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Possessing mathematical competence: Normative versus descriptive view

During the past two decades, the concept of mathematical competence has been actively discussed both in research and in practice. An important conceptual question that mostly gets neglected in the discussions relates to the basic assumptions concerning the possession of competence. Different positions regarding this question are adopted in different contexts, but the adopted position is barely ever made explicit, which easily leads to misunderstandings. This paper addresses two existing views on mathematical competence—the normative and the descriptive view—that differ in their assumptions regarding the possession of competence. The two views are discussed from a theoretical, practical, and empirical perspective.

Theoretical underpinnings of the two views

Regarding the issue of possessing competence, competence can generally be conceptualized as a binary concept, as a continuum, or as a combination of these two approaches. The fundamental assumption behind the first approach is that at a given time, a person is either competent or incompetent (Vitello et al., 2021). In order for someone to possess competence in a domain, they thus need to demonstrate some pre-defined minimum level of performance in that domain. When competence is regarded as a continuum, the stance taken instead is that a person can be more or less competent (Vitello et al., 2021). As a result of this assumption, any performance counts as an indicator of competence. If these two approaches are combined, then it is assumed that a competent/incompetent boundary exists, but different levels of performance can further be identified within both competent and incompetent performance areas (Vitello et al., 2021).

The latter, combined approach is the position taken by the view I will call here the normative view on mathematical competence. With the normative view I refer to the notion of mathematical competence originally introduced by the Danish KOM project at the early 2000s. In this project (for a more detailed account, see, e.g., Niss et al., 2016), mathematical competence was chosen as a label for a curriculum reform idea that emphasized the enactment of mathematics as the main goal of mathematics education, in contrast to the wide-spread practice of teaching principally content knowledge and procedural skills. Inherent in the enactment of mathematics is the necessity of basing actions on mathematical insight, which has been made explicit in the following definition: “*mathematical competence* is someone’s insightful

readiness to act appropriately in response to all kinds of *mathematical* challenges pertaining to given situations” (Niss & Højgaard, 2019, p. 12). Central to the competence concept of the KOM project is also the fact that competence is comprised of several *competencies* (e.g., modeling competency, reasoning competency, representation competency), each of which enables a person to master mathematical challenges of a specific type. Furthermore, the KOM project posits that each mathematical competency—and thus mathematical competence in general—can evolve in an unlimited manner (Niss & Højgaard, 2019). Hence, it is possible to distinguish lower and higher levels of competence in this normative view on mathematical competence. However, given the necessity of mathematical insight, there also exists a competent/incompetent boundary, which deals with the difference in the quality of action: actions that are not based on mathematical insight are not at all considered instances of mathematical competence in the normative view.

The competence concept of the KOM project has influenced the curriculum design in Denmark as well as in many countries outside of Denmark (Niss et al., 2016). However, in the school curricula, only the idea of the existence of different mathematical competencies has typically been adopted and further combined with the rather psychometric idea of competence as a continuum. This combination results in what I will call here the descriptive view on mathematical competence. Such a view is visible also in the German educational standards of mathematics, in which several competencies are first listed and different levels of each competency are then characterized by varying degrees of cognitive complexity. Regarding reasoning competency in upper secondary mathematics, for instance, a person who can only provide simple justifications based on performing procedural skills possesses a low level of reasoning competency (no matter the quality of mental action that produces the justifications), whereas a person who also manages mathematical proofs has reached a high level of reasoning competency (Kultusministerkonferenz, 2015).

Consequences of the two views for teaching and assessment

The normative and the descriptive view on mathematical competence lead to teaching and learning practices that differ in some fundamental respects. Within the descriptive view, the focus is, above all, on carrying out different types of mathematical practices that are connected to the different competencies. In other words, apart from symbolic manipulations, teaching and learning of mathematics should also entail practices related to mathematical reasoning and modeling, for example. At first sight, this supposes an enrich-

ment for mathematics education. However, the lack of any competent/incompetent boundary in the descriptive view allows practices related to any competency to get drilled and learnt without actual mathematical insight, in the same way as symbolic manipulations often do. If teaching practice is not reflected upon more closely, the descriptive view on mathematical competence may thus lead to learning environments in which students appear to be reasoning, modeling, or otherwise enacting mathematics as they complete tasks that address different types of mathematical practices, while they are in fact mentally engaged with “pseudo-analytical thought processes” (Vinner, 1997) only, irrespective of the task. From the normative view on mathematical competence instead, teaching should first and foremost foster insightful actions, regardless of the type and the complexity of the mathematical practices that are to be carried out.

Concerning the assessment of mathematical competence from the descriptive view, tests should contain, above all, tasks that address several types of mathematical practices. From the normative view, the aspect of mathematical insight is again paramount. In oral exams, the degree of insight behind an answer can be (relatively) easily verified with the help of follow-up questions. In written exams, however, the judgement of the degree of insight—and therefore of competence—can only be based on the written solutions of a student. Especially in the typical teacher-made tests, which often predominantly consist of relatively short tasks, possible training effects and the superficial solvability of tasks need to be taken into account when selecting tasks suitable for testing competence. From an objective perspective, a task may be complex—concerned with producing a mathematical proof, for example—but if due to the prior learning experiences it is likely that many students will simply reproduce a proof they have learnt by rote, then the task does not test mathematical competence in the normative sense.

The view on mathematical competence in the research on assessment

In the previous studies on assessment of mathematics, competence requirements have been examined especially in written test tasks. In the German-speaking context, such studies use competence frameworks that are based on the cognitive demand of the mathematical practices and of the mathematical content (see, e.g., Drüke-Noe, 2014; Siller et al., 2015). Similar to many school curricula, these studies seem to follow the descriptive view on mathematical competence: the focus of the studies is to examine the types and the complexity of mathematical practices that from an objective point of view need to be carried out in test tasks. No attention is paid to the quality with which the practices may be performed by the students.

Given that the normative view on mathematical competence actually precedes the descriptive view, as discussed above, it would be reasonable to finally start addressing the question of competence requirements in written test tasks from the normative view. Following the previous considerations in this paper, this requires a shift of attention from the objective complexity of tasks to the familiarity and the superficial solvability of tasks. Paying attention to the familiarity of tasks further means that the prior learning experiences of students must be taken into account. A first attempt to comply with such requirements is made in a current research project, in which competence requirements in written teacher-made tests of upper secondary mathematics in Germany, Sweden, and Finland are examined from the normative view (for more information on the theoretical and methodological underpinnings of the project, see Kröger & Büchter, in press).

References

- Drüke-Noe, C. (2014). Empirische Untersuchungen zur Aufgabenkultur in Klassenarbeiten neunter und zehnter Klassen im Fach Mathematik [Empirical investigations to task culture within written teacher-made tests in 9th and 10th grade in mathematics]. In J. Roth, & J. Ames (Eds.), *Beiträge zum Mathematikunterricht* (pp. 313–316). GDM. <http://dx.doi.org/10.17877/DE290R-231>
- Kröger, I., & Büchter, A. (in press). Mathematical competence and classroom assessment: A framework for task classification. *Proceedings of the 13th Congress of the European Society for Research in Mathematics Education*.
- Kultusministerkonferenz. (2015). *Bildungsstandards im Fach Mathematik für die Allgemeine Hochschulreife* (Beschluss vom 18.10.2012). https://www.kmk.org/fileadmin/Dateien/veroeffentlichungen_beschluesse/2012/2012_10_18-Bildungsstandards-Mathe-Abi.pdf
- Niss, M., Bruder, R., Planas, N., Turner, R., & Villa-Ochoa, J. A. (2016). Survey team on: Conceptualisation of the role of competencies, knowing and knowledge in mathematics education research. *ZDM*, 48, 611–632. <https://doi.org/10.1007/s11858-016-0799-3>
- Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics*, 102, 9–28. <https://doi.org/10.1007/s10649-019-09903-9>
- Siller, H.-S., Bruder, R., Hascher, T., Linnemann, T., Steinfeld, J., & Sattlberger, E. (2015). Competency level modelling for school leaving examination. In K. Krainer & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education* (pp. 2716–2723). Charles University in Prague, Faculty of Education and ERME.
- Vinner, S. (1997). The pseudo-conceptual and the pseudo-analytical thought processes in mathematics learning. *Educational Studies in Mathematics*, 34, 97–129. <https://doi.org/10.1023/A:1002998529016>
- Vitello, S., Grotorex, J., & Shaw, S. (2021). *What is competence? A shared interpretation of competence to support teaching, learning and assessment*. Cambridge University Press & Assessment.