The Population Dynamics of Endangered Blue Whales: Past, Present, and Future[1]

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Abstract. Blue whales (Balaenoptera musculus) are the largest animals that have ever lived on earth, but their populations were nearly driven to extinction due to industrial hunting at the beginning of the 20th century. Their numbers were estimated to be between 250,000 and 300,000 before the hunting, but this drastically declined over the years.

Continuous models of population dynamics are used to estimate the intrinsic growth rate and other demographic characteristics of their populations. The Euler-Lotka equation is used to determine the mean annual growth rate, and with the help of the piecewise exponential distribution as a life table model, simple formulas can be derived for the calculation of important demographic parameters such as the age structure, life expectancy, and maximum age.

The pre-exploitation abundance of Antarctic blue whales is found using the logistic function, assuming a minimum abundance of 1,000 in 1970, an intrinsic growth rate of 4.1%, and documented annual catches between 1904 and 1972. The estimated pre-exploitation abundance is forecast as 280,471 in 1904. Using the logistic model to forecast the population, it is calculated that it will take nearly 140 years for the population to recover to even half of its pre-exploitation abundance at current assumed rates.

To preserve the endangered blue whales, it is essential to monitor their populations continuously, develop effective conservation strategies, and reduce the anthropogenic pressures on the species. Through collaborative efforts and conservation measures, it may be possible to help these magnificent creatures recover and thrive once again.

Keywords: Population dynamics, Demographic models, Logistic function, Population forecasts, Piecewise exponential survivor function, Life Table, Euler-Lotka equation, SDGs.

1. Introduction

At the beginning of the 20th century, the population of blue whales was estimated to be between 250,000 and 300,000. However, industrial hunting led to a massive collapse of their populations. In the 1970s and 1980s, bans and moratoriums on commercial whaling were implemented, and these seem to have had some effect as the number of blue whales has increased from 5,000 (low point) in the 1990s to about 10,000 in 2010. This doubling in the considered 20 years corresponds to a growth rate of about 3.5% per year. Despite this increase, blue whales are currently classified as endangered by the IUCN (International Union for the Conservation of Nature).

This critical juncture in the history of blue whales connects directly to several Sustainable Development Goals (SDGs). SDG 14 - "Life Below Water" highlights the importance of conserving and sustainably using marine resources. The conservation of blue whales is a pivotal aspect of this goal, given their role in marine ecosystems. Moreover, the situation speaks to SDG 15 - "Life on Land" as the well-being of blue whales is intricately linked to both terrestrial and marine ecosystems.

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Despite some recovery since the implementation of whaling bans and moratoriums, the study reveals that the population is far from its pre-exploitation levels. In alignment with the United Nations' Sustainable Development Goals (SDGs), this research contributes to several key goals. Notably, it addresses SDG 14 - "Life Below Water," aiming to conserve and sustainably use marine resources, emphasizing the need for ongoing conservation efforts to ensure the long-term survival of these majestic creatures and their vital role in marine ecosystems. Furthermore, it relates to SDG 15 - "Life on Land," as blue whales' survival connects with terrestrial and marine ecosystems. Additionally, the study underscores the importance of SDG 12 - "Responsible Consumption and Production" by highlighting the impact of overexploitation and the need for sustainable practices. Moreover, it emphasizes the significance of SDG 13 - "Climate Action" since climate change impacts marine ecosystems, including the habitats of blue whales. By addressing these SDGs, this research contributes to a broader global effort to protect marine biodiversity, mitigate climate change, and promote responsible consumption and production.

Using a logistic function, the population dynamics of blue whales are analyzed in this article, focusing on past, present, and future. Assumptions about the mortality and fertility of blue whales are necessary to compute the intrinsic rate of growth, which gives the hypothetical growth rate of the whale population if the population could grow exponentially without limits, assuming no industrial hunting. The actual growth rate depends on environmental restrictions, such as food availability, and on industrial hunting. The logistic function is a mathematical model that describes how a population grows over time under the influence of limiting factors, such as limited resources or predation. The model is based on the idea that the rate of population growth is proportional to the current population size and the available resources. As the population size approaches the carrying capacity of the environment, the growth rate slows down and eventually levels off. This leads to an S-shaped curve that is typical for logistic growth.

The basis of the investigation is an article by Branch (2008), who estimated the mean annual growth rate of a whale population using the Euler-Lotka equation and a stochastic model. He assumed biologically plausible distributions for adult survival, calf survival, annual pregnancy rate, age at first parturition, and the proportion of births that are female. In my article, I adopted the variables and data from Branch (2008) but used a deterministic representation. The focus of my investigation is on the methodological presentation of the life table analysis and the stable model with additional metrics. Specifically, I used a simple life table model, the piecewise exponential distribution, which has also been used implicitly by Branch. This distribution allowed me to derive simple formulas for calculating important demographic parameters.

2. Demographic Assumptions

2.1 The Life Table

The piecewise exponential survival function is a suitable choice for modeling blue whale populations' demographics when there is limited information on mortality, and one is not interested in estimating the distribution of old age or maximum age. This function allows for the derivation of fundamental demographic parameters, and while continuous hazard functions provide similar results, they require estimating additional parameters that may not be practical for wild cetacean populations. However, a criticism of this approach is that the force of mortality function is not continuous, which can be addressed with continuous force of mortality functions like the Gompertz-Makeham, Gauss, or Lazarus formulas, as extensively described in Pflaumer (2022) by extending Branch's (2008) approach.

We use the data from Branch (2008) to construct the life table, assuming a mean survival rate of S = 0.963 for adults and S_1 = 0.84 for the first year. The following parameters are used:

The piecewise exponential survivor function derived from the given assumptions is:

$$l(x) = \begin{cases} l_1(x) = \exp(\ln S_1 \cdot x) = S_1^x & 0 \le x \le 1 \\ l_2(x) = S_1 \cdot \exp\left(-k \cdot (x-1)\right) & x > 1 \end{cases}$$

The force of mortality or hazard rate for this survivor function is:

$$\mu(x) = \begin{cases} \mu_1 = -\ln S_1 & 0 \le x \le 1 \\ \mu_2 = k & x > 1 \end{cases}$$

The functions used are depicted in Figure 1.

From the survivor function, we can calculate the life expectancy at birth (23.6 years) and median age (15 years), meaning that 50% of the whale population dies before reaching 15 years of age, with the following formulas:

Life Expectancy at Birth and Median Age

$$e(0) = \int_0^1 l_1(x)dx + \int_1^\infty l_2(x)dx = \frac{S_1 - 1}{\ln S_1} + \frac{S_1}{k}; \qquad x_{0.5} = 1 + \frac{\ln(2 \cdot S_1)}{k} \quad S_1 > 0.5$$

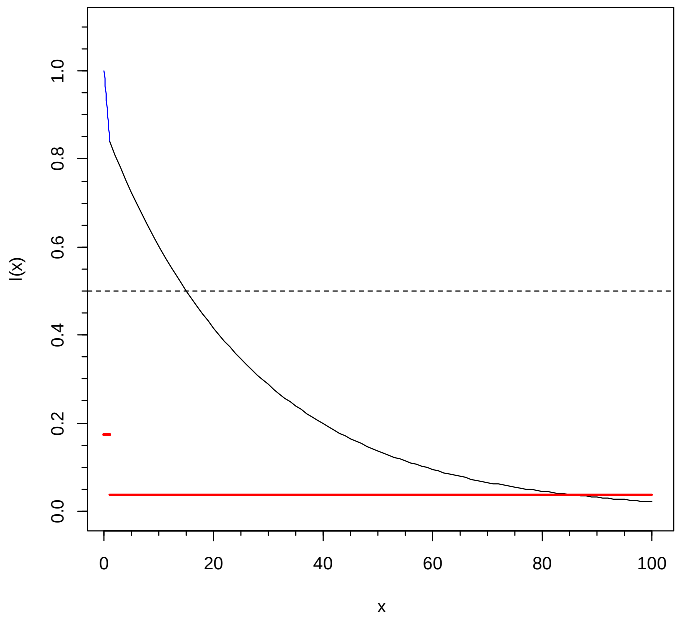

Fig. 1: Piecewise Survivor Function and Force of Mortality Function (red) (S_1=0.84, $\mu_1 = -\ln(0.84) = 0.174 \quad 0 \le x \le 1$, $\mu_2 = k = 0.037 \quad x > 1$)

Additional discussion and demographic parameters for the piecewise survivor function can be found in Pflaumer (2022).

2.2 Intrinsic rate of growth

Once the life table has been determined, a population model for blue whales can be formulated using the continuous stable model, which is well-known in demography. A brief overview of stable theory in demography is provided here, while detailed explanations and interpretations of the parameters can be found elsewhere, such as in Keyfitz (1968, 1977).
A population with an invariable age structure and a fixed rate of increase is referred to as a stable population. The stable age structure is given by:

$$c(x)dx = e^{-rx} \cdot l(x)dx \,.$$

The stable intrinsic growth rate r can be determined by solving the Euler-Lotka equation:

$$1 = \varphi(r) = \int_{\alpha}^{\beta} e^{-rx} l(x) m(x) dx \,,$$

where m(x) is the maternity function and $l(x)m(x)$ is the net maternity function.
The limits of the integral are the youngest fertile age α and the highest β. The characteristic function $\varphi(r)$ crosses the vertical axis at the net reproduction rate

$$\varphi(0) = \int_{\alpha}^{\beta} l(x) m(x) dx = R_0 \,.$$

In this case, assuming the piecewise survivor function, simple functions can be derived, with α representing the age of first parturition (α =10) and m representing the constant maternity rate (m=0.1971).

Characteristic function for the intrinsic growth rate is:

$$\Phi(r) = \int_{\alpha}^{\infty} e^{-rx} \cdot l(x) \cdot m(x)\, dx = \frac{m \cdot S_1 \exp\left(k \cdot (1-\alpha) - r \cdot \alpha\right)}{k + r}$$

Net reproduction rate is:

$$R_0 = \int_{\alpha}^{\infty} l(x) \cdot m(x)\, dx = \frac{m \cdot S_1 \exp\left(k \cdot (1-\alpha)\right)}{k}$$

Euler-Lotka equation (continuous version) is:

$$1 = \int_{\alpha}^{\infty} e^{-rx} \cdot l(x) \cdot m(x)\, dx = \frac{m \cdot S_1 \exp\left(k \cdot (1-\alpha) - r \cdot \alpha\right)}{k + r}$$

The equation cannot be explicitly solved for the intrinsic growth rate r. Therefore, only a numerical solution is possible using iterative methods such as the Newton-Raphson method or Regula Falsi.

Branch (2008) assumes an annual pregnancy rate between 1/3 and 1/2. The arithmetic mean is $5/12 \approx 0.4167$. Taking into account the proportion of births that are female (0.473), we arrive at a constant maternity rate of m=0.1971. We calculate the intrinsic rate of growth from the Euler-Lotka equation, which is given by the following formula:

$$1 = \frac{0.1971 \cdot 0.84 \cdot \exp\left(0.037 \cdot (1-10) - r \cdot 10\right)}{0.037 + r}$$

The numerical solution yields an intrinsic growth rate of r = 0.0414.

3. The Dynamic Logistic Model for Antarctic Blue Whales

Following Branch (2008a, p. 3), the pre-exploitation abundance of Antarctic blue whales is estimated using the logistic function (a S-shaped curve). It is arbitrarily assumed that the minimum abundance is 1,000 in 1970. r is the intrinsic growth rate.
The model is:

$$N_{1905} = K$$
$$N_{t+1} = N_t + r \cdot N_t \cdot \left(1 - \frac{N_t}{K}\right) - C_t,$$

where K is the carrying capacity (maximum value of the curve), assumed equal to pre-exploitation abundance, N the number of whales in the year t, and Ct is the catch in year t ,r is the intrinsic growth rate; The growth rate r is assumed to be 4.1% yearly. The value of K, representing the carrying capacity of the population, was determined in this model by assuming that the minimum population size in 1970 was 1000[2]. The annual catches are given in Table 1.

The estimated pre-exploitation abundance (K) was 280,471 in 1904. The population trend in Figure 1 shows a steady and sharp decline, interrupted only by the Second World War. From 1970, the population grew from the assumed minimum of 1000 to about 7150 in 2021. Growth rates, yearly population changes, and catches are illustrated in Figures 2 and 3.

2 Critics of the choice of 1000 as the minimum population size in 1970 may argue that this assumption is arbitrary or lacks a strong empirical basis. They may also argue that the value of K can have a significant impact on the results of the model, and that assumptions about K should be based on more robust data and analysis.To address these concerns, sensitivity analyses can be performed to investigate how the results of the model change with different assumptions about the value of K. This can help to identify the range of values for K that are most plausible given the available data and the assumptions of the model.

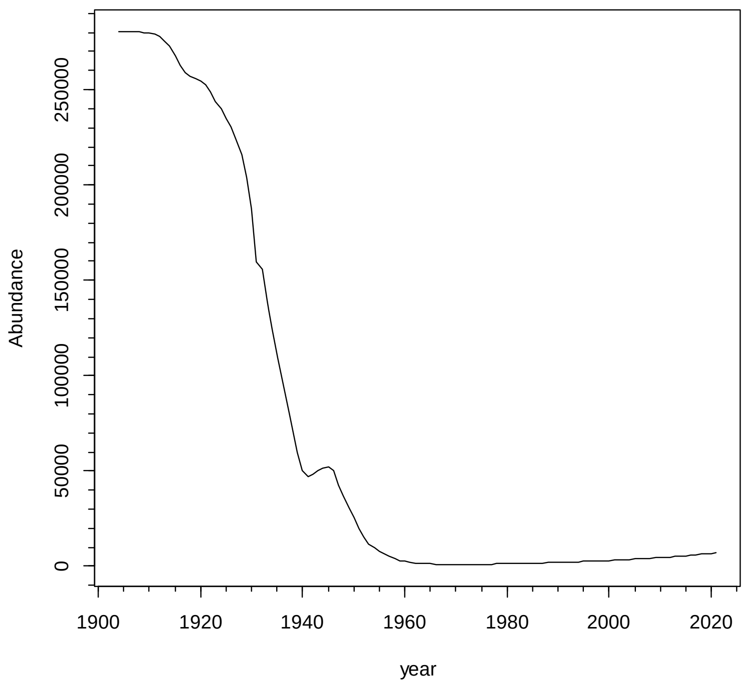

Figure 2: Estimated Abundance of Antarctic Blue Whales

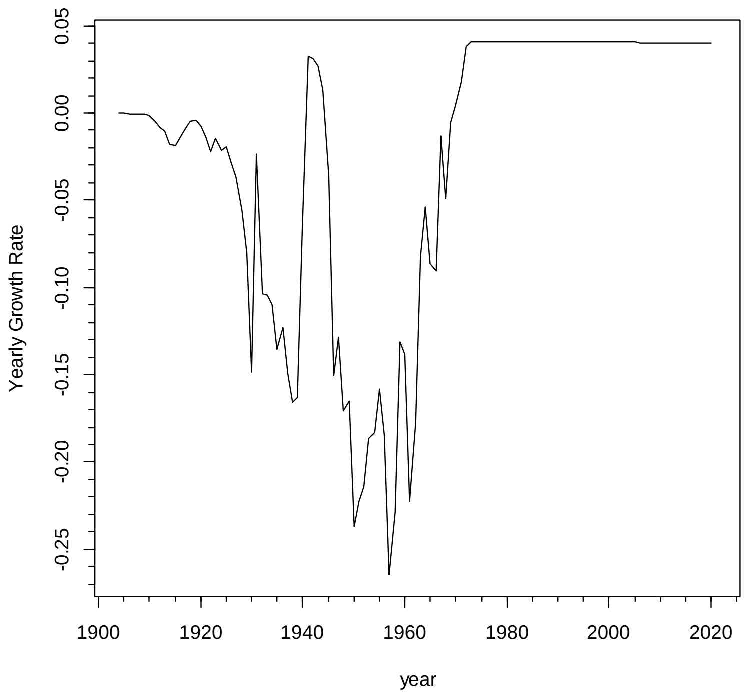

Figure 3: Estimated Yearly Growth Rates of Antarctic Blue Whales

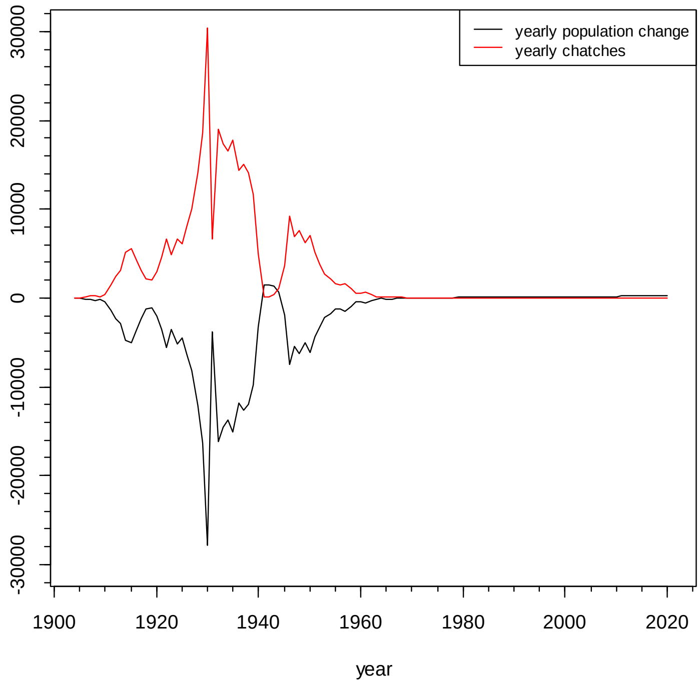

Figure 4: Estimated Yearly Population Changes and Yearly Catches of Antarctic Blue Whales
(see also Table 1)

We will use again the logistic model to forecast the population.

The forecast model is:

$$N_t = \frac{K}{1+b\cdot e^{-r\cdot t}} = \frac{280{,}471}{1+279{,}471\cdot e^{-0.041\cdot t}} \quad t=0,1,2... \quad (1970,1971,...)$$

$$b=\frac{K}{N_0}-1 \rightarrow b=\frac{280{,}471}{1{,}000}-1=279{,}471$$

$t_{K/2}=\frac{\ln b}{r}$; time until half saturation level is reached: $t_{140235.5}=137.4\,(\text{years})$ or in 2107

$t_{K-N_0}=2\cdot\frac{\ln b}{r}$; time until the population size is saturation level minus N_0:

$t_{280{,}471-1000}=274.8\,(\text{years})$ or in 2245

Figure 4 shows the population trajectory. At current estimated rates, it will take nearly 140 years for the population to recover to even half its pre-exploitation abundance. A recently accepted estimate of Antarctic blue whale abundance south of 60°S from three complete circumpolar surveys is from Branch (2007), who gives the number as 2,280 whales in 1997/98. The above model in Figure 1 predicts the abundance to be 2,800 in 1998.

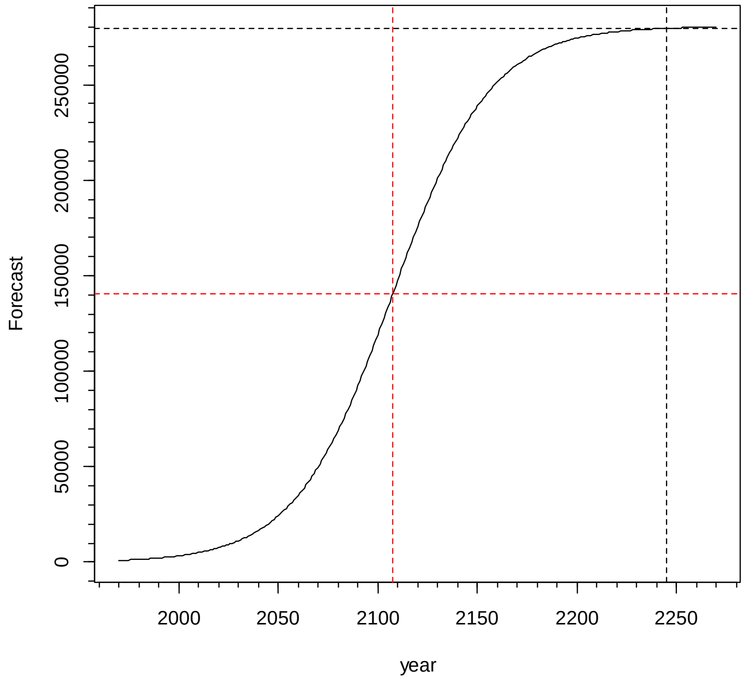

Figure 5: Population Projection of Antarctic Blue Whales (assuming 1970: 1000)

Table 1: Annual Catches of Antarctic Blue Whales

year	catches	*year*	catches	*year*	catches	*year*	catches
1904	11	*1922*	6694	*1940*	4973	*1958*	1082
1905	51	*1923*	4829	*1941*	63	*1959*	534
1906	111	*1924*	6629	*1942*	126	*1960*	481
1907	201	*1925*	6028	*1943*	346	*1961*	611
1908	244	*1926*	8143	*1944*	1047	*1962*	395
1909	176	*1927*	10006	*1945*	3603	*1963*	183
1910	422	*1928*	14130	*1946*	9234	*1964*	129
1911	1477	*1929*	18608	*1947*	6936	*1965*	164
1912	2391	*1930*	30365	*1948*	7641	*1966*	155
1913	3113	*1931*	6577	*1949*	6196	*1967*	58
1914	5125	*1932*	18961	*1950*	7057	*1968*	95
1915	5503	*1933*	17413	*1951*	5111	*1969*	47
1916	4356	*1934*	16578	*1952*	3851	*1970*	37
1917	3061	*1935*	17815	*1953*	2704	*1971*	23
1918	2143	*1936*	14414	*1954*	2171	*1972*	3
1919	1987	*1937*	15019	*1955*	1578		
1920	2955	*1938*	14110	*1956*	1504		
1921	4552	*1939*	11722	*1957*	1667		

Source: Branch (2008a, p. 6)

4. Methodological Remarks in Estimating the Pre-exploitation Carrying Capacity

In estimating the pre-exploitation carrying capacity of blue whale populations, we employed a dynamic logistic model—a widely used framework for understanding population dynamics. This model integrates key factors such as intrinsic growth rate, carrying capacity, and historical catch data to simulate the trajectory of blue whale populations over time.

Our approach involved conducting simulations with various choices of carrying capacity (K). By systematically varying the value of K, we explored how different levels of carrying capacity influence the population dynamics of blue whales. This trial-and-error procedure allowed us to identify the carrying capacity value that resulted in the minimum abundance of the blue whale population over time—a critical indicator of population sustainability.

To assess the accuracy of our model, we compared the minimum abundance obtained from simulations with historical counting or estimations of pre-exploitation abundance. This comparison served as a validation step, helping us determine the carrying capacity that best aligned with observed population levels prior to significant human exploitation.

Through this iterative process, we pinpointed the carrying capacity at which the blue whale population would reach its lowest point, indicating a balance between growth potential and environmental constraints.

The four graphs in Figure 6 depict the simulated population dynamics of blue whales under varying carrying capacity (K) values. Each graph represents a different scenario, providing insights into how different levels of carrying capacity impact the abundance and sustainability of blue whale populations over time. K not only influences the size of the minimum but also the time at which it occurs.

Graph 1 (K = 280,000): In this scenario, the population experiences a rapid decline and eventual extinction. The carrying capacity of 280,000 proves insufficient to sustain the population, highlighting the vulnerability of blue whales to low carrying capacity levels.

Graph 2 (K = 280,471): Although slightly higher than in the previous scenario, at a carrying capacity of 280,471, the blue whale population demonstrates a slow recovery from its minimum abundance of 1000. With a deterministic view, this scenario suggests a gradual increase in population size over time, eventually reaching its pre-exploitation level (see Figure 5). However, a stochastic view acknowledges the vulnerability of the population, as slight changes in influential parameters such as lower fertility rates and higher mortality rates could lead to the population dying out It's important to note that while the deterministic perspective offers a hopeful outlook for the population's recovery, stochastic factors introduce uncertainty and highlight the need for careful monitoring and conservation efforts to ensure the long-term survival of blue whales (see Branch, 2008).

Graph 3 (K = 281,000): Under a carrying capacity of 281,000, the population exhibits fluctuations but maintains relative stability over time. This scenario suggests that a slightly higher carrying capacity could support a more resilient blue whale population.

Graph 4 (K = 282,000): In this scenario, the population demonstrates steady growth and stability, indicating that a carrying capacity of 282,000 is sufficient to sustain a thriving blue whale population in the long term.

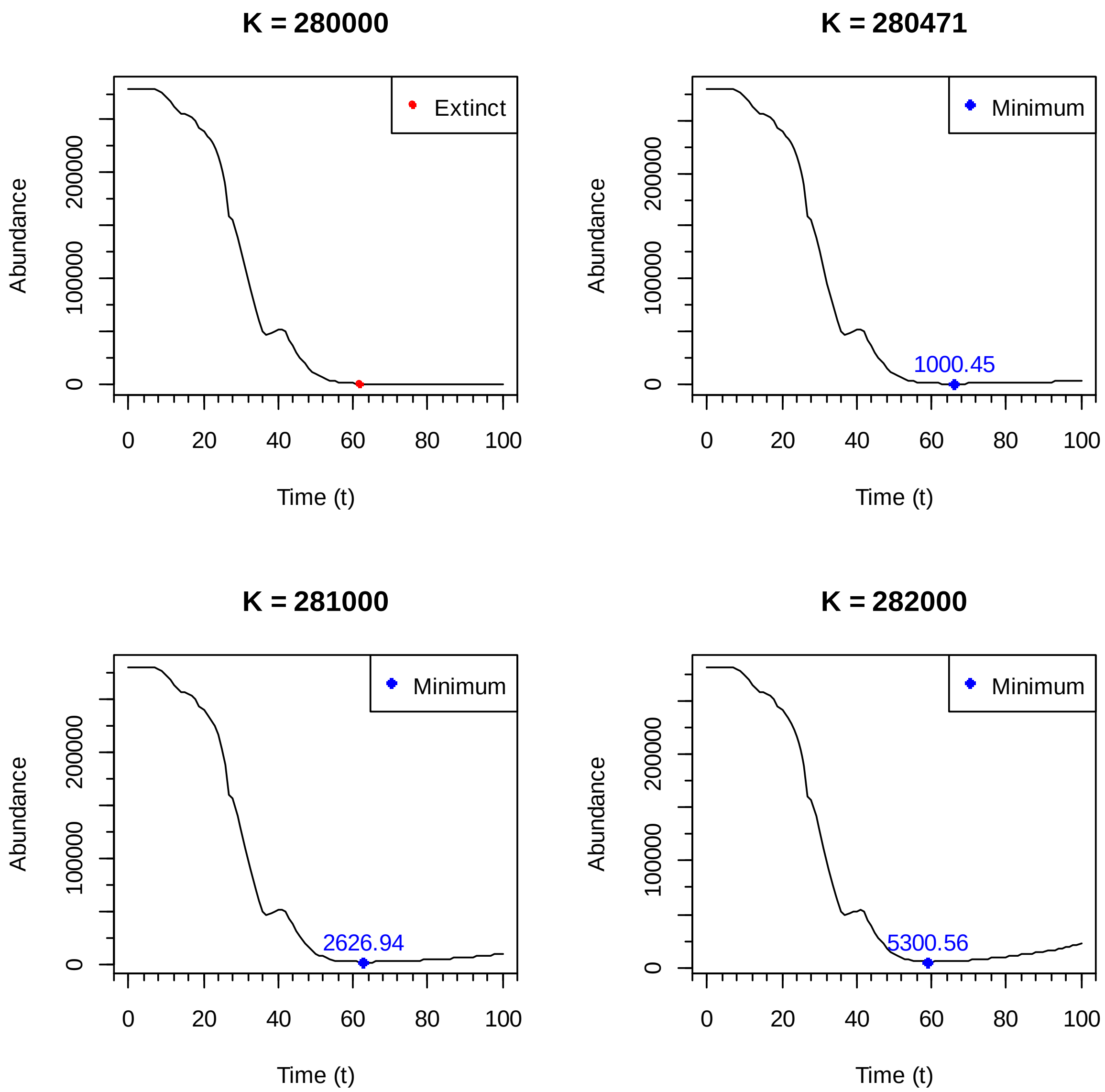

Fig. 6: Blue Whale Population Dynamics: Impact of Varying Carrying Capacity

5. Conclusion

In conclusion, our analysis underscores the profound and enduring impact of commercial whaling on the Antarctic blue whale population, which continues to struggle in its efforts to regain pre-exploitation numbers. Through comprehensive life table analysis and population model estimates, we have gained insights into the complex dynamics of this population, emphasizing the critical role of age structure and intrinsic growth rate in predicting future trends.

The projections from our logistic model present a sobering reality, suggesting that, at current estimated rates, it could take nearly 140 years for the population to recover to even half its pre-exploitation abundance. However, it's essential to acknowledge the stochastic nature of population dynamics, as highlighted in our interpretation of Graph 2 in Figure 6. While deterministic perspectives offer hope for gradual recovery, stochastic factors introduce

uncertainty. Slight changes in influential parameters, such as lower fertility rates and higher mortality rates, could significantly impact population outcomes, leading to extinction.
These findings emphasize the urgency of sustained monitoring and conservation initiatives to safeguard the survival of the Antarctic blue whale. As outlined in the introduction, this species holds ecological, cultural, and economic significance, making its preservation a moral imperative aligned with the Sustainable Development Goals (SDGs), particularly those concerning life below water (SDG 14) and biodiversity conservation (SDG 15).
Furthermore, our research contributes to broader sustainability goals by highlighting the interconnectedness of ecosystems and the need for concerted efforts to achieve a sustainable future. By integrating ecological insights with conservation efforts, we aim to address the challenges facing marine life and environmental conservation, fostering resilience and sustainability across various domains.

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Appendix: R-Code for Blue Whale Population Dynamics Simulation Using ChatGPT

```
#Simulation and Analysis of Blue Whale Population Dynamics
# with Varying Carrying Capacities

# Load necessary libraries
library(Hmisc)

# Define parameters
r <- 0.041
Tmax <- 100
K_values <- c(280000, 280471, 281000, 282000)

# Define catch data
C <- c(11, 51, 111, 201, 244, 176, 422, 1477, 2391, 3113, 5125, 5503, 4356, 3061, 2143, 1987, 2955, 4552,
6694, 4829, 6629, 6028, 8143, 10006, 14130, 18608, 30365, 6577, 18961, 17413, 16578, 17815, 14414, 15019,
14110, 11722, 4973, 63, 126, 346, 1047, 3603, 9234, 6936, 7641, 6196, 7057, 5111, 3851, 2704, 2171, 1578,
1504, 1667, 1082, 534, 481, 611, 395, 183, 129, 164, 155, 58, 95, 47, 37, 23, 3)
C <- c(C, rep(0, Tmax - length(C)))

# Run simulations
N_results <- matrix(0, nrow = Tmax + 1, ncol = length(K_values))
population_status <- rep("Survived", length(K_values))
min_abundance <- rep(NA, length(K_values))
min_time <- rep(NA, length(K_values))

for (i in seq_along(K_values)) {
  N <- numeric(length(t))
  N[1] <- K_values[i]

  for (h in seq_len(Tmax)) {
    new_N <- N[h] + r * N[h] * (1 - N[h] / K_values[i]) - C[h]

    # Check if population size becomes negative
    if (new_N <= 0) {
      population_status[i] <- "Extinct"
      break  # Terminate the simulation
    }

    # Ensure N doesn't go negative
    N[h + 1] <- max(new_N, 0)
  }

  N_results[, i] <- N

  # Find minimum abundance and corresponding time
  min_abundance[i] <- min(N)
  min_time[i] <- which.min(N) - 1
}

# Plot results
par(mfrow = c(2, 2))

for (i in seq_along(K_values)) {
  plot(0:Tmax, N_results[, i], type = "l", xlab = "Time (t)", ylab = "Abundance", main = paste("K =",
K_values[i]))
  minor.tick(nx = 5, ny = 2, tick.ratio = 0.5)

  if (population_status[i] == "Extinct") {
    # Add marker for extinction
    points(min_time[i], min_abundance[i], col = "red", pch = 20)
```

```
    legend("topright", legend = "Extinct", col = "red", pch = 20)
  } else {
    # Add marker for minimum abundance
    points(min_time[i], min_abundance[i], col = "blue", pch = 16)
    legend("topright", legend = "Minimum", col = "blue", pch = 16)

    # Add text label for minimum abundance
    text(min_time[i], min_abundance[i], label = round(min_abundance[i], digits = 2), pos = 3, col = "blue")
  }
}

# Find minimum abundance and corresponding time for survived populations
min_values <- data.frame(K = K_values, Minimum = min_abundance, t = min_time)

# Print results
print(min_values)
print(N_results)
```