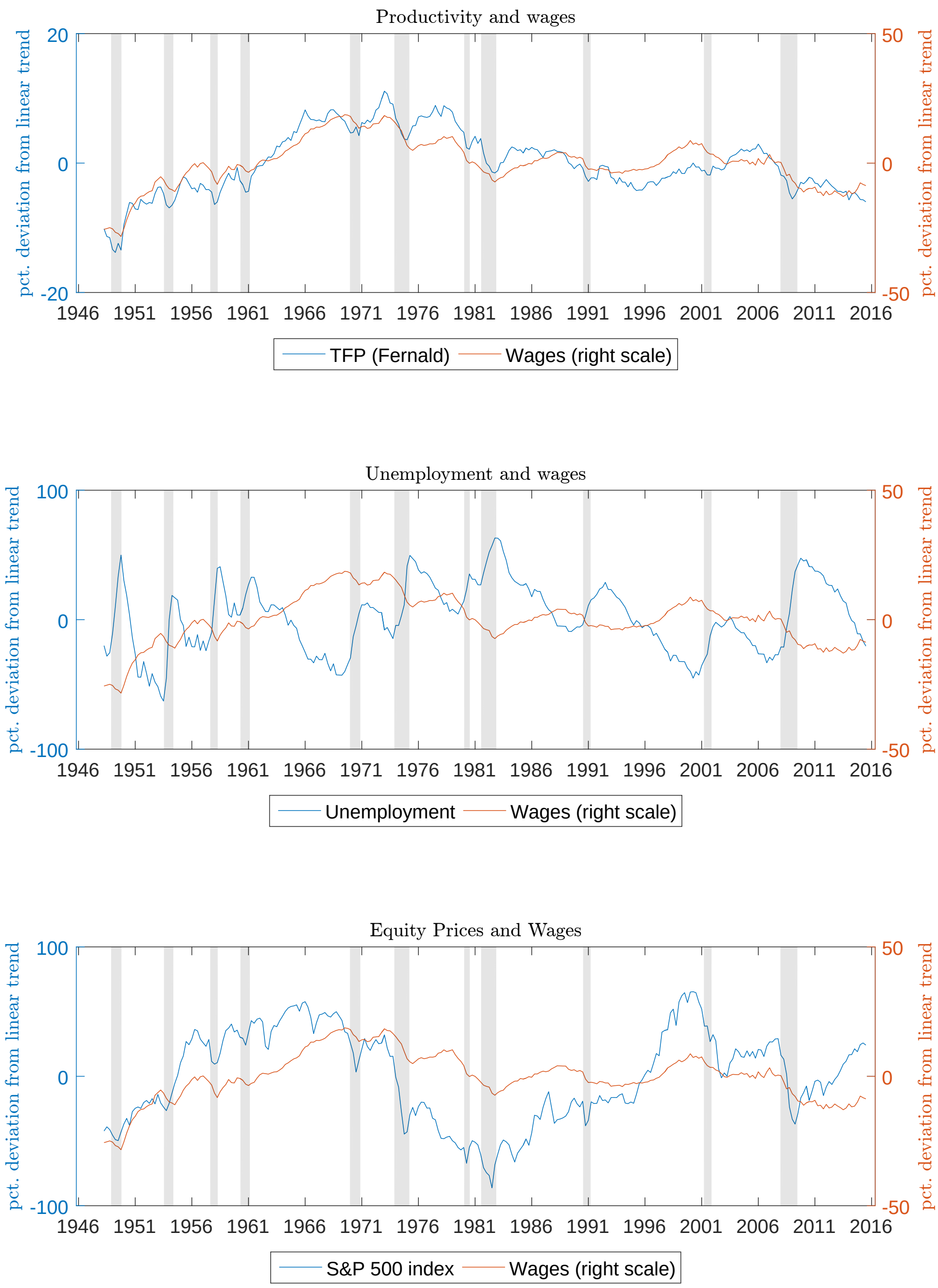


Figure 2.15: Wages vs productivity, unemployment and equity prices. Quarterly U.S. data. Grey bands denote NBER recessions.

Figure 2.16: Dividends vs productivity, unemployment and equity prices. Quarterly U.S. data. Grey bands denote NBER recessions.

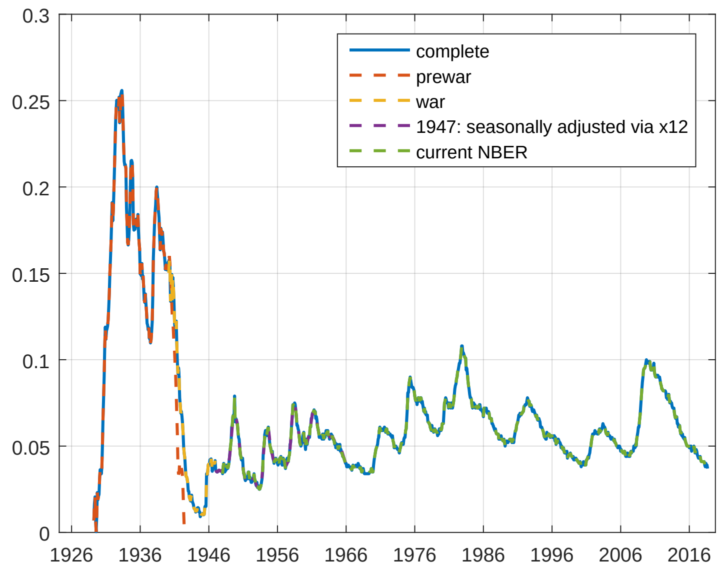


Figure 2.17: Composite unemployment rate for 1929-2018. Four different sources

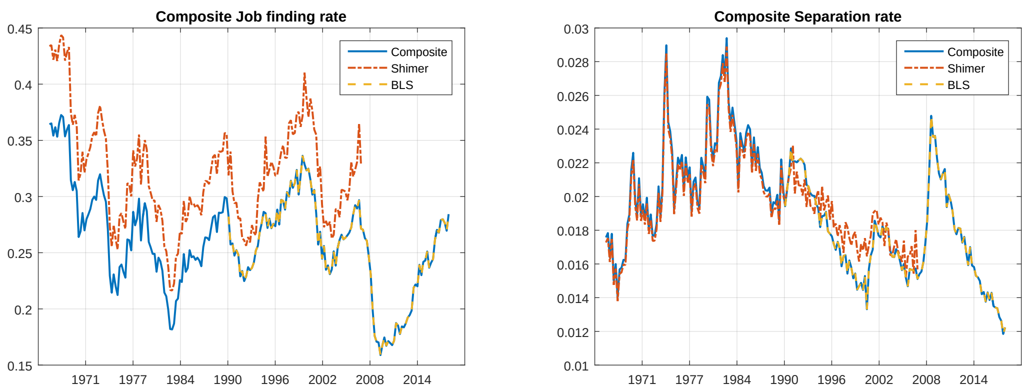


Figure 2.18: Composite job-finding and separation rate

2.B Derivations

2.B.1 Productivity adjustment

Assume that A_t scales the constant parameters $\{\mu_\epsilon, \psi_\epsilon, \kappa_1, \kappa_2, \tau_{eu}, b\}$ e.g. $b_t = bA_t$. Define the productivity-adjusted variables $c_t = \frac{C_t}{A_t}$, $w_t = \frac{W_t}{A_t}$ $d_t = \frac{D_t}{A_t}$, $p_t = \frac{P_t}{A_t}$, $\widetilde{\Psi}_t = \frac{\Psi_t}{A_t}$, $\widetilde{\Sigma}_t = \frac{\Sigma_t}{A_t}$. Define $\omega = \frac{1-\gamma}{1-\frac{1}{\psi}}$ and the productivity-adjusted bellman equation $\widetilde{v}_t \equiv \frac{V_t}{A_t}$. Adjusted for productivity growth, the model reads

$$
\begin{aligned}
\widetilde{v}_t =& \left\{(1-\beta)c_t^{1-\frac{1}{\psi}} + \beta\left\{\mathbb{E}_t[e^{(1-\gamma)(a_{t+1}-a_t)}(\widetilde{v}_{t+1})^{1-\gamma}]\right\}^{\frac{1}{\omega}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \\
c_t =& l_t(1-\pi_{eu,t})(e^{z_t}-\mu_\epsilon) + l_t\widetilde{\Psi}_t - \kappa_1 v_t - \kappa_2 q_t v_t \\
m_{t+1} \equiv& M_{t+1}\frac{A_{t+1}}{A_t} = \beta\left(\frac{c_{t+1}}{c_t}\right)^{-\frac{1}{\psi}}\left(\frac{A_{t+1}}{A_t}\right)^{(1-\gamma)}\left\{\frac{\widetilde{v}_{t+1}}{\mathbb{E}_t[(\frac{A_{t+1}}{A_t})^{1-\gamma}\widetilde{v}_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right\}^{\frac{1}{\psi}-\gamma}
\end{aligned}
$$

and the labour and production side

$$
\begin{aligned}
j_t \equiv& \frac{J_t}{A_t} = (1-\pi_{eu,t})(e^{z_t}-\mu_\epsilon) + \widetilde{\Psi}_t - (1-\pi_{eu,t})w_t + (1-\pi_{eu,t})\mathbb{E}_t m_{t+1} j_{t+1} \\
\widetilde{\Delta}_t \equiv& \frac{\Delta_t}{A_t} = [(w_t - b)(1-\pi_{eu,t})] + \mathbb{E}_t m_{t+1}\widetilde{\Delta_{t+1}}(1-\pi_{eu,t}-\pi_{ue,t}) \\
\mathbb{E}_t m_{t+1} j_{t+1} =& \frac{\kappa_1}{q_t} + \kappa_2 - \widetilde{\Theta}_t \\
\widetilde{\Theta}_t v_t q_t =& 0 \\
l_{t+1} =& l_t(1-\pi_{eu,t}) + \pi_{ue,t}(1-l_t) \\
w_t =& \Big[(1-\pi_{eu,t})\Big]^{-1}\Big\{\varrho_t\Big[(1-\pi_{eu,t})(e^{z_t}-\mu_\epsilon) + \widetilde{\Psi}_t + (1-\pi_{eu,t})\mathbb{E}_t m_{t+1} j_{t+1}\Big] \\
& + (1-\varrho_t)\Big[b(1-\pi_{eu,t}) - \mathbb{E}_t m_{t+1}\widetilde{\Delta}_{t+1}(1-\pi_{eu,t}-\pi_{ue,t})\Big]\Big\} \\
\pi_{eu,t} =& \left[1 + \exp\left(\frac{\mathbb{E}_t m_{t+1}\widetilde{\Sigma}_{t+1} + e^{z_t} - \mu_\epsilon + \tau_{eu} - b}{\psi_\epsilon}\right)\right]^{-1} \\
\widetilde{\Psi}_t =& -\psi_\epsilon[(1-\pi_{eu,t})\log(1-\pi_{eu,t}) + \pi_{eu,t}\log\pi_{eu,t}] - \pi_{eu,t}\tau_{eu}.
\end{aligned}
$$

Finally, risk-free rate and stock price read

$$\begin{aligned}\frac{1}{R^f_{t+1}} =&\mathbb{E}_t M_{t+1} = \mathbb{E}_t\Big[m_{t+1}\frac{A_t}{A_{t+1}}\Big]\\ p_t =&\underbrace{\left(\frac{\kappa_1 + q_t\kappa_2}{q_t} - \widetilde{\Theta_t}\right)}_{\mathbb{E}_t m_{t+1} j_{t+1}} l_{t+1}.\end{aligned}$$

Productivity states, z_t and x_t, and wage rigidity (2.16) are not affected by A_t.

2.B.2 Equity price and return

This section follows Wachter and Kilic (2018) and Petrosky-Nadeau et al. (2018) to derive the equity price and return. The firm takes wages and market tightness, therefore q_t, as given and maximizes its cum-dividend value P^c_t:

$$\begin{aligned}P^c_t &= \max_{\{v_{t+\tau}, l_{t+\tau}\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau}\\ s.t.\ l_{t+1} &= l_t(1-\pi_{eu,t}) + \pi_{ue,t}(1-l_t)\\ &= l_t(1-\pi_{eu,t}) + q_t v_t\\ & \quad q_t v_t \geq 0.\end{aligned}$$

Attach a Lagrange multiplier $\Theta_{t+\tau}$ to the non-negativity constraint of posted vacancies, $q_t v_t \geq 0$ because $q_t > 0$. Introduce law of motion's Lagrange multiplier $\Lambda_{t+\tau}$,

$$\begin{aligned}\max_{\{v_{t+\tau}, l_{t+\tau}\}} \ & M_t[l_t(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + l_t\Psi_t - l_t(1-\pi_{eu,t})W_t - \kappa_{1,t}v_t - q_t v_t \kappa_{2,t}\\ & - \Lambda_t(l_{t+1} - l_t(1-\pi_{eu,t}) - q_t v_t) + \Theta_t q_t v_t]\\ & + \mathbb{E}_t M_{t+1}[l_{t+1}(1-\pi_{eu,t+1})(e^{z_{t+1}}A_{t+1} - \mu_{\epsilon,t+1}) + l_{t+1}\Psi_{t+1}\\ & - l_{t+1}(1-\pi_{eu,t+1})W_{t+1} - \kappa_{1,t+1}v_{t+1} - q_{t+1}v_{t+1}\kappa_{2,t+1}\\ & - \Lambda_{t+1}(l_{t+2} - l_{t+1}(1-\pi_{eu,t+1}) - q_{t+1}v_{t+1}) + \Theta_{t+1}q_{t+1}v_{t+1}]\\ & + \dots\end{aligned}$$

First-order conditions with respect to v_t and l_{t+1}, using M_t =1, read

$$
\begin{aligned}
(v_t)\ & M_t[-\kappa_{1,t} - q_t\kappa_{2,t} + \Lambda_t q_t + \Theta_t q_t] = 0 \\
& -\kappa_{1,t} - q_t\kappa_{2,t} + \Lambda_t q_t + \Theta_t q_t = 0 \\
& \Lambda_t = \frac{\kappa_{1,t} + q_t\kappa_{2,t}}{q_t} - \Theta_t
\end{aligned}
$$

$$
\begin{aligned}
(l_{t+1})\ & -M_t\Lambda_t + \mathbb{E}_t M_{t+1}[(1-\pi_{eu,t+1})(e^{z_{t+1}}A_{t+1} - \mu_{\epsilon,t+1}) \\
& + \Psi_{t+1} - (1-\pi_{eu,t+1})W_{t+1} - \Lambda_{t+1}(-(1-\pi_{eu,t+1})) = 0 \\
& \mathbb{E}_t M_{t+1}[(1-\pi_{eu,t+1})(e^{z_{t+1}}A_{t+1} - \mu_{\epsilon,t+1}) \\
& + \Psi_{t+1} - (1-\pi_{eu,t+1})W_{t+1} + \Lambda_{t+1}(1-\pi_{eu,t+1})] = \Lambda_t
\end{aligned}
$$

The intratemporal first-order condition (v_t) is the free-entry condition and the intertemporal first-order condition (l_{t+1}) defines the continuation value of a filled vacancy, $\Lambda_t = \mathbb{E}_t M_{t+1} J_{t+1}$.

Expand the cum-dividend profits P_t^c using labour law of motion,

$$
\begin{aligned}
P_t^c =& l_t(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + l_t\Psi_t - l_t(1-\pi_{eu,t})W_t - \kappa_{1,t}v_t - q_t v_t\kappa_{2,t} \\
& - \Lambda_t(l_{t+1} - l_t(1-\pi_{eu,t}) - q_t v_t) + \Theta_t q_t v_t \\
& + \mathbb{E}_t M_{t+1}[l_{t+1}(1-\pi_{eu,t+1})(e^{z_{t+1}}A_{t+1} - \mu_{\epsilon,t+1}) + l_{t+1}\Psi_{t+1} - l_{t+1}(1-\pi_{eu,t+1})W_{t+1} \\
& - \kappa_{1,t+1}v_{t+1} - q_{t+1}v_{t+1}\kappa_{2,t+1} - \Lambda_{t+1}(l_{t+2} - l_{t+1}(1-\pi_{eu,t+1}) - q_{t+1}v_{t+1}) \\
& + \Theta_{t+1}q_{t+1}v_{t+1}] + \dots
\end{aligned}
$$

By the FOCs of (v_t), the terms $-\kappa_t v_t - q_t v_t \kappa_{2,t} + \Theta_t q_t v_t$ and $-\Lambda(-q_t v_t)$ cancel out in each period,

$$
\begin{aligned}
P_t^c =& [l_t(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + l_t\Psi_t - l_t(1-\pi_{eu,t})W_t] - \Lambda_t(l_{t+1} - l_t(1-\pi_{eu,t}))] \\
& + \mathbb{E}_t M_{t+1}[l_{t+1}(1-\pi_{eu,t+1})(e^{z_{t+1}}A_{t+1} - \mu_{\epsilon,t+1}) + l_{t+1}\Psi_{t+1} - l_{t+1}(1-\pi_{eu,t+1})W_{t+1} \\
& - \Lambda_{t+1}(l_{t+2} - l_{t+1}(1-\pi_{eu,t+1}))] + \dots
\end{aligned}
$$

By the FOC of (l_{t+1}), we know $\Lambda_t l_{t+1} = \mathbb{E}_t M_{t+1}[l_{t+1}(1-\pi_{eu,t+1})(e^{z_{t+1}}A_{t+1} - \mu_{\epsilon,t+1}) + l_{t+1}\Psi_{t+1} - l_{t+1}(1-\pi_{eu,t+1})W_{t+1} + l_{t+1}\Lambda_{t+1}(1-\pi_{eu,t+1})]$, so we get the price cum-dividend

$$P_t^c = l_t \left[(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t - (1-\pi_{eu,t})W_t + \Lambda_t(1-\pi_{eu,t}) \right],$$

with $\Lambda_t = -\Theta_t + \frac{\kappa_{1,t}+q_t\kappa_{2,t}}{q_t}$. Dividing by A_t yields

$$p_t^c = l_t\Big[(1-\pi_{eu,t})(e^{z_t} - \mu_\epsilon) + \widetilde{\Psi_t} - (1-\pi_{eu,t})w_t + \frac{\kappa_1 + q_t\kappa_2}{q_t}(1-\pi_{eu,t}) - (1-\pi_{eu,t})\widetilde{\Theta_t}\Big]$$

The price ex-dividend then follows from P_t^c, the first-order condition with respect to v_t and labour's law of motion,

$$\begin{aligned}
P_t &= P_t^c - D_t \\
&= l_t(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + l_t\Psi_t - l_t(1-\pi_{eu,t})W_t + \Lambda_t l_t(1-\pi_{eu,t}) \\
&- l_t(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) - l_t\Psi_t + l_t(1-\pi_{eu,t})W_t + \Omega_t \\
&= \Lambda_t l_t(1-\pi_{eu,t}) + \kappa_{1,t}v_t + q_t v_t \kappa_{2,t} \mid FOC\ (v_t) \\
&= \Lambda_t l_t(1-\pi_{eu,t}) + \Lambda_t q_t v_t + \Theta_t q_t v_t \mid LOM\ l_{t+1} \\
P_t &= \Lambda_t l_{t+1} + \Theta_t q_t v_t = \Lambda_t l_{t+1} = \left(\frac{\kappa_{1,t} + q_t\kappa_{2,t}}{q_t} - \Theta_t\right) l_{t+1} \\
p_t &= \left(\frac{\tilde{\kappa}_{1t} + q_t\kappa_2}{q_t} - \widetilde{\Theta_t}\right) l_{t+1},
\end{aligned}$$

where we used the Kuhn-Tucker condition $\Theta_t q_t v_t = 0$. The Lagrange multiplier Λ_t equals expected vacancy-posting costs per matched worker which, by free-entry, is equal to a match's expected return of a worker, $E_t m_{t+1} j_{t+1}$.

The realized return of equity follows,

$$
\begin{aligned}
R^s_{t+1} =& \frac{P_{t+1} + D_{t+1}}{P_t} \\
=& \frac{1}{\Lambda_t l_{t+1}} \Big\{ \Lambda_{t+1} l_{t+2} + l_{t+1}(1-\pi_{eu,t+1})(e^{z_{t+1}} A_{t+1} - \mu_{\epsilon,t+1}) + l_{t+1}\Psi_{t+1} \\
& - l_{t+1}(1-\pi_{eu,t+1}) W_{t+1} - \kappa_{1,t+1} v_{t+1} - q_{t+1} v_{t+1} \kappa_{2,t+1} \Big\} \\
=& \frac{\Lambda_{t+1}}{\Lambda_t} \Big[\frac{l_{t+2}}{l_{t+1}} + (1-\pi_{eu,t+1})(e^{z_{t+1}} A_{t+1} - \mu_{\epsilon,t+1}) + \Psi_{t+1} \\
& - (1-\pi_{eu,t+1}) W_{t+1} - \frac{\kappa_{1,t+1} v_{t+1}}{l_{t+1}} - \frac{q_{t+1} v_{t+1} \kappa_{2,t+1}}{l_{t+1}} \Big] \\
=& \frac{1}{\Lambda_t} \Big\{ \Big(\frac{\kappa_{1,t+1} + q_{t+1}\kappa_{2,t+1}}{q_{t+1}} - \Theta_{t+1} \Big) \Big[(1-\pi_{eu,t+1)} + \frac{q_{t+1} v_{t+1}}{l_{t+1}} \Big] \\
& \quad + (1-\pi_{eu,t+1})(e^{z_{t+1}} A_{t+1} - \mu_{\epsilon,t+1}) \\
& + \Psi_{t+1} - (1-\pi_{eu,t+1}) W_{t+1} - \frac{\kappa_{1,t+1} v_{t+1}}{l_{t+1}} - \frac{q_{t+1} v_{t+1} \kappa_{2,t+1}}{l_{t+1}} \Big\} \\
=& \frac{1}{\frac{\kappa_{1,t} + q_t \kappa_{2,t}}{q_t} - \Theta_t} \Big\{ (1-\pi_{eu,t+1})(e^{z_{t+1}} A_{t+1} - \mu_{\epsilon,t+1}) + \Psi_{t+1} - (1-\pi_{eu,t+1}) W_{t+1} \\
& + \frac{\kappa_{1,t+1} + q_{t+1}\kappa_{2,t+1}}{q_{t+1}} (1-\pi_{eu,t+1}) - \Theta_{t+1}[(1-\pi_{eu,t+1)} + \frac{q_{t+1} v_{t+1}}{l_{t+1}}] \Big\} \mid KKT \\
=& \frac{A_{t+1}}{A_t} \frac{1}{\frac{\tilde{\kappa}_{1,t} + q_t \kappa_2}{q_t} - \widetilde{\Theta_t}} \Big\{ (1-\pi_{eu,t+1})(e^{z_{t+1}} - \mu_\epsilon) + \widetilde{\Psi_{t+1}} - (1-\pi_{eu,t+1}) w_{t+1} \\
& + \frac{\tilde{\kappa}_{1,t+1} + q_{t+1}\kappa_2}{q_{t+1}} (1-\pi_{eu,t+1}) - \widetilde{\Theta_{t+1}} (1-\pi_{eu,t+1}) \Big\} \\
=& \frac{A_{t+1}}{A_t} \frac{(1-\pi_{eu,t+1})(e^{z_{t+1}} - \mu_\epsilon) + \widetilde{\Psi_{t+1}} - (1-\pi_{eu,t+1}) w_{t+1} + (1-\pi_{eu,t+1}) \widetilde{\Lambda_{t+1}}}{\widetilde{\Lambda_t}} \\
=& \frac{(1-\pi_{eu,t+1})(e^{z_{t+1}} A_{t+1} - \mu_{\epsilon,t+1}) + \Psi_{t+1} - (1-\pi_{eu,t+1}) W_{t+1} + \mathbb{E}_{t+1} M_{t+2} J_{t+2} (1-\pi_{eu,t+1})}{\mathbb{E}_t M_{t+1} J_{t+1}}.
\end{aligned}
$$

2.B.3 Bargaining: wages and separations

This section derives i) the surplus (value) functions that enter the Nash bargaining, ii) the separation probability, and iii) the wage equation. The wage equation takes a more complicated form, because we do not substitute out the expected surplus functions. Last, the section asks whether the wage equation can be simplified to the textbook Nash wage: *in a flexible wage model*, we can work with the simplification; in a model with time-varying bargaining power, the simplification might lead to large forecasting errors.

The worker surplus The family's consumption is given by

$$C_t = W_t l_t(1-\pi_{eu,t}) - T_t + b_t(\pi_{eu,t} l_t + 1 - l_t) + s_t D_t + s_t P_t - s_{t+1} P_t + \widetilde{B}_t - \frac{1}{R_t}\widetilde{B}_{t+1}.$$

An additional worker raises the family's consumption by

$$\frac{\partial C_t}{\partial l_t} = W_t(1-\pi_{eu,t}) + b_t(\pi_{eu,t} - 1).$$

We work with a transformation of the value function, $\widetilde{V}_t \equiv (1-\beta)(C_t - C_t^H)^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t[\widetilde{V}_{t+1}^{\omega}])^{\frac{1}{\omega}}$ with $\omega = \frac{1-\gamma}{1-\frac{1}{\psi}}$and $\tilde{V}_t = V_t^{1-\frac{1}{\psi}}$. Marginal rates of substitution are unaffected by the transformation, $\frac{\partial V_t/\partial l_t}{\partial V_t/\partial C_t} = \frac{(\partial V_t/\partial \widetilde{V}_t)(\partial \widetilde{V}_t/\partial l_t)}{(\partial V_t/\partial \widetilde{V}_t)(\partial \widetilde{V}_t/\partial C_t)} = \frac{(\partial \widetilde{V}_t/\partial l_t)}{(\partial \widetilde{V}_t/\partial C_t)}$.

The utility value of an additional worker in the household is given by

$$\frac{\partial \tilde{V}_t}{\partial l_t} = (1-\beta)\frac{1}{1-1/\psi}(C_t - C_t^H)^{-\frac{1}{\psi}}\frac{\partial C_t}{\partial l_t} + \beta\frac{1}{\omega}(E_t[\tilde{V}_{t+1}^{\omega}])^{\frac{1}{\omega}-1}\mathbb{E}_t[\omega \tilde{V}_{t+1}^{\omega-1}\frac{\partial \tilde{V}_{t+1}}{\partial l_{t+1}}\frac{\partial l_{t+1}}{\partial l_t}]$$

Using $l_{t+1} = l_t(1-\pi_{eu,t}) + (1-l_t)\pi_{ue,t}$,

$$\begin{aligned}\frac{\partial \tilde{V}_t}{\partial l_t} =& (1-\beta)\frac{1}{1-1/\psi}(C_t - C_t^H)^{-\frac{1}{\psi}}\frac{\partial C_t}{\partial l_t} \\ &+ \beta\frac{1}{\omega}(E_t[\tilde{V}_{t+1}^{\omega}])^{\frac{1}{\omega}-1}\mathbb{E}_t[\omega \tilde{V}_{t+1}^{\omega-1}\frac{\partial \tilde{V}_{t+1}}{\partial l_{t+1}}(1-\pi_{eu,t}-\pi_{ue,t})].\end{aligned}$$

The marginal value of a worker in terms of goods reads

$$\begin{aligned}\Delta_t \equiv \frac{\frac{\partial \tilde{V}_t}{\partial l_t}}{\frac{\partial \tilde{V}_t}{\partial C_t}} =& \frac{(1-\beta)\frac{1}{1-1/\psi}(C_t - C_t^H)^{-\frac{1}{\psi}}\frac{\partial C_t}{\partial l_t} + \beta\frac{1}{\omega}(\mathbb{E}_t[\tilde{V}_{t+1}^{\omega}])^{\frac{1}{\omega}-1}\mathbb{E}_t[\omega \tilde{V}_{t+1}^{\omega-1}\frac{\partial \tilde{V}_{t+1}}{\partial l_{t+1}}(1-\pi_{eu,t}-\pi_{ue,t})]}{(1-\beta)(1-\frac{1}{\psi})(C_t - C_t^H)^{\frac{-1}{\psi}}} \\ =& \frac{(1-\beta)\frac{1}{1-1/\psi}(C_t - C_t^H)^{-\frac{1}{\psi}}[W_t(1-\pi_{eu,t}) + b_t(\pi_{eu,t}-1)]}{(1-\beta)(1-\frac{1}{\psi})(C_t - C_t^H)^{\frac{-1}{\psi}}}\end{aligned}$$

and after some rearranging,

$$
\begin{aligned}
=&[W_t(1-\pi_{eu,t})+b_t(\pi_{eu,t}-1)]+\frac{\beta\mathbb{E}_t[\tilde{V}_{t+1}^{\omega-1}\Delta_{t+1}(C_{t+1}-C_{t+1}^H)^{\frac{-1}{\psi}}](1-\pi_{eu,t}-\pi_{ue,t})}{(C_t-C_t^H)^{\frac{-1}{\psi}}(\mathbb{E}_t[\tilde{V}_{t+1}^{\omega}])^{\frac{\omega-1}{\omega}}}\\
=&[(W_t-b_t)(1-\pi_{eu,t})]+\mathbb{E}_t[M_{t+1}\Delta_{t+1}](1-\pi_{eu,t}-\pi_{ue,t})
\end{aligned}
$$

The firm surplus The firm's value of a job follows from the cum dividend value optimization. Start with the recursive formulation,

$$
\begin{aligned}
P_t^c=&l_t(1-\pi_{eu,t})(e^{z_t}A_t-\mu_{\epsilon,t})+l_t\Psi_t-l_t(1-\pi_{eu,t})W_t\\
&-\kappa_{1,t}v_t-q_tv_t\kappa_{2,t}+\Theta_tq_tv_t+\mathbb{E}_tM_{t+1}P_{t+1}^c
\end{aligned}
$$

s.t. $l_{t+1}=l_t(1-\pi_{eu,t})-q_tv_t$.

The first-order condition with respect to v_t reads

$$
\begin{aligned}
-\kappa_{1,t}-q_t\kappa_{2,t}+\Theta_tq_t+\mathbb{E}_t\frac{\partial(M_{t+1}P_{t+1}^c)}{\partial l_{t+1}}q_t=0&\\
\frac{\kappa_{1,t}+q_t\kappa_{2,t}}{q_t}-\Theta_t=\mathbb{E}_t\frac{\partial(M_{t+1}P_{t+1}^c)}{\partial l_{t+1}}.&
\end{aligned}
$$

Differentiating P_t^c with respect to l_t

$$
\begin{aligned}
J_t\equiv\frac{\partial P_t^c}{\partial l_t}=&(1-\pi_{eu,t})(e^{z_t}A_t-\mu_{\epsilon,t})+\Psi_t-(1-\pi_{eu,t})W_t+\mathbb{E}_t\frac{\partial M_{t+1}P_{t+1}^c}{\partial l_{t+1}}\frac{\partial l_{t+1}}{\partial l_t}\\
=&(1-\pi_{eu,t})(e^{z_t}A_t-\mu_{\epsilon,t})+\Psi_t-(1-\pi_{eu,t})W_t\\
&+(1-\pi_{eu,t})\left[\frac{\kappa_{1,t}+q_t\kappa_{2,t}}{q_t}-\Theta_t\right]\\
J_t=&(1-\pi_{eu,t})(e^{z_t}A_t-\mu_{\epsilon,t})+\Psi_t-(1-\pi_{eu,t})W_t+(1-\pi_{eu,t})\mathbb{E}_tM_{t+1}J_{t+1}
\end{aligned}
$$

The firm's outside option is the separation cost $\tau_{eu,t}$, which are part of the option value Ψ_t.

Nash bargaining Assume Nash bargaining with workers' bargaining power ρ_t,

$$
\arg\max_{\pi_{eu,t},W_t}\Delta^{\rho_t}J_t^{1-\rho_t}.
$$

The FOCs of the bargaining with respect to $\pi_{eu,t}$ and W_t read

$$\begin{aligned}
(\pi_{eu,t})\ \rho_t\Delta_t^{\rho_t-1}J_t^{1-\rho_t}(-W_t+b_t-E_tM_{t+1}\Delta_{t+1})&\\
+(1-\rho_t)\Delta^{\rho_t}J_t^{-\rho_t}(-e^{z_t}A_t+\mu_{\epsilon,t}+\frac{\partial\Psi_t}{\partial\pi_{eu,t}}+W_t-\mathbb{E}_tM_{t+1}J_{t+1})&=0\\
(W_t)\ \rho_t\Delta_t^{\rho_t-1}J_t^{1-\rho_t}(1-\pi_{eu,t})+(1-\rho_t)\Delta^{\rho_t}J_t^{-\rho_t}(-(1-\pi_{eu,t}))&=0
\end{aligned}$$

with $\frac{\partial\Psi_t}{\partial\pi_{eu}}=-\psi_{\epsilon,t}[-\log(1-\pi_{eu,t})+\log(\pi_{eu,t})]-\tau_{eu,t}$.
Combining FOCs yields the surplus sharing rule

$$\begin{aligned}
\rho_t\Delta_t^{\rho_t-1}J_t^{1-\rho_t}&=(1-\rho_t)\Delta^{\rho_t}J_t^{-\rho_t}\\
\rho_tJ_t&=(1-\rho_t)\Delta_t.
\end{aligned}$$

Separation rate Starting with the FOC $(\pi_{eu,t})$, solve for $\pi_{eu,t}$,

$$\begin{aligned}
&\rho_t\Delta_t^{\rho_t-1}J_t^{1-\rho_t}(-W_t+b_t-\mathbb{E}_tM_{t+1}\Delta_{t+1})\\
&+\rho_t\Delta_t^{\rho_t-1}J_t^{1-\rho_t}\Big\{-e^{z_t}A_t+\mu_{\epsilon,t}-\psi_{\epsilon,t}[-\log(1-\pi_{eu,t})+\log(\pi_{eu,t})]\\
&-\tau_{eu,t}+W_t-\mathbb{E}_tM_{t+1}J_{t+1}\Big\}=0\\
\Leftrightarrow&(b_t-\mathbb{E}_tM_{t+1}\Delta_{t+1})+\Big\{-e^{z_t}A_t+\mu_{\epsilon,t}-\psi_{\epsilon,t}[-\log(1-\pi_{eu,t})+\log(\pi_{eu,t})]\\
&-\tau_{eu,t}-\mathbb{E}_tM_{t+1}J_{t+1}\Big\}=0\\
\Leftrightarrow&(b_t-\mathbb{E}_tM_{t+1}\Delta_{t+1})+\Big(-e^{z_t}A_t+\mu_{\epsilon,t}\\
&-\tau_{eu,t}-\mathbb{E}_tM_{t+1}J_{t+1}\Big)=\psi_{\epsilon,t}[\log(\frac{\pi_{eu,t}}{1-\pi_{eu,t}})]\\
\Leftrightarrow&(-e^{z_t}A_t+\mu_{\epsilon,t}-\tau_{eu,t}+b_t-\mathbb{E}_tM_{t+1}\Sigma_{t+1})=\psi_{\epsilon,t}[\log(\frac{\pi_{eu,t}}{1-\pi_{eu,t}})]\\
\Leftrightarrow&\exp[\frac{-e^{z_t}A_t+\mu_{\epsilon,t}-\tau_{eu,t}+b_t-\mathbb{E}_tM_{t+1}\Sigma_{t+1}}{\psi_{\epsilon,t}}]=\frac{\pi_{eu,t}}{1-\pi_{eu,t}}
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow\pi_{eu,t}&=\frac{\exp[\frac{-e^{z_t}A_t+\mu_{\epsilon,t}-\tau_{eu,t}+b_t-\mathbb{E}_tM_{t+1}\Sigma_{t+1}}{\psi_{\epsilon,t}}]}{1+\exp[\frac{-e^{z_t}A_t+\mu_{\epsilon,t}-\tau_{eu,t}+b_t-\mathbb{E}_tM_{t+1}\Sigma_{t+1}}{\psi_{\epsilon,t}}]}\\
\Leftrightarrow\pi_{eu,t}&=\frac{1}{\exp[\frac{e^{z_t}A_t-\mu_{\epsilon,t}+\tau_{eu,t}-b_t+\mathbb{E}_tM_{t+1}\Sigma_{t+1}}{\psi_{\epsilon,t}}]+1}
\end{aligned}$$

Wage equation Next, start with the surplus sharing rule and solve for W_t, using the definition of value functions

$$
\begin{aligned}
&\varrho_t J_t = (1-\varrho_t)\Delta_t \\
\Leftrightarrow &\varrho_t[(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t - (1-\pi_{eu,t})W_t + (1-\pi_{eu,t})\mathbb{E}_t M_{t+1}J_{t+1}] \\
&= (1-\varrho_t)[(W_t - b_t)(1-\pi_{eu,t}) + \mathbb{E}_t[M_{t+1}\Delta_{t+1}](1-\pi_{eu,t}-\pi_{ue,t})] \\
\Leftrightarrow &\varrho_t[(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t + (1-\pi_{eu,t})\mathbb{E}_t M_{t+1}J_{t+1}] \\
&- (1-\varrho_t)[(-b_t)(1-\pi_{eu,t}) + \mathbb{E}_t[M_{t+1}\Delta_{t+1}](1-\pi_{eu,t}-\pi_{ue,t})]] \\
&= (1-\varrho_t)W_t + \varrho_t(1-\pi_{eu,t})W_t \\
\Leftrightarrow &\varrho_t[(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t + (1-\pi_{eu,t})\mathbb{E}_t M_{t+1}J_{t+1}] \\
&- (1-\varrho_t)[(-b_t)(1-\pi_{eu,t}) + \mathbb{E}_t[M_{t+1}\Delta_{t+1}](1-\pi_{eu,t}-\pi_{ue,t})] = W_t(1-\pi_{eu,t},)
\end{aligned}
$$

and finally,

$$
\begin{aligned}
W_t = \frac{1}{1-\pi_{eu,t}}\{&\varrho_t[(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t + (1-\pi_{eu,t})\mathbb{E}_t M_{t+1}J_{t+1}] \\
&+ (1-\varrho_t)[b_t(1-\pi_{eu,t}) - \mathbb{E}_t[M_{t+1}\Delta_{t+1}](1-\pi_{eu,t}-\pi_{ue,t})]\}. \qquad (2.15)
\end{aligned}
$$

This is equation (2.15) used in the main text and the solution algorithm.

Simplify the wage equation? It is common to simplify the wage equation via the surplus sharing rule, but this only works when the surplus sharing rule holds in expectations about the continuation values, $(1-\varrho_t)\mathbb{E}_t M_{t+1}\Delta_{t+1} =$

$\varrho_t \mathbb{E}_t M_{t+1} J_{t+1}$. For argument's sake, assume this holds. Then,

$$\begin{aligned}
W_t^N =& \frac{1}{1-\pi_{eu,t}} \{\varrho_t[(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t + (1-\pi_{eu,t})\mathbb{E}_t M_{t+1} J_{t+1}] + \\
& (1-\varrho_t)[(b_t(1-\pi_{eu,t}) - \mathbb{E}_t[M_{t+1}\Delta_{t+1}](1-\pi_{eu,t}-\pi_{ue,t})]\} \\
& \text{use } \; (1-\varrho_t)\mathbb{E}_t M_{t+1}\Delta_{t+1} = \varrho_t \mathbb{E}_t M_{t+1} J_{t+1} \\
=& (1-\pi_{eu,t})^{-1}\{\varrho_t[(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t + (1-\pi_{eu,t})\mathbb{E}_t M_{t+1} J_{t+1}] \\
& + (1-\varrho_t)(b_t(1-\pi_{eu,t}) - (1-\pi_{eu,t}-\pi_{ue,t})\mathbb{E}_t M_{t+1} J_{t+1}\varrho_t\}, \\
& \text{define } \tilde{z}_t \equiv (1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t, \\
W_t^N =& (1-\pi_{eu,t})^{-1}\{\varrho_t[\tilde{z}_t + \pi_{ue,t}\mathbb{E}_t M_{t+1} J_{t+1}] + (1-\varrho_t)(b_t(1-\pi_{eu,t})\} \\
=& (1-\pi_{eu,t})^{-1}\{\varrho_t[\tilde{z}_t + \pi_{ue,t}(\frac{\kappa_{1,t}}{q_t} + \kappa_{2,t} - \Theta_t)] + (1-\varrho_t)(b_t(1-\pi_{eu,t})\} \\
& \text{use } \; \pi_{ue,t} = q_t\theta_t \\
=& (1-\pi_{eu,t})^{-1}\{\varrho_t[\tilde{z}_t + (\kappa_{1,t}\theta_t + \kappa_{2,t}q_t\theta_t - \Theta_t q_t\theta_t)] + (1-\varrho_t)(b_t(1-\pi_{eu,t})\}, \\
& \text{by complementary slackness,} \\
=& (1-\pi_{eu,t})^{-1}\{\varrho_t[(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t + (\kappa_{1,t}\theta_t + \kappa_{2,t}q_t\theta_t)] \\
& + (1-\varrho_t)(b_t(1-\pi_{eu,t})\}.
\end{aligned}$$

which deviates from the textbook Nash wage, $\varrho_t(e^{z_t}A_t + \kappa_{1,t}\theta_t) + (1-\varrho_t)b_t$, by endogenous separations, training costs and a different timing of separations. Concerning complementary slackness, for $v_t > 0$, the multiplier Θ_t is zero. For $v_t \leq 0$, this multiplier becomes non-zero, v_t is set to 0 and the wage reduces to $W_t^N = (1-\pi_{eu,t})^{-1}\{\varrho_t[(1-\pi_{eu,t})(e^{z_t}A_t - \mu_{\epsilon,t}) + \Psi_t] + (1-\varrho_t)(b_t(1-\pi_{eu,t})$.

Crucially, non-standard assumptions about the wage setting or a time-varying bargaining power can lead to $(1-\varrho_t)\mathbb{E}_t M_{t+1}\Delta_{t+1} \neq \varrho_t \mathbb{E}_t M_{t+1} J_{t+1}$. Then, the bargained wage is not equal to the standard Nash wage, $W_t \neq W_t^N$, i.e. we have to use (2.15) or households make considerable forecasting errors every period. Exemplary, Wachter and Kilic (2018) assume that bargaining only determines 5% of the wage; the remaining 95% are insulated from the labour market tightness. When bargaining over the 5%, the parties must take this friction into account, or else they make a large forecasting error. Wachter and Kilic (2018) do not seem to acknowledge this. Equation (2.15) has an implication for the model solution: we cannot substitute the expected surplus functions for one another. Hence, we have to "carry" and update both surplus functions in the model solution.

2.B.4 Acceptable wages

Non-flexible wages can result in non-acceptable wages for firms and households. Following Hall (2005b) wages have to lie between the firm's (upper bound) and household's (lower bound) reservation wages. Households do not accept wages that turn their surplus of employment negative. The household's reservation wage is determined by $\Delta_t = 0$,

$$W_t(1-\pi_{eu,t}+b_t\pi_{eu,t}+\mathbb{E}_t M_{t+1}\Delta_{t+1}(1-\pi_{ue,t})) = b_t + \mathbb{E}_t M_{t+1}\Delta_{t+1}\pi_{ue,t}$$
$$\underline{W_t} = \frac{b_t(1-\pi_{eu,t}) - \mathbb{E}_t M_{t+1}\Delta_{t+1}(1-\pi_{eu,t}-\pi_{ue,t})}{1-\pi_{eu,t}}$$

Firms do not accept wages that turn their surplus of a job negative. The firm's reservation wage is determined by $J_t = 0$,

$$(1-\pi_{eu,t})(e^{Z_t}A_t - \mu_{\epsilon,t}) + \Psi - (1-\pi_{eu,t})W_t + (1-\pi_{eu,t})\mathbb{E}_t M_{t+1}J_{t+1} = 0$$
$$\overline{W_t} = \frac{(1-\pi_{eu,t})(e^{Z_t}A_t - \mu_{\epsilon,t}) + \Psi + (1-\pi_{eu,t})\mathbb{E}_t M_{t+1}J_{t+1}}{1-\pi_{eu,t}}$$

When the bargaining power is a function of the persistent component of growth the wage bounds become occasionally binding in expectations and in the policy function of wages. In both cases I restrict wages to fall in between $[\underline{W_t}, \overline{W_t}]$.

2.C Time series matching

To match data series without a non-linear filter, I exploit the orthogonality of two shocks in my models: Employment is a predetermined variable with policy function $\widehat{l_{t+1}}(l_t, z_t, x_t)$. The *iid* innovation to productivity growth $\epsilon_{a,t}$ only influences the policy function through expectations. The realization of $\epsilon_{a,t}$ does not affect employment. The *iid* innovation (together with z_t and x_t) affect those variables which grow with trend, e.g. consumption and output scale with productivity growth, $c_tA_t = C_t$ and $y_tA_t = Y_t$. Figure 2.19 illustrates the three shocks' effects on TFP. $\epsilon_{a,t}$ pushes the economy to a new balanced growth path within an instant, but the realization of $\epsilon_{a,t}$ carries no information for the following $\epsilon_{a,t+j}$. In contrast, the LRR shock

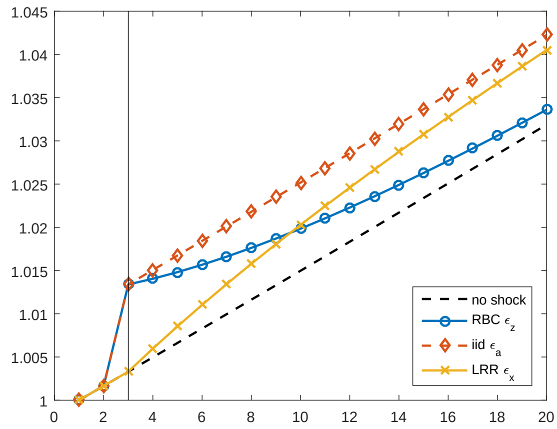


Figure 2.19: Three shocks and their effect on aggregate productivity $e^{z_t} A_t$

and the RBC shock carry information for the following $\{x_{t+j}, z_{t+j}\}$ because they are autoregressive processes. I exploit this property when I match the employment and output series: x_t and z_t affect the employment rate (and the output growth rate) while $\epsilon_{a,t}$ does not.

The figures of matched series are derived by the following three steps:

i) starting from employment in April 1929, I interpolate the labour policy function to solve

$$\hat{z}_t = \min_{z_t} \|l_{t+1}^{data} - \widehat{l_{t+1}}(l_t^{data}, z_t)\|.$$

Intuitively, given employment today I ask which level of TFP is necessary today to make the empirical employment tomorrow the optimal choice. This way, I can estimate a time series for the cyclical component z_t. For the LRR component proceed equivalently and find $\hat{x}_t$.

ii) $l_t, l_{t+1}, \hat{x}_t$ and $\hat{z}_t$ imply a growth rate for output, $\Delta \log Y_t^1$,

$$\begin{aligned}
\Delta \log Y_t^1 &= g_a + \widehat{x_{t-1}} + \Delta \log \hat{y}_t \\
\Delta \log \hat{y}_t &= \log \hat{y}(l_t, \hat{z}_t, \hat{x}_t) - \log \hat{y}(l_{t-1}, \hat{z}_{t-1}, \hat{x}_{t-1}) \\
g_a + \widehat{x_{t-1}} &= \log A_t - \log A_{t-1} - \sigma_a \epsilon_{a,t},
\end{aligned}$$

where $\hat{y}(.)$ denotes the policy function of detrended output. Now, I compute the difference between this implied growth rate and the empirical monthly growth rate (linearly interpolation from quarterly data) and attribute the difference to the *iid* growth innovation $\epsilon_{a,t}$.

$$\frac{\Delta Y_t^{data} - \Delta \log Y_t^1}{\sigma_a} = \epsilon_{a,t}.$$

One can think of $\epsilon_{a,t}$ as measurement error.

iii) Set a starting point of unity for long-run productivity $A_0 = 1$ and use $\epsilon_{a,t}$, $\hat{x}_t$ and the constant growth rate g_a to compute a model-generated time series for A_t. For any variable, first compute the detrended variable, e.g. $c(l_t, \hat{x}_t, \hat{z}_t)$. If this variable scales with A_t, compute the scaled $C_t = A_t c(l_t, \hat{x}_t, \hat{z}_t)$.

2.D Related models

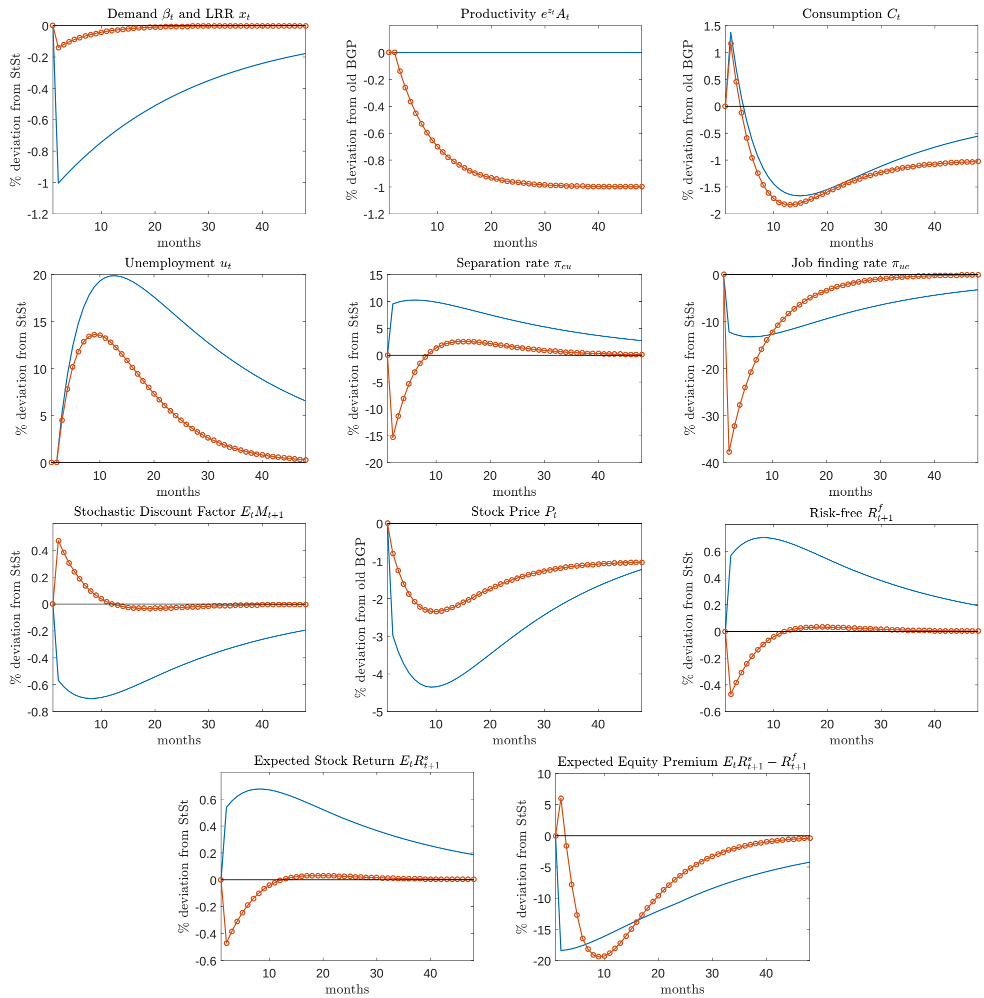


Figure 2.20: IRFs in additional models. The blue, solid line is the response to to a β_t shock. The dotted, red line is the response to an LRR shock, ϵ_x, in the time-varying κ_t model. The LRR shock reduces long-term TFP by 1%; the demand shock reduces β_t by 1%. After the initial shock β_t and x_t follow their laws of motion without further innovations. Variables that grow with trend, e.g. Y, C, P, are plotted in deviations from a balanced growth path, which did not experience the shock ("old BGP"). Variables that do not grow with trend are plotted in percentage deviations from steady state (StSt).

2.D.1 LRR and time-varying vacancy-posting costs

This version of the LRR model (Section 2.4) adds one alternation: Vacancy-posting costs $\kappa_{1,t}$ are now state-dependent, i.e. $\kappa_{1,t} = A_t \widetilde{\kappa_{1,t}}$ with

$$\widetilde{\kappa_{1,t}} = \bar{\rho} exp[-x_t \alpha_{\kappa,x}].$$

Parameter	Value	Parameter	Value
γ	10	ψ	1.5
β	0.9991	g_a	0.0015
σ_a	0.0022	σ_x	0.2855
ρ_x	0.8604	$\bar{\rho}$	0.0463
α_x	278.3001	ι	0.8560
b	0.9197	μ_ϵ	0.0476
ψ_ϵ	0.3091	$\alpha_{\kappa,x}$	97.2000
τ_{eu}	-1.9115	κ_2	1.4409
κ_1	0.9695		

Table 2.9: Parametrization of LRR model with state-dependent vacancy-posting costs

Wage rigidity (2.16) is still active and necessary to solve the Shimer puzzle.

Table 2.9 shows the model parametrization. Figure 2.20 depicts the impulse responses to an $\epsilon_{x,t}$ shock. When adverse news is revealed, unemployment rises, the job-finding rate falls and output falls as expected. As outlined above, the worker's surplus can rise because strong wage rigidity causes high wages in bad states. Countercyclical vacancy-posting costs work in the same direction: in a bust, vacancy-posting costs rise and fewer vacancies are posted. Hence, labour market tightness is low which raises the worker's surplus and dominates the effect of lower productivity on separations. In essence, it is very difficult to find a model whith (i) productivity driven by news shocks, (ii) endogenous separations, (iii) the correct cyclicality of labour transition rates, and (iv) a high volatility of unemployment.

2.D.2 Time-varying discount factor

In a stylized model without productivity shocks, Hall (2017) assumes that the SDF follows a Markov chain. To amplify the volatile SDF's effect on investment, Hall assumes rigid wages. In this related variant of my model, I assume that productivity is *iid*

$$\log A_{t+1} - \log A_t = g_a + \sigma_a \epsilon_{a,t+1}$$

and shocks to the time-discount factor drive the economy,

$$\frac{\beta_{t+1} - \beta}{\beta} = \zeta_t,$$

with ζ_t being percentage deviations of β_t from its steady state value β. When ζ_t rises, β_t goes up, households become more patient, increasing savings and decreasing consumption. I assume that ζ_t follows an AR(1)-process: $\zeta_t = \rho_\zeta \zeta_{t-1} + \sigma_\zeta \epsilon_{\zeta,t}$ with $\epsilon_{\zeta,t}$ *iid* standard normal. The time-varying β_t is a stand-in for the household's desire to save and invest, which changes in response to uncertainty, news, sentiment, or other forces. In a demand-driven New-Keynesian model, β_t would be interpreted as a demand shock. In my supply-driven framework, it is a patience shock to investment good supply.

I experimented with wage rigidity in the form of $\rho_t = \bar{\rho} e^{-\zeta_t \alpha_\zeta}$, but the impatience shock and a small surplus calibration are strong enough to solve the Shimer puzzle. Thus, I choose $\alpha_\zeta = 0$.

Table 2.10 shows the model's preliminary parametrization and Figure 2.20 displays impulse responses to a β-shock.

Parameter	Value	Parameter	Value
γ	10	ψ	1.5
β	0.9991	g_a	0.0015
σ_a	0.0050	σ_ζ	0.0040
ρ_ζ	0.9631	$\bar{\rho}$	0.1559
α_ζ	0	ι	0.8560
b	0.8049	μ_ϵ	0.0690
ψ_ϵ	0.9000	τ_{eu}	-0.0812
κ_1	0.9695	κ_2	1.4409

Table 2.10: Parametrization of time-varying discount factor model.

Contrary to a New-Keynesian model, a contractionary demand shock ($\beta_t \uparrow$) causes an immediate dip of consumption followed by above average employment and consumption. In this flexible price model, less patient households cause a recession via lower investment. Consider this ($\beta_t \downarrow$) case: the immediate reaction to the shock is a surge of consumption as households reduce savings. The expected value of the SDF inherits the autoregressive β_t's response. The discounted firm surplus falls and so does the job-finding rate. Lower investment and a higher separation rate lead to a surge of unemployment. When the shock is revealed, the market prices in lower future employment and a higher vacancy-filling rate; stock prices fall. Counter-factually, the equity premium and the risk-free rate are both counter-cyclical.

This recession is more similar to the ones outlined in Hall (2017) and Campbell and Cochrane (1999): consumption is low, marginal utility is large and the discount factor is low (discount rates are high) which depresses the value an employer attributes to a new match. However, the counter-cyclical equity premium is a deficiency of this particular model. Habits may help in this dimension.

Figure 2.21 plots matched series. Contrary to a New-Keynesian model, β_t must fall at the onset of a recession. The risk-free rate, more or less the reciprocal of β_t, is counter-factually volatile. The model has an average risk premium of 2% in simulations, but it does not track equity returns over the time series. Finally, the model features return predictability, although the volatility of the PE10 is not matched.

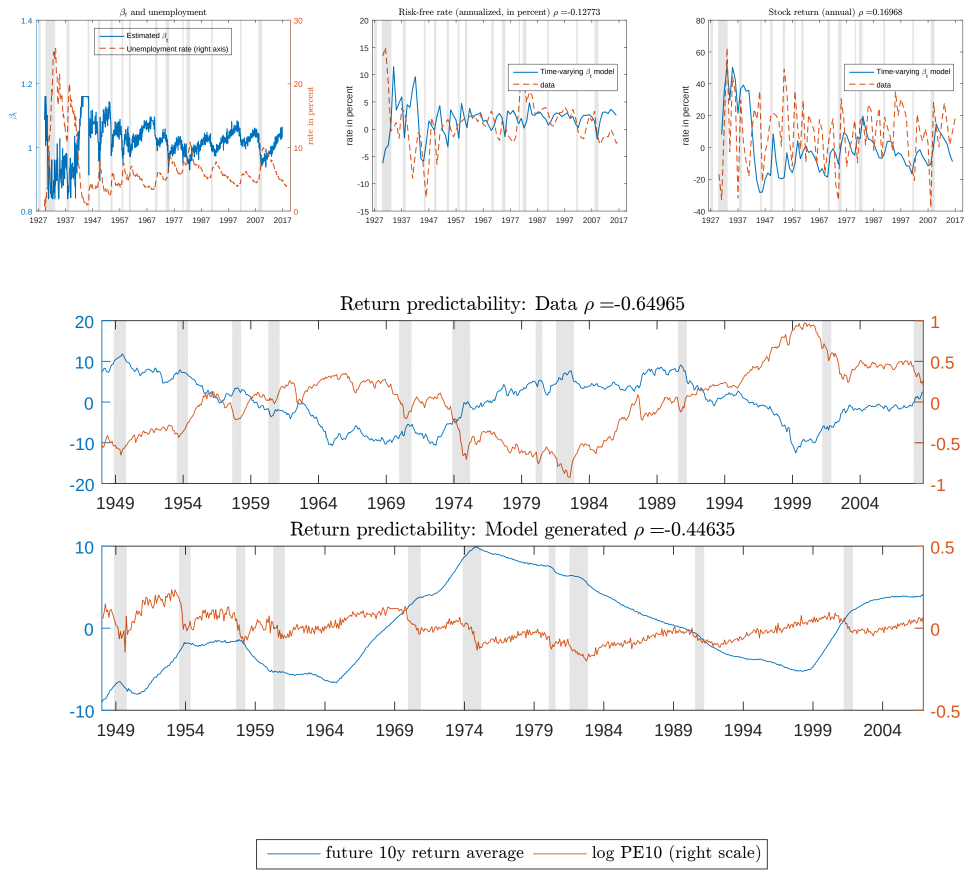


Figure 2.21: Matched series of the time-varying discount factor model.

2.E Additional figures

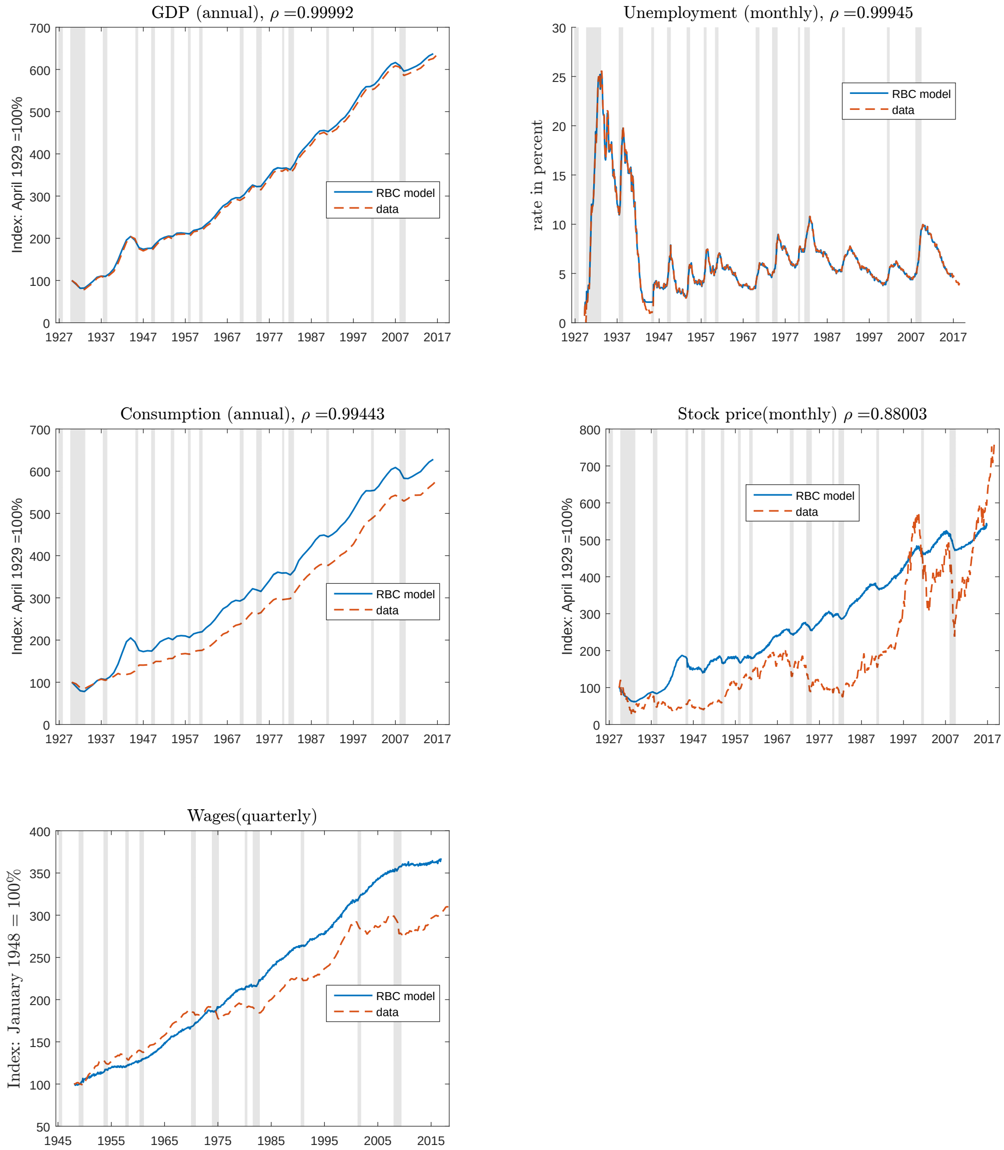


Figure 2.22: Matched time series of the RBC model: Forecasts with trend growth.

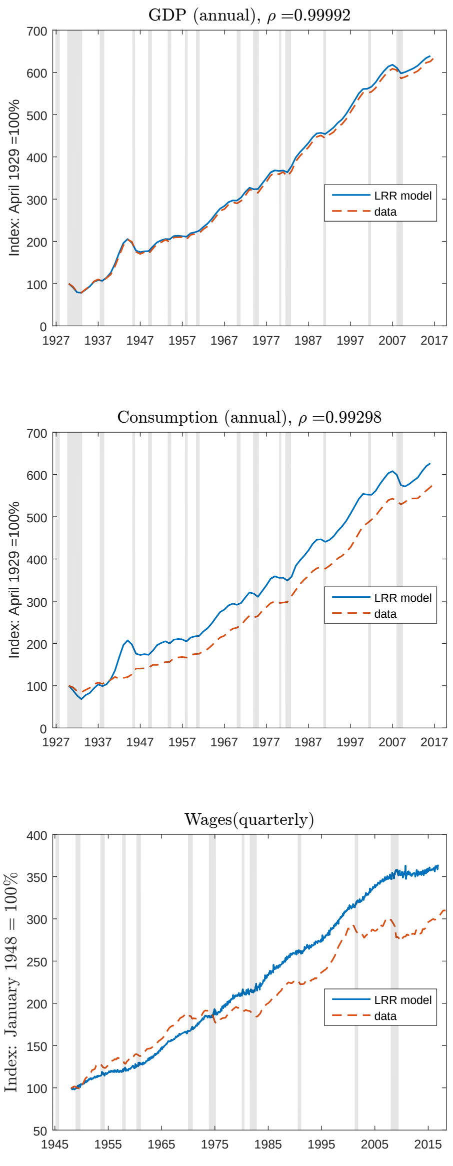


Figure 2.23: Matched time series of the LRR model: Forecasts with trend growth.

2.F Numerical solution and estimation

2.F.1 Global solution

Pseudo code I follow Petrosky-Nadeau et al. (2018)'s (henceforth PNZK) projection algorithm to solve the productivity-adjusted model (Appendix 2.B.1). Contrary to the original algorithm, instead of using a non-linear function solver, I solve the model iteratively by slowly updating the value functions, $v(S)$, $E_t m' j(S)$, $E_t m' \tilde{\Delta}(S)$. Depending on the wage setting rule the surplus sharing rule might not hold. Hence, I keep track of the expected discounted value of the worker's surplus share separately. My code also solves the exogenous disaster risk model by Wachter and Kilic (2018): state λ denotes time-varying disaster risk, ξ the iid severity of a disaster and κ_1 is the state of vacancy-posting costs after an exogenous disaster.

Define grids of the endogenous state $l \in G_l$, and discretized exogenous processes, $x, z, \lambda, \kappa_1, \xi$. Denote the joint transition function of exogenous states $P(x', z', \lambda', \kappa_1', \xi', \epsilon_a' | x, z, \lambda, \kappa_1, \xi, \epsilon_a) = P(x', z', \lambda', \kappa_1', \xi', \epsilon_a' | x, z, \lambda, \kappa_1)$. The last equality follows from the independence of ξ' and a' of their past realizations. Denote the vector of state variables $S = \{l, x, z, \lambda, \kappa_1\}$ with subset $S_{exo} = \{x, z, \lambda, \kappa_1\}$. S' denotes the state vector in the next period. Note that expectations about future periods are functions of the current state vector S.

Guess value functions $\widehat{v_0}(S)$, $\widehat{\mathbb{E}_t m' j_0'}(S)$ and $\widehat{\mathbb{E}_t m' \tilde{\Delta}_0'}(S)$. For better legibility define

$$
\begin{aligned}
\widehat{\mathbb{E}_t m' j_0'}(S) &\equiv \mathcal{J}_0(S) \\
\widehat{\mathbb{E}_t m' \tilde{\Delta}_0'}(S) &\equiv \mathcal{W}_0(S).
\end{aligned}
$$

In iteration $i = 1, 2, \ldots,$

1. Use $\mathcal{J}_{i-1}(S)$ in the free-entry condition to find $\widetilde{q(S)}$. Force $q(S)$ to fall within its domain, i.e. $q(S) \in [0,1]$. If the non-negativity constraint of vacancies binds ($\widetilde{q(S)} < 0$), set $v(S) = 0$ and $\theta(S) = 0$. Calculate vacancy posting $v(S)$ and $\theta(S)$.

2. Calculate the expected discounted match surplus $\mathbb{E}_t m' \tilde{\Sigma}(S) = \mathcal{J}_{i-1}(S) + \mathcal{W}_{i-1}(S)$ and the endogenous separation rate $\pi_{eu}(S)$.

3. Calculate consumption via the aggregate resource constraint

$$c(S) = (1-\pi_{eu}(S))l(e^z - \mu_\epsilon) + l\widetilde{\Psi(S)} - v(S)\kappa_1 - v(S)q(S)\kappa_2.$$

   Note that l and κ_1 are state variables.

4. Derive next period's employment via the law of motion:

$$l'(S) = (1-\pi_{eu}(S))l + \pi_{ue}(S)(1-l).$$

5. For all gridpoints, compute the expected value

$$\begin{aligned}
&\mathcal{D}(S) = \mathbb{E}_t\left[\left(\frac{A'}{A}\right)^{1-\gamma} \widehat{v_{i-1}}(S')^{1-\gamma}\right] \\
&= \int \left(\frac{A'}{A}\right)^{(1-\gamma)} \widehat{v_{i-1}}(S')^{1-\gamma} P(S'_{exo}, \xi', a'|S_{exo})(dx' \times dz' \times d\lambda' \times d\xi' \times da') \\
&= e^{(1-\gamma)g_a + (1-\gamma)^2 \frac{1}{2}\sigma_a^2} \\
&\quad \times \int e^{(1-\gamma)(x_t + \delta'\xi')} \widehat{v_{i-1}}(S')^{1-\gamma} P(S'_{exo}, \xi'|S_{exo})(dx' \times dz' \times d\lambda' \times d\xi')
\end{aligned}$$

   where I interpolate to approximate $\widehat{v_{i-1}}(S')$. I use monomials/GH-quadrature borrowed from Judd et al. (2014) to approximate the integral.

6. Use $\mathcal{D}(S)$ and $c(S)$ to derive a new guess for the value function

$$\widehat{v_i}(S) = \left\{(1-\beta)c(S)^{(1-1/\psi)} + \beta\mathcal{D}(S)^{1/\omega}\right\}^{\frac{1}{1-1/\psi}}$$

7. Interpolate $\mathcal{J}_{i-1}(S')$ and $\mathcal{W}_{i-1}(S')$ over next period's productivity and employment l' collected in S'.
   Force $q(S')$ into its domain and derive $v(S'), \theta(S'), \pi_{eu}(S'), w(S'), c(S')$.[17]

8. Compute the stochastic discount factor, m'.

[17] To follow Wachter and Kilic (2018) one needs a reference wage under tightness insulation. To obtain $w(S')$ under tightness insulation I interpolate $\theta(S)$ over employment to find the labour market tightness at the fix point $l' = l, x = 0, z = 0, \lambda = E[\lambda], \xi = 0$. Wherever a function is evaluated at the future state vector S', one has to carefully account for disaster states S' with a high $\kappa_1 = \overline{\kappa}$.

9. Derive a new guess using the investment first-order condition

$$\mathcal{J}_i(S) = \mathbb{E}_t m' \Big\{ (1-\pi_{eu}(S'))(e^{z'} - \mu_\epsilon - w(S')) \\ + \widetilde{\Psi}(S') + (1-\pi_{eu}(S')) \Big[\frac{\kappa_1'}{q(S')} + \kappa_2 - \widetilde{\Omega(S')} \Big] \Big\},$$

where $\widetilde{\Omega(S')}$ is the Lagrange multiplier of the firm's investment problem, $\widetilde{\Omega(S')} = \kappa_1' + \kappa_2 - \widehat{\mathbb{E}_t m' j_{i-1}'}(S')$.

10. Derive a new guess via the definition of the worker's surplus share

$$\mathcal{W}_i(S) = \mathbb{E}_t m' \{ (w(S') - b)(1-\pi_{eu}(S')) + \mathcal{W}_{i-1}(S')(1 - \pi_{eu}(S') - \pi_{ue}(S')) \}$$

11. Check for convergence: If $\max[|\widehat{v_i}(S) - \widehat{v_{i-1}}(S)|, |\mathcal{J}_i(S) - \mathcal{J}_{i-1}(S)|, |\mathcal{W}_i(S) - \mathcal{W}_{i-1}(S)|] > \epsilon_{vf}$, slowly update the value functions

$$\begin{aligned} \widehat{v_i}(S) &= \lambda_v \widehat{v_i}(S) + (1-\lambda_v)\widehat{v_{i-1}}(S), \\ \mathcal{J}_i(S) &= \lambda_p \mathcal{J}_i(S) + (1-\lambda_p)\mathcal{J}_{i-1}(S) \\ \mathcal{W}_i(S) &= \lambda_p \mathcal{W}_i(S) + (1-\lambda_p)\mathcal{W}_{i-1}(S). \end{aligned}$$

with smoothing parameters λ_v and $\lambda_p \in (0,1]$. In my experience, the algorithm demands a slow updating of surplus functions but is quite robust to larger step sizes of the value function, $\lambda_p < \lambda_f$. Return to step 1.

Improvements This code deviates from PNZK by (i) using iteration instead of a non-linear function solver, (ii) Gauss-Hermite quadrature, and (iii) precomputation of expectations.

Iteration makes the algorithm more robust to bad first guesses and kinks in the value functions. PNZK solve the model in three steps: first, they solve a linearized version of the model; second, they set the lower bound of employment to 60% and solve globally; last, they extend the employment grid to 5% for the final global solution. My algorithm is much more robust to bad first guesses: it solves the model robustly with all-ones matrices as first guesses. The algorithm is quick when the updating weights are set well. I set $\lambda_p = 0.005 < \lambda_f = 0.5$, i.e. the utility value function is updated quickly while the discounted surplus functions are updated slowly. The non-negativity constraint of vacancies creates a kink in the policy functions jeopardizing stability. The utility function inherits the kink but the discontinuity is less pronounced; so, speedier updating can save time.

Gauss-Hermite quadrature (Judd et al., 2014) allows to solve the model with a number of different shocks but keeps the curse of multidimensionality in check.

Precomputation and vectorization of expectation nodes speeds up the solution significantly and comes at a negligible memory cost. Denote Gauss-Hermite evaluation nodes by η_j and weights by ω_j. Nodes are vectors with two entries, $\eta_j = (\eta_{x,j}, \eta_{z,j})'$, while ω_j is the probability of each node. For clarity assume away time-varying disaster risk. As an example, in the computation of $\mathcal{D}(S)$, I approximate the integral

$$\int \widehat{v_{i-1}}(S')^{1-\gamma} P(S'_{exo}, \xi'|S_{exo})(dx' \times dz'), \tag{2.24}$$

with Gauss-Hermite quadrature,

$$\sum_i \widehat{v_{i-1}}(l', \underbrace{\rho_x x + \sigma_a \sigma_x \eta_{x,j}}_{x'_j}, \underbrace{\rho_z z + \sigma_z \eta_{z,j}}_{z'_j})^{1-\gamma} \omega_j. \tag{2.25}$$

The unknowns in this expression are $\widehat{v_{i-1}}$ and l', which depends on the value function $\widehat{E_t m' j_i}$. For every x and z one knows the respective x' and z' nodes *a priori*. Suppose there are $n_l \times n_x \times n_z = \mathcal{N}$ grid points and N_j Gauss-Hermite nodes. Before the projection algorithm starts, I store three matrices with dimension $(\mathcal{N} \times N_j)$: An x'_j matrix, a z'_j matrix and an $\left(\frac{A'}{A}\right)^{1-\gamma}$ matrix.

Whenever I interpolate, I do not need to compute the nodes again. Instead, I pass a vectorized l' together with the matrices to the Matlab function `griddedInterpolant.m`. The $\left(\frac{A'}{A}\right)^{1-\gamma}$ matrix together with ω_j speeds up computation of $m'(S)$ and $\mathcal{D}(S)$. Precomputation and vectorization are the major time savers: with bad first guesses (all-ones matrices) my algorithm solves the LRR model in 3.5 minutes and the RBC model in 7.5 minutes on a regular desktop computer.

Log normal distribution In the solution, there is no need to evaluate Gauss-Hermite nodes for ϵ_a, because this shock is *iid* and only influences the solution in expectations. By property of the log normal distribution, $E_t e^{g_a+\sigma_a\epsilon_{a,t+1}} = e^{g_a+\frac{1}{2}\sigma_a^2}$. For iid $\epsilon_{a,t+1}$ its expected value is independent of realizations of other (stochastic) variables. Hence,

$$\begin{aligned}
v_t(S_t) = & \max_{s_{t+1},\widetilde{B}_{t+1}} \Big\{ (c_t - c_t^H)^{1-\frac{1}{\psi}} \\
& + \beta (e^{(1-\gamma)(g_a+(1-\gamma)\frac{1}{2}\sigma_a^2)} \mathbb{E}_t[(e^{(1-\gamma)[x_t+\delta_{t+1}\xi_{t+1}]})(v_{t+1}(S_{t+1}))^{1-\gamma}])^{\frac{1}{\omega}} \Big\}^{\frac{1}{1-\frac{1}{\psi}}} \\
= & \max_{s_{t+1},\widetilde{B}_{t+1}} \Big\{ (c_t - c_t^H)^{1-\frac{1}{\psi}} \\
& + \beta e^{(1-\psi)(g_a+(1-\gamma)\frac{1}{2}\sigma_a^2)} (\mathbb{E}_t[e^{(1-\gamma)[x_t+\delta_{t+1}\xi_{t+1}]} v_{t+1}(S_{t+1})^{1-\gamma}])^{\frac{1}{\omega}} \Big\}^{\frac{1}{1-\frac{1}{\psi}}}.
\end{aligned}$$

The stochastic discount factor can be expressed as

$$\begin{aligned}
m_{t+1} = & M_t \frac{A_{t+1}}{A_t} \\
= & \beta \left(\frac{c_{t+1} - c_{t+1}^H}{c_t - c_t^H} \right)^{-\frac{1}{\psi}} v_{t+1}^{\frac{1}{\psi}-\gamma} \times \\
& e^{(1-\frac{1}{\psi})(g_a+x_t)+(1-\gamma)\frac{1}{2}\sigma_a^2(\gamma-\frac{1}{\psi})} e^{(1-\gamma)(\sigma_a\epsilon_{a,t+1}+\delta_{t+1}\xi_{t+1})} \times \\
& \left\{ E_t \left[e^{\delta_{t+1}\xi_{t+1}(1-\gamma)} v_{t+1}^{1-\gamma} \right] \right\}^{\frac{\gamma-\frac{1}{\psi}}{1-\gamma}}.
\end{aligned}$$

2.F.2 Simulated method of moments

Let d denote data and ζ denote the N parameters to estimate. Generalized Method of Moments (Hansen, 1982) demands that the model generated moments $m(d|\zeta)$ used to identify parameters ζ are known analytically which is not the case here. I resort to the Simulated Method of Moments (SMM or Moment of Simulated Methods(MSM)) proposed by McFadden (1989), Lee and Ingram (1991) and Duffie and Singleton (1993). SMM substitutes the analytical solution of model-generated moments by the mean across S simulations $m(d|\zeta) = \frac{1}{S}\sum_s m(d_s|\zeta)$. SMM estimates the parameters $\hat{\zeta}$ which minimize the distance between simulation mean $m(d|\zeta)$ and empirical moments $m(d)$,

$$\hat{\zeta} = \arg\min_{\zeta} ||m(d|\zeta) - m(d)||.$$

To enforce scale invariance, I minimize the percentage deviations $e(d,d|\zeta) = \frac{m(d|\zeta)-m(d)}{m(d)}$, which gives the SMM estimator

$$\hat{\zeta} = \arg\min_{\zeta} e(d,d|\zeta)^T W e(d,d|\zeta).$$

The most common approach to choose the weighting matrix W is as follows: i) start with the identity matrix, run the estimation and find the optimum $\hat{\zeta}_1$. ii) Solve the model once with parameters $\hat{\zeta}_1$ and run S simulations. Calculate the errors $e(d,d|\hat{\zeta}_1)$ and estimate the variance-covariance matrix $W_2^{-1} = \frac{1}{N} e(d,d|\hat{\zeta}_1) e(d,d|\hat{\zeta}_1)^T$. iii) Run the estimation again using W_2 as the weighting matrix

$$\hat{\zeta}_2 = \arg\min_{\zeta} e(d,d|\zeta)^T W_2 e(d,d|\zeta).$$

I use Matlab's `fmincon.m` function as the minimization routine, which among other minimizers tend to get stuck at local minima. To counteract this I use Matlab's `MultiStart.m` function, which passes multiple starting points to parallel runs of `fmincon.m` and conducts a global search for a minimum. The global search is computationally costly and only used in the first step of my SMM estimation. `MultiStart.m` is handy when parameter values are completely unknown. With a sensible prior, I find that a simple grid search often outperforms `MultiStart.m`.

Tables 2.11 and 2.12 show estimation results of the RBC and LRR models. The first column are the targets $m(d)$, the second column are mean moments across simulations after the second minimization step $m(d|\zeta_2)$. Define the unit-free estimation error of simulation s as

$$e(d,d_s|\zeta_2) \equiv \frac{m(d_s|\zeta_2) - m(d)}{m(d)}.$$

Column three shows the average estimation errors across simulations, $\frac{1}{S}\sum_s e(d,d_s|\zeta_2)$. If the weighting matrix is the identity matrix, the squared sum of this column equals the SMM criterion. Column four shows the standard deviations of the unit-free errors,

$$[\frac{1}{S}\sum_{s} e(d,d_s|\zeta_2)^2 - (\frac{1}{S}\sum_{s} e(d,d_s|\zeta_2))^2]^{1/2}.$$

The tables shed light on the fit of the model to the estimation targets and the way the two-stage SMM treats different targets. SMM aims to minimize the distance between target and mean moments measured as the mean log error. Some moments, exemplary the autocorrelation of output growth, are difficult to match because of simulation noise. For the same ζ_2, simulated autocorrelations differ strongly, resulting in a large standard deviation of the log error. For comparison, for the same ζ_2, the simulated standard deviations of unemployment growth are more densely distributed around their mean, resulting in a small standard deviation of the log error. In the two-stage SMM, the weighting matrix puts stronger weight on moments that can be matched more precisely.

	Moments		Estimation errors	
	Target	Mean simulated	Mean log error	Std log error
$\sigma(\Delta Y)$	0.0232	0.0262	0.1311	0.1223
$\rho(\Delta Y)$	0.1900	0.1997	0.0512	0.5776
$\sigma(\Delta u)$	0.0768	0.0751	-0.0228	0.0723
$\rho(\Delta^q Y, \Delta^q u)$	-0.5780	-0.5385	-0.0684	0.0977
$\sigma(\Delta^a W)$	0.0270	0.0216	-0.2002	0.0787
$\sigma(\Delta^q \pi_{eu})$	0.0667	0.0703	0.0534	0.0669
$\rho(\Delta^q Y, \Delta^q \pi_{eu})$	-0.4400	-0.4532	0.0299	0.1179
Criterion SMM			0.06881	

Table 2.11: SMM estimation of the RBC model.

	Moments		Estimation errors	
	Target	Mean simulated	Mean log error	Std log error
$\sigma(\Delta Y)$	0.0232	0.0235	0.0129	0.0985
$\rho(\Delta Y)$	0.1900	0.2118	0.1148	0.5840
$\sigma(\Delta u)$	0.0768	0.0736	-0.0417	0.0793
$\rho(\Delta^q Y, \Delta^q u)$	-0.5780	-0.6100	0.0554	0.0768
$\sigma(\Delta^a W)$	0.0270	0.0271	0.0023	0.0949
$\sigma(\Delta^q \pi_{eu})$	0.0667	0.0645	-0.0340	0.0917
$\rho(\Delta^q Y, \Delta^q \pi_{eu})$	-0.4400	-0.3785	-0.1398	0.1415
Criterion SMM			0.03886	

Table 2.12: SMM estimation of the LRR model.

CHAPTER 3

The Equity Premium and Unemployment: A Case for Habits

Chapter Abstract

This paper presents a Diamond-Mortenssen-Pissarides model with slow-moving habits and capital adjustment costs. The framework solves both the equity premium and the Shimer puzzle and produces a high correlation of employment and stock prices. Habits amplify the conditional variance of the stochastic discount factor; capital and employment adjustment costs raise the volatility of equity prices. Together, these mechanisms robustly generate a large equity premium. Unlike other attempts to solve the two puzzles that rely on excessive consumption volatility or demand a global solution, this model is parametrized to post-war U.S. data and solved with perturbation.

3.1 Introduction

It is well known that the canonical real business cycle model struggles to explain the equity premium, we observe empirically. Since Shimer (2005), if not before, it is known that the canonical Diamond-Mortensen-Pissarides (DMP) framework struggles to explain the large volatility of unemployment. My prior attempts to build a framework that solves both puzzles were unsatisfactory: an endogenous disaster model, that solves the equity premium puzzle, generates disasters far too frequently. Long-run risk in line of Bansal and Yaron (2004) has proven to be too weak to raise the volatility of the stochastic discount factor sufficiently.

This paper augments the DMP framework with Campbell and Cochrane (1999)-habit formation and capital adjustment costs. A Hagedorn and Manovskii (2008) parametrization of the worker's outside options solves the Shimer puzzle. When matched to the empirical unemployment series, the model is effective in matching equity prices and returns and reproduces the striking correlation of equity prices and unemployment.

This paper builds on Kehoe et al. (2019) and Chapter 2, testing whether popular extensions of the canonical DMP model suffice to solve both the equity premium and the Shimer puzzle. Independently, both find that the long-run risk model in line of Bansal and Yaron (2004) and Croce (2014) does not generate enough volatility in unemployment or equity returns. The endogenous disaster model by Petrosky-Nadeau et al. (2018) solves the equity premium puzzle only under the assumption that consumption volatility is about twice as high as its post-war average and yields disasters far too frequently.

General equilibrium models that explain asset market data have to overcome a number of issues known since Rouwenhorst (1995) at the latest. Firstly, risk-averse households smooth consumption, reducing the volatility of marginal utility. Intuitively, households in an economy that allows them to smooth consumption with capital and employment need little motivation, in form of an equity premium, to hold risky equity. Importantly, households must perceive consumption as risky, but the model must not overestimate the consumption volatility, i.e. the conditional volatility of consumption is large, but the observed volatility of consumption is low. Secondly, to yield a risk premium, the equity return needs to be sufficiently risky. Hence, the stock price, which reflects the value of capital and employment, needs to be volatile. In this paper, habits address the former and capital adjustment costs the latter. Slow-moving habits keep the volatility of the risk-free rate low, but generate a time-varying risk aversion: investors, who consume close to their habit, become more risk-averse even for a low parameter of risk aversion. The time-varying risk aversion thus raises the volatility of marginal utility in response to small perturbations of consumption. In this environment, investors demand risk premia for risky assets. Now, the equity return needs to be a sufficiently risky asset: in the canonical business cycle model with capital adjustment costs, prices are driven by the volatility of capital adjustment costs, or Tobin's marginal q. Merz and Yashiv (2007) show that, when employment is frictional, its adjustment costs enter the stock price just like capital's adjustment costs enter as Tobin's marginal q. In Chapter 2, models without capital showed an insufficient volatility of equity prices. Building on this insight, I let capital adjustment costs and volatile hiring costs jointly generate a volatility of equity returns in line with empirical estimates.

This paper is closely related to Kehoe et al. (2019) and Bai and Zhang (2021). The former augment the DMP framework with Campbell-Cochrane habits and on-the-job human capital. Building on Hall (2017), they show that the volatility of the stochastic discount factor, generated by habits, can raise the unemployment volatility without relying on a small surplus calibration (Ljungqvist and Sargent, 2017). Still, their model does not match the empirical fact that the volatility of unemployment is about eight times higher than that of consumption. In contrast, this model uses the more standard physical capital and meticulously matches this ratio. My model is solved using perturbation, which I view as a major advantage for model builders in the future. Bai and Zhang (2021) augment the DMP model with capital adjustment costs and recursive preferences (Epstein and Zin, 1989). Building on Petrosky-Nadeau et al. (2018), their target moments stem from a comprehensive international, historical panel; consumption and unemployment volatility targets far exceed the post-war U.S. moments. Hence, the model generates disasters endogenously, which solves the equity premium puzzle. Dupraz et al. (2019), Kehoe et al. (2019), and Chapter 2 independently show that the closely related Petrosky-Nadeau et al. (2018)

model generates disasters far too frequently.

This paper is organized as follows. The next section presents the model. Section 3.3 parametrizes the model, presents baseline results and works out the major mechanisms that solve the puzzles. Section 3.4 matches the model to the unemployment time series.

3.2 Model

The model extends the canonical Diamond-Mortensen-Pissarides framework by external habits and capital adjustment costs. Habits raise risk aversion when consumption approaches habit, which motivates a return premium for risky assets. Search frictions and convex capital adjustment costs increase the volatility of Tobin's q, making equity a risky investment.

Preferences The representative consumer has preferences

$$V_t = E_t \sum_{j=0}^{\infty} \frac{(C_{t+j} - X_{t+j})^{1-\gamma} - 1}{1-\gamma}, \tag{3.1}$$

where X_t denotes an external "keeping-up-with-the-Joneses" habit. Following Campbell and Cochrane (1999), denote the surplus consumption ratio

$$S_t \equiv \frac{C_t - X_t}{C_t}. \tag{3.2}$$

Habits increase risk aversion vis-à-vis standard CRRA utility when surplus consumption is low, which raises the volatility of marginal utility for low values of γ: keeping habit X_t fixed, the coefficient of relative risk aversion is $\frac{-C_t V_{cc,t}}{V_{c,t}} = \frac{\gamma}{S_t}$, i.e. a low surplus consumption ratio raises relative risk aversion. For $S_t = 1$, utility defaults to standard power utility.

The stochastic discount factor (SDF) is the marginal rate of substitution,

$$M_{t+1} = \frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t} = \beta \left(\frac{S_{t+1}}{S_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}. \tag{3.3}$$

Assume that surplus consumption follows the AR(1)-process

$$\log S_{t+1} = (1-\rho_s)\log(\bar{S}) + \rho_s \log(S_t) + \lambda(S_t)\left[\Delta \log(C_{t+1}) - g_a\right], \tag{3.4}$$

where $\bar{S}$ denotes the surplus consumption ratio in steady state and ρ_s is the persistence of habits. $\lambda(S_t)$ determines the sensitivity of habits with respect to innovations of consumption growth.

Following Campbell and Cochrane (1999), assume the sensitivity function

$$\lambda(S_t) = \frac{1}{\bar{S}}\sqrt{1 - 2(\log S_t - \log \bar{S})} - 1 \tag{3.5}$$

The seminal paper by Campbell and Cochrane (1999) assumes a specific functional form for $\bar{S}$ which keeps the interest rate constant. Here, $\bar{S}$ is a free parameter. In the model estimation the empirical volatility of the risk-free rate serves as the target to estimate $\bar{S}$.

Production The homogeneous output good is produced with the normalized CES-production function

$$Y_t = A_t e^{z_t}\left[\alpha\left(\frac{K_t}{K_{0,t}}\right)^{\eta} + (1-\alpha)N_t^{\eta}\right]^{\frac{1}{\eta}}. \tag{3.6}$$

Parameter α is the capital share of production, η captures the elasticity of substitution between capital and labour via $\eta = \frac{\sigma-1}{\sigma}$. N_t denotes the number of productive matches in period t. Following Klump and de La Grandville (2000), parameter $K_{0,t}$ normalizes capital input, which grows with the rate of technological growth while the number of matches cannot exceed unity.

Productivity $A_t e^{z}{}_t$ grows at a constant rate g_a and has an autoregressive, cyclical component, z_t,

$$\begin{aligned} \Delta \log A_{t+1} &= \overline{\Delta \log A} = g_a \\ z_{t+1} &= (1-\rho_z)\bar{z} + \rho_z z_t + \sigma_z \epsilon_{z,t+1}, \end{aligned} \tag{3.7}$$

where $\epsilon_{z,t+1}$ is an *iid* standard normal variable.

Firms own the capital stock, which evolves according to the law of motion

$$K_{t+1} = (1-\delta)K_t + \Phi(K_t, I_t), \tag{3.8}$$

where $\Phi(K_t, I_t)$ denotes the capital installation function: an investment expenditure of I_t units raises the capital stock by $\Phi_t \le I_t$ units. The installation function is increasing and concave, $\Phi_I(.) > 0$ and $\Phi_{II}(.) < 0$, i.e. higher investment leads to a larger capital stock but at a decreasing rate. In other words we have convex adjustment costs, $\mathfrak{C}_t \equiv I_t - \Phi_t$. Following Jermann (1998), assume the functional form,

$$\Phi(K_t, I_t) = \left[a_1 + a_2 \frac{1}{1 - \frac{1}{\nu}} \left(\frac{I_t}{K_t} \right)^{1-\frac{1}{\nu}} \right] K_t. \tag{3.9}$$

$a_1 = \frac{e^{\overline{\Delta \log A}} - (1-\delta)}{1-\nu}$ and $a_2 = \left[e^{\overline{\Delta \log A}} - (1-\delta) \right]^{\frac{1}{\nu}}$ set capital adjustment costs to zero on the balanced growth path.

Parameter $\nu > 0$ controls the curvature of the installation function; the lower ν, the quicker capital adjustment costs rise when investment increases. For $\nu \to \infty$, capital installation costs vanish, $\lim_{\nu \to \infty} \Phi(K_t, I_t) = I_t$ since $\lim_{\nu \to \infty} a_1 = 0$ and $\lim_{\nu \to \infty} a_2 = 1$. Without adjustment costs, every unit of net investment raises the capital stock by an additional constant unit, $\lim_{\nu \to \infty} \Phi_I(.) = 1$ and $\lim_{\nu \to \infty} \Phi_{II}(.) = 0$.

Investment and matching At the beginning of a period there are l_t firm-worker matches in the economy. Let u_t denote the mass of unemployed workers and normalize the population to unity, $u_t + l_t = 1$. Separation occurs before production, i.e. $N_t = (1 - \pi_{eu}) l_t$. This timing is slightly different from the canonical DMP model but allows for a straightforward introduction of endogenous separations, exemplary see Jung and Kuhn (2014).

The timing is as follows: at the beginning of a period, l_t matches exist. The aggregate states of productivity are revealed and a constant π_{eu} fraction of matches is separated. The remaining firm-worker pairs bargain over wages. Next, matches produce output which is used for consumption and investment into capital and vacancies. Finally, vacancies and unemployed workers are matched in a frictional market. The sum of productive matches and new matches determines next period's employment,

$$\begin{aligned} l_{t+1} &= l_t(1 - \pi_{eu}) + \pi_{ue,t}(1 - l_t) \\ &= N_t + q_t v_t. \end{aligned} \tag{3.10}$$

Firms discount profits with the representative agent's stochastic discount factor. The representative firm takes the probability of a match q_t as given and chooses vacancies, employment, investment, and capital to maximize its cum-dividend stock price

$$\begin{aligned}
P_t^c &= \max_{\{v_{t+\tau}, l_{t+\tau+1}, I_{t+\tau}, K_{t+\tau+1}\}_{\tau=0}^{\infty}} E_t \sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau} && (3.11)\\
s.t.\ l_{t+1} &= l_t(1-\pi_{eu}) + q_t v_t \\
K_{t+1} &= (1-\delta)K_t + \Phi(K_t, I_t) \\
D_t &= Y_t - N_t W_t - \kappa_{1,t} v_t - q_t v_t \kappa_{2,t} - I_t \\
& \quad K_0,\ l_0 \text{ given.} && (3.12)
\end{aligned}$$

Firms pay posting costs $\kappa_{1,t}$ per vacancy, v_t. They pay additional training costs $\kappa_{2,t}$ if the vacancy is matched to an unemployed worker. Appendix 3.D examines the firm problem with a non-negativity constraint of vacancies approximated with a penalty function. Quantitatively, the penalty function has a negligible effect on policy functions and is thus omitted in the main text's model.

Combining the first-order conditions for investment and capital yields the investment Euler equation

$$\frac{\partial V_t}{\partial C_t} \frac{1}{a_2} \left(\frac{I_t}{K_t} \right)^{\frac{1}{\nu}} = \mathbb{E}_t \frac{\partial V_t}{\partial C_{t+1}} \left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \frac{1}{a_2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^{\frac{1}{\nu}} (1-\delta+a_1) + \frac{1}{\nu-1} \frac{I_{t+1}}{K_{t+1}} \right]$$

or

$$\underbrace{\frac{1}{a_2} \left(\frac{I_t}{K_t} \right)^{\frac{1}{\nu}}}_{\equiv Q_t^K} = \mathbb{E}_t M_{t+1} \left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \frac{1}{a_2} \left(\frac{I_{t+1}}{K_{t+1}} \right)^{\frac{1}{\nu}} (1-\delta+a_1) + \frac{1}{\nu-1} \frac{I_{t+1}}{K_{t+1}} \right]. \quad (3.13)$$

The Euler equation equates the utility of foregone consumption today to the expected utility of the marginal product of capital net depreciation tomorrow. Q_t^K denotes the shadow price of capital. Relative to the no-adjustment cost case ($\nu \to \infty$, $a_1 = 0$, $a_2 = 1$), convex costs have three effects on optimal investment: i) an additional unit of capital comes at a higher costs in terms of consumption goods when the adjustment of the capital stock deviates from steady state; ii) the shadow price of capital determines the value of the undepreciated capital stock carried to the next period; iii) higher investment in t saves capital adjustment costs in $t+1$. In terms of the asset pricing equation, $\mathbb{E}_t M_{t+1} R^K_{t+1} = 1$, we can express (3.13) as

$$R^K_{t+1} = \frac{\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \frac{1}{a_2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^{\frac{1}{\nu}}(1-\delta+a_1) + \frac{1}{\nu-1}\frac{I_{t+1}}{K_{t+1}}}{\frac{1}{a_2}\left(\frac{I_t}{K_t}\right)^{\frac{1}{\nu}}},$$

i.e. if capital were traded like equity, the rate R^K_{t+1} would be its return.

The first-order conditions for vacancies and employment yield the employment Euler equation,

$$\frac{\kappa_t}{q_t} = \underbrace{\mathbb{E}_t M_{t+1}\left[(1-\pi_{eu,t+1})\left(\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + \frac{\kappa_{t+1}}{q_{t+1}}\right)\right]}_{\mathbb{E}_t M_{t+1} J_{t+1}}, \tag{3.14}$$

where $\kappa_t \equiv \kappa_{1,t} + q_t \kappa_{2,t}$ denotes the expected costs of a vacancy. The left-hand side is the cost of creating one job, i.e. the cost of filling a vacancy with certainty. It equals the shadow price of employment. The right-hand side is the expected discounted value of a job for the firm where the firm's value of a match at the beginning of a period reads

$$J_t \equiv \frac{\partial P^c_t}{\partial l_t} = (1-\pi_{eu})\left[\frac{\partial Y_t}{\partial N_t} - W_t + \mathbb{E}_t M_{t+1} J_{t+1}\right]. \tag{3.15}$$

In terms of the asset pricing equation, the employment Euler (3.14) can be expressed as $\mathbb{E}_t M_{t+1} R^N_{t+1} = 1$ with

$$R^N_{t+1} = \frac{(1-\pi_{eu,t+1})\left(\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + \frac{\kappa_{t+1}}{q_{t+1}}\right)}{\frac{\kappa_t}{q_t}}.$$

Unemployed workers and vacancies meet according to the Cobb-Douglas matching function

$$\Xi_m(u_t, v_t) = \xi_m u_t^{\alpha_m} v_t^{1-\alpha_m} \quad \xi_m > 0,\ \alpha_m \in (0,1). \tag{3.16}$$

Defining labour market tightness $\theta_t \equiv \frac{v_t}{u_t}$, the job-finding rate $\pi_{ue,t}$ and vacancy-filling rate q_t are given by

$$\begin{aligned} q_t &= \xi_m \theta^{\alpha_m - 1} \\ \pi_{ue,t} &= \xi_m \theta^{\alpha_m} = q_t \theta_t. \end{aligned}$$

Wages The representative family earns wage W_t for each employed worker and a benefit b_t for each unemployed. The family pays lump sum taxes T_t. It trades shares s_t of the mutual fund and bonds $\tilde{B}_t$. Shares trade at price P_t and pay dividend D_t; bonds have a return R_t^f. The family's consumption reads

$$C_t = W_t l_t (1 - \pi_{eu,t}) - T_t + b_t (1 - l_t + \pi_{eu,t} l_t) + s_t (D_t + P_t) - s_{t+1} P_t + \widetilde{B}_t - \frac{1}{R_{t+1}^f} \widetilde{B}_{t+1}.$$

The family takes lump sum taxes and dividends as given. Using the Envelope condition and employment's law of motion (3.10), we find the family's value of an additional worker,

$$\Delta_t \equiv \frac{\partial V_t(S_t)/\partial l_t}{(C_t - X_t)^{-\gamma}} = [(W_t - b_t)(1 - \pi_{eu})] + \mathbb{E}_t M_{t+1} \Delta_{t+1} (1 - \pi_{eu} - \pi_{ue,t}).$$

An additional worker raises household income by the wage, but the household foregoes the outside option. The last term is the present value of entering the next period with an additional match.

We assume that Nash bargaining determines the wage and let $\varrho \in (0,1)$ denote the worker's bargaining power.[1] The wage reads

$$\begin{aligned} W_t =& \arg\max_{W_t} \Delta_t^{\varrho} J_t^{1-\varrho} \\ =& (1 - \pi_{eu})^{-1} \Bigg\{ \varrho \left[(1 - \pi_{eu}) \left(\frac{\partial Y_t}{\partial N_t} + \mathbb{E}_t M_{t+1} J_{t+1} \right) \right] \\ &+ (1 - \varrho) \left[b_t (1 - \pi_{eu}) - \mathbb{E}_t M_{t+1} \Delta_{t+1} (1 - \pi_{eu} - \pi_{ue,t}) \right] \Bigg\}. \end{aligned} \tag{3.17}$$

Using the surplus sharing rule (first-order condition of (3.17)) and the va-

[1] As shown by Stole and Zwiebel (1996), when capital is a predetermined variable, the marginal product of labour is decreasing and a large firm may strategically overemploy to reduce wages in an intrafirm bargaining model. Cahuc et al. (2008) examine this mechanism in a matching model and find that overemployment is not a major concern for the macroeconomy.

cancy first-order condition of the firm problem (3.11), the wage reduces to the usual expression

$$W_t = \rho\left[\frac{\partial Y_t}{\partial N_t} + \frac{\kappa_t\theta_t}{1-\pi_{eu}}\right] + (1-\rho)b_t. \tag{3.18}$$

The Nash wage is a convex combination of the marginal product of labour, saved vacancy-posting costs, and the worker's outside option. Contrary to Chapter 2, I do not assume wage stickiness, i.e. ρ is constant. Hence, the surplus sharing rule holds in expectations, $\rho\mathbb{E}_t M_{t+1} J_{t+1} = (1-\rho)\mathbb{E}_t M_{t+1}\Delta_{t+1}$, and we can reduce the wage to the simple expression (3.18).

Returns The risk-free interest rate is

$$R^f_{t+1} = \left(\mathbb{E}_t M_{t+1}\right)^{-1}. \tag{3.19}$$

See Bai and Zhang (2021) for a derivation of the equity return and price,[2]

$$\begin{aligned}
P_t &= Q^N_t l_{t+1} + Q^K_t K_{t+1} \\
Q^N_t &= \mathbb{E}_t\left[M_{t+1}\frac{\partial P^c_{t+1}}{\partial l_{t+1}}\right] = \left(\frac{\kappa_t}{q_t}\right) \\
Q^K_t &= \mathbb{E}_t\left[M_{t+1}\frac{\partial P^c_{t+1}}{\partial K_{t+1}}\right] = \underbrace{\frac{1}{a_2}\left(\frac{I_t}{K_t}\right)^{\frac{1}{\nu}}}_{=\frac{1}{\partial\Phi(K_t,I_t)/\partial I_t}}.
\end{aligned} \tag{3.20}$$

The stock price rises with employment and capital and their respective shadow prices, Q^N_t and Q^K_t. In Tobin (1969) and Hayashi (1982) stock prices are only a function of capital and its installation cost, Q^K_t. Merz and Yashiv (2007) document that under frictional labour markets, the shadow price of employment, Q^N_t, enters equity prices and returns. Stock prices now fluctuate with employment and capital and their respective installation costs.

[2] The proof follows a guess-and-verify method: Assume $P_{t+1} = Q^N_{t+1} l_{t+2} + Q^K_{t+1} K_{t+2}$. Apply the Euler theorem to the linear homogeneous functions $\Phi()$ and $Y()$ and use the firm's first-order conditions to show $P_t = Q^N_t l_{t+1} + Q^K_t K_{t+1}$.

Finally, the equity return is the weighted sum of the input's returns,

$$\begin{aligned} R^s_{t+1} =& \frac{P_{t+1} + D_{t+1}}{P_t} \\ =& \frac{Q^K_t K_{t+1}}{Q^N_t l_{t+1} + Q^K_t K_{t+1}} R^K_{t+1} + \frac{Q^N_t l_{t+1}}{Q^N_t l_{t+1} + Q^K_t K_{t+1}} R^N_{t+1}. \end{aligned}$$

Aggregation and productivity adjustment In the aggregate, bonds are in zero net supply $\tilde{B}_t = 0\ \forall t$. The representative family holds the mutual fund, $s_t = 1\ \forall t$. Taxes are used to pay unemployment benefits, so $T_t = b_t(\pi_{eu} l_t + u_t)$. In general equilibrium, all goods are either consumed or invested into vacancies or physical capital. Hence, aggregate consumption reduces to

$$C_t = W_t l_t (1 - \pi_{eu}) + D_t = Y_t - \kappa_{1,t} v_t - q_t v_t \kappa_{2,t} - I_t. \tag{3.21}$$

In Appendix 3.A the model is adjusted for productivity growth. Constants that scale with trend are denoted with a time index e.g. $b \equiv \frac{b_t}{A_t}$. Let lowercase letters denote productivity-adjusted variables, e.g. $c_t \equiv \frac{C_t}{A_t}$. The capital stock is an exception to this rule: K_{t+1} is a pre-determined variable in $t+1$ and its detrended form k_{t+1} should be pre-determined as well.[3] For robustness to stochastic growth, assume $k_{t+1} \equiv \frac{K_{t+1}}{A_t}$.

The model is solved with third-order perturbation using Dynare. The following section parametrizes and simulates the model and studies the quantitative importance of habits and capital adjustment costs.

3.3 Quantitative results

This section parametrizes the model to post-war U.S. data and shows simulation results. The model is parametrized to match the mean and volatility of equity and bond returns. A Hagedorn and Manovskii (2008) calibration of the worker's outside option solves the Shimer (2005) puzzle. Finally, the mechanisms that solve the equity premium puzzle are examined in detail.

[3] With stochastic growth, A_{t+1} is a random variable in period t and $\frac{K_{t+1}}{A_{t+1}}$ would inherit this randomness.

3.3.1 Parametrization

Table 3.1 summarizes parametrization and simulation. The model is in monthly frequency and parametrized to key macroeconomic moments of the U.S. economy from 1950 to 2018. I use the Simulated Method of Moments (SMM) to estimate the model. Policy functions are non-linear because of habits and the matching function's curvature (see Appendix 3.D) and we are interested in the risk premium. Hence, the model cannot be accurately solved with first-order perturbation and a simple Kalman filter or maximum likelihood cannot estimate the model. The Generalized Method of Moments (GMM) is popular in the asset pricing literature. GMM matches analytical solutions or approximations of moments to their empirical counterpart. In this general equilibrium model, there exist no closed-form solutions for model-generated moments and second-order approximations of the mean and volatility, using Taylor expansions, have proven to be very bad approximations of the simulation moments. Consequently, I use the Simulated Method of Moments (SMM) which essentially substitutes the closed-form solutions for simulation moments.

Parameters $\{\gamma, \delta, \rho_z, \varrho\}$ are parametrized to conventional values. Following Campbell and Cochrane (1999), set $\gamma = 2$. The depreciation rate is $\delta = 0.008$ (Smets and Wouters, 2007; Jermann, 1998). The autoregressive parameter of productivity is a conventional $0.975^{\frac{1}{3}}$ (Leduc and Liu, 2016).

The bargaining power of workers is set to $\varrho = 0.15$. I abstain from estimating ϱ in the SMM because it is not a key parameter in this model. Firstly, Ljungqvist and Sargent (2017) show that the low elasticity of wages plays a negligible role in solving the Shimer puzzle. Secondly, Chapter 2 shows that the pro-cyclicality of dividends plays a negligible role in solving the equity premium puzzle. Petrosky-Nadeau et al. (2018) claim that a low bargaining power leads to pro-cyclical dividends and solves the equity premium puzzle. Their proposed mechanism goes as follows: low worker bargaining power generates wage inertia in a downturn because wages are generally low compared to the outside option and workers do not accept wage cuts that bring wages even closer to the outside option. From the firm's perspective, dividends are revenue minus investment minus wages. In a downturn, revenue and investment fall, but rigid wages remain stable and decrease dividends. Investors dislike low dividends in adverse times and demand a risk premium for equity. While this is conceptually correct, Chapter 2 shows that, quantitatively, the immediate dividends matter little for the correlation of marginal utility and equity returns. In a general equilibrium model, forward-looking equity prices are the major driver of the equity return and those prices will be pro-cyclical no matter the immediate dividends. In short, the bargaining power's value affects the model quantitatively, but it does not drive the results of this paper. Including it in the SMM complicates the estimation without altering the qualitative findings. Finally, the bargaining power $\varrho = 0.15$ implies a elasticity of wages with respect to TFP of 0.3, not too far from the estimate of 0.449 by Hagedorn and Manovskii (2008).[4]

Productivity growth g_a sets the mean annual growth rate of consumption to 1.8%. Chirinko and Mallick (2017) estimate the elasticity of substitution between capital and labour to be 0.4, which determines η. Following the World Inequality Database, I set the capital share to $\alpha = 0.26$.[5] In a robustness check (Table 3.2), I show that results hold for a larger capital share.

Sedláček (2016) estimates the elasticity of the matching function to be $\alpha_m = 0.76$. The matching efficiency, ξ_m, is calibrated to match the steady state vacancy-filling rate. Davis et al. (2013) estimate a daily vacancy-filling rate of 5%. At 20 business days per month the daily estimate aggregates to a 64.15%, which is my target vacancy-filling rate. The constant separation rate is set to its 1967-2018 average, $\pi_{eu} = 1.92\%$. The average job-finding rate is 26.05%, which determines the vacancy-posting costs, κ_1. Training costs equal one month's wages, consistent with estimates by Barron et al. (1999). Finally, $\bar{z}$ normalizes the marginal product of labour to unity and K_0 equals the steady state capital stock.

SMM estimates the parameters $\{\sigma_z, b, \beta, \rho_s, \overline{S}, \nu\}$ with the following targets: the volatility of quarterly consumption growth and the relative volatility of unemployment growth; the mean and standard deviation of the annual

[4]Following Hagedorn and Manovskii (2008), log quarterly wages and TFP are HP-filtered ($\lambda = 1600$). Then, wages are regressed on TFP.

[5]wid.world: Labour and capital share of national income. Annual U.S. data 1960-2018. The labour (capital) share is defined as the ratio of pure labour (capital) income and 70(30)% of mixed income over factor price national income. I treat profits like mixed income and distribute 70% of profits to labour and 30% to capital.

risk-free rate; the mean equity premium and the standard deviation of equity returns. The justification behind setting these targets is as follows: the volatility of consumption, which is closely linked to output but more important for asset prices, estimates σ_z. The target unemployment volatility estimates the outside option b (Hagedorn and Manovskii, 2008).

The mean risk-free rate serves as an SMM target to estimate time preference β. Unconventionally, I estimate β in the SMM, not in the calibration to account for precautionary savings: habits make households more risk averse on average because they fear consumption close to habit. This fear motivates prudent households to save as a precaution: the risk-free rate in a higher (≥ 3) order simulation is considerably lower than the risk-free rate in the deterministic steady state or lower order simulation.

The surplus consumption ratio in steady state, $\overline{S}$, and β affect the mean and the volatility of the risk-free rate. High surplus consumption reduces the need to save, raising the risk-free rate and reducing its standard deviation. A lower β increases both the mean and the volatility of the rate. Hence, I include the standard deviation of the risk-free rate to estimate $\overline{S}$. The opposite effects of $\overline{S}$ and β on the volatility discriminate between the two parameters in the estimation.

Finally, the lower the adjustment cost parameter ν, the more convex the capital adjustment costs. Hence, a large ν reduces the equity premium and the standard deviation of equity returns. Examining the Jacobian of the SMM target function, the persistence of habits, ρ_s, raises the standard deviation of equity returns and reduces the equity premium. I include the standard deviation of equity returns and the equity premium as targets to estimate the remaining ν and ρ_s.

3.3.2 Baseline results

The model is solved with third-order perturbation and simulated with pruning.[6] Table 3.1 summarizes the simulation results. The high outside option of workers solves the Shimer puzzle, i.e. the volatility of unemployment is

[6]Following Born and Pfeifer (2014), I simulate the model with no shocks for 2000 periods and pick the last observation as the estimate of the ergodic mean. Then, I draw 100×240 random realizations of $\epsilon_{z,t}$ and simulate the model 100 times for 240 years, starting at the ergodic mean. The first 120 years are discarded as a burn-in phase.

comparable to U.S. data. Habits and capital accumulation costs generate a risk premium close to the empirical estimate. The simulation matches the mean risk-free rate, but overshoots its volatility by 50%. This is a well-known problem, related to the "equity volatility puzzle" (Campbell, 2003): models that match the relative standard deviation of equity returns to consumption, e.g. the Cochrane (1991) model, must not imply an equally volatile risk-free rate. The model's risk-free rate is strongly linked to consumption growth and without a monetary policy-maker, the risk-free rate fluctuates too much. The volatility of equity returns is close to its empirical targets. Overall, the model outperforms the endogenous disaster risk model and the long-run risk model studied in Chapter 2 in terms of asset pricing statistics.

3.3.3 Mechanisms

This section demonstrates why habits and capital adjustment costs solve the equity premium puzzle. A robustness analysis supports the interpretation of the mechanisms.

A large equity premium arises when an asset is negatively correlated with the stochastic discount factor and the volatilities of the asset and the discount factor are sufficiently large. The expected equity return is

$$\mathbb{E}_t[R^s_{t+1} - R^f_{t+1}] = -(R^f_{t+1})Cov_t\left[M_{t+1}, R^s_{t+1}\right] \tag{3.22}$$

and the covariance reads

$$Cov_t\left[M_{t+1}, R^s_{t+1}\right] = \rho_t\left[M_{t+1}, R^s_{t+1}\right]\sigma_t(M_{t+1})\sigma_t(R^s_{t+1}), \tag{3.23}$$

where σ_t and ρ_t denote the volatility and correlation coefficient conditional on the information set in t. In Chapter 2, a DMP model parametrized to post-war data and a DMP model with long-run productivity risk, along the lines of Bansal and Yaron (2004) and Croce (2014), both fail to generate a sufficiently large volatility of the stochastic discount factor. Although the correlation coefficient is close to -1, households do not perceive the economy as risky enough to demand a sizeable premium. How can we raise the conditional volatility of marginal utility? Petrosky-Nadeau and Zhang (2013) assume a high volatility of TFP which gives rise to disaster

	Parametrization			
	Parameter	Value	Target	Source
γ	Intertemporal substitution	2	-	Campbell and Cochrane (1999)
ρ_z	TFP persistence	$0.975^{\frac{1}{3}}$	-	Leduc and Liu (2016)
g_a	constant growth	0.0015	$\overline{\Delta^a Y} = 1.8\%$	U.S. Data 1948-2017
η	production fun. parameter	-1.5	substitution elasticity = 0.4	Chirinko and Mallick (2017)
α	capital share parameter	0.26	Labour share = 75%	U.S. Data 1960-2017 (wid.world)
δ	capital depreciation	0.008	-	Smets and Wouters (2007) Jermann (1998)
ρ	bargaining power	0.15	-	-
α_m	elasticity w.r.t. job-seeker	0.76	-	Sedláček (2016)
ξ_m	matching efficiency	0.3234	$\overline{q} = 64.15\%$	Davis, Faberman, Haltiwanger (2013)
π_{eu}	separation probability	0.0192	-	U.S. Data 1967-2018
κ_1	vacancy-posting cost	1.018	$\overline{\pi_{ue}} = 26.05\%$	U.S. Data 1967-2018
κ_2	training cost	0.948	one monthly wage	Barron et al. (1999)
$\overline{z}$	steady state TFP	0.2453	$\overline{\partial Y/\partial N} = 1$	-
K_0	production fun. normalizer	26.848	$K_0 = \overline{k}$	-
σ_z	TFP volatility	0.0167		estimated with SMM
b	worker outside option	0.8000		
β	time discount	1.0009		
ρ_s	habit persistence	0.9142		
$\overline{S}$	consumption surplus ratio	0.2458		
ν	adjustment cost parameter	2.0962		
	SMM/Simulation results			
		Simulation	Target	Source
$\sigma(\Delta^q C)$	consumption volatility	0.83 %	0.85 %	all SMM targets: U.S. data 1948-2017
$\frac{\sigma(\Delta^q u)}{\sigma(\Delta^q C)}$	unemployment volatility	9.20	9.03	
r^f	mean risk-free rate	1.92%	2.30%	
$\sigma(r^f)$	volatility risk-free rate	4.04%	2.44%	
$r^s - r^f$	equity risk premium	5.02%	4.80%	
$\sigma(r^s)$	equity return volatility	9.85%	11.21%	

Table 3.1: Baseline parametrization and simulation results. The parameters in the top section are determined in steady state calibration. Parameters below are estimated with Simulated Method of Moments (SMM) with targets in the bottom section. Unless otherwise stated, U.S. data is taken from Chapter 2. All moments are in percent. Consumption and unemployment volatility are measured as the standard deviation of the quarterly growth rates. Returns are annual. Following Petrosky-Nadeau and Zhang (2013), the data's equity risk premium is adjusted downwards to account for leverage. $\overline{x}$ denotes the steady-state value of x_t. $\Delta^q x$ denotes the quarterly growth rate of x.

risk. However, the model's simulation results are inconsistent with post-war data as demonstrated independently in Dupraz et al. (2019), Kehoe et al. (2019), and Chapter 2. Instead of raising the volatility of consumption, one could raise the coefficient of risk aversion until the puzzle is solved, but that would lead to an implausibly high coefficient (Mehra and Prescott, 1985). Habits keep the coefficient at a modest level but change how households value innovations of consumption.

Essentially, Campbell and Cochrane (1999) habits amplify risk aversion in adverse times which raises the conditional volatility, $\sigma_t(M_{t+1})$. Varying C_t, while keeping habit X_t fixed, the local coefficient of relative risk aversion is $\frac{-C_t V_{cc,t}}{V_{c,t}} = \frac{\gamma}{S_t}$, i.e. when surplus consumption is low, households become more risk averse. The economy can be characterized by "boring" business cycle fluctuations around steady state, but investors fear cyclical fluctuations because they are used to consumption at steady state level. In the Campbell-Cochrane partial-equilibrium model,[7] the standard deviation of the innovation to the log stochastic discount factor reads

$$\sigma_m^{CC} = \frac{\gamma\sigma_c}{\bar{S}^{cc}}\sqrt{1 - 2(\log S_t - \log \bar{S}^{cc})}.$$

At the upper bound of surplus consumption, σ_m^{cc} equals $\gamma\sigma_c$, which is the volatility under CRRA utility with constant relative risk aversion. However, when surplus consumption falls towards zero, the right-hand side term can grow without bound. In summary, habits raise marginal utility in adverse states vis-à-vis CRRA utility. This raises risk aversion and the volatility of the stochastic discount factor and solves the main problem of Chapter 2.

Revisiting equation (3.23), the DMP model generates a correlation coefficient close to -1 and habits introduce a large conditional volatility of the SDF. Only the conditional volatility of equity returns remains as a possible pitfall

[7] Campbell and Cochrane (1999) assume that consumption follows a random walk and assume a specific functional form of $\bar{S}$

$$\log C_{t+1} - \log C_t = g_a + \sigma_c \epsilon_{c,t+1}, \quad \epsilon_{c,t+1} \sim N(0,1)$$
$$\bar{S}^{cc} = \sigma_c \sqrt{\frac{\gamma}{1-\rho_s}}.$$

In this paper's general equilibrium model, consumption growth does not follow the random walk and $\bar{S}$ is a free parameter.

on the path towards solving the equity premium puzzle. This volatility is driven by the volatilities of dividends and equity prices, $R^s_{t+1} = \frac{P_{t+1}}{P_t} + \frac{D_{t+1}}{P_t}$. In the general equilibrium model, dividends are net transfers from firms to households, rather than profit shares determined in a stockholder's meeting; the model-generated dividends are small, can become negative in a recession, and have a low volatility. As such, the volatility of the stock price has to drive the volatility of returns. Repeat the stock price,

$$\begin{aligned}
P_t &= Q^N_t l_{t+1} + Q^K_t K_{t+1} \\
Q^N_t &= \mathbb{E}_t\left[M_{t+1}\frac{\partial P^c_{t+1}}{\partial l_{t+1}}\right] = \left(\frac{\kappa_t}{q_t}\right) \\
Q^K_t &= \mathbb{E}_t\left[M_{t+1}\frac{\partial P^c_{t+1}}{\partial K_{t+1}}\right] = \frac{1}{a_2}\left(\frac{I_t}{K_t}\right)^{\frac{1}{\nu}}.
\end{aligned}$$

When labour and capital adjustment are frictional, the firm's value equals employed labour and capital times their respective installation costs, or shadow prices. The volatilities of these shadow prices together drive the volatility of equity prices and ultimately equity returns. Matching frictions raise the volatility of Q^N_t. In a boom, firms want to hire because the expected surplus is large, but finding workers is hard because the market is so tight with vacancies. This is reflected in a high Q^N_t. If employment were traded like a stock, the price of the employment stock would be large. In a recession, the expected surplus is low, market tightness is low, and the vacancy-filling rate is large. It is easy to find workers, which reduces the value of the employment stock. Similarly, capital adjustment costs raise the volatility of Q^K_t. In a boom, firms invest strongly relative to their capital stock, which is costly. The price of investment rises, increasing the shadow price of available capital. In a recession, Q^K_t falls as investment costs vanish; investment goods are abundant and few firms decide to renew. A caveat of the textbook q theory of investment is that, in order to match asset prices, the investment-capital ratio must be very volatile and the capital adjustment costs need to be very sensitive. In this paper, the burden of matching the stock price volatility is shared by employment and capital frictions which improves the model's goodness-of-fit vis-à-vis a model without capital (Chapter 2) or the textbook q theory (c.f. Merz and Yashiv, 2007).

	Target	Baseline	$\bar{S}=1,\ \rho_s=0$	$\nu=4.2$	$b=0.5$	$\gamma=1$	$\beta=0.999$	$\alpha=1/3$
$\sigma(\Delta^q C)$	0.85	0.86	1.71	0.69	0.75	0.99	0.91	0.74
$\frac{\sigma(\Delta^q u)}{\sigma(\Delta^q C)}$	9.03	9.39	2.20	31.35	1.45	4.17	60.03	17.86
r^f	2.30	2.15	2.34	2.19	1.95	0.53	3.60	2.07
$\sigma(r^f)$	2.44	3.72	2.03	2.83	3.75	2.78	4.73	3.19
$r^s - r^f$	4.80	5.09	1.83	2.11	4.35	3.15	6.10	3.60
$\sigma(r^s)$	11.21	9.52	6.13	6.57	8.38	7.54	11.00	8.29
Baseline parameters			$\bar{S}=0.25,\ \rho_s=0.91$	$\nu=2.1$	$b=0.8$	$\gamma=2$	$\beta=1.001$	$\alpha=0.26$

Table 3.2: Robustness checks. Simulation results for different parametrizations. All moments are in percent. Consumption and unemployment volatility are measured as the standard deviation of the quarterly growth rates. Returns are annual.

Table 3.2 quantitatively checks whether the expectations about habits and capital adjustment costs outlined above hold:

Habits With habits, households smooth consumption relative to their habit to avoid a low consumption surplus. When surplus consumption always equals unity, the utility function defaults to power utility. Without habit, the household has less incentive to smooth consumption and the standard deviation of consumption almost doubles. Unemployment's volatility decreases which is in line with Kehoe et al. (2019): they show that habits can generate a large volatility of the expected discounted surplus that translates into volatile hiring. Habits motivate precautionary savings: without habits, the equilibrium risk-free rate rises and its volatility falls. The equity premium and the standard deviation of equity returns fall. Habits can motivate the risk premium, but frictions amplify the standard deviation of returns.

Capital adjustment costs Doubling ν reduces the capital installation friction. Now, households can smooth consumption more easily (lower $\sigma(\Delta^q C)$). When capital is the major instrument to smooth consumption, households let unemployment fluctuate more. The equity premium falls by half and the standard deviation of equity returns falls as expected. As stated above, a high correlation between return and SDF does not suffice for a premium when the asset is not sufficiently risky.

Outside option The foremost effect of a lower outside option of workers is to reduce unemployment volatility. The fundamental surplus (Ljungqvist and Sargent, 2017) is large and small TFP innovations do not translate into large unemployment fluctuations. The outside option has effects on equity prices: a lower outside option weakens the worker's bargaining position. Firms can extract more profits from a match and an investment into equity becomes less risky and more profitable. As a consequence, equilibrium equity prices are larger and the equity premium falls.

Risk aversion A reduction of the parameter of risk aversion by half raises the standard deviation of consumption. Lower risk aversion naturally reduces the equity premium. Reducing γ affects the risk-free rate via a precautionary savings effect and an intertemporal substitution effect. Firstly, less risk-averse agents save less as precaution: lower precautionary savings raise the equilibrium risk-free rate r^f. Secondly, the steady state growth rate g_a is positive and the utility function does not distinguish risk aversion from intertemporal substitution; because the reciprocal of risk aversion is the elasticity of intertemporal substitution, the elasticity rises. With $g_a > 0$ households would prefer to shift consumption from the future to the present, i.e. they increase their bond supply. To equate bond supply and demand, the risk-free rate must fall. On average, the substitution effect dominates and the risk-free rate is lower in comparison to the benchmark case.

Patience When β is reduced, households save less and consumption volatility goes up. First, impatient households supply fewer bonds, which raises the average risk-free rate. Second, impatient households invest less into equity. Equity is itself invested into capital and vacancies, hence the steady state stock of capital and employment is lower and more volatile. We see a higher unemployment volatility and riskier equity. Equation (3.22) shows that a higher risk-free rate and more volatile equity returns drive the equity premium upwards.

Capital share of income Raising the capital share of income from $\alpha = 0.26$ to $\alpha = 1/3$, reduces the standard deviation of consumption and returns. Qualitatively, the results do not depend on parameter α: the model still solves the puzzles. The higher α raises the relative standard deviation of unemployment; this would demand a re-estimation of parameter b.

Simulation results show that the model jointly solves the equity premium puzzle and the Shimer puzzle. The next section takes the parametrized model to time series data and asks the following questions. Does the model reproduce the correlation structure between unemployment and asset prices? Can the model reproduce not only moments, but the time series of key macroeconomic variables?

3.4 Matched series

Section 3.3.2 shows that the model can, in terms of simulation moments, jointly solve the equity premium puzzle and the Shimer puzzle. However, empirically, unemployment and stock prices strongly co-move and this correlation initially motivated the connection of labour market frictions and asset pricing. To examine whether the model generates the same correlation structure, I match the parametrized model to unemployment data and show a number of business cycle time series. The model-generated asset prices are strongly correlated with unemployment and resemble the data, but model-generated consumption and the interest rate are too volatile.

Appendix 3.B describes the algorithm to derive the TFP estimate from the empirical unemployment series and the model's policy function of employment. Essentially, starting from January 1950, the algorithm asks: given today's unemployment rate (and other state variables excluding TFP), what level of TFP is necessary today such that the unemployment rate tomorrow becomes an optimal solution? Using backed-out TFP today, the algorithm updates the state variables and moves forward in time. Finally, it computes all remaining variables of the model.

Figure 3.1 compares the simulated times series to data: two major findings stand out. First, the simulation matches stock prices and the equity premium quite well. Second, the model strongly overestimates the volatility of consumption, even though consumption volatility was an estimation target. The risk-free rate inherits the overshooting of consumption.

The simulated unemployment rate perfectly matches the empirical unemployment rate, i.e. unemployment always lies in the codomain of the policy function. The model is effective in matching equity prices and the equity premium and replicates the correlation structure of employment and equity prices. The model-generated recessions follow the data: at the onset of a recession (grey bands), equity prices and the equity premium fall; unemployment rises. In the recession, equity returns are large because investors, who consume close to their habit, demand a large premium to hold the risky asset.[8] The equity price is a jump variable. In anticipation of better times it recovers quickly, while capital adjustment costs and search frictions slow down the recovery of employment. In summary, the DMP model with habits and capital adjustment costs clearly outperforms the models examined in Chapter 2 in the dimension of asset prices and their correlation with labour market data.

It is obvious from Figure 3.1 that this exercise overestimates the volatility of consumption and the risk-free rate. Technically, the policy function of employment is too linear. It takes substantial innovations of TFP (last panel) to replicate the swings of the unemployment rate. The volatile TFP then leads to overly volatile output and consumption.

The risk-free rate can be expressed as

$$r^f_{t+1} = -\log\beta + \gamma\mathbb{E}_t\left(\Delta\log S_{t+1} + \Delta\log C_{t+1}\right) - \frac{\sigma_t(\log M_{t+1})^2}{2}.$$

It is determined by time preference, the growth rates of habit and consumption, and precaution. Hence, the risk-free rate inherits the excessive volatility of consumption. I believe that a monetary authority, together with monetary frictions, could improve the goodness-of-fit in this dimension. Freund and Rendahl (2020), for instance, explore a DMP model with monetary frictions.

In the future, how could we improve the model's goodness-of-fit in terms of consumption volatility? i) Maybe we need to add supplementary (productivity or policy) shocks that affect consumption but are orthogonal to stock prices and employment? In contrast to this idea, Angeletos et al. (2020) show evidence for a single common driver of business cycles and Chari et al. (2007) show that different shocks often translate into the same wedges in first-order conditions. ii) If we take the theory literally, then the marginal worker, who is almost indifferent between working and the outside option, is also the investor who sets asset prices and the consumer who determines aggregate consumption. In this model, consumption is too volatile because

[8] *Buy when there is blood in the streets, even if the blood is your own.* - Baron Rothschield

employment is so closely linked to consumption. Introducing heterogeneity could be a step forward. Firstly, volatile consumption of the marginal worker need not translate into volatile aggregate consumption. Secondly, equity is held by the top decile of the wealth distribution; the marginal worker's consumption growth should not be the direct determinate of stock prices. A New-Keynesian model with search frictions and heterogeneity would address these inconsistencies and the risk-free rate's excess volatility. Finally, note that the disaster risk literature does not address the problem of excessive consumption volatility. In fact, it is common practice to parametrize a disaster risk model to very volatile, international historic data, e.g. Barro and Ursúa (2008), Petrosky-Nadeau and Zhang (2013), Bai and Zhang (2021) or Wachter and Kilic (2018). A high consumption volatility is then interpreted as a feature, rather than the model's failure to match post-war U.S. data. I do not support this view: a model that regularly predicts disasters in post-war U.S. time series is not a good representation of this economy.[9]

Last, Figure 3.2 depicts return predictability: empirically, a price-to-earnings ratio below average is a good predictor for higher future returns. To erase the effect of cyclical volatility, the figure plots the ratio of price to earnings over the past ten years (PE10) and the average future return in the following ten years. The correlation between PE10 and returns is -34% in the model and -59% in the data, i.e. empirically the PE10 still has more predictive power than the model's PE10. However, the model clearly outperforms the models studied in Chapter 2.

[9] Disasters are typically defined as cumulative drops of GDP or consumption by at least 10-15%. In U.S. data 1950-2018 only the cumulative effect of two oil crises is a disaster.

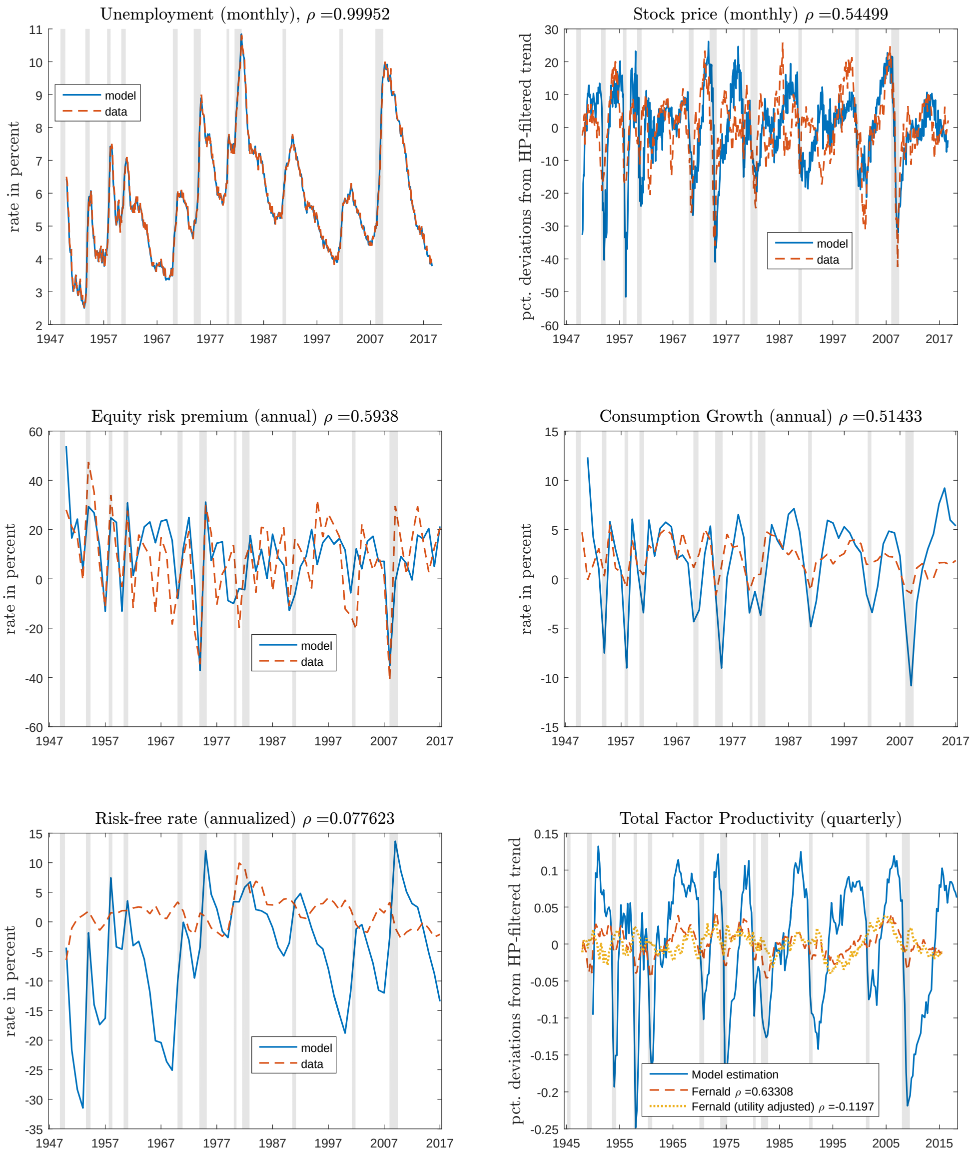


Figure 3.1: Matched time series. ρ denotes the correlation coefficient of model-generated and data time series. Grey bands denote NBER recessions. Stock prices ($\lambda = 129,600$) and TFP ($\lambda = 100,000$) are HP-filtered log deviations from trend.

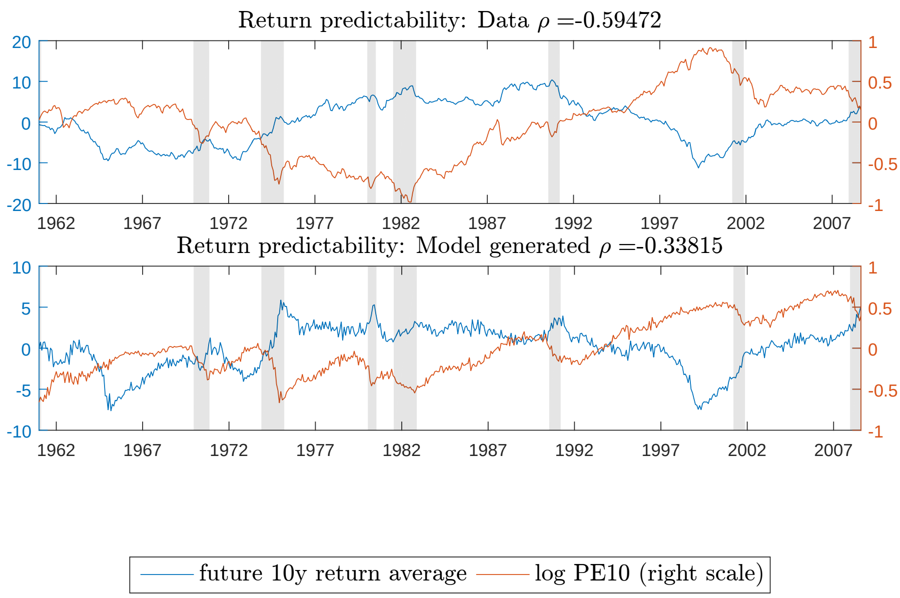


Figure 3.2: Matched series: Return predictability. The price earnings ratio (PE10) is the ratio of the current price to average earnings over the past ten years (also called Cyclically Adjusted PE Ratio (CAPE) or Shiller PE Ratio). I compute log PE10 as $\log \frac{P}{D+1}$ to avoid explosive PE10 for dividends close to zero. Future 10 year returns are the geometrical average of annual returns over the next ten years. The figure shows absolute deviations from mean. ρ denotes the correlation coefficient of PE10 and future returns for data and model respectively.

3.5 Conclusion

Including habits and capital adjustment costs in the DMP framework has proven to robustly solve the equity premium puzzle and the Shimer puzzle. In simulations, the model yields a large equity premium while maintaining the standard deviation of the risk-free rate at a moderate level. When matched to U.S. post-war data, the model reproduces the striking correlation of unemployment and stock prices and tracks the time series of equity prices and the premium. Recessions are described well by the model.

The model falls short of tracking the empirical time series of the risk-free interest rate and consumption. Moreover, this representative agent theory treats the consumer, the worker, and the investor as one household: therefore consumption, equity prices, and interest rates are too strongly linked to one another. Building on these findings, a heterogeneous agent New-Keynesian model with labour and capital frictions could kill two birds with one stone: First, including a monetary friction and a Taylor rule will probably increase the goodness-of-fit of the interest rate and consumption. Second, a heterogeneous agent model can destroy the strong link between workers and investors, which is counter-factual given high degrees of wealth inequality and counter-intuitive in the sense that a marginal worker, who is almost indifferent between a job and unemployment, belongs to the same family that possesses all wealth in this economy.

Appendix

3.A Productivity adjustment

The parameters $\{\kappa_{1,t}, \kappa_{2,t}, b_t\}$ grow with technology A_t and are otherwise constants, e.g. $b_t = bA_t$. Define the productivity-adjusted variables $c_t = \frac{C_t}{A_t}$, $w_t = \frac{W_t}{A_t}$, $d_t = \frac{D_t}{A_t}$, $p_t = \frac{P_t}{A_t}$, $\phi_t(k_t, i_t) = \frac{\Phi(K_t, I_t)}{A_t}$, $y_t = \frac{Y_t}{A_t}$, $i_t = \frac{I_t}{A_t}$. Employment l_{t+1} and K_{t+1} are control variables in period t, i.e. they are predetermined in period $t+1$. While employment does not grow with TFP, the capital stock does. To keep the productivity-adjusted capital stock predetermined in period $t+1$, assume $k_{t+1} = \frac{K_{t+1}}{A_t}$ and the normalizing constant $K_0 = \frac{K_{0t+1}}{A_t}$. Dividing by A_t instead of A_{t+1}, gives the solution robustness to stochastic growth: it keeps k_{t+1} predetermined if A_{t+1} is a random variable. Here, A_t grows at a constant rate $\Delta\log(A_t) = \overline{\Delta\log(A)} = g_a \;\forall t$. Adjusted for productivity growth, the model reads

$$
\begin{aligned}
m_{t+1} &\equiv M_{t+1}\frac{A_{t+1}}{A_t} = \beta e^{\Delta\log A_{t+1}}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\left(\frac{S_{t+1}}{S_t}\right)^{-\gamma} \\
\frac{1}{R^f_{t+1}} &= \mathbb{E}_t M_{t+1} = \mathbb{E}_t\left[m_{t+1}e^{-\Delta\log A_{t+1}}\right] \\
c_t &= y_t - \kappa_1 v_t - q_t v_t \kappa_2 - i_t \\
l_{t+1} &= l_t(1-\pi_{eu}) + \pi_{ue,t}(1-l_t) \\
k_{t+1} &= (1-\delta)k_t e^{-\Delta\log A_t} + \phi_t(k_t, i_t) \\
\phi_t(k_t, i_t) &= \left[a_1 + \left(\frac{a_2}{1-\frac{1}{\nu}}\frac{i_t}{k_t}e^{\Delta\log A_t}\right)^{1-\frac{1}{\nu}}\right]k_t e^{-\Delta\log A_t} \\
a_1 &= \frac{e^{\overline{\Delta\log A}} - (1-\delta)}{1-\nu} \\
a_2 &= \left[e^{\overline{\Delta\log A}} - (1-\delta)\right]^{\frac{1}{\nu}} \\
\frac{\partial Y_t}{\partial K_t} &= \frac{y_t}{k_t}e^{\Delta\log A_t}\frac{\alpha\left(\frac{k_t}{K_0}\right)^{\eta}}{\left[\alpha\left(\frac{k_t}{K_0}\right)^{\eta} + (1-\alpha)N_t^{\eta}\right]} = \frac{\partial y_t}{\partial k_t}e^{\Delta\log A_t} \\
\frac{\partial Y_t}{\partial N_t}\frac{1}{A_t} &= \frac{y_t}{N_t}\frac{(1-\alpha)(N_t)^{\eta}}{\left[\alpha\left(\frac{k_t}{K_0}\right)^{\eta} + (1-\alpha)N_t^{\eta}\right]} = \frac{\partial y_t}{\partial N_t}.
\end{aligned}
$$

Notice that the un-adjusted marginal product of capital is time invariant but the marginal product of employment grows with A_t. The job creation curve and capital Euler equation read

$$\frac{\kappa_1}{q_t} + \kappa_2 = \mathbb{E}_t M_{t+1}\left[(1-\pi_{eu})\left(\frac{\partial y_t}{\partial N_t} - w_{t+1} + \frac{\kappa_1}{q_{t+1}} + \kappa_2\right)\right]$$

$$\frac{1}{a_2}\left(\frac{i_t}{k_t}\right)^{\frac{1}{\nu}} e^{\frac{1}{\nu}\Delta\log A_t} = \mathbb{E}_t M_{t+1}\left[\frac{\partial y_{t+1}}{\partial k_{t+1}} + \frac{1}{a_2}\left(\frac{i_{t+1}}{k_{t+1}}\right)^{\frac{1}{\nu}} e^{\left(\frac{1}{\nu}-1\right)\Delta\log A_{t+1}}(1-\delta+a_1) + \frac{1}{\nu-1}\frac{i_{t+1}}{k_{t+1}}\right].$$

Finally the stock price reads,

$$p_t = \frac{1}{a_2}\left(\frac{i_t}{k_t}\right)^{\frac{1}{\nu}} e^{\frac{1}{\nu}\Delta\log(A_t)} k_{t+1} + \left(\frac{\kappa_1 + q_t\kappa_2}{q_t}\right) l_{t+1}.$$

3.B Estimating the productivity series

The algorithm estimates a time series for TFP by interpolating employment's policy function to match the unemployment time series. Alternatively one could assume a discrete Markov chain for z_t and match the data by picking the z_t nodes that best describe the data. This Markov chain approach would deliver an inferior match of unemployment. Alternatively, one could use a non-linear filter to estimate the model. A standard Kalman filter is not applicable because of the higher order solution.

The state vector, comprises employment l_t, the capital stock k_t, surplus consumption S_{t-1}, consumption c_{t-1} and TFP z_t. Equation (3.4) shows that S_t is not a state variable in t, but its lag and lagged consumption are. Let $\widehat{l_{t+1}}(l_t, k_t, S_{t-1}, c_{t-1}, z_t)$ denote the policy function of employment. In order to initiate the algorithm, I set $S_{01/1950}$, $c_{01/1950}$ and $k_{01/1950}$ to their steady state values. Starting values "wash out" quickly, so the algorithm is quite robust to different starting values. The computation of the matched series follows two steps: Starting from employment in January 1950, interpolate the labour policy function to solve

$$\widehat{z_t} = \min_{z_t} \| l_{t+1}^{data} - \widehat{l_{t+1}}(l_t^{data}, k_t, S_{t-1}, c_{t-1}, z_t) \|.$$

Unless l_{t+1}^{data} is not in the codomain of the policy function at the state vector in t, the algorithm perfectly matches the employment time series. Then, update the remaining three state variables:

$$\begin{aligned} k_{t+1} &= \widehat{k_{t+1}}(l_t^{data}, k_t, S_{t-1}, c_{t-1}\hat{z}_t) \\ S_t &= \widehat{S_t}(l_t^{data}, k_t, c_{t-1}, S_{t-1}, \hat{z}_t) \\ c_t &= \widehat{c_t}(l_t^{data}, k_t, c_{t-1}, S_{t-1}, \hat{z}_t). \end{aligned}$$

The policy functions of the capital stock and surplus consumption determine the state vector in the following period. Finally, given the state vector, we can easily compute all other control variables. Some controls, e.g. consumption, grow with trend which is added after the simulation.

Intuitively, given today's employment and capital and yesterday's consumption and habit, I ask which level of TFP is necessary today to make the empirical employment tomorrow the optimal choice. This way, I can estimate a time series for TFP. There is no guarantee that the estimated series follows an AR(1)-process; so this exercise should be interpreted with care. Relative to Chapter 2, the model has only one exogenous shock, which allows to match only one time series. I choose the unemployment rate to estimate productivity for three reasons: the unemployment rate does not grow on the balanced growth path, empirically it has no trend and the rate is bounded between zero and unity both in the model and the data. We do not need to filter the unemployment series and can use only seasonally-adjusted data in the estimation.

3.C Welfare-improving consumption destruction

This section applies Ljungqvist and Uhlig (2015)'s critique of Campbell and Cochrane (1999) to the main text's model. They show that a social planner can raise welfare via a one-time, non-marginal destruction of consumption in the endowment model by Campbell and Cochrane. A one-time fast comes at the cost of a contemporaneous utility loss, but instantaneously reduces the slow-moving habit. Households can enjoy a large consumption surplus ratio after the fast, which can dominate the immediate effect in terms of welfare. Consistent with the critique, in my framework, a social planner may raise utility by destroying consumption. The finding does not carry over to the productive inputs; a destruction of capital or employment is not welfare improving. The result shows that welfare analysis of policy is difficult in this framework because policies may have an undesired effect on habit and a counter-intuitive effect on welfare.

I start by laying out the Ljungqvist and Uhlig (2015) critique. For illustrative purpose, assume a pure endowment economy and assume away any shock $\epsilon_{z,t} = 0 \ \forall t$. In periods $t < 0$, the economy is in steady state. Deviating from the main text, let lowercase letters denote the logarithm. For convenience, repeat habit's law of motion,

$$s_{t+1} = (1-\rho_s)\bar{s} + \rho_s s_t + \lambda(s_t)\left[c_{t+1} - c_t - g_a\right].$$

Consider a destruction of consumption goods $\psi < 0$ at time 0. Let $\hat{c}_t$ denote log consumption according to the policy function, i.e. without the destruction in period 0, households would consume $\hat{c}_0$:[10]

$$c_0 = \hat{c}_0 + \psi < \hat{c}_0 \tag{3.24}$$

$$c_t = \hat{c}_t \quad \forall t \geq 1 \tag{3.25}$$

$$s_0 = \bar{s} + \lambda(\bar{s})\psi < \bar{s} \tag{3.26}$$

$$s_t = \bar{s} - \rho_s^{t-1}\psi\left[\lambda(\bar{s} + \lambda(\bar{s})\psi) - \rho_s\lambda(\bar{s})\right] > \bar{s}, \quad t \geq 1. \tag{3.27}$$

The fast reduces consumption in period 0 and immediately reduces surplus consumption (3.26). In the following periods, consumption follows the

[10]In period $t = 1$, $s_1 = (1-\rho_s)\bar{s} + \rho_s s_0 + \lambda(s_0)[\hat{c}_1 - \hat{c}_0 - \psi - g_a] = (1-\rho_s)\bar{s} + \rho_s s_0 - \psi\lambda(s_0)$.

policy function and equals endowment (3.25). Surplus consumption follows its law of motion, and for $t \geq 1$ surplus consumption exceeds the steady state surplus (3.27). Households derive utility from the surplus consumption; in terms of welfare, the excess surplus in $t \geq 1$ can overcompensate the utility loss by the consumption destruction in period $t = 0$.

Campbell and Cochrane (1999) choose habit's law of motion (3.4) and the sensitivity function (3.5) such that habit increases for a marginal consumption good but never decreases for a marginal reduction of consumption near the steady state. This also implies that a marginal increase in consumption will always be welfare improving. As shown by Ljungqvist and Uhlig (2015), this does not hold for non-marginal reductions of consumption, ψ. This argument applies to the endowment Campbell-Cochrane model, where households consume an exogenous stream of consumption goods. Can a consumption reduction be welfare improving in the production-based model of the main text, too?

Yes. Figure 3.3 shows the impulse response to a five percent destruction of consumption in $t = 0$. Unlike in the endowment economy, consumption does not immediately jump to its steady state. Instead it slowly reverts back to steady state. The fast persistently reduces habit, X_t, which allows the consumption surplus, S_t, to rise above its steady state level. Since households derive utility from the surplus, their felicity, $\frac{(S_t C_t)^{1-\gamma}-1}{1-\gamma}$, rises above steady state. Depending on the size of the shock, the discount factor, growth and risk aversion, the higher felicity in $t \geq 1$ can overcompensate the low felicity in $t = 0$ in terms of welfare. In general equilibrium, the fast raises the SDF leading to more investment into vacancies and capital and higher output.

Does this result carry over to the productive factors, capital and employment? No. Figure 3.4 plots the impulse response to a 10% destruction of capital or employment. As expected, consumption and habit fall in response to the destruction. Unlike before, the reduction of surplus consumption is not reverted immediately. It takes a longer period to recover, especially following the destruction of capital. Hence, felicity never exceeds the steady state level. In general equilibrium, households react by dissaving: the SDF falls and so do employment, capital and output.

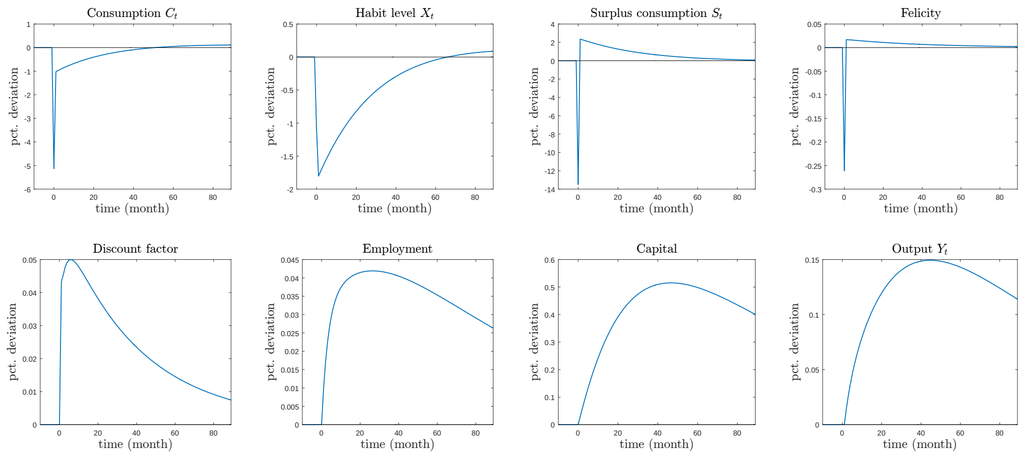


Figure 3.3: Impulse response to a 5% consumption destruction. All other shocks are set to zero: $\psi_t = 0 \; \forall t \neq 0$ and $\epsilon_{z,t} = 0 \; \forall t$.

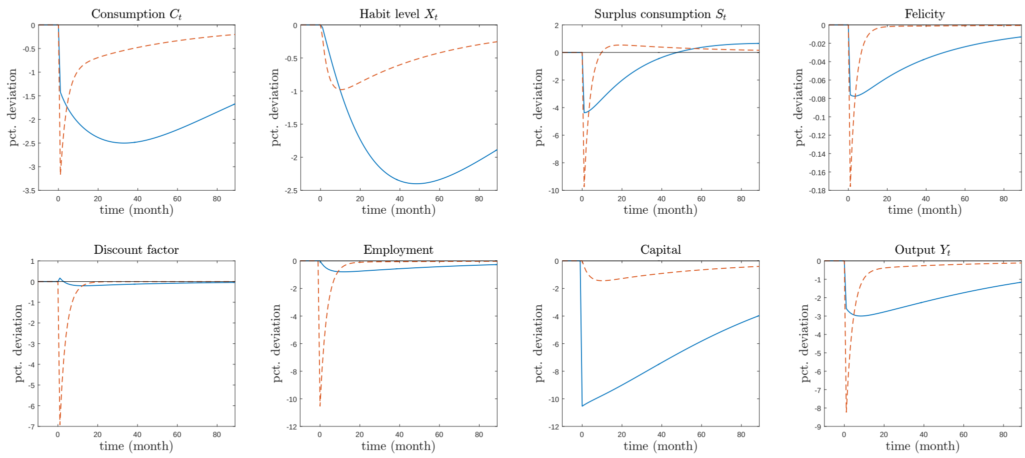


Figure 3.4: Impulse response to a 10% destruction of capital (solid line) or employment (dashed line). All other shocks are set to zero: $\psi_t = 0 \; \forall t \neq 0$ and $\epsilon_{z,t} = 0 \; \forall t$.

3.D Non-negativity of vacancies

Firms should not be able to endogenously separate matches to "earn" vacancy posting costs. In a global solution of the DMP model, Petrosky-Nadeau and Zhang (2017) assume a hard inequality constraint on vacancies. But, a hard constraint introduces a non-differentiability and perturbation methods will fail to solve the model. This section demonstrates how to approximate the constraint with a penalty function and shows that the penalty function has a negligible effect on the policy function of employment. Fortunately, with a higher order approximation of the model, negative vacancy posting becomes very rare, compared to first- and second-order approximations.

Following Den Haan and De Wind (2012), to approximate the constraint $x \geq 0$ the penalty function enters the firm problem as

$$Z(x) = \frac{\zeta_1}{\zeta_0 x} e^{-\zeta_0 x} + \zeta_2 x.$$

Parameter ζ_0 controls the curvature of the function. Exemplary, for $\zeta_2 = 0$,

$$\lim_{\zeta_0 \to \infty} Z(x) = \begin{cases} = \infty & \text{for } x < 0 \\ = 0 & \text{for } x \geq 0. \end{cases} \tag{3.28}$$

For $\zeta_0 \to \infty$ the penalty function implements a non-negativity constraint.

Global solution As a prior note, Petrosky-Nadeau and Zhang (2017) assume a den Haan et al. (2000) matching function, which guarantees $q_t \geq 0$. The non-negativity constraint of vacancies $v_t \geq 0$ is then equivalent to $q_t v_t \geq 0$. The job creation curve in Petrosky-Nadeau and Zhang (2017) reads:

$$\frac{\kappa_t}{q_t} - \lambda_t = \mathbb{E}_t M_{t+1} \underbrace{\left[(1 - \pi_{eu}) \left(\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + \frac{\kappa_{t+1}}{q_{t+1}} - \lambda_{t+1} \right) \right]}_{J_{t+1}},$$

with the Karush-Kuhn-Tucker conditions

$$v_t q_t \geq 0, \quad \lambda_t \geq 0, \quad \lambda_t v_t q_t = 0.$$

Intuitively, the marginal cost of creating a job (adjusted by the multiplier) equals the firm's value of a job. When the constraint becomes binding, the multiplier acts like additional value of a job or discount to the posting costs.

Pitfalls When we introduce a non-negativity constraint in the DMP model, we have to account for equilibrium values of the vacancy-filling rate. First, why do Petrosky-Nadeau and Zhang (2017) implement the constraint as

$q_t v_t \geq 0$? With the constraint $v_t \geq 0$, the job creation curve would read

$$\frac{\kappa_t}{q_t} - \frac{\lambda_t}{q_t} = \mathbb{E}_t M_{t+1} J_{t+1}.$$

When the constraint binds, $\lambda_t > 0$ but in equilibrium $q_t \to 0$. The left-hand-side explodes. Implementing the constraint as $q_t v_t \geq 0$ alleviates this.

Second, in my model's perturbation solution with a penalty function $q_t v_t \geq 0$, the job-creation curve would read

$$\frac{\kappa_t}{q_t} - \left(\zeta_1 e^{-\zeta_0 q_t v_t} - \zeta_2\right) = \mathbb{E}_t M_{t+1} J_{t+1}.$$

When vacancies are negative the penalty grows exponentially and should fulfil the same task as the multiplier: it ought to raise the additional value of a job relative to the posting costs. But when $v_t < 0$, the equilibrium job-finding rate $q_t < 0$, as well. Then, the penalty does the opposite of the KKT multiplier. Hence, neither $v_t > 0$ nor $q_t v_t \geq 0$ are good constraints for this problem.

Solution Given that the Cobb-Douglas function and the perturbation solution used here do not ensure $q_t \geq 0$, I assume a non-negativity constraint on employment's growth rate beyond the constant separation rate $-\pi_{eu}$. Employment's law of motion reads

$$l_{t+1} = l_t(1 - \pi_{eu}) + q_t v_t$$

and reaches its minimum growth rate at $v_t = 0$,

$$\min_{v_t} \log l_{t+1} - \log l_t = -\pi_{eu}.$$

In terms of a constraint,

$$\underline{l}_{t+1} \equiv \log l_{t+1} - \log l_t + \pi_{eu} \geq 0,$$

i.e. the growth rate of employment must not fall below $-\pi_{eu}$. Let function $Z(\underline{l}_{t+1})$ denote a penalty function for endogenous separations. Following

Den Haan and De Wind (2012), assume the particular form

$$Z(\underline{l}_{t+\tau}) = \frac{\zeta_1}{\zeta_0} e^{-\zeta_0 \underline{l}_{t+\tau}} + \zeta_2 \underline{l}_{t+\tau}. \tag{3.29}$$

Parameter ζ_0 controls the curvature of the function. Exemplary, for $\zeta_2 = 0$,

$$\lim_{\zeta_0 \to \infty} Z(\underline{l}_{t+\tau}) = \begin{cases} = \infty & \text{for } \underline{l}_{t+\tau} < 0 \\ = 0 & \text{for } \underline{l}_{t+\tau} \geq 0. \end{cases} \tag{3.30}$$

For $\zeta_0 \to \infty$ the penalty function implements a non-negativity constraint. Such a hard inequality constraint, as implemented in global solutions by Petrosky-Nadeau and Zhang (2017) and Chapter 2, introduces a kink (non-differentiability) at $\underline{l}_{t+\tau} = 0$. For moderately large ζ_0 the penalty function maintains differentiability and perturbation techniques can solve the model.

Let the firm maximize its cum-dividend stock price minus the penalty,

$$\begin{aligned}
P_t^{c,Z} &= \max_{\{v_{t+\tau}, l_{t+\tau+1}, I_{t+\tau}, K_{t+\tau+1}\}_{\tau=0}^{\infty}} \mathbb{E}_t \sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau} - Z(\underline{l}_{t+\tau}) \\
s.t.\ l_{t+1} &= l_t(1-\pi_{eu}) + q_t v_t \\
K_{t+1} &= (1-\delta)K_t + \Phi(K_t, I_t) \\
D_t &= Y_t - N_t W_t - \kappa_{1,t} v_t - q_t v_t \kappa_{2,t} - \Phi(K_t, I_t) \\
\underline{l}_{t+1} &= \log l_{t+1} - \log l_t + \pi_{eu} \\
& \quad K_0,\ l_0 \text{ given.}
\end{aligned}$$

Combining the FOCs for investment and capital yields the investment Euler equation, which remains unaffected by $Z()$,

$$\frac{1}{a_2}\left(\frac{I_t}{K_t}\right)^{\frac{1}{\nu}} = \mathbb{E}_t M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \frac{1}{a_2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^{\frac{1}{\nu}}(1-\delta+a_1) + \frac{1}{\nu-1}\frac{I_{t+1}}{K_{t+1}}\right].$$

Together the FOCs for vacancies and employment yield the employment

Euler equation or job creation curve

$$\frac{\kappa_t}{q_t} - \frac{1}{l_{t+1}}\left[-\zeta_1 e^{\zeta_0 \underline{l}_{t+1}} - \zeta_2\right] =$$
$$\mathbb{E}_t M_{t+1}\left[(1-\pi_{eu})\left(\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + \frac{\kappa_{t+1}}{q_{t+1}} - \frac{1}{l_{t+1}}\left[\zeta_1 e^{-\zeta_0 \underline{l}_{t+2}} - \zeta_2\right]\right)\right].$$

The penalty function appears as two wedges in the job creation curve, which equalizes marginal costs and the firm's value of creating a job. Consider the case $\underline{l}_{t+1} < 0$: The penalty in brackets on the left-hand side grows exponentially and reduces the left-hand side. The penalty acts similar to a Karush-Kuhn-Tucker multiplier in the global solution and raises the firm value of the job relative to its costs. In case $\underline{l}_{t+1} > 0$, the term in brackets reduces to a constant. In steady state $l_{t+1} = l_t$ and $\underline{l}_{t+1} = \pi_{eu}$.

Calibration I set ζ_1 to unity and $\zeta_2 = \zeta_1 e^{-\zeta_0 \pi_{eu}}$ such that the wedge in the job creation curve cancels out in steady state. The remaining parameter, ζ_0, governs the concavity of the the penalty function. If it is too large, the value function becomes non-differentiable. I choose $\zeta_0 = 30$, which is large enough to significantly affect the job creation curve but maintains differentiability.

Policy functions and simulation Figure 3.5 compares the solution of four models: (i) the baseline model in the main text, (ii) the baseline model with penalty function, (iii) a first-order solution and (iv) a second-order solution of the main text model Panels a) and c) compare vacancy posting as a function of employment and TFP. Panels b) and d) compare the employment policy functions. Steady states lie roughly at the intersection of the policy functions. Recall that the steady state of TFP is $\bar{z} > 0$. In the vicinity of the steady state, policy functions of model (i) closely resemble policies of model (ii), i.e. in the vicinity of the steady state the penalty function has a negligible effect on vacancy posting and employment. If we take the penalty function model (ii) as the benchmark case, the third order perturbation beats the first- and second-order perturbations in terms of accuracy. Although model (i)'s vacancy-posting policy deviates from model (ii) for smaller employment states, it does not fall below zero. The first- and second order solutions appear to be more prone to negative vacancy posting for very large l_t and very low z_t.

Panels e) and f) compare example simulations of models (i)-(iv). All economies are simulated with the same series of errors $\{\epsilon_{z,t}\}_{t=1}^{100}$. Differences in vacancy posting are small. The difference between employment in model (i) and (iv) only amounts to less than one percentage point at most.

In summary, the main text model and the model with a penalty function return similar policy functions for employment and vacancies. The third order approximation, used in the main text, beats the first- and second order approximations in terms of accuracy. Den Haan and De Wind (2012) discuss that the penalty function's qualitative effect is comparable to a hard inequality constraint. But, quantitatively the penalty differs from a hard constraint, especially in peaks and troughs. Hence, I neglect the non-negativity constraint in the main text model.

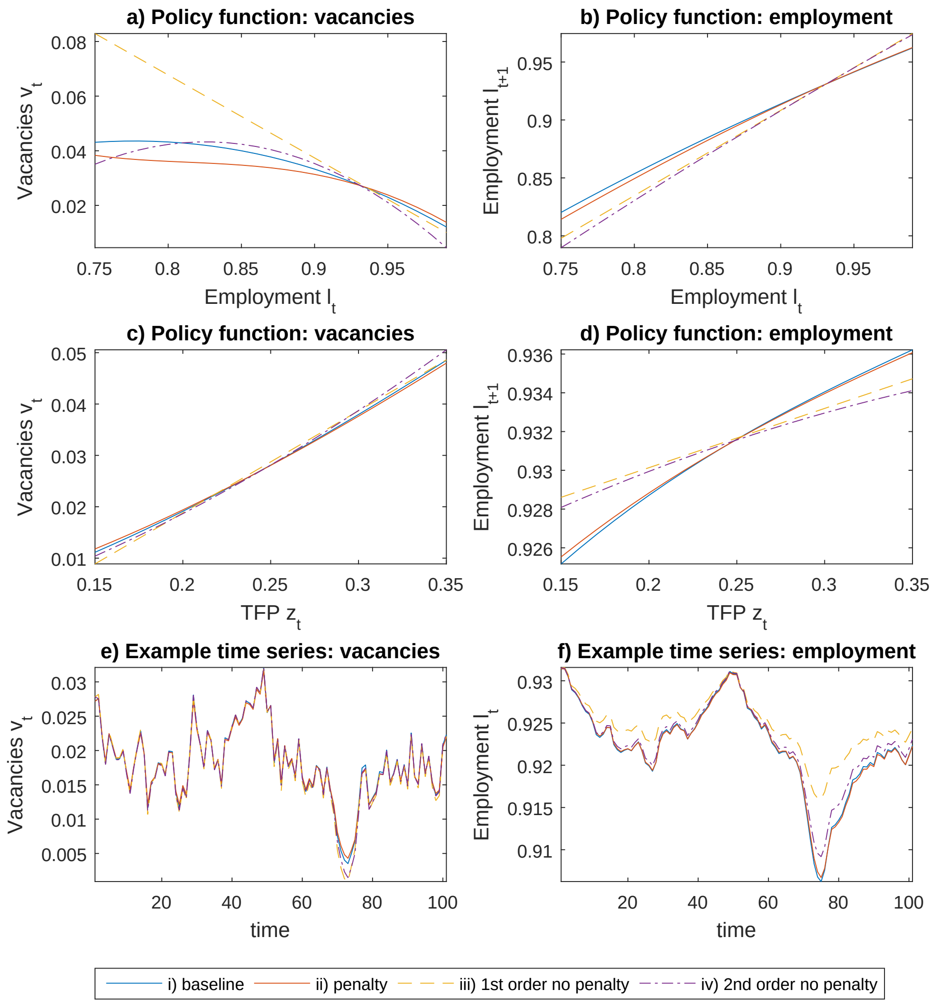


Figure 3.5: Vacancy posting and employment policy functions: (i) the baseline model, (ii) the baseline model with penalty function (3.29), (iii) a first-order solution, (iv) a second-order solution of the model. In Panels a) and b), all states except employment are fixed at steady state. In Panels c) and d) all states except TFP are fixed at steady state. Models (i) and (ii) are solved with a third order perturbation. Panels e) and f) are example simulations of the four model solutions using the same error time series.

Concluding Remarks

This thesis presents three self-contained essays on optimal inheritance taxation, unemployment, and asset prices.

Chapter 1 rationalizes inheritance tax deductions for mature family-owned firms. Under the assumptions that a small family-owned firm is not marketable and hiring an outside manager is not possible or profitable, the firm will be dissolved if there is no intra-family succession. In this case, workers loose match-specific human capital, suffering earnings losses. Inheritance tax deductions for business assets let heirs internalize these earnings losses and incentivize firm succession.

Chapters 2 and 3 study the co-movement of asset prices and labour market flows, developing a framework that consistently solves the equity premium puzzle and the unemployment volatility puzzle. Chapter 2 presents a DMP model with endogenous separations and wage rigidity and finds that neither cyclical fluctuations nor long-run productivity risk suffice to solve the equity premium puzzle.

Chapter 3 introduces capital adjustment costs and slow-moving habits into the DMP framework. This framework robustly solves the unemployment volatility puzzle and generates an equity risk premium comparable to empirical estimates.

In summary, the thesis contributes to multiple branches of literature, showcasing the possibilities that frictional labour markets offer in understanding the economy.

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