Tackling the Challenge of Aging Populations: The Impact of Increasing Life Expectancy and Low Fertility on the Old-Age Dependency Ratio[1]

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Abstract: The old-age dependency ratios are indicators of the number of elderly people who are generally economically inactive compared to the number of people of working age. They significantly affect the financial burden of social public pension schemes, making it essential to analyze the influence of mortality on this ratio. In this paper, the Gompertz model is used to investigate the effect of mortality and fertility on the old-age dependency ratio, with a focus on the impact of changes in life expectancy. Elasticity formulas are derived to analyze this effect, and the results indicate that an increase in life expectancy leads to a considerable rise in the old-age dependency ratio.
Keywords: Actuarial models, Demographic changes, Gompertz distribution, Social security, Life table.

1. Introduction

The old-age dependency ratio is an important demographic indicator that reflects the proportion of elderly people who are not in the labor force and dependent on those who are working. It has become important in analyzing the financial burden of social pension insurance, as it indicates how many potential retirees a potential worker has to support. Its development significantly affects the financial burden of social pension insurance, making it essential to analyze the influence of mortality on the old-age dependency ratio. The Gompertz model is a suitable model for analyzing this influence since it provides a good approximation for low-mortality life tables. According to the United Nations, the old-age dependency ratio is projected to increase significantly in the coming decades. By 2050, it is expected to reach 37% globally, meaning that there will be nearly four elderly people for every ten people of working age. This increase is primarily due to the aging of the baby boomer generation and declining birth rates in many countries.

The Gompertz model is a well-known model of demography that was proposed by Benjamin Gompertz in 1825. It states that the mortality intensity exponentially increases with age in adulthood. It has been much applied in life table analysis and in insurance mathematics using various modifications. Due to declining children and youth mortality, it has again become essential in order to describe "modern" life tables with low mortality. The model allows us to fully describe the present and future life tables in industrialized countries using only two parameters that are easy to estimate from data.

In this paper, we investigate the influence of mortality and fertility on the old-age dependency ratio using the Gompertz model. We derive approximation formulas for the old-age dependency ratio and its elasticity with respect to life expectancy. Elasticities are computed to analyze the influence of a change in life expectancy on the old-age dependency ratio. We also investigate the dependence of elasticities on the population growth rate and the life expectancy. Finally, we will demonstrate that in a stable population with a negative growth

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rate, where fertility is below replacement level, the elasticities are significantly higher than in a stationary population.

2. Analyzing the Effects of Life Expectancy

An increase in the old-age dependency ratio causes an increase in the premium, when the pensions are constant, or a decrease in the pensions, when the premiums stay constant, all else being equal, assuming, for example, that there is no change in the population growth rate or the age classes used to define the old-age dependency ratio. Therefore, it is important to analytically analyze the influence of mortality on the old-age dependency ratio. This will be done hereafter with the Gompertz model since it provides a good approximation for low-mortality life tables (see Pollard, 1991). Especially the effect of changes in the life expectancy on the ratio is analyzed. An increase in the life expectancy will undoubtedly raise the dependency ratio, but by how much?

The old-age dependency ratio is a demographic indicator that measures the number of elderly individuals (as defined by us, aged 60 and older) relative to the working-age population (as defined by us, those aged 20 to 60); often we find the age 65 instead of 60:

$$OADR = \frac{\int_{60}^{\infty} l(x)dx}{\int_{20}^{60} l(x)dx} = \frac{\int_{60}^{\infty} \exp\left(-e^{k\cdot(x-m)}\right)dx}{\int_{20}^{60} \exp\left(-e^{k\cdot(x-m)}\right)dx} = \frac{e_{60}\cdot l_{60}}{e_{20}\cdot l_{20} - e_{60}\cdot l_{60}} = \frac{1}{\frac{e_{20}\cdot l_{20}}{e_{60}\cdot l_{60}} - 1},$$

where $l(x) = \exp\left(e^{-k\cdot m} - e^{k\cdot(x-m)}\right) \approx \exp\left(-e^{k\cdot(x-m)}\right)$ is the survivor function of the Gompertz distribution with m >> k > 0 ; m is the modal value and k is the growth rate of the exponential force of mortality function. For a detailed presentation of the Gompertz distribution, see, e.g., Pollard (1991, 1998), Carriere (1992, 1994) or Pflaumer (2018).

The mean or life expectancy at birth is

$\mu = e_0 = m - \frac{\gamma}{k}$ with $\gamma = 0.57722....$ the Euler-Mascheroni constant.

The variance is $\sigma^2 \approx \frac{\pi^2}{6}\cdot\frac{1}{k^2}$.

Thus, the reciprocal value of k can be regarded as a dispersion parameter.

Typical values for low mortality life tables fall generally within the range of 85 to 90 for m, and 0.09 to 0.11 for k. For instance, using the German life table 2019/2021 for females with a life expectancy of 83.4 years, a fit with the Gompertz distribution yields values of m=89.04 and k=0.1143. This results in an estimated life expectancy of 83.95 years. The use of only two parameters of the Gompertz distribution is sufficient to describe the entire life table and obtain good approximations for the life table parameters of empirical tables. The survivor and density functions of this German life table are presented in the appendix.

The life expectancy at age x can be approximated by

$$e(x) = -\frac{\frac{\gamma + k\cdot(x-m) - \exp\left(k\cdot(x-m)\right)}{k}}{\exp\left(e^{-k\cdot m} - e^{k\cdot(x-m)}\right)}$$ (cf. Carriere, 1992 and 1994).

Substituting the values of the life expectancy and the survivor function at age x in the transformed formula of the old-age dependency ratio, provides a good approximation of the ratio when the modal age m is greater than 70:

$$OADR_1 \approx \frac{1}{\dfrac{\gamma - k\cdot(m-20) - \exp\left(-k\cdot(m-20)\right)}{\gamma - k\cdot(m-60) - \exp\left(-k\cdot(m-60)\right)} - 1}.$$

If the modal value m is still increasing, then the ratio finally tends to

$$OADR_{hat} = \frac{1}{\dfrac{\gamma - k\cdot(m-20)}{\gamma - k\cdot(m-60)} - 1} = \frac{m - \dfrac{\gamma}{k} - 60}{40} = \frac{e_0 - 60}{40}.$$

Figure 1 displays a 3-dimensional plot of the OADRs, which depend on both k and e(0). The OADR is primarily determined by life expectancy and is less affected by k. An increase in life expectancy leads to a considerable rise in OADR, while increasing k only slightly decreases OADR. The numerical results of OADR for different e(0) and k values are presented in the appendix.

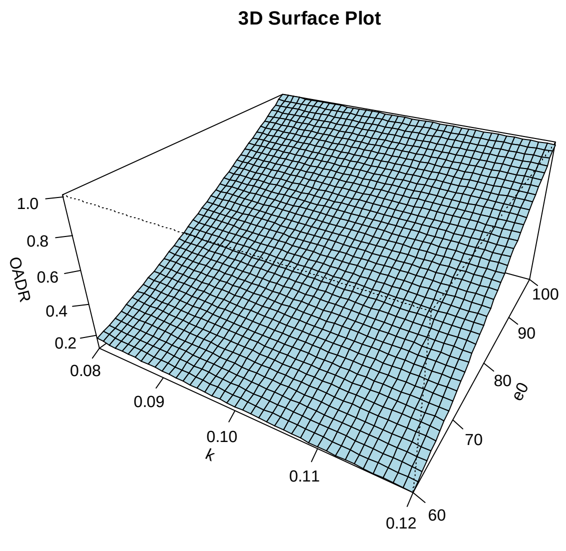

Figure 1. Surface plot of OADR as a function of e(0) and k (see also the values in the appendix)

Table 1 presents the old-age dependency ratios for a fixed value of k=0.1, calculated through numerical integration. The difference between the exact and approximate values is negligible when the modal ages are high. Moreover, even the simple approximation formula provides satisfactory results for very old ages.

Table 1. Old-age dependency ratios and elasticities: Exact and approximate values (k=0.1)

e0	m	OADR	OADR1	OADRhat	Elast	Elasthat
65	70.8	0.220	0.229	0.125	5.18	13
70	75.8	0.314	0.318	0.25	4.42	7
75	80.8	0.418	0.419	0.375	3.86	5
80	85.8	0.528	0.529	0.5	3.43	4
85	90.8	0.644	0.644	0.625	3.09	3.4
90	95.8	0.762	0.762	0.75	2.83	3
95	100.8	0.883	0.883	0.875	2.62	2.71
100	105.8	1.005	1.005	1	2.44	2.5

To analyze the influence of a change in the life expectancy on the old-age dependency ratio, elasticities are computed. Elasticity is the ratio of the percent change in one variable to the percent change in another variable. Mathematically, elasticity is defined as

$$\varepsilon\left(\mathrm{OADR}, e_0\right) = \frac{d\mathrm{OADR}}{de_0} \cdot \frac{e_0}{\mathrm{OADR}} .$$

Using the above approximation formula leads to a rather complicated formula

$$\varepsilon_1\left(\mathrm{OADR}_1, e_0\right) = \frac{k \cdot e^{\gamma + e_0 \cdot k}\left(40 \cdot k \cdot e^{\gamma + e_0 \cdot k} - e^{20 \cdot k}\left(e^{40 \cdot k}\left(e_0 \cdot k - 20k + 1\right) - e_0 \cdot k + 60k - 1\right)\right)}{\left(40 \cdot k \cdot e^{\gamma + e_0 \cdot k} + e^{20 \cdot k}\left(1 - e^{40 \cdot k}\right)\right) \cdot \left(k \cdot e^{\gamma + e_0 \cdot k}\left(e_0 - 60\right) + e^{60 \cdot k}\right)} \cdot e_0 ,$$

where

$$e_0 = m - \frac{\gamma}{k} .$$

An easy-to-use elasticity formula is obtained using the simple approximation

$$\varepsilon_{hat}\left(\mathrm{OADR}_{hat}, e_0\right) = \frac{d\left(\frac{e_0 - 60}{40}\right)}{de_0} \cdot \frac{e_0}{\frac{e_0 - 60}{40}} = \frac{e_0}{e_0 - 60} .$$

The elasticity provides insight into the proportional change in the old-age dependency ratio (OADR) in response to a 1 percent increase in the life expectancy. Specifically, at age 85 (e(0)=85), it indicates that such an increase in life expectancy leads to approximately a 3 percent rise in the OADR, as shown in Table 1.

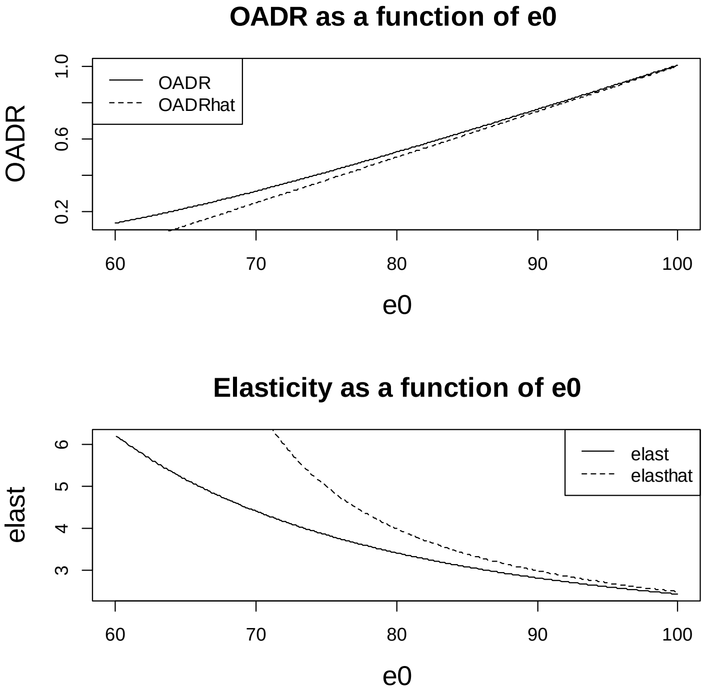

Figure 2. OADR and Elasticity as a function of e(0) with k=0.1; dotted lines represent approximations with the simple formulas

Figure 2 and Table 1 illustrate the elasticities (elast) of the old-age dependency ratio (OADR) as a function of life expectancy. The results indicate that a 1% increase in life expectancy would result in approximately a 3% increase in the OADR for a stationary population with low mortality.

3. Stable Populations and Low Fertility

A population with an unchanging age structure and a fixed rate of increase is called a stable population (see, e.g., Keyfitz, 1977). To determine the old-age dependency ratio in a stable population with a growth rate of r, one must compute

$$OADR(r) = \frac{\int_{60}^{\infty} e^{-r \cdot x} l(x)dx}{\int_{20}^{60} e^{-r \cdot x} l(x)dx}.$$

This expression can be approximated based on Keyfitz (1977) by

$$OADR(r) \approx OADR(0) \cdot e^{-T \cdot r},$$

where T is the difference between the mean age of the two generations in the age classes 20 to 60 and 60 to ω.

Using the simple approximation of the old-age dependency ratio, for example, results in the following elasticity formula:

$$\varepsilon\left(\mathrm{OADR}, e_0\right) \approx \frac{e_0}{e_0 - 60} \cdot e^{-T \cdot r} .$$

The elasticities are now dependent on the growth rate r and the life expectancy e(0). In a stable population with a positive (negative) growth rate, the elasticities are lower (higher) than in a stationary population by the factor $e^{-T \cdot r}$. For example, at life expectancy e(0)=85, the elasticity is initially 3.09 (see Table 1). If we consider a negative growth rate due to low fertility of -1% and a generation difference of 30 years (T=30), the factor $e^{-T \cdot r}$ is approximately 1.35, resulting in an increased elasticity of about 4 under these specific demographic conditions and generation difference.

4. Conclusion

The paper emphasizes the importance of studying the profound impact of an aging population on social welfare policies and the economy. A mere 1% increase in life expectancy translates to approximately a 3% rise in the old-age dependency ratio within a stationary population exhibiting low mortality. This effect is even more pronounced in stable populations with low fertility rates, falling below replacement levels. As the population ages persistently, it is imperative for policymakers to effectively address the requirements of both the elderly and the younger generations, thereby fostering sustainable economic growth and upholding social welfare. Neglecting this balance could entail consequential economic and social repercussions for generations to come.

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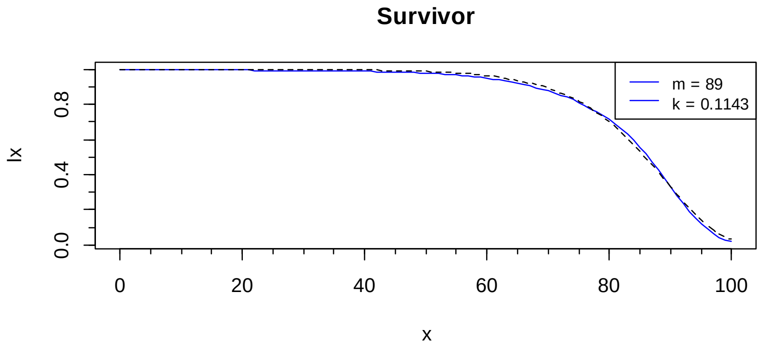
Appendix:



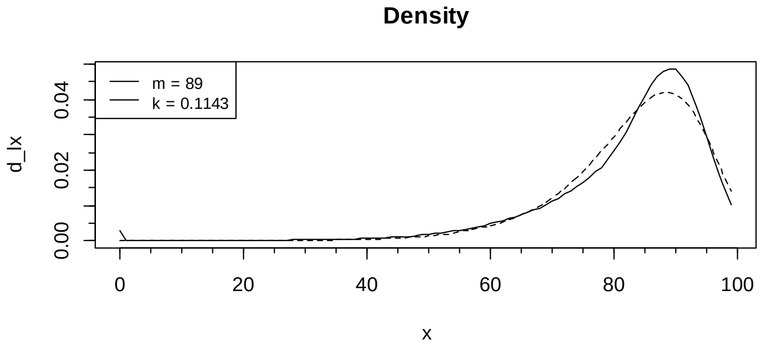


Figure A1. Comparison between actual (solid lines) and estimated (dotted lines) survival and death density functions (German life table 2019/2021 for females: e(0)=84; OADR=0.61)

Table A1. Old-age dependency rations as a function of k and e(0)

k/e(0)	**60**	**65**	**70**	**75**	**80**	**85**	**90**	**95**	**100**
0.08	0.180	0.259	0.349	0.447	0.553	0.663	0.777	0.895	1.014
0.082	0.175	0.254	0.344	0.444	0.549	0.660	0.775	0.893	1.013
0.084	0.170	0.250	0.340	0.440	0.546	0.658	0.773	0.891	1.011
0.086	0.166	0.245	0.336	0.437	0.543	0.656	0.771	0.890	1.010
0.088	0.161	0.241	0.333	0.433	0.541	0.653	0.770	0.889	1.009
0.09	0.157	0.237	0.329	0.430	0.538	0.651	0.768	0.887	1.008
0.092	0.153	0.233	0.326	0.428	0.536	0.650	0.767	0.886	1.008
0.094	0.150	0.230	0.323	0.425	0.534	0.648	0.765	0.885	1.007
0.096	0.146	0.227	0.320	0.422	0.532	0.646	0.764	0.884	1.006
0.098	0.143	0.223	0.317	0.420	0.530	0.645	0.763	0.884	1.006
0.1	0.140	0.220	0.314	0.418	0.528	0.644	0.762	0.883	1.005
0.102	0.137	0.217	0.312	0.416	0.527	0.642	0.761	0.882	1.005
0.104	0.134	0.215	0.309	0.414	0.525	0.641	0.760	0.882	1.004
0.106	0.131	0.212	0.307	0.412	0.524	0.640	0.760	0.881	1.004
0.108	0.128	0.209	0.305	0.410	0.522	0.639	0.759	0.881	1.003
0.11	0.126	0.207	0.303	0.408	0.521	0.638	0.758	0.880	1.003
0.112	0.123	0.205	0.301	0.407	0.520	0.637	0.758	0.880	1.003
0.114	0.121	0.202	0.299	0.405	0.519	0.637	0.757	0.879	1.003
0.116	0.118	0.200	0.297	0.404	0.518	0.636	0.756	0.879	1.002
0.118	0.116	0.198	0.295	0.403	0.517	0.635	0.756	0.879	1.002
0.12	0.114	0.196	0.294	0.401	0.516	0.634	0.756	0.878	1.002