Probabilistic Perspectives on Sex Ratio at Birth Dynamics[1]

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Abstract: Predicting trends in Sex Ratio at Birth (SRB) is crucial in demographic research, shedding light on evolving population dynamics. This study conducts a thorough investigation into the selection and evaluation of optimal forecasting models for SRB data. Utilizing historical SRB records from selected countries, we meticulously assess various models, including Autoregressive Integrated Moving Average (ARIMA), Autoregressive (AR), and White Noise models. Our empirical analysis reveals the prominence of the AR(2) model in capturing intricate SRB dynamics. Additionally, we explore the White Noise model's role in understanding and predicting SRB fluctuations. Our findings emphasize the AR(2) model's efficacy, attributed to its parsimonious complexity, empirical validation, theoretical alignment, and superior statistical performance. Extending projections to 2070 for Germany, our study not only offers foresight into future SRB trends but also contributes a robust methodology to the broader field of time series analysis.

Keywords: Demographic Trends, Autoregressive Models, Forecasting Methodology, White Noise Model.

1. Introduction

The sex ratio at birth (SRB) remains a complex demographic phenomenon influenced by diverse factors. Despite extensive research, a definitive explanation remains elusive (Pavić, 2009). This paper explores SRB fluctuations using time series models, analyzing data from 39 countries[2] with the R statistical software. Our aim is to propose models for presenting and forecasting SRB trends.

The choropleth map 1 displays SRB values across regions, representing the number of newborn boys per 100 newborn girls. Color-coded ranges depict varying SRB values:

Light Green (103-105): This range illustrates SRB values between 103 and 105, with regions such as the United States, South America, and most African countries falling into this category. These areas exhibit a slightly male-biased SRB, with approximately 103 to 105 newborn boys for every 100 newborn girls.

Dark Green (105-107): The dark green range signifies SRB values between 105 and 107. European countries, North African countries, the Near East region, and Australia belong to this range. These regions also show a slightly higher male-biased SRB, with approximately 105 to 107 newborn boys per 100 newborn girls.

Blue (107-110): The blue range indicates SRB values between 107 and 110, primarily represented by India and China. In these countries, the SRB is significantly higher, with approximately 107 to 110 newborn boys for every 100 newborn girls. This denotes a considerable gender imbalance in favor of male births.

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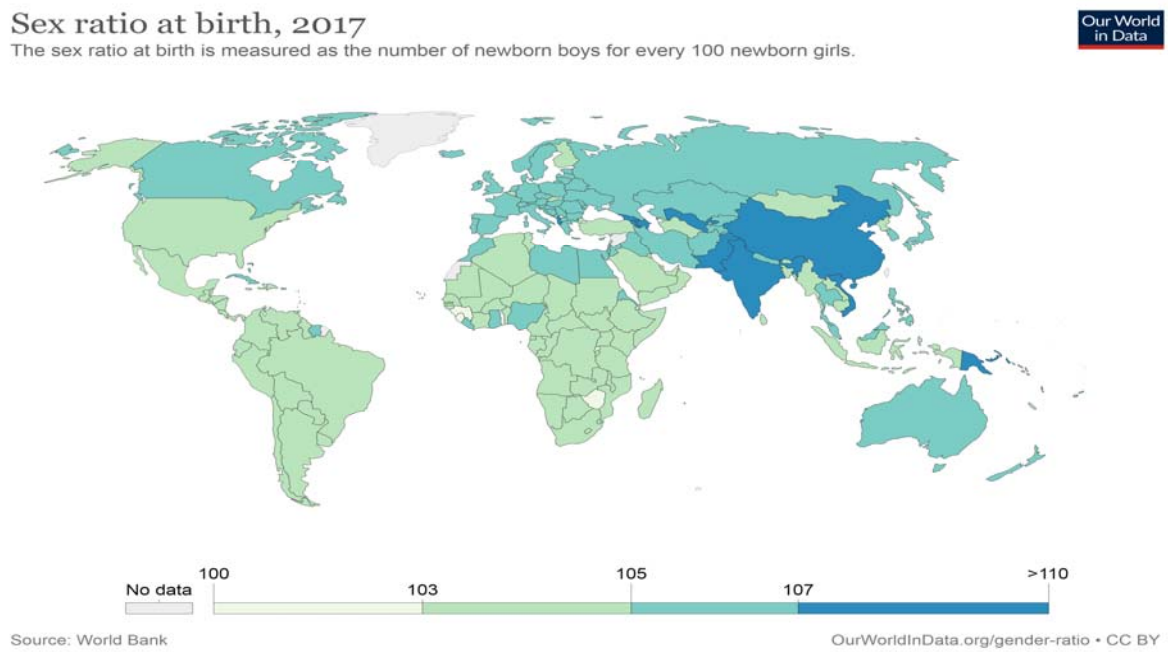
[2] Australia, Austria, Belarus, Belgium, Bulgaria, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Greenland, Hungary, Iceland, Ireland, Israel, Italy, Japan, Latvia, Lithuania, Luxembourg, Netherlands, New Zealand, Norway, Poland, Portugal, Russia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Taiwan, United Kingdom, Ukraine, USA.



Map 1: Sex Ratio at Birth (SRB) - Global Distribution of Male Births per Female Births

The choropleth map serves as a powerful tool, visually revealing the variations in SRB across diverse regions and providing significant insights into its global distribution. Its utility lies in facilitating the swift identification of areas exhibiting potential gender imbalances. This visualization highlights regions where issues related to gender equality and family planning practices may necessitate additional attention and targeted interventions.

1.1 Defining Primary and Secondary Sex Ratio at Birth

The primary sex ratio at birth refers to the proportion of male and female embryos conceived, which is typically considered to be 1:1. The secondary sex ratio at birth, on the other hand, pertains to the ratio of male to female live births. In a study conducted by Orzack et al. (2015), data was analyzed from a range of sources, including 3- to 6-day-old embryos, induced abortions, fetal membrane samples, miscarriages, and live births.

Summarized Main Findings: The study's findings provide valuable insights into the shifts and patterns of primary and secondary sex ratios at various stages of pregnancy:

- Primary Sex Ratio at Conception: At the point of conception, the distribution of male and female embryos is nearly equal, maintaining a 1:1 ratio.
- Mid-Pregnancy Male Bias: As pregnancy progresses, there is a gradual increase in male bias attributed to excess female mortality. This bias continues until mid-pregnancy.
- Steady Mid-Pregnancy Ratio: From mid-pregnancy until the end of the seventh month, the sex ratio remains relatively constant at around 1.28 (male to female).
- Late-Pregnancy Shift: The final two months of pregnancy witness a decline in the sex ratio due to increased male mortality.

1.2 Gini´s White Noise Model

In Gini's (1908) formulation, the concept of "White Noise" is employed to describe the sex ratio at birth (SRB). This notion suggests that in the absence of dominant factors or sets of factors influencing SRB, the appropriate time-series model would exhibit characteristics resembling random noise. Gini's observation is particularly relevant for short-term series where no prominent influencing factors are at play. This concept serves as a foundation for understanding SRB fluctuations in cases where no discernible patterns or trends are apparent. Moreover, the intricacies of real-world SRB data often reveal deviations from the idealized "White Noise" scenario. These deviations manifest as increasing or decreasing trends, and occasionally singular effects, such as noticeable SRB spikes following periods of conflict[3]. To address these complexities, advanced modeling techniques are essential. Among these, the Autoregressive Integrated Moving Average (ARIMA) models offer a potent framework. By encompassing various patterns arising from the interplay of different factors, ARIMA models allow us to meticulously dissect and comprehend the influences that contribute to SRB fluctuations.

Our empirical analysis demonstrates the versatility of ARIMA models in capturing the diverse dynamics of SRB across multiple countries. Intriguingly, our findings also reveal that for a significant number of countries, the "White Noise" model stands as an appropriate choice.

2. Sex Ratio at Birth as a Binomial Process

Fluctuations in the sex ratio at birth (SRB) result from both random and systematic influences. If only random influences were present, we could explain SRB fluctuations through the binomial model.
In the binomial model: N represents the sample size (number of births). p is the probability of success (probability of a male B_M or female birth B_F). X represents the number of male or female births.

$$P(X = x) = \binom{N}{x} \cdot p^x \cdot (1 - p)^{N-x}$$

Assumptions:
The probability of success is the same for each birth.
Births are independent replications.

However, there are additional variations to consider, including Lexis Variation, Poisson Variation, and Markov Variation. This is particularly relevant in the context of sex ratio at birth (SRB), where these variations refer to different sources of variability that can impact the

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3 Other Possible Singular Effects:
Wars and Conflicts: Typically lead to an increase in SRB due to stress-induced biological responses favoring male fetuses.
Technological Advances (Prenatal Sex Determination): Likely lead to an increase in SRB due to selective abortions of female fetuses.
Economic Prosperity: Can lead to a decrease in SRB due to improved healthcare and nutrition, reducing gender-selective effects.
Natural Disasters: May lead to a decrease in SRB due to stress and adverse conditions affecting male embryos.
Social Policies and Gender Equality: Can lead to a decrease in SRB by reducing gender-based discrimination and bias.
Medical Interventions: Can lead to an increase in SRB by enabling gender-selective treatments or interventions.
Cultural and Religious Practices: May lead to both increases or decreases in SRB depending on specific practices and beliefs.
Migration and Demographic Shifts: Can influence SRB depending on the gender composition of migrating populations.
Environmental Factors: Can lead to changes in SRB based on the impact of environmental stressors on fetal development.
Healthcare Accessibility: Can impact SRB by affecting the survival rates of male and female fetuses.

distribution of male and female births within a population. In many studies, the agreement with the binomial distribution is only modest. Even in cases where the agreement seems substantial, it's possible that various effects have overlapped, rendering them indistinguishable. By considering Lexis, Poisson, and Markov variations (see, e.g. James (2000), researchers gain a comprehensive understanding of the multifaceted factors that influence the sex ratio at birth.

Lexis Variation:
Lexis variation pertains to the differences in SRB observed among different subpopulations within a larger population. It highlights how SRB can vary based on factors such as geographical location, ethnicity, socioeconomic status, and other demographic characteristics. Lexis variation can be influenced by cultural norms, societal practices, and regional preferences for sons or daughters. Researchers study Lexis variation to better comprehend the demographic dynamics and gender-related implications in different subsets of the population.

Poisson Variation:
Poisson variation, also known as family-level variation, refers to the fluctuations in SRB that occur within individual families. It explores the variability in SRB among siblings within the same family, where the SRB may differ from one birth to another. Poisson variation occurs due to random biological factors and chance events during conception and gestation processes. Factors like genetic influences, maternal health conditions, and environmental exposures may contribute to Poisson variation in SRB. It does not imply any intentional gender selection or sex-specific practices within families but reflects inherent variability in sex determination during conception.

Markov Variation:
Markov variation refers to the dependency or correlation between the sex ratio at birth in successive time periods. It explores how the SRB in one time period might influence or be influenced by the SRB in the following time period. Markov variation can result from various factors, including societal changes, policies, and demographic trends that affect the sex ratio at birth over time. Understanding Markov variation helps researchers identify temporal patterns and potential long-term shifts in SRB.

Assuming the binomial distribution, the variance of the SRB estimator is approximately (see Casella; Berger, 2002, p.242):

$Var\left(\frac{B_M}{B_F}\right) = Var\left(\frac{\hat{p}}{1-\hat{p}}\right) \approx \frac{p}{N\cdot(1-p)^3} \approx \frac{4}{N}$, where $N=B_M+B_F$ is the number of births. If p is close to 0.5, then $Var\left(\frac{B_M}{B_F}\right) \approx \frac{4}{N}$.

The smaller the number of births, the greater the fluctuations in SRB, all else being equal. Statistically, $\frac{\hat{p}}{1-\hat{p}}$ is an estimator for the odds ratio $\frac{p}{1-p}$.

Due to the dominance of numerous (independent) influences on SRB, the white noise model is sufficient to explain the temporal development of SRB, provided there are no systematic trends. SRB fluctuates randomly around the mean μ:

$$SRB_t = \mu + u_t$$

with $E\left(u_t = 0\right)$

$$E\left(u_t u_s\right)=\begin{cases}\sigma^2 & t=s\\ 0 & otherwise\end{cases}$$

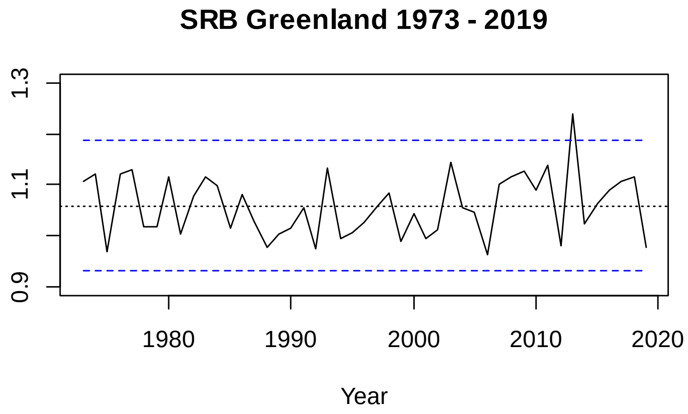
where μ, the mean function, reflects the deterministic part, and u_t represents the stochastic part of the SRB.

Example: Binomial Process: Sex Ratio at Birth in Greenland (N = 967; p = 0.5143; SRB = 1.059) [4]
Figure 1 (Top) displays the SRB between 1973 and 2019 (source: Statbank Greenland; bank.stat.gl/pxweb/en/Greenland).

The Ljung-Box test is used to check for the presence of autocorrelation in the data. A low p-value indicates that there is evidence of autocorrelation, meaning that the data points are not independent, and there might be some underlying patterns or trends affecting the SRB over time.

The observed p-values from the Ljung-Box test indicate that the sex ratio at birth (SRB) exhibits evidence of following a binomial process. The p-values obtained from the test (p = 0.3998/0.3628, df = 24) suggest that there is no significant autocorrelation in the SRB data. In other words, the data points appear to be relatively independent, supporting the assumption that the fluctuations in SRB can be explained by the binomial model.
The absence of significant autocorrelation implies that the SRB values at different time points do not depend on each other and are not influenced by any systematic trends over time. Therefore, the binomial process adequately captures the random variations in the sex ratio at birth, making it a suitable model for explaining the observed fluctuations in SRB over the given time span (1973-2019) in Greenland.
Overall, the findings from the Ljung-Box test provide empirical support for the suitability of the binomial model in explaining the temporal dynamics of SRB and further validate the randomness in sex ratio fluctuations at birth.

[4] N = 967 represents the average annual number of births between 1973 and 2019.



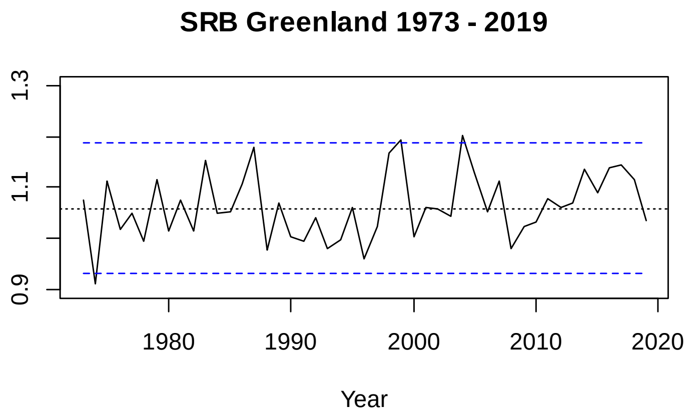


Fig. 1: Sex Ratio at Birth in Greenland from 1973-2019 (Top) [p=0.5143, N=967] and Simulated Sex Ratio at Birth from 1973-2019 (Bottom) [p=0.5143, N=967]
p-values=0.3998 (Top) and 0.3628 (Bottom) with df=24.

3. Trends and Peaks after Wars

Before embarking on an exploration of historical trends and unique patterns in sex ratio at birth (SRB) that emerge following significant events, it is vital to acknowledge the substantial body of research that extensively addresses the influences molding SRB. These comprehensive investigations span a wide spectrum of factors, including the intricate interplay of biological, environmental, cultural, and socioeconomic determinants. For instance, researchers have scrutinized the role of maternal age, health conditions, and lifestyle choices in shaping SRB. Furthermore, studies have probed the impact of environmental factors such as pollution and variations in climate. Sociocultural dynamics, exemplified by son preference and gender bias, have also been recognized as influential determinants. Additionally, socioeconomic status and economic development have demonstrated links to SRB disparities. Recognizing these multifaceted influences, our focus transitions to analyzing historical trends and SRB patterns. In particular, we explore the extent to which pivotal historical events, demographic shifts, and modern complexities contribute to the observed trends and peaks in SRB. Guided by insights gleaned from renowned researchers, including Düsing (1884), Tschuprov (1915), Fisher (1930), Mackenroth (1953), Jöckel and Pflaumer (1981), Chahnazarian (1988), James (1987), Gellatly (2009), Grech and Mamo (2014), Bethmann and Kvasnicka (2014), Strahlenschutzkommision (2014), Scalone and Rettaroli (2015), Ritchie and Roser (2020), we embark on an investigation into how wars, shifts in fertility, and contemporary factors contribute to the intricate tapestry of observed SRB trends and peaks.

3.1 Historical Trends during the Demographic Transition from High to Low Fertility

Increasing Trends (e.g., Sweden):

- In countries like Sweden, historical data reflects an increasing trend in SRB during the transition from high to low fertility.
- Socioeconomic improvements have been associated with decreased stillbirth rates, which tend to be higher among male neonates.
- Birth distribution by birth order and parental ages also contribute to the SRB trend, as SRB decreases with increasing parity and the age of the father.
- The socioeconomic and demographic factors collectively contribute to the observed increase in SRB.

Decreasing Trends (e.g., France 19th century):

- During the 19th century in France, a decreasing SRB trend was observed.
- Attempts at abortion during this period were found to affect relatively more male neonates.
- The prevalence of abortion attempts and their selective impact on male neonates explain the observed decreasing SRB trend.

3.2 Contemporary Declining Trends in Industrialized Countries

Nutritional Habits and Environmental Hazards:

- Contemporary trends in industrialized countries reveal declining SRBs influenced by factors such as nutritional habits, including slimming diets.
- Environmental hazards, like chemical substances in makeup and baby powder, have also been linked to declining SRBs.
- Ecological factors, including climate and air pollution, play a role in shaping SRB trends.
- The interplay of changing nutritional habits, exposure to environmental hazards, and ecological conditions contribute to the contemporary declining SRB trends.

3.3 Increase during and after Wars

Impact of Wars on SRB:

- Wars have been associated with unique SRB patterns, characterized by increases during and after wartime periods.
- The tremendous stress experienced by women during wars, along with high levels of estrogen and testosterone, can elevate the probability of male births.
- Nonprogrammed copulation and elevated coital rates during wartime contribute to more conceptions occurring early or late in the menstrual cycle.
- Birth parity also influences Sex Ratio at Birth (SRB), as many couples tend to have their first child during or after wars, given that SRB is highest for the first child.
- Families with more sons are likely to have a higher number of sons still alive, leading to more male births in the next generation, within the framework of Lexis Variation.
- The combined effects of stress-induced hormonal changes, altered copulation patterns, and birth parity influence the observed SRB increases during and after wars.

3.4 Examples

Figures 2 shows examples of SRB trends:

- Sweden (1749-2014): An increasing trend with a peak after World War II.
- The United States (1933-2014): A decreasing trend with a peak during and after World War II.
- France (1806-2014): A decreasing trend during the 19th century, with peaks following World Wars I and II, and a continuing decrease since the 1960s.
- Norway: Exhibits white noise, despite being a long series with 100 observations.

These examples emphasize the intricate interplay between historical events, demographic shifts, and societal factors in shaping the sex ratio at birth (SRB). The varied trends observed across different countries and time periods underscore the complexity of SRB dynamics, highlighting the necessity for nuanced analyses beyond simplistic models.

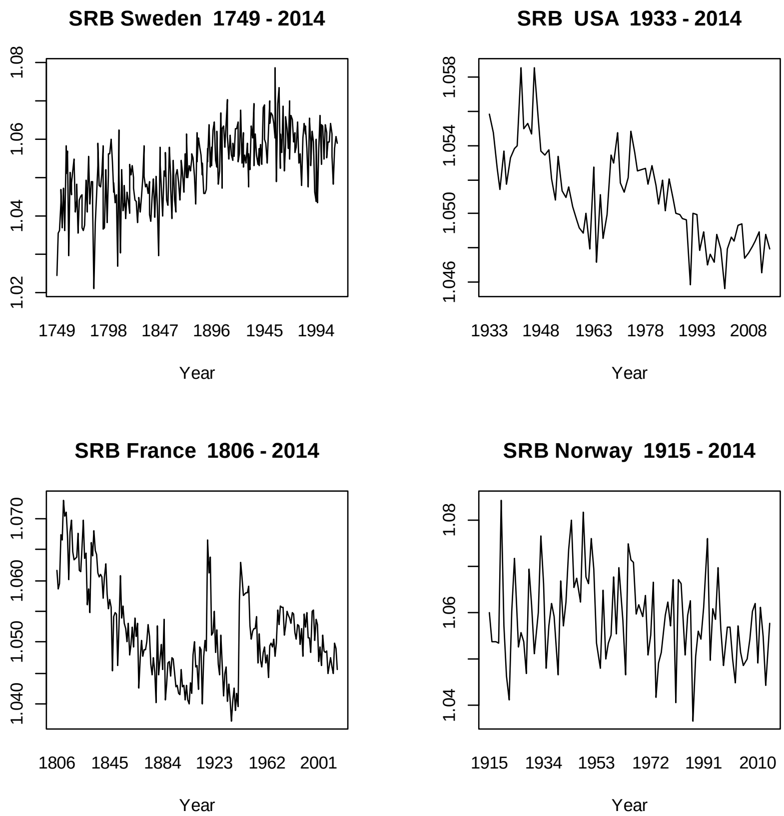


Fig. 2: Trends of the Sex Ratio at Birth in four Countries

4. Empirical Analysis

4.1 Used Models

In our pursuit of deeper insights into the sex ratio at birth (SRB) dynamics, this chapter engages in an empirical analysis utilizing specific models to uncover patterns and trends. The models employed include:

4.1.1 White Noise Model (No Dominating Influence Factor)

The notion of the SRB being governed by the same probability laws as random phenomena was postulated by Gini (1908). The White Noise Model stands as a fundamental approach for situations where no single influencing factor dominates the sex ratio at birth. The formula capturing this model elucidates the inherent randomness present within the SRB data:

$$SRB_t = \mu + u_t$$
with $E(u_t = 0)$
$$E(u_t u_s) = \begin{cases} \sigma^2 & t = s \\ 0 & otherwise \end{cases}$$

where μ, the mean function, reflects the deterministic part, and u_t represents the stochastic part of the SRB.

4.1.2 Autoregressive Models ARIMA(p,d,q) (for Trend Factors)

As we embark on the investigation of potential trend factors influencing SRB, Autoregressive Models ARIMA(p,d,q) come to the fore. These models offer a mechanism to comprehend the impact of historical trends on SRB. The formula encapsulates the complexities of these models, which aid in capturing the temporal evolution of sex ratio dynamics.

The here proposed models are known as ARIMA(p,d,q) models in time series analysis (see, e.g., Chatfield and Xing, 2019). The acronym ARIMA stands for Auto-Regressive Integrated Moving Average**,** where:

- **p** is the number of autoregressive terms,
- **d** is the number of nonseasonal differences needed for stationarity,
- **q** is the number of moving average terms.

The ARIMA(p,d,q) model for forecasting the sex ratio at birth (SRB) can be expressed as:

$$SRB_t^d = c + \phi_1 SRB_{t-1}^d + \phi_2 SRB_{t-2}^d + ...\phi_p SRB_{t-p}^d + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + ... + \theta_{t-q} \varepsilon_{t-q},$$

where:

SRB_t^d is the diffenced sex ratio at birth at time t in the time series.
c is a constant term.
$\phi_1, \phi_{2,...} \phi_p$ are the autoregressive coefficients for lag 1, lag 2, and so on up to lag p.
$\theta_1, \theta_{2,...} \theta_q$ are the moving average coefficients for lag 1, lag 2, and so on up to lag q.
ε_t represents the white noise error term at time t.

The d parameter represents the number of differences needed to make the SRB time series stationary. It's used to transform the original SRB time series into SRB_t^d a stationary series $\Delta^d SRB_t$, where Δ is the difference operator. The series may need to be differenced more than once in order to achieve stationarity. In this representation, the ARIMA model combines autoregressive (AR) terms, moving average (MA) terms, and differencing to model the patterns and relationships in the sex ratio at birth data. The predictors on the right hand side include both lagged values of SRB and lagged errors.

Commonly employed models include:

1. ARIMA(p,0,q) with a constant (using the original observations)

$$SRB_t = c + \phi_1 SRB_{t-1} + \phi_2 SRB_{t-2} + ...\phi_p SRB_{t-p} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + ... + \theta_{t-q} \varepsilon_{t-q} .$$

2. AR(2,0,0) Model with a constant or Autoregressive Model of Order 2 (AR(2)) (using the original observations)

$$SRB_t = c + \phi_1 \cdot SRB_{t-1} + \phi_2 \cdot SRB_{t-2} + \varepsilon_t .$$

3. ARIMA(1,1,0) using the first differences of the original observations

$$\Delta SRB_t = c + \phi_1 \cdot \Delta SRB_{t-1} + \varepsilon_t \quad \text{or}$$
$$\left(SRB_t - SRB_{t-1}\right) = c + \phi_1 \cdot \left(SRB_{t-1} - SRB_{t-2}\right) + \varepsilon_t .$$

These models use past observations and their differences to forecast future SRB values, and the parameter values are estimated based on the historical data. The ARIMA(2,0,0) model considers the original SRB values directly, while the ARIMA(1,1,0) model focuses on the changes between consecutive SRB values. Modeling the sex ratio at birth with an ARIMA(1,1,0) model must assume that c=0 in the long run because the SRB cannot exhibit a long-lasting trend; after a while, the SRB will revert to its usual value around 1.05.

4.2 Data Basis and Model Selection

4.2.1 Data

Our investigation relies on Annual SRB observations until 2014, sourced from The Human Mortality Database (mortality.org). This dataset includes birth and death time-series data spanning various periods and countries. Data for Germany are obtained from Statistisches Bundesamt, and information is analyzed until 2019.

4.2.2 Criteria for Model Selection

The pivotal task of model selection is approached methodically, considering several criteria:

- The p-value of the Ljung-Box test (with lag 24) is leveraged to ascertain white noise characteristics.
- Visual analyses of Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) charts guide the determination of AR(p) model orders.
- Parameter estimation is executed rigorously.
- Estimates are evaluated based on p-values and Akaike Information Criterion (AIC).

4.2.3 Detailed Results

For a comprehensive perspective, detailed results are presented, both for the entire time span and the narrower time span of 1964-2014. These results are encapsulated in Tables 1 and 2, offering insights into the model selection process. Supplementary to these tables, appendix materials comprise SRB time series graphs and ACF/PACF charts for each country, providing a visual aid in understanding the model choices. Tables A1 and A2 in the appendix list the results by country in alphabetical order.

Table 1: Results for the Entire Time Span

Country	Observations	p-(Ljung-Box)	Model	Mean SRB
Ukraine	68	0.000	AR1/AR3	1.060
Australia	152	0.184	AR1/WN	1.053
Austria	144	0.000	AR2	1.057
Chile	99	0.000	AR2	1.046
Denmark	180	0.000	AR2	1.054
France	209	0.000	AR2	1.052
Germany*	136	0.000	AR2	1.058
Hungary	65	0.000	AR2	1.061
Poland	57	0.000	AR2	1.063
Portugal	127	0.000	AR2	1.065
Russia	56	0.000	AR2	1.055
Spain	107	0.000	AR2	1.075
Taiwan	104	0.000	AR2	1.071
United Kingdom	92	0.000	AR2	1.054
USA	82	0.000	AR2	1.051
Netherlands	163	0.000	AR3	1.056
Sweden	266	0.000	AR3	1.052
Belgium	175	0.000	AR4	1.052
Finland	148	0.000	AR4/AR2	1.052
Italy	151	0.000	AR4/AR2	1.059
Japan	136	0.000	AR4/AR2	1.049
Bulgaria	64	0.176	WN	1.060
Canada	91	0.790	WN	1.056
Czech Republic	68	0.367	WN	1.058
Estonia	55	0.833	WN	1.059
Greece	33	0.494	WN	1.067
Iceland	176	0.232	WN	1.053
Ireland	65	0.254	WN	1.058
Israel	32	0.616	WN	1.055
Latvia	55	0.466	WN	1.055
Lithuania	55	0.851	WN	1.052
Luxembourg	65	0.722	WN	1.061
New Zealand	66	0.202	WN	1.054
Norway	100	0.296	WN	1.059
Slovenia	32	0.272	WN	1.058
Belarus	56	0.022	WN*	1.060
Slovakia	65	0.025	WN*	1.055
Switzerland	144	0.000	WN*	1.052

Germany 1964-2018; WN* (White Noise Model with $p<0.05$)

Table 2: Results for 1964 – 2014

Country	Observ.	p-(Ljung-Box)	Model	Mean SRB
Belgium	51	0.000	AR1	1.054
Italy	49	0.012	AR1	1.060
Portugal	49	0.021	AR1	1.066
Taiwan	51	0.000	AR1	1.077
Ukraine	50	0.000	AR1	1.060
USA	51	0.000	AR2	1.050
France	51	0.000	AR2	1.051
Hungary	51	0.000	AR2	1.059
Japan	51	0.000	AR2	1.058
Poland	51	0.001	AR2	1.062
Russia	51	0.000	AR2	1.055
Spain	51	0.000	AR2	1.065
United Kingdom	50	0.000	AR2	1.055
Australia	48	0.084	WN	1.055
Bulgaria	47	0.127	WN	1.060
Canada	48	0.414	WN	1.055
Chile	42	0.335	WN	1.044
Czech Republic	51	0.956	WN	1.056
Denmark	51	0.687	WN	1.056
Estonia	50	0.874	WN	1.059
Finland	49	0.724	WN	1.047
Germany	51	0.094	WN	1.056
Greece	33	0.494	WN	1.067
Iceland	50	0.111	WN	1.050
Ireland	51	0.233	WN	1.059
Israel	32	0.616	WN	1.055
Lithuania	50	0.834	WN	1.052
Luxembourg	51	0.592	WN	1.062
Netherlands	49	0.812	WN	1.051
New Zealand	50	0.071	WN	1.054
Norway	51	0.807	WN	1.057
Slovakia	51	0.824	WN	1.053
Slovenia	32	0.272	WN	1.058
Sweden	51	0.180	WN	1.058
Switzerland	51	0.136	WN	1.056
Belarus	51	0.002	WN*	1.060
Latvia	50	0.008	WN*	1.055
Austria	51	0.570	WN/AR1	1.054

WN* (White Noise Model with $p<0.05$)

A graphical examination of ARIMA modeling, utilizing the Auto Correlation Function (ACF) and Partial Auto Correlation Function (PACF), is illustrated in Figures 3 and 4, as shown in this example. Refer to the graphical analysis provided in the appendix for the remaining countries.

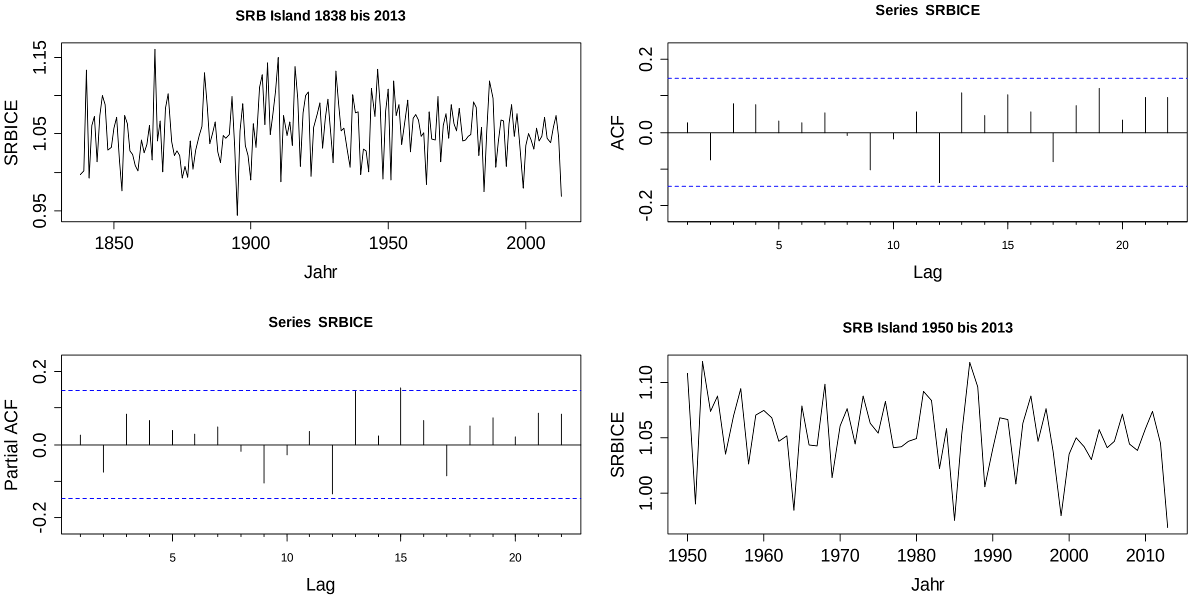


Fig. 3: White Noise Model for Iceland 1838 to 2013 with ACF and PACF and 1950 to 2013 (Average SRB: 1.05)

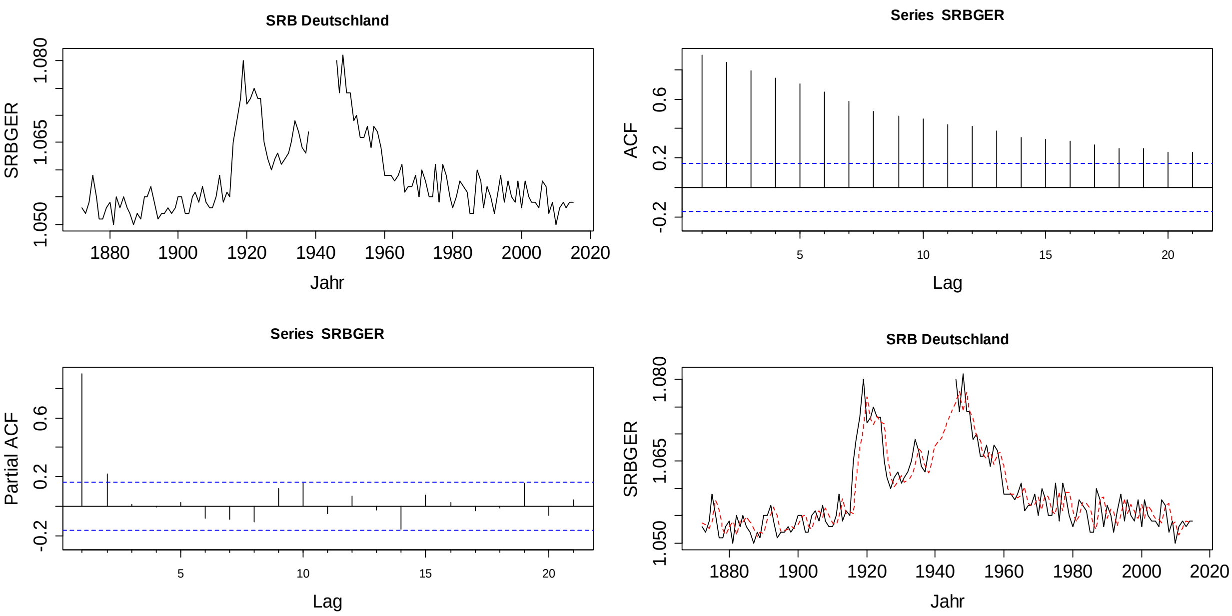


Fig. 4: AR(2)-Model for Germany 1872 to 2019 with ACF, PACF and Predicted SRB (red); mean=1.0582; model: $SRB_t = 0.0803 + 0.6858 \cdot SRB_{t-1} + 0.2383 \cdot SRB_{t-2}$; $\hat{\sigma} = 0.00316$

4.3 Summary of Results

4.3.1 Whole Time Span Analysis

An overarching summary reveals compelling findings:

- The significance of most estimators.
- An observation that ARIMA(p,0,d) models yield unsatisfactory results.
- White Noise (WN) models are frequently observed in countries with smaller numbers of SRB observations, especially those with fewer entries. However, countries with 90 or more SRB observations, such as Iceland, Switzerland, Norway, and Canada, also exhibit characteristics consistent with White Noise, challenging the notion that WN models are solely prevalent in countries with fewer SRB.
- A prominent AR(2) model adequacy is observed across many countries, such as Austria, Chile, Denmark, France, Germany, Hungary, Poland, Portugal, Russia, Spain, Taiwan, the UK, and the USA.
- Noteworthy cases of AR(3) models, including the Netherlands and Sweden, further underscore the diverse model choices.
- Countries such as Belgium, Finland, Italy, and Japan stand out with AR(4) models.

4.3.2 Shorter Time Span Analysis

A nuanced view of the results for the shorter time span (1964-2014) emerges:

- The prevalence of White Noise Models for short-term periods in 26 of the 39 countries.
- Countries like Belgium, Italy, Portugal, Taiwan, and Ukraine exhibit AR(1) models.
- France, Hungary, Japan, Poland, Russia, Spain, the UK, and the USA manifest the suitability of AR(2) models.
- Notably, there are instances where the preference for a White Noise model defies a small p-value in the Ljung-Box test, exemplified by Belarus and Latvia.

4.3.3 Analyzing Sex Ratio at Birth (SRB) Data: Model Selection and Observation Lengths

In this section, we thoroughly investigate the analysis of sex ratio at birth (SRB) data, focusing on the selection of appropriate models in accordance with observation lengths. The provided frequency table showcases the distribution of countries across different observation length categories, along with the corresponding utilization of models for analysis. The table structure is as follows:
Rows categorize observation lengths: "0-49," "50-99," and "100+."
Columns represent applied models: "WN" (White Noise), "AR1" to "AR4" (Autoregressive Order 1 to 4).

Table 3: Frequeny Table by Observation Lenght and Selected Model

Observation lenghts	WN	AR1	AR2	AR3	AR4	Sum
0-49	3					3
50-99	12	1	5			18
100+	3	1	8	2	4	18
Sum	18	2	13	2	4	39

The intersections reveal the count of countries falling under specific observation lengths and modeled using the respective techniques. This table succinctly illustrates the alignment between observation length and model choice in SRB analysis across diverse countries.
The trends revealed in the table highlight a strategic approach to model selection. Longer observation lengths tend to lead to the application of Autoregressive (AR) models, with AR2 being the most common. Conversely, for shorter observation spans, the prevalent choice is the White Noise (WN) model. This deliberate alignment underscores the careful consideration given to data availability, resulting in a nuanced comprehension of sex ratio dynamics. Autoregressive (AR2) models and the White Noise (WN) model emerge as the predominant choices.

The comprehensive analysis of Sex Ratio at Birth (SRB) data reveals diverse model preferences across countries and observation lengths. While Autoregressive (AR2) models dominate longer spans, White Noise (WN) models are prevalent in shorter periods, highlighting a strategic alignment between observation length and model choice for a nuanced understanding of SRB dynamics.

5. Mathematical Proofs and Model Selection

The AR(2) model with a parameter value of c greater than 0 is expressed as a linear nonhomogeneous second-order difference equation, excluding disturbances, as discussed in Pflaumer (1992).

$SRB_t = c + \phi_1 \cdot SRB_{t-1} + \phi_2 \cdot SRB_{t-2}$ with $1 - \phi_1 - \phi_2 \neq 0$.

Solving the associated characteristic equation

$$\lambda^2 - \phi_1 \cdot \lambda - \phi_2 = 0$$

leads us to the general solution of the difference equation, which exhibits two distinct real roots if $\phi_1^2 > 4 \cdot \phi_2$:

$$SRB_t = C_1 . \lambda_1^t + C_2 . \lambda_2^t + \frac{c}{1 - \phi_1 - \phi_2} .$$

If $|\lambda_1| < 1$ and $|\lambda_2| < 1$ then

$$\lim_{t \to \infty} SRB_t = \frac{c}{1 - \phi_1 - \phi_2} .$$

The point forecast tends to converge towards the mean of SRB_t. By incorporating the initial conditions of SRB_0 (the penultimate observed value) and SRB_1 (the last observed value), we can ascertain the specific solution for the AR(2) model of Germany, as depicted in Figures 4 and 5, through the resolution of additional equations

$SRB_0 = C_1 + C_2 + \frac{c}{1 - \phi_1 - \phi_2}$ and $SRB_1 = C_1 \cdot \lambda + C_2 \cdot \lambda + \frac{c}{1 - \phi_1 - \phi_2}$ which yields

$$SRB_t = -0.00357248 \cdot 0.94^t - 0.00062563 \cdot (-0.252)^t + 1.0582 \quad \text{for } t = 0,1,2,....$$

using the initial values (last observed values) 1.054 and 1.055 ($\lim_{t \to \infty} SRB_t = 1.0582$).

Figure 5 illustrates both the deterministic and stochastic AR(2) models. In the absence of random disturbances, the process would gradually converge to its mean of 1.0582. However, random shocks prevent the Sex Ratio at Birth from approaching its mean; the SRB exhibits random cycles around its mean, as demonstrated in a simulation assuming disturbances with a mean of μ=0 and a standard deviation of σ=0.003127.

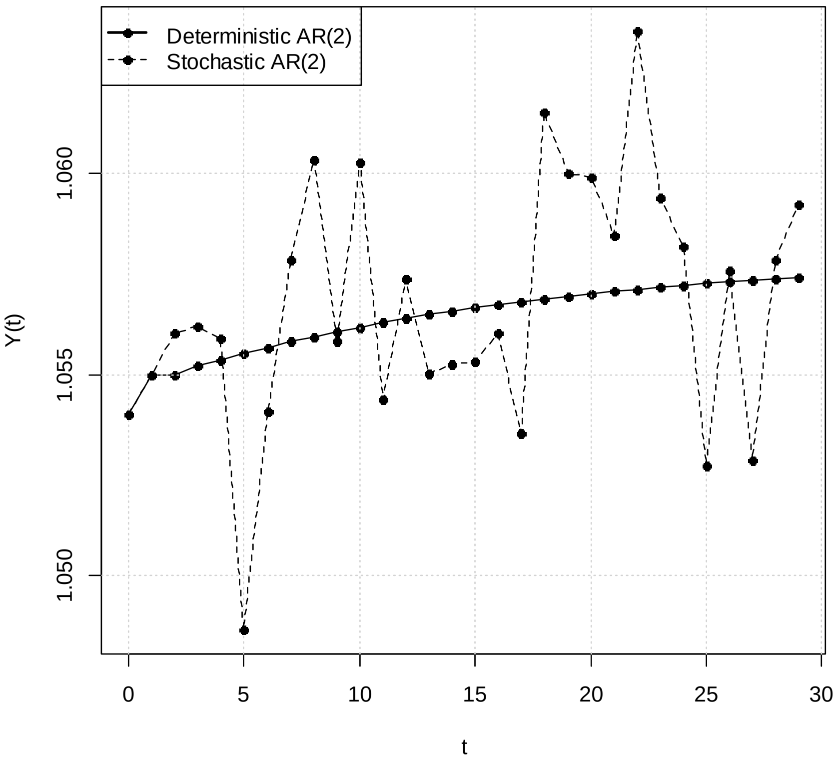


Fig. 5: Deterministic and Stochastic AR(2) Models of the German SRB

$SRB_t = 0.0803 + 0.6858 \cdot SRB_{t-1} + 0.2383 \cdot SRB_{t-2} = -0.00357248 \cdot 0.94^t - 0.00062563 \cdot (-0.252)^t + 1.0582$ for $t = 0,1,2,....$

Starting values: 1.054 and 1.055; σ=0.003127

The equation of the first differences $\Delta SRB_t = \phi_1 \cdot \Delta SRB_{t-1}$ with $|\phi_1| < 1$ leads to the homogenous second order difference equation $SRB_t = (1+\phi_1) \cdot SRB_{t-1} - \phi_1 \cdot SRB_{t-2}$ with the general solution (roots of the characteristic equations are $\lambda_1 = 1$ and $\lambda_2 = \phi_1$)

$$SRB_t = C_1 + C_2 \cdot (\phi_1)^t$$

with $\lim_{t\to\infty} SRB_t = C_1 = \dfrac{\phi_1 \cdot SRB_0 - SRB_1}{\phi_1 - 1}$.

If $SRB_0 = SRB_1$ then we get $\lim_{t\to\infty} SRB_t = SRB_1$, which is the naïve forecast.

The long-term point forecast of the AR(2) or ARIMA(2,0,0) model represents the mean of the stochastic process. In contrast, point forecasts of ARIMA(1,1,0) models tend to converge towards the last observation. Notably, AR(2) incorporates all past values, while ARIMA(1,1,0) relies solely on the last observation, which can be particularly critical if the last observation is an extreme outlier.

The selection of the AR(2) model is guided by several key considerations. Emphasizing the principle of parsimony, we aim to balance capturing essential SRB data trends while avoiding unnecessary complexity. The AR(2) model aptly encapsulates the dynamic nature of SRB data and aligns well with its tendency to converge towards a value near 1 over time. We acknowledge and address the limitations of alternative models, emphasizing the sensitivity of ARIMA(1,1,0) to outliers and recognizing the potential complexity introduced by AR(p) models with p > 2, while empirical analysis underscores the infrequent occurrence and potential randomness of these models. Additionally, the assessment of ARIMA(p,d,q) models reveals the less suitability of their estimators for capturing nuanced SRB fluctuations. The unique property of the AR(2 model to tend towards the mean further strengthens its suitability for robust forecasting, considering all past values to capture intrinsic SRB dynamics effectively.

6. Forecasting Germany's Sex Ratio at Birth (2020-2070) with ARIMA Models

In this chapter, we explore the application of ARIMA models to forecast Germany's sex ratio at birth (SRB) for the period 2020 to 2070. Pioneering the use of ARIMA modeling for SRB in Germany, Jöckel and Pflaumer (1981) initiated this line of research. Their analysis covered SRBs from 1872 to 1978 and identified potential model choices, including ARIMA(1,1,0) or ARIMA(1,1,1).

Table 4 presents the estimated results of distinct ARIMA models for SRB (scaled by *1000), drawing insights from the work of Jöckel and Pflaumer (1981). Notably, these models encompass ARIMA(1,1,0) and ARIMA(1,1,1). The estimators of autoregressive (AR) and moving average (MA) terms, along with other pertinent statistics, facilitate the assessment of modeling alternatives.

Table 4: Estimated Results of the ARIMA (p, d, q) Models for the Sex Ratio at Birth (the t-statistics of the estimates are in bracket; SRB*1000)

	ARIMA (1,1,0)	ARIMA(1,1,1)
AR(1)	- 0.36	- 0.62
	(-3.93)	(-3.01)
MA(1)	-	-0.29
		(-1.18)
Sigma	3.25	3.23

Source: Jöckel and Pflaumer (1981)

Our focus shifts to forecasting SRB for the years 2020 to 2070, based on historical data spanning 1872 to 2019. The analysis of the SRB time series, as well as the autocorrelation function (ACF) and partial autocorrelation function (PACF) depicted in Figure 4, guides the model selection process. Notably, the PACF exhibits prominent peaks at lags 1 and 2, hinting at the potential suitability of an AR(2) model. The choice of forecasting model plays a pivotal role in accurately capturing the sex ratio at birth (SRB) dynamics. Our selection process leads us to prioritize the AR(2) model for several key reasons: The AR(2) model strikes an optimal balance between complexity and predictive accuracy. It effectively captures SRB trends by considering the influence of the two most recent observations while avoiding unnecessary intricacies. Our analysis of historical SRB data substantiates the AR(2) model's efficacy. It demonstrates consistent and robust performance, aligning well with observed trends.

The AR(2) model's behavior resonates with the underlying theoretical understanding of SRB dynamics. It leverages both recent and relevant past observations to provide accurate forecasts. The AR(2) model outperforms alternative models in terms of statistical metrics such as the Akaike Information Criterion (AIC). Its AIC values consistently indicate better performance in capturing SRB variations (see table 5). In essence, the AR(2) model's parsimonious complexity, empirical validation, theoretical alignment, and statistical superiority collectively position it as the optimal choice for forecasting SRB dynamics.

Table 5: Estimators of Different Models

ARIMA	d	mean	ar1	ar2	ma1	sigma	AIC
(2,0,0)	0	1.0582	0.6858	0.2383		0.003127	-1277.41
(1,1,0)	1		-0.275			0.003191	-1269.66
(0,1,1)	1				-0.2851	0.003186	-1270.02
(1,1,1)	1		-0.0839		-0.2068	0.003197	-1268.1
ARIMA	d		s.e.ar1	s.e.ar2	s.e.ma1	s.e.Int.	
(2,0,0)	0		0.0793	0.0796		0.0031	
(1,1,0)	1		0.079				
(0,1,1)	1				0.0778		
(1,1,1)	1		0.3082		0.3047		

Figure 6 depicts the AR(2) forecasts of SRB in Germany for the years 2020 to 2070, supplemented by 80% and 95% prediction intervals for the mean. The point forecast underscores a gradual SRB increase from 1.054 (2018) and 1.055 (2019) to 1.058 by 2070. However, the 95% prediction interval underscores a broader range, extending between 1.045 and 1.072. The influence of peaks after World War I and II on the trends is evident in the graph.

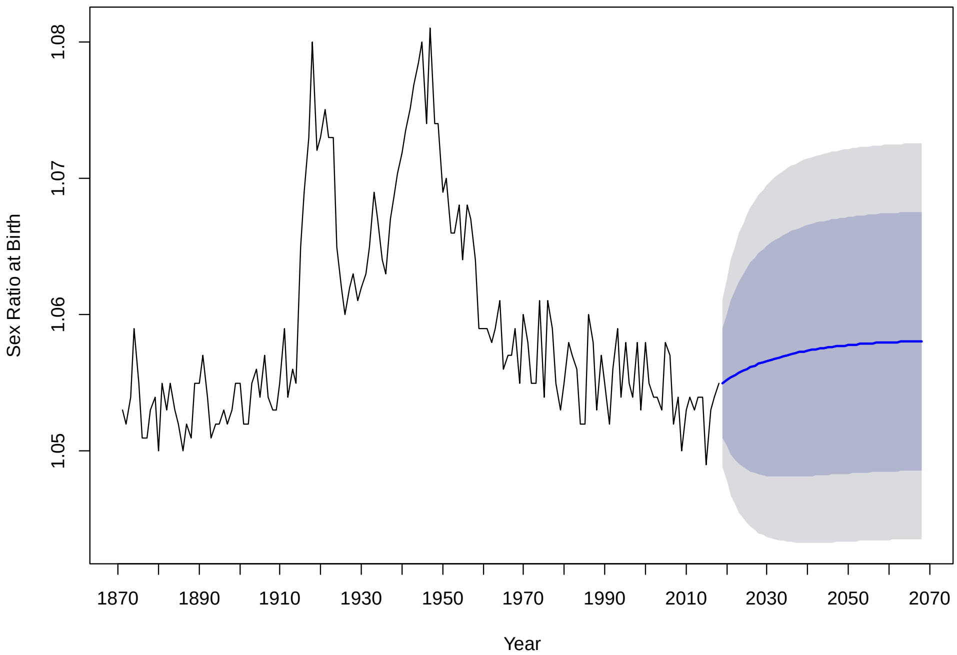


Fig. 6: Forecasts of German SRB between 2020 and 2070 with 80% and 90% Prediction Intervals for the Mean using an AR(2) Model

Table 6 presents key parameters related to the sex ratio at birth (SRB) across various countries, along with the outcomes of an ARIMA(2,0,0) model applied to this data. The parameters offer valuable insights into the behavior of the SRB time series and shed light on how the ARIMA(2,0,0) model captures its patterns.

Table 6: Cyclical Behavior of Sex Ratio at Birth (SRB) Across Countries: ARIMA(2,0,0) or AR(2) Model Insights (see Table 1)

Country	Mean	c	ar1	arl	lambda1	lambda2
Austria	1.057	0.528	0.294	0.207	0.625	-0.331
Chile	1.046	0.257	0.349	0.405	0.835	-0.485
Denmark	1.054	0.623	0.168	0.241	0.582	-0.414
France	1.052	0.129	0.569	0.308	0.908	-0.339
Germany	1.058	0.08	0.668	0.237	0.94	-0.252
Hungary	1.061	0.265	0.318	0.432	0.835	-0.517
Poland	1.063	0.286	0.367	0.364	0.814	-0.447
Portugal	1.065	0.49	0.385	0.155	0.631	-0.246
Russia	1.055	0.72	0.372	0.56	0.957	-0.585
Spain	1.075	0.042	0.622	0.339	0.971	-0.349
Taiwan	1.071	0.102	0.722	0.183	0.921	-0.199
UK	1.054	0.121	0.473	0.412	0.921	-0.447
USA	1.051	0.132	0.431	0.443	0.915	-0.484
Mean	*1.059*	*0.313*	*0.441*	*0.330*	*0.835*	*-0.392*

- Country: This column specifies the country under consideration.
- Mean: The mean SRB for the country represents its long-term average sex ratio at birth.
- c: This coefficient represents the constant term of the ARIMA(2,0,0) model.
- ar1: Representing the autoregressive (AR) term of lag 1, this coefficient illustrates how the previous value of SRB influences the present.
- ar2: This coefficient corresponds to the autoregressive term of lag 2, conveying the influence of the SRB two time steps ago.
- lambda1: Emerging as one of the solutions from the ARIMA(2,0,0) model, this value contributes to the reduction of deviations from the mean in the sex ratio. Its positive magnitude serves as an indicator of stability within the sex ratio data.
- lambda2: Another solution of the ARIMA(2,0,0) model, lambda2 is responsible for longer-term behavior. A negative value with an absolute magnitude less than 1 suggests the presence of damped cycles or oscillations in the SRB data.

The significance of the absolute values of λ_1 and λ_2 being less than 1 is pivotal. This indicates that the ARIMA(2,0,0) model renders the SRB time series stationary across all selected countries. Stationarity implies that the statistical properties of the series remain consistent over time, without exhibiting significant trends. Furthermore, the negative value of λ_2 implies the existence of damped oscillations or cycles in the SRB data. These cycles demonstrate that the sex ratio at birth, while fluctuating cyclically around its mean, gradually dampens in amplitude. The positive values of λ_1 play a crucial role in diminishing deviations from the mean in the sex ratio at birth (SRB) data. This effect is essential for reducing the extent of deviations over time.

Acknowledging the inherent uncertainties linked to long-term projections, we conducted a simulation study using the AR(2) model and estimated parameters. Employing 100,000 simulations and assuming a normally distributed error term (ut) with a mean of 0 and standard deviation of 0.03127 (see table 5), we generated a distribution of potential SRB trajectories.

Figure 7 presents the outcome of this simulation endeavor, showcasing 100,000 simulated SRB trajectories in Germany from 2020 to 2070. This visual representation of various trajectories underscores the inherent variability and the need to account for multiple potential outcomes in long-term forecasting.

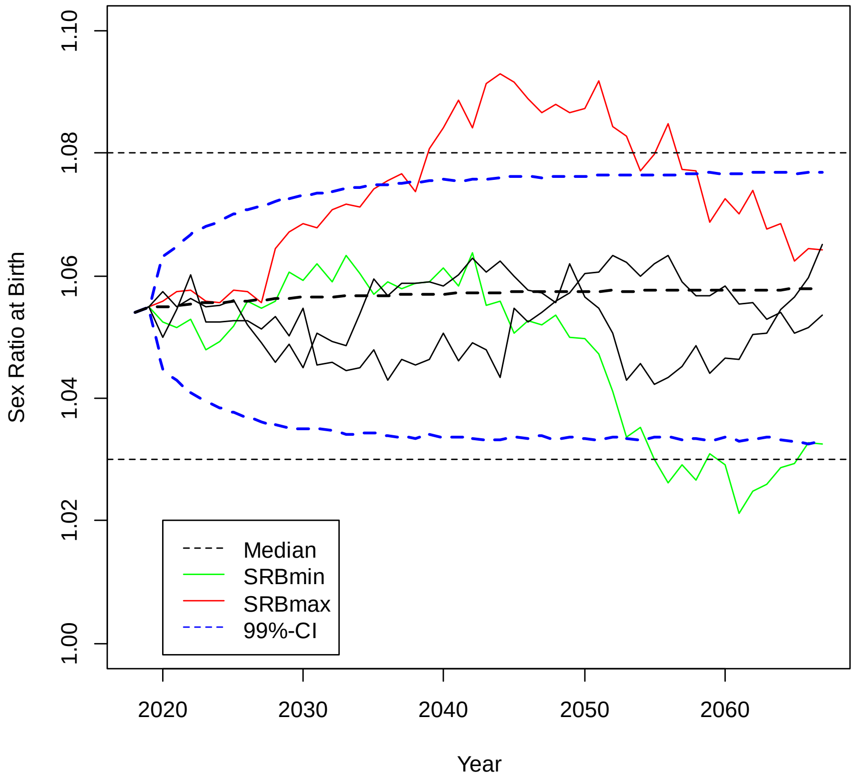


Fig. 7: Results of 100,000 Simulated Trajectories of the SRB in Germany from 2020 to 2070
Remarks: Simulation with R; seed (3004) Starting values: 1.055, 1.054; model:

$$SRB_t = 0.0803 + 0.6858 \cdot SRB_{t-1} + 0.2383 \cdot SRB_{t-2}, \; u_t \sim N\left(0, 0.003127^2\right)$$

Figure 7 provides a visual representation of the simulation study's outcomes, shedding light on the potential range of trajectories for Germany's sex ratio at birth (SRB) from 2020 to 2070. This comprehensive depiction is accompanied by a 99% confidence interval, encapsulating the diversity of possible SRB trajectories and their associated uncertainties.
Displayed within Figure 7 are four distinct series, each representing a unique simulated SRB trajectory. These trajectories collectively capture the spectrum of possible outcomes, underscoring the inherent variability in long-term forecasting. The corresponding 99% confidence intervals provide a robust framework to understand the potential range of SRB values by 2070.

The median trajectory, represented by the dotted black line, offers valuable insights into the central tendency of the simulation outcomes. As evidenced in Figure 7, the median trajectory exhibits a slight increase over the forecast period, with a median SRB of 1.058 by 2070. This underscores a gradual upward trend in the projected SRB values.
The depiction of diverse trajectories serves as a reminder of the complexity inherent in long-term forecasting. The black series, represented by "Sim No. 1" and "Sim No. 100,000," highlights the range of possible individual paths that SRB could follow. The green series,

portraying SRB with the lowest observed values (below 1.03), stands in contrast to the red series, which showcases SRB with the highest observed values (near 1.09).

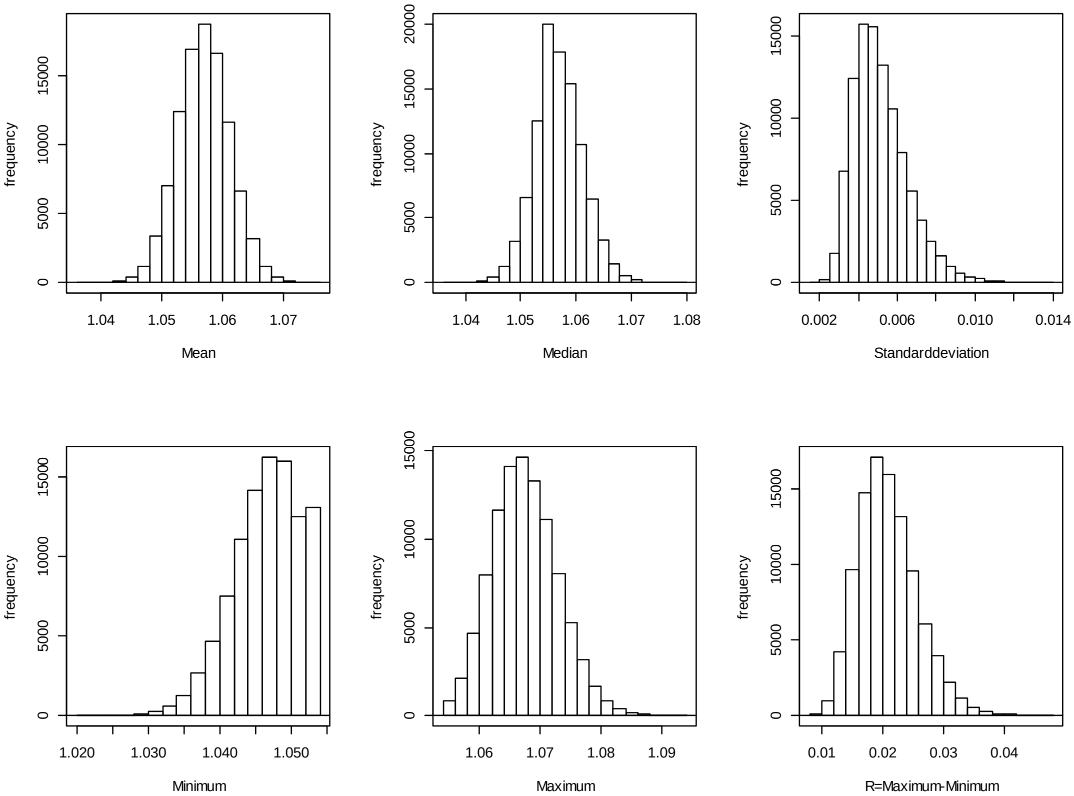


Fig. 8: Simulated Means, Medians, Standard Deviations, Minima, Maxima, Ranges

Figure 8 illustrates various statistical measures derived from previous simulation: means, medians, standard deviations, minima, maxima, and ranges. The distributions of means and medians appear to be relatively symmetrical, while the distributions of the other parameters are skewed to the left with varying degrees of intensity. The mean of all means is 1.569 with a standard deviation of 0.0042, and the median similarly stands at 1.0569 with a standard deviation of 0.0043.

In summary, this chapter showcases the application of ARIMA models to forecast Germany's SRB from 2020 to 2070. Jöckel and Pflaumer's paper laid the foundation for this research direction. The AR(2) model emerges as a robust choice, and our simulations offer valuable insights into the potential range of outcomes. This analysis enhances our understanding of SRB dynamics and the intricacies of long-term forecasting within the realm of demographic research.

7. Conclusion

The intricacies of sex ratio at birth (SRB) are a complex interplay of biological and socio-economic factors, their combined influence contributing to the dynamic nature of SRB dynamics. As observed by Gini (1908), when no singular or dominant factor asserts control, the appropriate time-series model often reflects the "White Noise" pattern, particularly evident in short-term series. This fundamental understanding has paved the way for more nuanced modeling approaches.

In instances where trend factors or unexpected spikes arise, an autoregressive model devoid of differencing may be suitable to capture the SRB behavior. Notably, the AR(2)-Model frequently emerges as a robust choice for describing SRB variations, encompassing the multifaceted trends and oscillations present in the data.

Despite deviations from its mean, SRB consistently exhibits a compelling tendency to revert to its intrinsic equilibrium, a phenomenon observed across most countries where the natural sex ratio hovers around 1.05. However, when contemplating forecasts over extended horizons, the intricacies of SRB dynamics render predictions unreliable. While we can assert that the Sex Ratio at Birth will likely hover within the approximate interval of 1.03 to 1.08 across most countries, the inherent complexities make precise forecasting a challenging endeavor.

Our study emphasizes the importance of comprehensive modeling approaches that consider the complex interplay of factors shaping SRB dynamics. As we explore the intricacies of SRB fluctuations, we are reminded of the complex interplay of factors contributing to the shaping of this essential demographic indicator.

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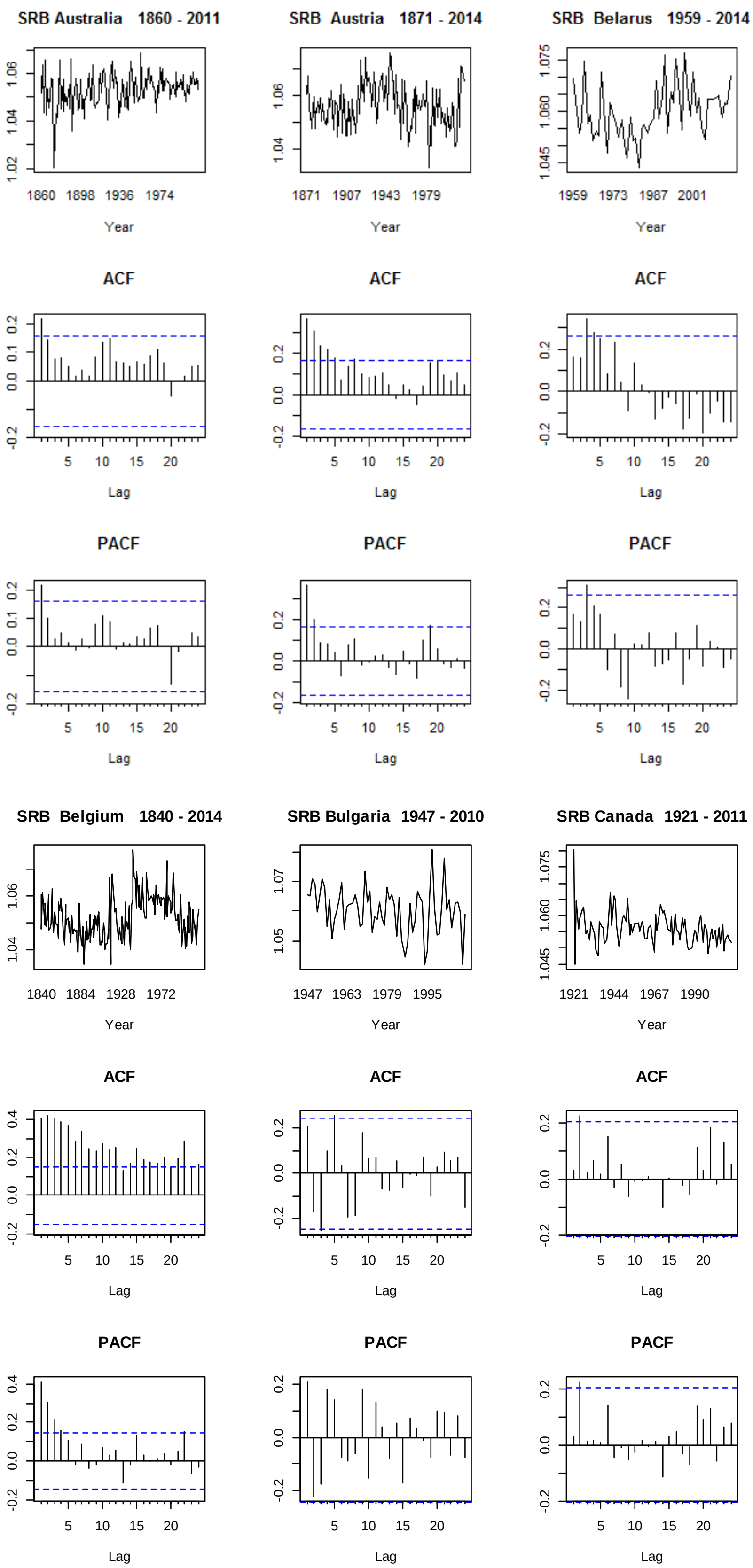
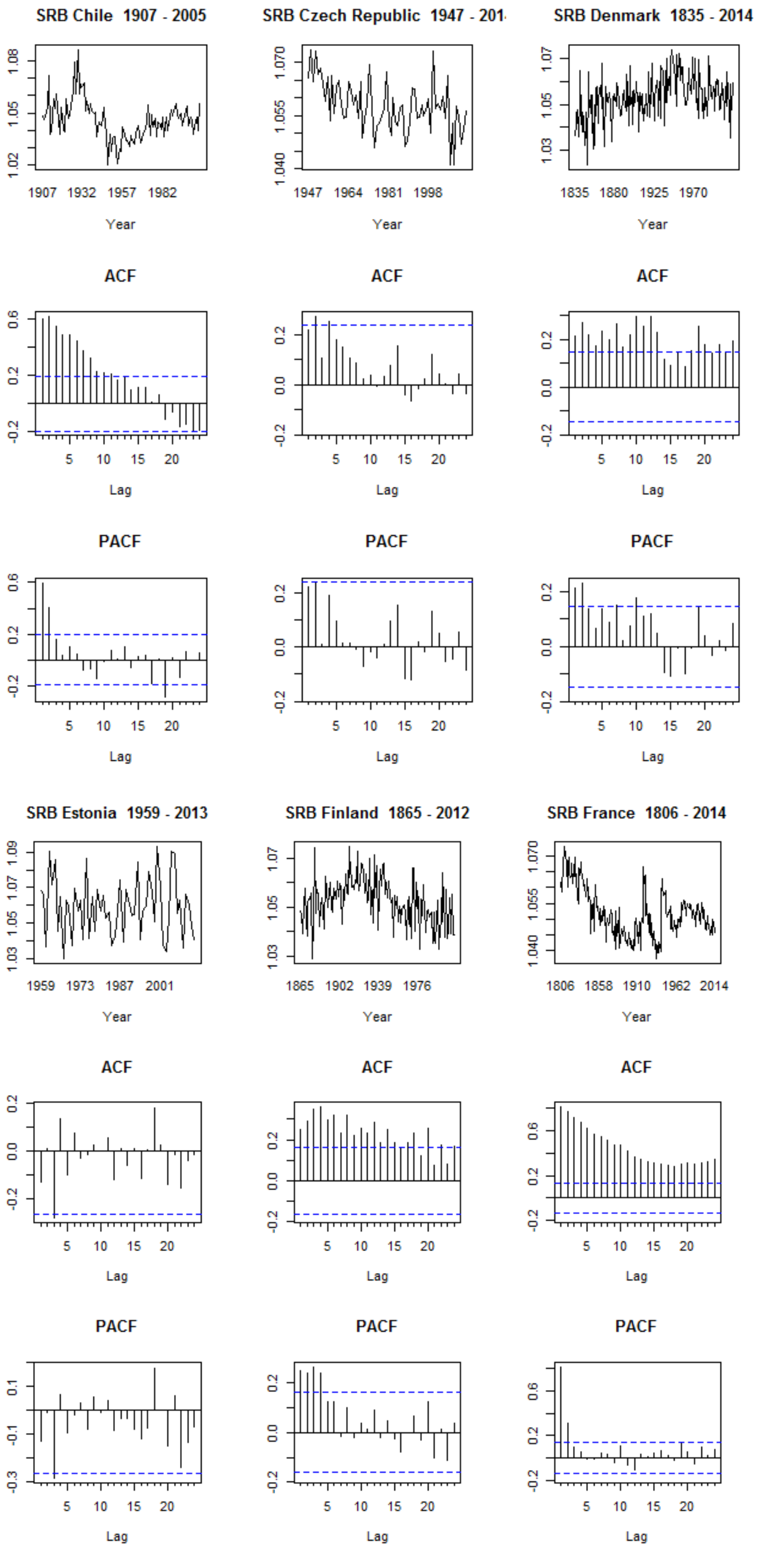
Appendix:

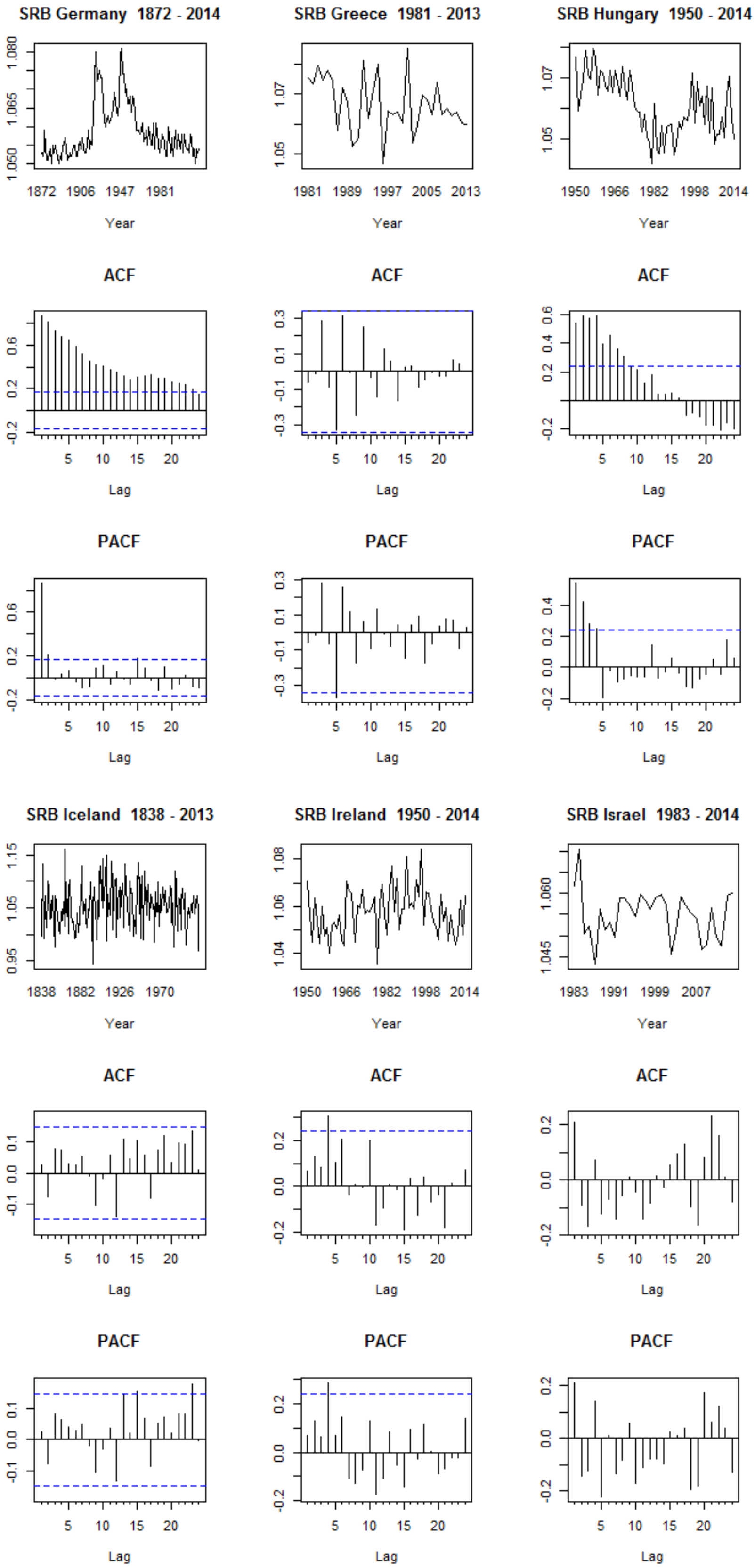
Plots of Sex Ratio at Birth (SRB), Autocorrelation Functions (ACFs), and Partial Autocorrelation Functions (PACFs) for the Studied Countries

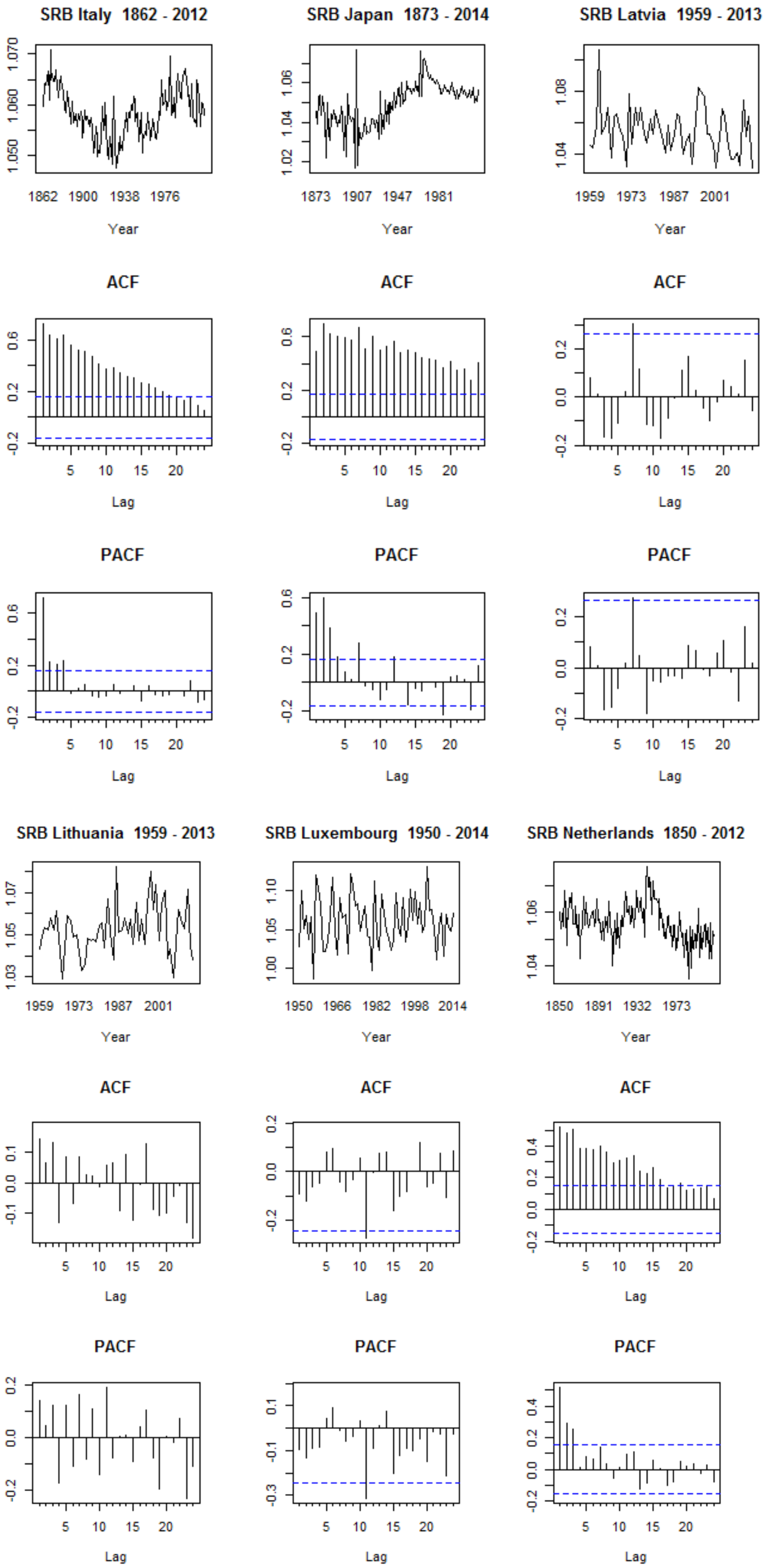
The analysis covers a total of 38 countries[5], including Australia, Austria, Belarus, Belgium, Bulgaria, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Latvia, Lithuania, Luxembourg, Netherlands, New Zealand, Norway, Poland, Portugal, Russia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Taiwan, United Kingdom, Ukraine, and the USA.

[5] Greenland see Fig. 1

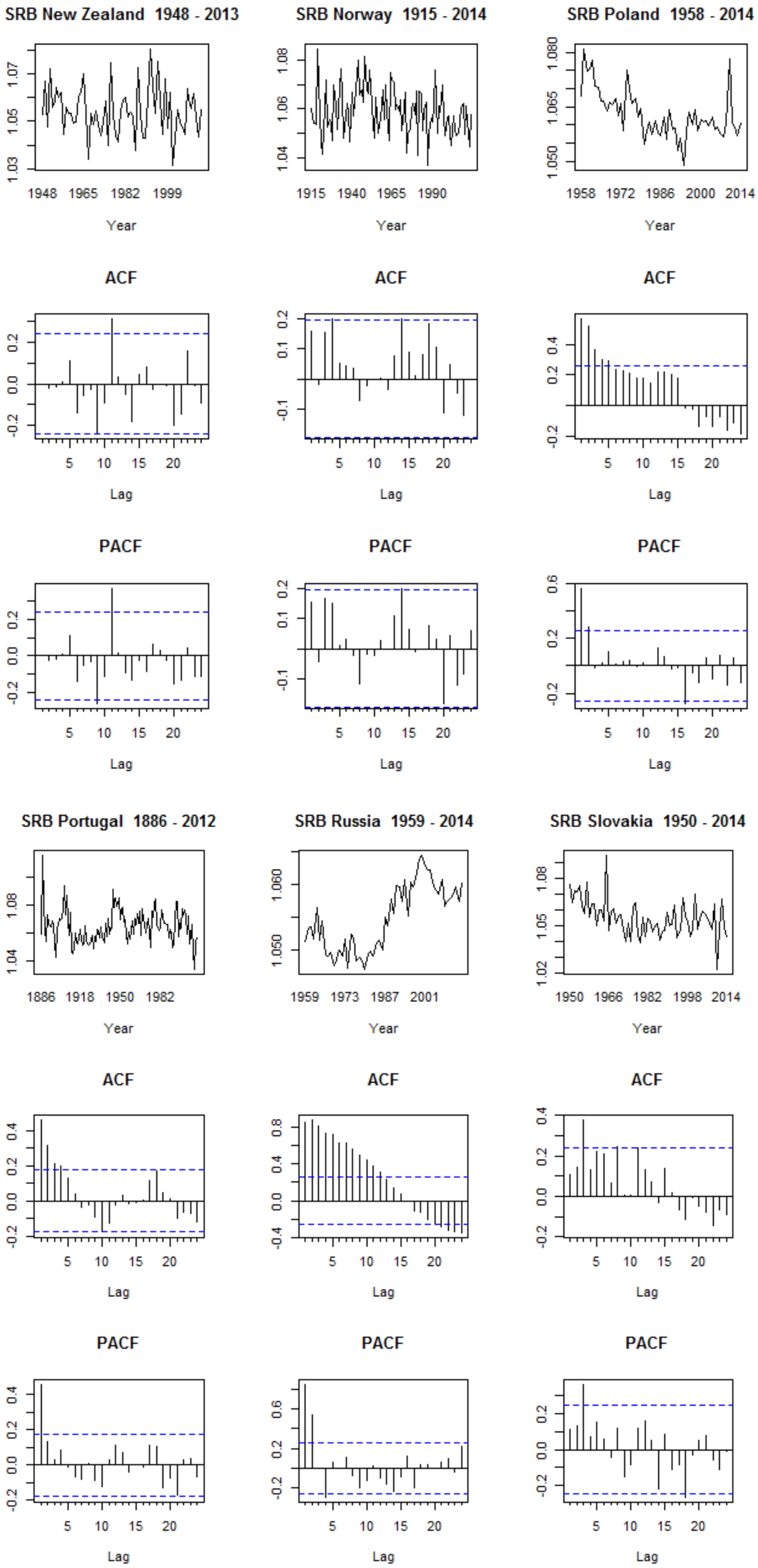

SRB Australia 1860 - 2011
SRB Austria 1871 - 2014
SRB Belarus 1959 - 2014
Year
ACF
PACF
Lag
SRB Belgium 1840 - 2014
SRB Bulgaria 1947 - 2010
SRB Canada 1921 - 2011
Year
ACF
PACF
Lag

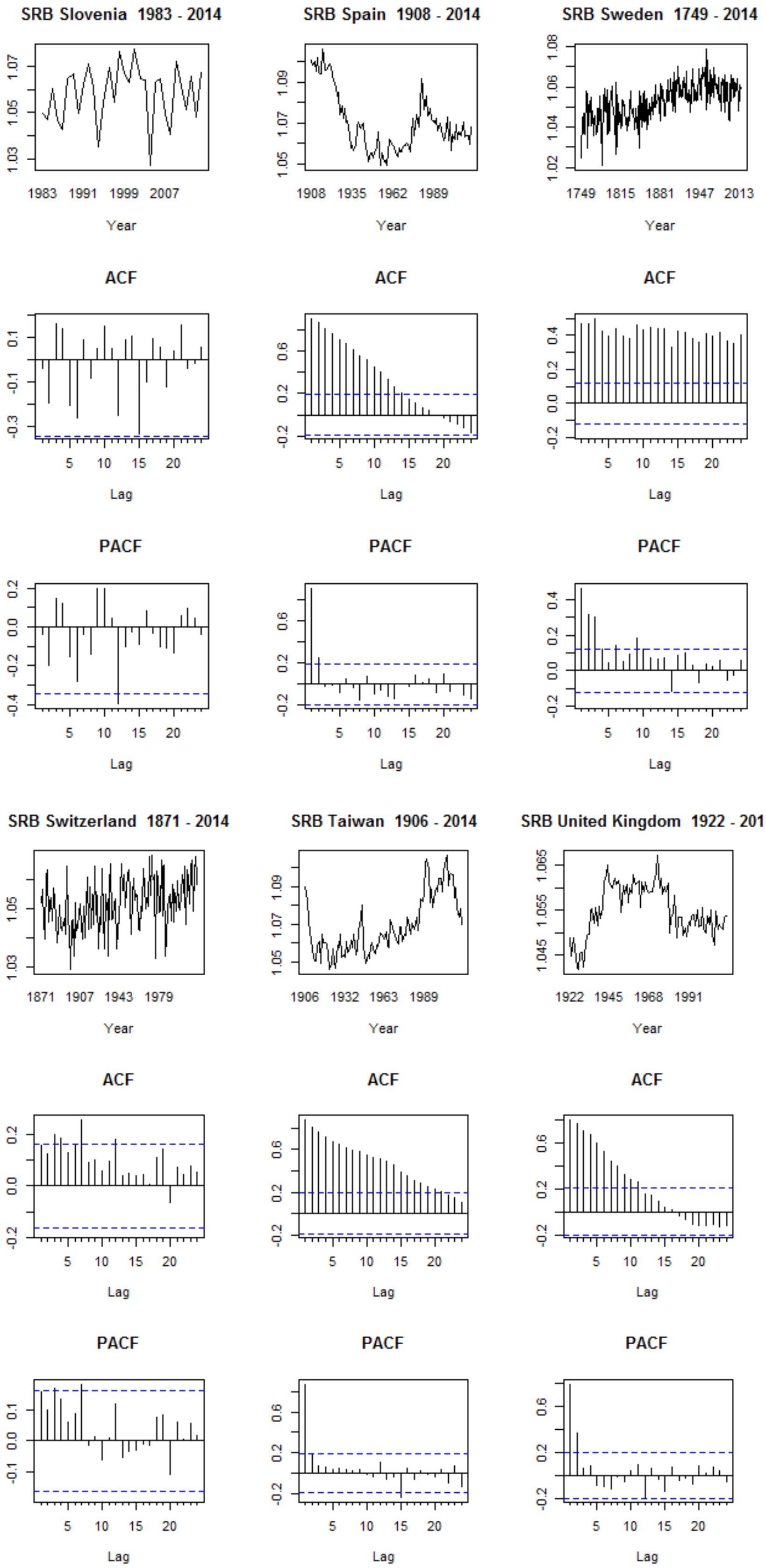

SRB Chile 1907 - 2005
SRB Czech Republic 1947 - 201
SRB Denmark 1835 - 2014
Year
ACF
Lag
PACF
SRB Estonia 1959 - 2013
SRB Finland 1865 - 2012
SRB France 1806 - 2014


SRB Germany 1872 - 2014
SRB Greece 1981 - 2013
SRB Hungary 1950 - 2014
Year
ACF
PACF
Lag
SRB Iceland 1838 - 2013
SRB Ireland 1950 - 2014
SRB Israel 1983 - 2014
Year
ACF
PACF
Lag



SRB Italy 1862 - 2012
SRB Japan 1873 - 2014
SRB Latvia 1959 - 2013
Year
ACF
PACF
Lag
SRB Lithuania 1959 - 2013
SRB Luxembourg 1950 - 2014
SRB Netherlands 1850 - 2012


SRB New Zealand 1948 - 2013
SRB Norway 1915 - 2014
SRB Poland 1958 - 2014
Year
ACF
PACF
Lag
SRB Portugal 1886 - 2012
SRB Russia 1959 - 2014
SRB Slovakia 1950 - 2014
Year
ACF
PACF
Lag


SRB Slovenia 1983 - 2014
SRB Spain 1908 - 2014
SRB Sweden 1749 - 2014
Year
ACF
PACF
Lag
SRB Switzerland 1871 - 2014
SRB Taiwan 1906 - 2014
SRB United Kingdom 1922 - 201

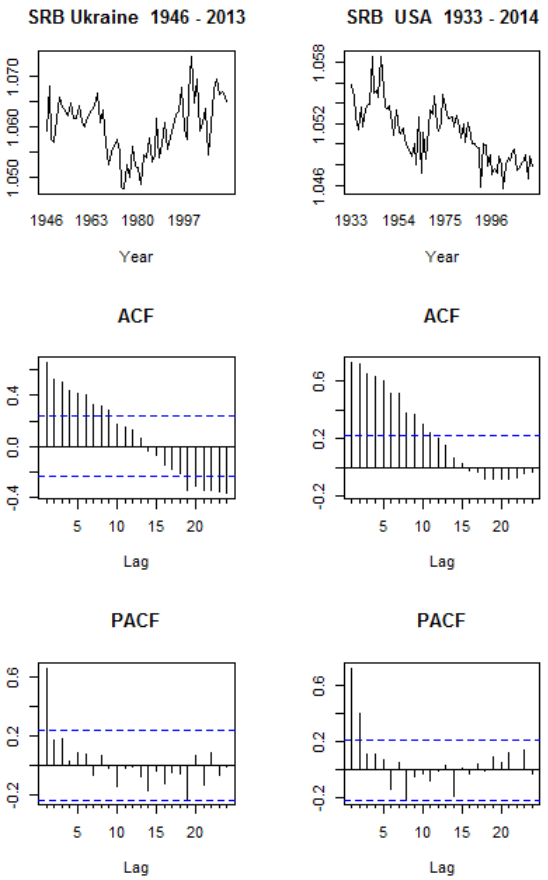


Table A1: Detailed Results for the Entire Time Span

Country	Observ.	p-(Ljung-Box)	Model	Mean	ar1	ar2	ar3	ar4	sigma2est	s.e. ar1	s.e. ar2	s.e. ar3	s.e. ar4
Australia	152	0.184	AR1/WN	1.053	0.2169				4.43E-05	0.079			
Austria	144	0.000	AR2	1.057	0.2936	0.2068			5.20E-05	0.0812	0.0822		
Belarus	56	0.022	WN*	1.060					5.39E-05				
Belgium	175	0.000	AR4	1.052	0.1852	0.1979	0.1829	0.1573	4.04E-05	0.0747	0.0749	0.0756	0.0752
Bulgaria	64	0.176	WN	1.060					5.25E-05				
Canada	91	0.790	WN	1.056					2.34E-05				
Chile	99	0.000	AR2	1.046	0.3494	0.4050			6.60E-05	0.0917	0.0917		
Czech Rep.	68	0.367	WN	1.058					4.42E-05				
Denmark	180	0.000	AR2	1.054	0.1681	0.2407			7.66E-05	0.0727	0.073		
Estonia	55	0.833	WN	1.059					2.57E-04				
Finland	148	0.000	AR4/AR2	1.052	0.0558	0.1501	0.2444	0.2525	6.68E-05	0.0794	0.0769	0.0778	0.0803
France	209	0.000	AR2	1.052	0.5688	0.3083			1.66E-05	0.0657	0.0658		
Germany	136	0.000	AR2	1.058	0.6883	0.2365			1.12E-05	0.0829	0.0833		
Greece	33	0.494	WN	1.067					7.96E-05				
Hungary	65	0.000	AR2	1.061	0.3183	0.4319			4.88E-05	0.1153	0.1152		
Iceland	176	0.232	WN	1.053					1.51E-03				
Ireland	65	0.254	WN	1.058					9.21E-05				
Israel	32	0.616	WN	1.055					2.89E-05				
Italy	151	0.000	AR4/AR2	1.059	0.4607	0.0808	0.0835	0.2408	8.80E-06	0.0785	0.0867	0.0869	0.0786
Japan	136	0.000	AR4/AR2	1.049	-0.1110	0.4318	0.3930	0.1720	4.88E-05	0.0844	0.078	0.0775	0.0848
Latvia	55	0.466	WN	1.055					2.19E-04				
Lithuania	55	0.851	WN	1.052					1.25E-04				
Luxembourg	65	0.722	WN	1.061					9.73E-04				
Netherlands	163	0.000	AR3	1.056	0.2912	0.1909	0.2658		3.08E-05	0.0752	0.0773	0.0758	
New Zealand	66	0.202	WN	1.054					9.75E-05				
Norway	100	0.296	WN	1.059					8.63E-05				
Poland	57	0.000	AR2	1.063	0.3670	0.3644			2.48E-05	0.1232	0.1324		
Portugal	127	0.000	AR2	1.065	0.3846	0.1551			1.04E-04	0.0907	0.0986		
Russia	56	0.000	AR2	1.055	0.3720	0.5600			4.36E-06	0.1095	0.1098		
Slovakia	65	0.025	WN*	1.055					1.23E-04				
Slovenia	32	0.272	WN	1.058					1.32E-04				
Spain	107	0.000	AR2	1.075	0.6219	0.3387			2.67E-05	0.091	0.0923		
Sweden	266	0.000	AR3	1.052	0.2282	0.2409	0.3091		5.31E-05	0.0587	0.0587	0.0591	
Switzerland	144	0.000	WN*	1.052					7.13E-05				
Taiwan	104	0.000	AR2	1.071	0.7218	0.1831			4.83E-05	0.0968	0.0972		
United King.	92	0.000	AR2	1.054	0.4735	0.4116			8.94E-06	0.0939	0.0954		
Ukraine	68	0.000	AR1/AR3	1.060	0.6584				1.75E-05	0.0895			
USA	82	0.000	AR2	1.051	0.4308	0.4433			2.67E-06	0.0982	0.0996		

Table A2: Detailed Results for 1964 – 2014

Country	Observ.	p-(Ljung-Box)	Model	Mean	ar1	ar2	s.e. ar1	s.e. ar2	Sigma2est
Australia	48	0.084	WN	1.055					1.42E-05
Austria	51	0.570	WN/AR1	1.054					6.54E-05
Belarus	51	0.002	WN*	1.060					5.48E-05
Belgium	51	0.000	AR1	1.054	0.3763		0.128		3.99E-05
Bulgaria	47	0.127	WN	1.060					6.36E-05
Canada	48	0.414	WN	1.055					1.43E-05
Chile	42	0.335	WN	1.044					3.63E-05
Czech Rep.	51	0.956	WN	1.056					4.18E-05
Denmark	51	0.687	WN	1.056					5.71E-05
Estonia	50	0.874	WN	1.059					2.52E-04
Finland	49	0.724	WN	1.047					7.34E-05
France	51	0.000	AR2	1.051	0.3814	0.2899	0.135	0.1354	6.28E-06
Germany	51	0.094	WN	1.056					7.22E-06
Greece	33	0.494	WN	1.067					7.99E-05
Hungary	51	0.000	AR2	1.059	0.2544	0.4273	0.1273	0.1278	4.63E-05
Iceland	50	0.111	WN	1.050					9.84E-04
Ireland	51	0.233	WN	1.059					9.65E-05
Israel	32	0.616	WN	1.055					3.10E-05
Italy	49	0.012	AR1	1.060	0.4644		0.1259		1.36E-05
Japan	51	0.000	AR2	1.058	0.1975	0.5466	0.1136	0.1158	1.76E-05
Latvia	50	0.008	WN*	1.055					1.84E-04
Lithuania	50	0.834	WN	1.052					1.39E-04
Luxembourg	51	0.592	WN	1.062					9.10E-04
Netherlands	49	0.812	WN	1.051					2.78E-05
New Zealand	50	0.071	WN	1.054					1.10E-04
Norway	51	0.807	WN	1.057					7.21E-05
Poland	51	0.001	AR2	1.062	0.2571	0.2517	0.136	0.1376	2.23E-05
Portugal	49	0.021	AR1	1.066	0.3919		0.1313		8.94E-05
Russia	51	0.000	AR2	1.055	0.4084	0.5244	0.1194	0.1196	4.39E-06
Slovakia	51	0.824	WN	1.053					1.09E-04
Slovenia	32	0.272	WN	1.058					1.38E-04
Spain	51	0.000	AR2	1.065	0.4117	0.4113	0.1262	0.1298	2.81E-05
Sweden	51	0.180	WN	1.058					3.56E-05
Switzerland	51	0.136	WN	1.056					6.85E-05
Taiwan	51	0.000	AR1	1.077	0.8483		0.0698		5.14E-05
United King.	50	0.000	AR2	1.055	0.4007	0.4114	0.1265	0.1274	8.89E-06
Ukraine	50	0.000	AR1	1.060	0.6882		0.1009		1.97E-05
USA	51	0.000	AR2	1.050	0.4525	0.3485	0.1366	0.137	2.22E-06

Germany 1964-2018; WN* (White Noise with p<0.05)