Simple Martingale Betting versus Labouchere Betting at Roulette: Simulations and Statistical Evaluations[1]

Peter Pflaumer
Department of Statistics, Technical University of Dortmund

Abstract: This paper investigates Labouchere betting systems through extensive simulations and a comprehensive comparison with martingale betting strategies. The analysis focuses on the statistical profit distribution of a Labouchere round, revealing it as a mixed random variable where the continuous part is approximated by the Gumbel-Gompertz distribution. The intricate nature of Labouchere's profit distribution is examined through simulations and statistical modeling, providing insights into its characteristics and significance. The study extends to practical considerations, presenting simulated outcomes under unlimited stakes, further enriching the understanding of Labouchere betting dynamics. The paper contributes to both theoretical and applied aspects of Labouchere betting, shedding light on its complexities and implications for players and researchers alike.

Keywords: Labouchere betting, Gumbel-Gompertz distribution, simulation, martingale, risk analysis, betting strategies, roulette, statistical modeling.

1. Labouchere Betting

The Labouchere betting system[2], also known as the Cancellation or Split Martingale system, is a negative progression strategy employed in gambling. Players initiate the process by forming a sequence of numbers, often presented as a list, to represent their desired profit goal. In each betting round, the player stakes an amount equal to the sum of the first and last numbers in the sequence. If the bet is successful, those numbers are eliminated; if it loses, the sum is appended to the end of the sequence. The iterative process continues until the entire sequence is cleared, achieving the profit goal, or until the player decides to stop (refer to the example in Table 1 or additional examples in the Appendix). Despite providing a structured approach to betting, Labouchere, like other systems, does not alter the fundamental odds of the game.

The Labouchere betting system is not exclusive to roulette; it can be applied to various casino games featuring even-money bets or bets with near 50-50 odds. While frequently discussed in the context of roulette (e.g., betting on red or black, odd or even), the Labouchere system can be adapted for other games such as baccarat, craps (specific bets), or even sports betting with binary outcomes. The key is to apply the system to games where bets have relatively equal chances of winning or losing, allowing players to adjust their sequence of numbers based on outcomes.

Several mathematical papers have explored aspects of this intricate betting system (see Downton, 1980; Ethier, 2010; Zubrilina, 2018; Han and Wang, 2019). For a simulation study, refer to Billings and Del Barco (2017). Illustrative presentations of Martingale Doubling and Labouchere Systems can be found in Rodrigues and Mendes (2018), along with their corresponding R codes.

---

[1] This article is a revised written version of an accepted poster presentation originally intended for “The 18th International Conference on Gambling & Risk Taking,” held in Las Vegas, Nevada, on May 23-25, 2023.

[2] The English politician, writer, and publisher Henry Du Pre Labouchere (1831-1912) popularized the system, see, e.g., Ethier (2010, p. 313). Labouchere devised a method of playing at Monte Carlo which became famous as “Labby´s System” (Pearson, 1936, p. 46).

Table 1: Example of the Labouchere system with the initial list {1,2,3,4}

Coup number	Bet Size	Result	List	profit (gain, loss)	cumulative profit
1	5	Win	2,3	5	5
2	5	Loss	2,3,5	-5	0
3	7	Loss	2,3,5,7	-7	-7
4	9	Loss	2,3,5,7,9	-9	-16
5	11	Win	3,5,7	11	-5
6	10	Loss	3,5,7,10	-10	-15
7	13	Win	5,7	13	-2
8	12	Win	empty	12	10

2. Some Important Roulette Martingale Formulas

We consider an unbiased roulette wheel, assuming that each number on the wheel is equally probable. Our analysis is confined to the European version of roulette with 37 numbers (including a single zero). However, the results can seamlessly apply to the American version with 38 numbers (including a double zero). Additionally, in our assumptions, if zero appears, all bets on simple chances (red or black, even or odd, low or high) result in a loss. Hence, the probability p of a loss on each coup is 19/37.

The expected profit or gain from a one-unit bet on a simple chance (red or black numbers, even or odd numbers, and low (1–18) or high (19–36) numbers) is given by:

$$E(g) = 1 \cdot (1-p) - 1 \cdot p = 1 - 2p = -0.027027 .$$

The variance is calculated as:

$$Var(g) = 1^2 \cdot (1-p) + (-1)^2 \cdot p - (1-2p)^2 = 4p - 4p^2 = 4p \cdot (1-p) = 0.99927 .$$

Formulas for the martingale strategy can be easily derived (see, e.g., Pflaumer, 2019).

The distribution of the gain or profit G_i in the i-th martingale round follows a two-point distribution:

$P\left(G_i = -\left(2^n - 1\right)\right) = p^n$ and $P\left(G_i = 1\right) = 1 - p^n$

with the expected value and the variance:

$$E(G_i) = -\left(2^n - 1\right) \cdot p^n + 1 \cdot \left(1 - p^n\right) = 1 - \left(2p\right)^n ,$$

$$Var(G_i) = \left(4p\right)^n - \left(2p\right)^{2n} ,$$

where n is the maximum number of coups in a martingale round that results in either a win of 1 or a loss of $2^n - 1$. The maximum bet size is 2^{n-1}.

The expected number of coups within a martingale round is given by:

$$E(m) = \frac{1 - p^n}{1 - p} .$$

We can calculate the expected total stake within a martingale round:

.

$$E(bet) = \frac{1-(2p)^n}{1-2p} = \frac{E(G)}{E(g)}.$$

Finally, we mention the expected bet of a coup within a martingale round, which is:

$$E(bet_c) = \frac{E(bet)}{E(m)} = \frac{1-p}{1-p^n} \cdot \frac{1-(2p)^n}{1-2p}.$$

We can calculate the probability of winning a martingale round:

$$probwin = 1 - p^n.$$

In conclusion, the expected value of a bet per coup is:

$$E(bet_c) = \frac{1-p}{1-p^n} \cdot \frac{1-(2p)^n}{1-2p} = \frac{E(G)}{E(g) \cdot E(m)} \qquad p \neq 0.5.$$

And the expected profit from a single-unit bet is:

$$E(g) = \frac{E(G)}{E(bet)} = \frac{E(G)}{E(bet_c) \cdot E(m)} = (1-2p).$$

Ethier (2010, p. 279) notes that this ratio is not coincidental, showing that all systems have this property (see Ethier, 2010, p. 298 ff). "All betting systems lead ultimately to the same mathematical expectation of gain per unit amount wagered" (Epstein, 2009, p. 52).

3. Comparing Martingale and Labouchere Betting

3.1 Simulation of One Million Labouchere Sequences

Simulations were conducted using the statistical software package R to simulate one million rounds betting strategies. In the Martingale presentation, the initial bet was set at one unit, signifying a sequence of n=5 doublings to reach the maximum bet of 16. For Labouchere, five standardized sequences were chosen: {1}, {0.5, 0.5}, {0.25, 0.25, 0.25, 0.25},{0.1, 0.2, 0.3, 0.4}and $\left\{\frac{3}{25}, \frac{4}{25}, \frac{5}{25}, \frac{6}{25}, \frac{7}{25}\right\}$[3]. A successful sequence in Labouchere resulted in a gain of one unit. The maximum bet size for both strategies was capped at 16 units. To enhance the interpretability of the parameters, the analytical results of Martingale, derived from corresponding formulas, were compared with the simulation results of Labouchere. In this context, a "standardized sequence" in the Labouchere betting system is a predefined list of numbers where each number represents a fraction or percentage of the total profit goal. The sum of the numbers in the list should equal 1 or 100%, making it a clear and systematic plan for managing bets. The findings are detailed in Table 2.

[3] The list $\{3,4,5,6,7\}$ is the sequence played by Labouchere himself.

Table 2: Important parameters (Simulation 1 million rounds)

	Martingale	Labouchere				
		Lab1	Lab2	Lab3	Lab4	Lab5
list		1	0.5,0.5	0.25…0.25	0.1,0.2..0.4	3/25,4/25..7/25
n	**5**	-	-	-	-	-
min. bet/coup	1	1	0.5+0.5	0.25+0.25	0.1+0.4	10/25
max. bet/coup	16	16	16	16	16	16
p	19/37	19/37	19/37	19/37	19/37	19/37
probwin	0.964	0.974	0.974	0.973	0.973	0.973
EG	-0.143	-0.245	-0.339	-0.449	-0.444	-0.487
EGc	-0.072	-0.079	-0.076	-0.05	-0.049	-0.043
VarG	35.259	66.353	76.695	89.481	89.409	94.337
SdvG	5.938	8.146	8.758	9.459	9.456	9.713
no. coups	1,982,157	3,082,140	4,485,997	8,978,818	8,984,422	11,334,123
Ebet	5.278	9.052	12.553	16.625	16.443	18.03
Ebetc	2.663	2.937	2.798	1.852	1.83	1.591
Eg	-0.027	-0.027	-0.027	-0.027	-0.027	-0.027
max.loss/round	31	136	171	191	196.1	185.4
coups per round						
min(*m*)	1	1	1	2	2	3
Q1(*m*)		1	1	4	4	4
Median(*m*)		1	3	5	5	7
Em (mean)	1.982	3.082	4.486	8.979	8.984	11.33
Q3(*m*)		4	6	11	11	15
max(*m*)	5	32	41	61	69	83

E(G): Expected gain of a round; E(bet): Expected bet of a round; E(Gc): Expected gain of a coup in a round; E(m): Expected number of coups of a round; E(betc): Expected bet per coup of a round; p: Probability of a simple chance at Roulette; probwin[4]: Probability to win a round; Eg: Expected profit from a single-unit bet; Var(G): Variance of a round; Sdv(G): Standard Deviation of a round; E(m): Expected number of coups of a round; Q1 and Q3: 1st and 3rd Quartile.

The analysis of Table 2 underscores key insights into the comparative performance of Labouchere and Martingale betting systems. Labouchere demonstrates a marginally higher probability of winning rounds, yet at the cost of increased risk, evident in heightened variance and maximum loss. Importantly, both the Martingale and Labouchere systems ultimately converge to the same negative mathematical expectation of gain, denoted as E(g) per unit amount wagered.

Martingale stands out for its efficiency, demanding the fewest coups for a set number of rounds, with an expected number of coups (Em) at 1.982. In contrast, Labouchere's effectiveness fluctuates with list structure and length, resulting in more coups and a higher average bet. The expected number of coups for Labouchere ranges from 3.082 to 11.330 for varying list structures. Breaking down the risk aspects reveals Martingale's smaller risk parameters in terms of variance and maximum loss, though its doubling strategy poses potential rapid and substantial losses. Labouchere, with a more gradual approach, remains exposed to prolonged sequences, particularly with extensive lists. Decision-making hinges on

[4] $probwin = 1 - \left(\frac{19}{37}\right)^5 = 0.9643$

individual risk tolerance, bankroll, and preferences. While Martingale may have lower risk metrics per round, when considering a fixed time duration, such as one hour of play, the potential intensity of betting and exposure to risk increase due to the faster progression of bets. In contrast, Labouchere's more gradual approach means that, within the same time frame, the number of rounds played may be fewer, resulting in a lower overall exposure to risk during that period.

Martingale adopts an aggressive stance, whereas Labouchere offers a slower progression, highlighting the trade-off between speed and prolonged exposure to risk. The time factor is crucial, given Martingale's rapid escalation in bet sizes and Labouchere's potential for an extended number of coups, emphasizing the need for a nuanced understanding of both average bet size and time commitment associated with each system.

Figure 1 and Figure 2 illustrate the distribution of the loss (probability=2.7%) and the distribution of the number of coups per round for a particular Labouchere sequence. Note that the win is one unit, with a probability of 97.3 %.

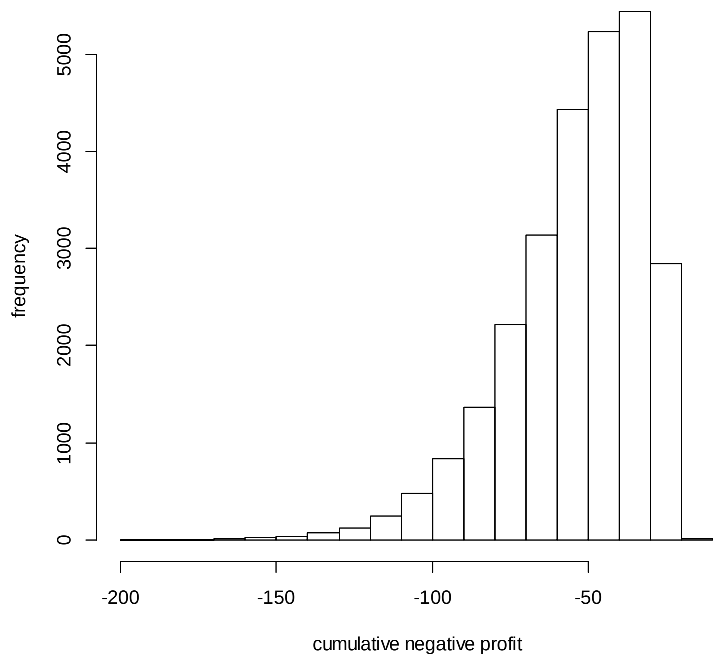

Figure 1: Histogram of the cumulative negative profit using Labouchere betting with the list {0.1, 0.2, 0.3, 0.4}; minimum=-196, maximum=-16; mean=-53.5

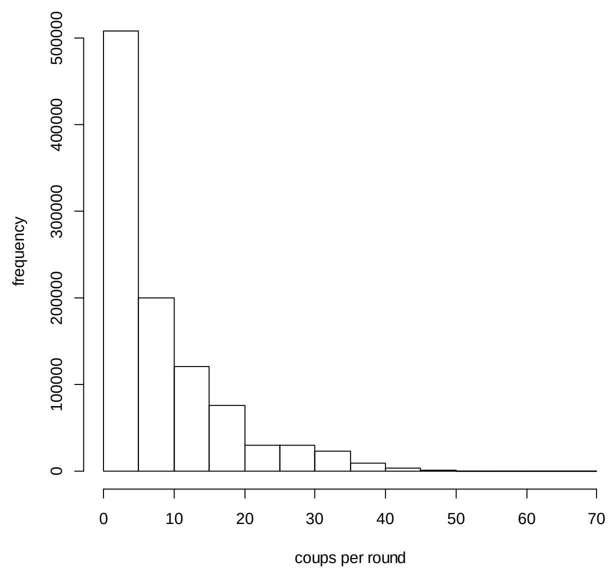

Figure 2: Number of coups per round using Labouchere betting with the list {0.1, 0.2, 0.3, 0.4}; minimum=2, maximum=69; mean=9

3.2 Simulations Assuming Unlimited Stakes

We now assume that there is no house limit, and the player has unlimited capital. In this case the player definitely wins one unit after completing a Labouchere round. The list is {0.5,0.5}. Our results are based on 100,000 simulated Labouchere rounds or 483,005 coups. The results in Table 3 show the mean and the quantiles of the coups. For example, 50% of all rounds produce a win of one unit (median) after three or fewer coups. The maximum number of coups to successfully complete one round is in this simulation 145.

Table 3: Quantiles of the number of coups and mean to win 1 unit

Min.	Q1	Median	Mean	Q3	Max.
1	1	3	4.83	6	145

The betting distribution is illustrated in Figure 3. The modal bet is one unit. The mean of the bets is 16.9 and is much higher than the median bet of 2 because of the possibility of very high bets. The maximum bet is 73,848 units.

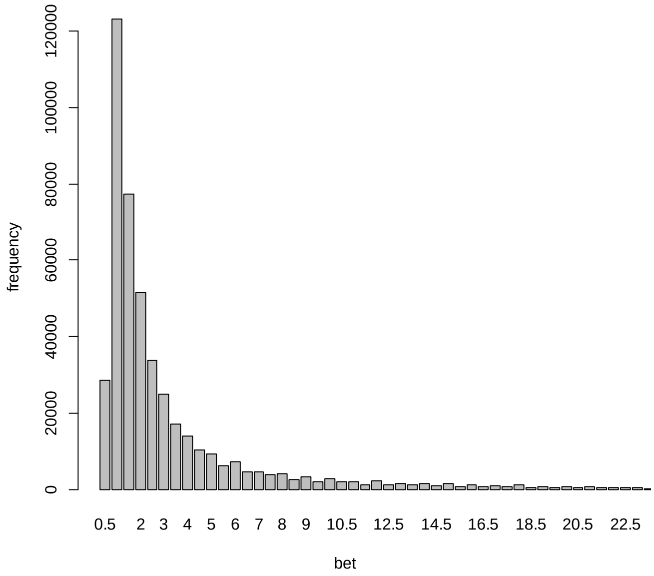

Figure 3: Bar plot of bets using Labouchere betting with the list {0.5,0.5}; 483,005 coups; minimum=0.5, Q1=1, median=2, Q3=4, maximum=73,848; mean=16.9 , sdv=446.4

Figure 4 shows the sequence of the longest Labouchere round of the 100,000 simulated rounds. The number of coups is 145. The highest loss of a coup is 60,601.5, whereas the highest gain is 73,848. The player needs a capital reserve of at least 271,792 units to finish this round, which finally guarantees a profit of 1 unit.

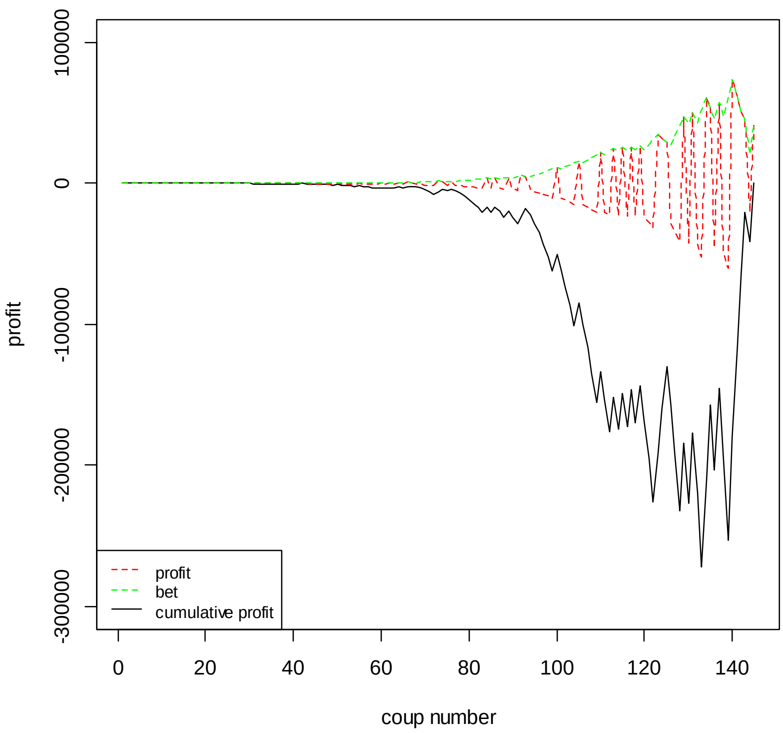

Figure 4: Longest Labouchere betting sequence with the highest capital investment of the 100,000 simulated rounds (list {0.5,0.5}); maximum cumulated negative profit: 271,792.

4. The Profit Distribution of a Labouchere Round as a Mixed Random Variable

The distribution of the gain or profit G_i in the i-th martingale round follows a two-point distribution:

$P\left(G_i = -\left(2^n - 1\right)\right) = p^n$ and $P\left(G_i = 1\right) = 1 - p^n$

with the expected value and the variance

$E(G_i) = -\left(2^n - 1\right) \cdot p^n + 1 \cdot \left(1 - p^n\right) = 1 - \left(2p\right)^n$,

$Var(G_i) = \left(4p\right)^n - \left(2p\right)^{2n}$ (see, e.g., Pflaumer, 2019)

The profit distribution of a Labouchere round is more complicated. We describe the Labouchere profit distribution as a mixed type distribution that has both a continuous and a discrete component in its probability distribution. Figure 5 shows the simulated empirical distribution of a Labouchere bet with the list {0.5, 0.5} (see Table 2). The profit distribution has both a discrete and an almost continuous distribution or a probability histogram (see Weld and Leemis, 2017). The profit is 1 with a probability of 0.974; the probability of a loss is 0.026 (corresponds to the area in the probability histogram).

Let the random variable X be the profit of a Labouchere round. Using our simulation results, X is modeled by

$$X \sim \begin{cases} C \text{ with probability } p_1 \\ D \text{ with probability } 1 - p_1 \end{cases}$$

where $f_C\left(x\right) =$ density of the negative outcome (x<0) and $f_D\left(x\right) = P\left(X = x_0\right)$; $x_0 > 0$.

In our example, $x_0 = 1$.

Since $f_C\left(x\right)$ is a left-skewed distribution, we use a Gumbel-Gompertz distribution to smooth the probability histogram (see, e.g., Pflaumer, 2018).

The distribution function is

$F(x) = 1 - \exp\left(-\exp\left(k \cdot \left(x - m\right)\right)\right)$ or

with the parameters $k > 0$ and $m < 0$.

An alternative representation is

$F(x) = 1 - \exp\left(-\frac{A}{k} \cdot e^{k \cdot x}\right)$ with $A = k \cdot e^{-k \cdot m}$.

Properties of $F(x)$:

1. $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to \infty} F(x) = 1$.
2. $F(0) = P\left((X \le 0\right) =$ 1-exp(- exp(-k·m)) ≈ 1 for large negative values of m.

The parameter m represents the mode.

The survivor function $l(x)$ and the density function $f(x)$ are

$l(x) = 1 - F(x) = \exp\left(-\exp\left(k \cdot \left(x - m\right)\right)\right)$,

$$f(x) = \frac{dF(x)}{dx} = \exp\left(-\exp\left(k \cdot \left(x - m\right)\right)\right) \cdot k \cdot \exp\left(k\left(x - m\right)\right).$$

The median is $x_{0,5} \approx m + \frac{\ln\left(\ln 2\right)}{k}$.

The expected value, variance and skewness can only be calculated approximately:

$\mu \approx \mathrm{m} - \frac{\gamma}{\mathrm{k}}$ with $\gamma = 0.577221566...$ (Euler-Mascheroni constant),

$$\sigma^2 \approx \frac{\pi^2/6}{\mathrm{k}^2} = \frac{1.6449341}{\mathrm{k}^2} ,$$

$$\mathrm{g}_1 \approx -1.1395415.$$

In general, the survivor function *l*(*x*) of the Gumbel-Gompertz distribution is used as a life table model with *x* > 0, *k* > 0, and *m* >>k> 0 (see, e.g., Pflaumer, 2018).

We choose the Labouchere system with the list {0.5, 0.5} as an example of the procedure with one million simulation rounds and a maximum bet of 16 (see Lab2 in Table 2). The mixed type profit distribution and its parameters can be seen in Figure 5 and Table 4, where 973,958 rounds ended with a win of one unit and the rest with a loss (summarized as a histogram). The distribution of the 26,042 rounds that ended with a loss (negative profit) is illustrated in Figure 6. The relevant parameters of the negative profits are given in Table 5. The data are presented in the Appendix.

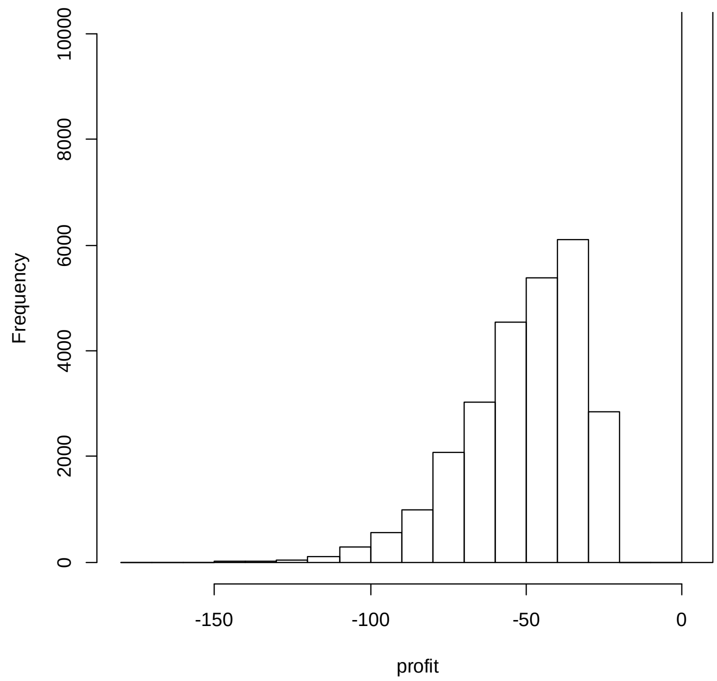

Figure 5: Mixed type profit distribution
The frequency of profit=1 is 97.4%
Labouchere betting with list {0.5; 0.5}

Table 4: Parameters of the mixed profit distribution

Min.	Q1	Median	Mean	Q3	Max	Sdv
-171	1	1	-0.3393	1	1	8.76

Probability of negative profit: 2.6%

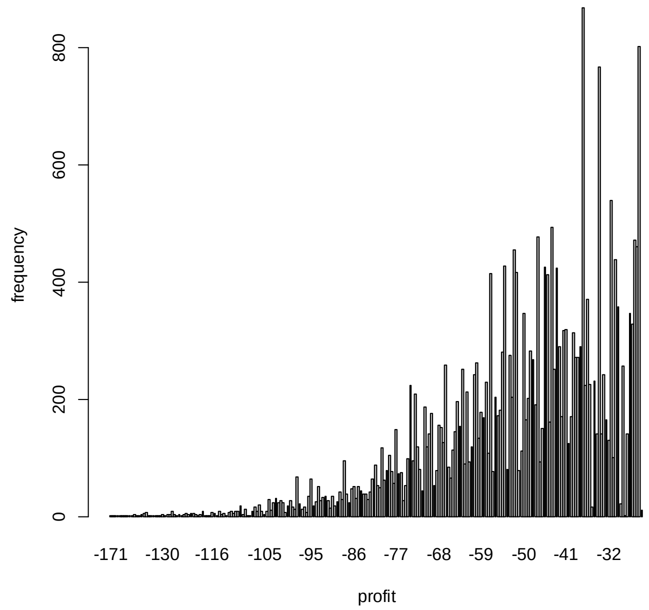

Figure 6: Bar plot of the negative profit
Labouchere betting with list {0.5, 0.5}

Table 5: Parameters of the negative profits from Figure 6

Min.	Q1	Median	Mean	Q3	Max	Sdv
-171	-61.5	-47	-50.43	-36.5	-22	19.21

The negative profits of the mixed distribution are smoothed by a Gumbel-Gompertz distribution. Nonlinear least squares estimation of the model using the NLS package of the R statistical software language yields the following results:

Parameters	Estimate	Std. Error	*t* value
M	-41.666	0.039	-1077.1
k	0.0637	0.0002	369.70

The obtained value for parameter A is calculated as $A = k \cdot e^{-k \cdot m}$, resulting in A = 0.905.

The cumulative distribution of the negative profits *X* can be well approximated by a Gumbel-Gompertz distribution with only two parameters

$$F(x) = 1 - \exp\left(-\exp\left(0.0637 \cdot \left(x + 41.666\right)\right)\right) = 1 - \exp\left(-\frac{0.905}{0.0637} \cdot e^{0.0637 \cdot x}\right),$$

$$= 1 - \exp\left(-14.207 \cdot e^{0.0637 \cdot x}\right)$$

which can be seen from Figure 7. The pseudo $R^2 = 0.9996$.

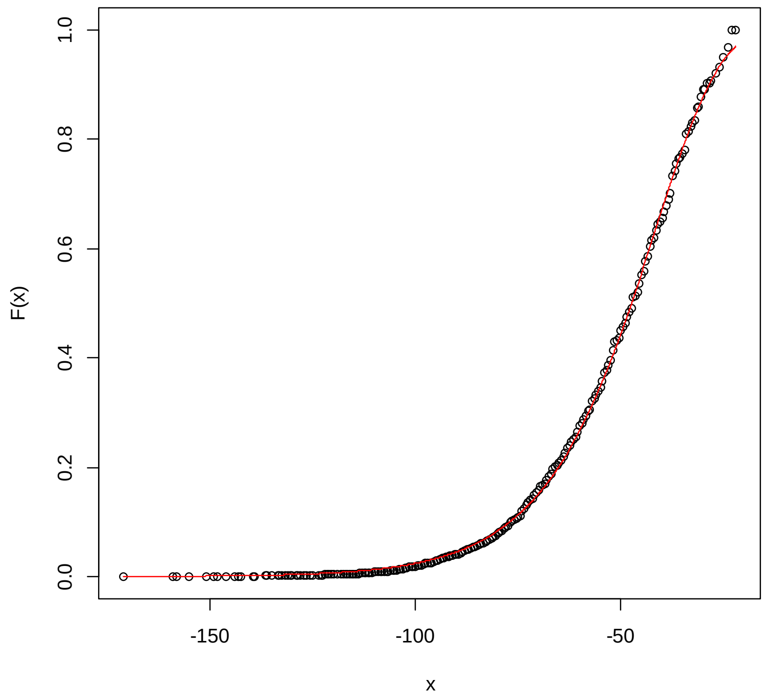

Figure 7: Estimated (red) and actual values (black) of $F(x)$ of the cumulative loss distribution

Figure 8 shows the relative frequencies and estimated density of negative profits. The discrete probability of profit x is estimated by the area $x + dx$ below the density function. For example, the probability that the profit will be between -100 and -50 is calculated by the integral $\int_{-100}^{-50} f_C(x)\,dx = 0.421$. In contrast, the sum of the corresponding relative frequencies is 0.431 (see the Appendix). However, the density function considerably underestimates the probability of small negative profits.

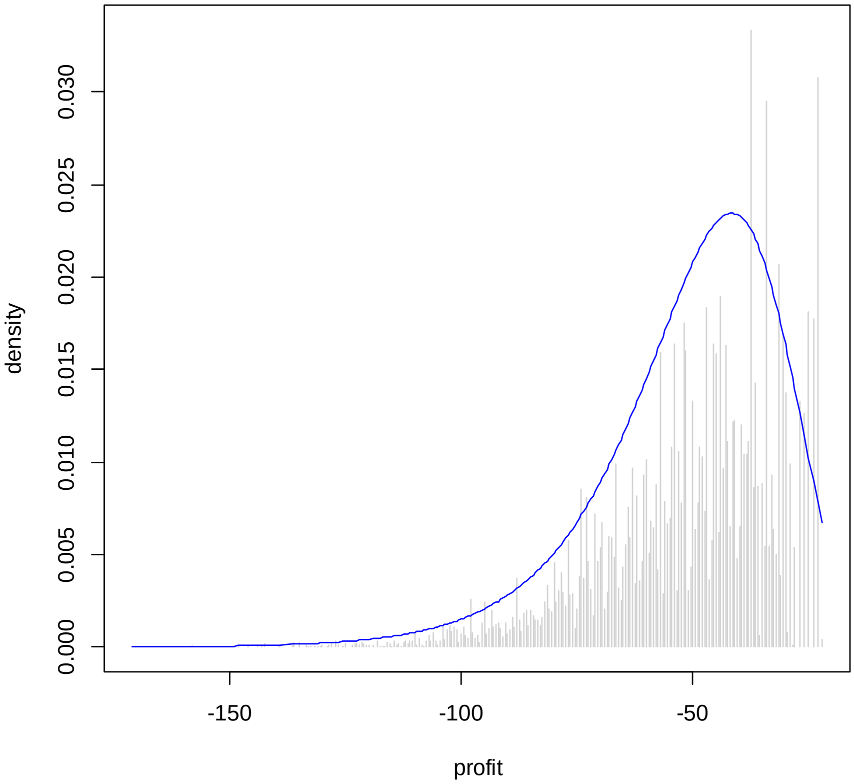

Figure 8: Relative frequencies and density $f_C(x)$ of the Gumbel-Gompertz distribution with m = -41.666 and k = 0.0637

The relevant parameters can be calculated with the Gumbel-Gompertz formula for $f_C(x)$. We obtain:

E(*X*)= -50.73
Var(*X*)=405.39
$\sigma(X) = 20.13$
Median=-47.42
Mode *m* = -41.666
(c.f. also the nearly identical results in Table 5).
With $\mathrm{E}(\mathrm{X}) = \mathrm{E}(\mathrm{X}^2) = 1$ for the discrete part $f_D(x) = P(X = x_0)$ of the mixed distribution, we can calculate (rounded numbers) the expected profit and the variance of a Labouchere round.

$\mathrm{E}(\mathrm{G}) = 0.0260 \cdot (-50.73) + 0.974 \cdot 1 = -0.345$.

$\mathrm{E}(\mathrm{G}^2) = 0.026 \cdot \left(405.39 + (-50.73)^2\right) + 0.974 \cdot 1 = 78.43$.

$Var(G) = E(G^2) - (E(G))^2 = 78.42$.

$\sigma(G) = 8.86$.

(cf. also the results in Table 2 below Lab2)

The calculated parameters do not much differ from the simulated parameters (cf. Table 2 below Lab2).

5. Playing Labouchere and Martingale Strategies for the Same Fixed Time

The comparison between Martingale and Labouchere strategies becomes intriguing when considering a fixed time duration. Despite Martingale's doubling strategy showing smaller risk metrics than Labouchere with moderately increasing bets (refer to Table 2), an interesting paradox emerges.
If you play a fixed time of either Labouchere or Martingale, you can play more rounds of Martingale. In our example, the average number of coups in a Martingale round is 1.982 (with a maximum bet of 16), while in a Labouchere round, it is 8.984 when playing the sequence {1, 2, 3, 4} or {0.1, 0.2, 0.3, 0.4}. This implies that while playing 1 Labouchere round, you can play, on average, 4.53 Martingale rounds.

Table 6 presents the expected values and standard deviations of the gain or profit for 30, 100, 1000, and 10000 Labouchere rounds , along with the corresponding Martingale rounds (136, 453, 4533, and 45238) that can be played in the same amount of time.

Table 6: Profit parameters for {0.1, 0.2, 0.3, 0.4} Labouchere and Martingale strategies

Game	NL	EGL	SigmaGL	LossProbL	NM	EGM	SigmaGM	LossProbM
1	*30*	-13.32	51.8	0.6015	*136*	-19.45	69.2	0.6107
2	*100*	-44.4	94.6	0.6806	*453*	-64.82	126.4	0.696
3	*1000*	-444	299	0.9312	*4533*	-648.19	399.8	0.9475
4	*10000*	-4440	945.6	0.9999987	*45328*	-6481.9	1264.2	0.9999999

Game: The game number.
NL: Number of Labouchere rounds played.
EGL: Expected gain for Labouchere after the specified number of rounds: $EGL = NL \cdot (-0.444)$.
SigmaGL: Standard deviation of gain for Labouchere after the specified number of rounds:
$SigmaGL = \sqrt{NL \cdot 89.409}$.
LossProbL: Probability that the gain is negative for Labouchere after the specified number of rounds.
NM: Number of Martingale rounds played.
EGM: Expected gain for Martingale after the specified number of rounds: $EGM = NM \cdot (-0.143)$.
SigmaGM: Standard deviation of gain for Martingale after the specified number of rounds:
$SigmaGM = \sqrt{NL \cdot 35.259}$.
LossProbM: Probability that the gain is negative for Martingale after the specified number of rounds.

The results reveal that Martingale is now riskier than Labouchere. Expectations are lower, and standard deviations are larger. As demonstrated earlier, the distributions of a Martingale round and a Labouchere round are distinct. The distribution of a Martingale round can be determined analytically, whereas the distribution of a Labouchere round requires simulation. Despite these differences, the gain or profit distributions for both alternatives can be approximated by a normal distribution when the number of rounds played is sufficiently large, following the central limit theorem. This assumption is based on the idea that individual rounds are independent identically distributed random variables. Figure 9 illustrates these distributions.

Using the normal approximation, we can calculate the loss probabilities shown in Table 6, which are higher for Martingale play. However, increased risk also implies the potential for achieving larger profits. For instance, in Game 2, the probability of having a profit greater than 100 is 9.6% for Martingale and 6.3% for Labouchere.

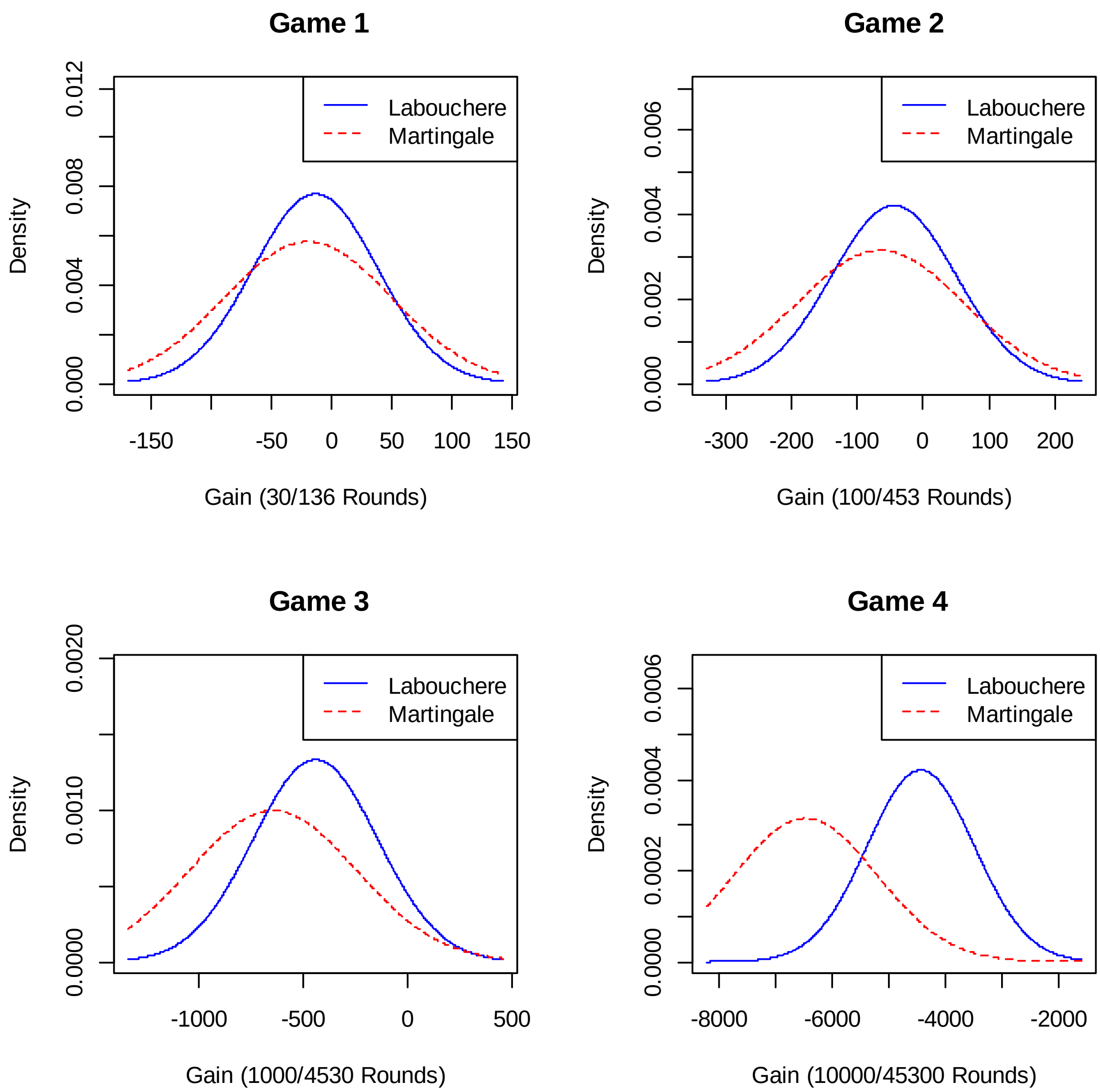

Figure 9: Profit Distributions after the specified number of rounds

It is important to note that the conclusions drawn in this chapter are based on a specific example with defined parameters. However, the principles discussed can be generalized to other instances of Martingale and Labouchere games, especially when considering higher limits or alternative Labouchere sequences. The dynamics of risk, reward, and the interplay between individual round characteristics and overall playtime remain consistent, providing a foundation for understanding these betting strategies across various settings. The specific outcomes may vary, but the broader insights into risk management and strategic considerations hold relevance in a broader context.

6. Conclusion

In conclusion, the comparison between the Martingale and Labouchere systems highlights distinct characteristics that players must carefully weigh. The Martingale system, characterized by a doubling strategy after each loss, presents a potential for rapidly escalating bet sizes. In the face of a prolonged losing streak, not only can individual losses become substantial, but the time required for recovery may also extend.

On the contrary, the Labouchere system, employing a more moderate progression in bet sizes, introduces a different dynamic. However, it comes with the trade-off of requiring a larger number of coups to complete a sequence. Players embracing the Labouchere system may find themselves spending more time at the betting table to achieve their profit goals or recover losses.

Essentially, the comparison underscores a crucial trade-off: Martingale offers rapid but potentially substantial losses, while Labouchere provides a more gradual yet potentially prolonged betting experience. This choice necessitates a thoughtful consideration not only of the average bet size but also of the associated time commitment and exposure to risk inherent in each system. Nevertheless, both the Martingale and Labouchere systems ultimately converge to the same negative mathematical expectation of gain per unit amount wagered.

From a statistical perspective, a Martingale round at roulette can be expressed through a two-point distribution, allowing for analytical derivation of formulas. In contrast, a Labouchere round is best characterized by a mixed random variable, involving a discrete spike at each win equal to the sum of the initial numbers of the chosen sequence. The distribution of losses in Labouchere rounds approximates a left-skewed pattern, often modeled by the continuous Gumbel-Gompertz distribution. Due to the intricate structure of the Labouchere random variable, analytical formulas are limited, and simulations play a primary role in understanding its dynamics.

Literature

Billings, J; Del Barco, S (2017): An Investigation into Labouchere´s Betting System to Improve Odds of Favorable Outcomes to Generate a Positive Externality Empirically,
arXiv: General Finance, DOI:10.3844/jmssp.2017.186.196, Corpus ID: 55724377.

Downton, F (1980): A Note on Labouchere Sequences, Journal of the Royal Statistical Society, Series A (General), Vol. 143, No. 3 (1980), pp. 363-366.

Ethier, S N (2010): The Doctrine of Chances: Probabilistic Aspects of Gambling, Heidelberg- New York.

Han, Y; Wang, G (2019): Expectation of the Largest Bet Size in the Labouchere system, Electron. Commun. Probab. 24 , no. 11, 1–10.

Pflaumer, P (2018): Demographic Applications of the Gompertz Distribution, Kempten.

Pflaumer, P (2019): A Statistical Analysis of the Roulette Martingale System: Examples, Formulas and Simulations with R, paper presented at the 17th International Conference on Gambling & Risk Taking. May 27-30, 2019, Las Vegas.
https://digitalscholarship.unlv.edu/cgi/viewcontent.cgi?article=1630&context=gaming_institute

Pearson, H (1936): Labby, The Life and Character of Henry Labouchere, London p. 46.

R Core Team (2020). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

Rodriguez, A; Mendes, B: (2018): Probability, Decisions and Games, A Gentle Introduction Using R, Wiley.

Zubrilina, N (2018): On the Expected Value of the Maximal Bet in the Labouchere system, preprint arXiv:1808.06642, 2018.

Weld, C; Leemis, L (2017): Modeling Mixed Type Random Variables, Proceedings of the 2017 Winter Simulation Conference, eds. W. K. V. Chan, A. D’Ambrogio, G. Zacharewicz, N. Mustafee, G. Wainer, and E. Page, pp. 1595-1606.

Appendix

Data of Figures 6 to 8 and Table 6

profit x	frequency	rel. Freq.	cum. freq.	profit x	frequency	rel. freq.	cum. freq.
-171	1	0.0000	0.0000	-81	53	0.0020	0.0724
-159	1	0.0000	0.0001	-80.5	49	0.0019	0.0743
-158	2	0.0001	0.0002	-80	118	0.0045	0.0788
-155	1	0.0000	0.0002	-79.5	63	0.0024	0.0812
-151	1	0.0000	0.0002	-79	79	0.0030	0.0842
-149	2	0.0001	0.0003	-78.5	105	0.0040	0.0883
-148	1	0.0000	0.0003	-78	77	0.0030	0.0912
-146	1	0.0000	0.0004	-77.5	57	0.0022	0.0934
-144	2	0.0001	0.0005	-77	149	0.0057	0.0991
-143	2	0.0001	0.0005	-76.5	73	0.0028	0.1020
-142.5	4	0.0002	0.0007	-76	75	0.0029	0.1048
-139.5	1	0.0000	0.0007	-75.5	27	0.0010	0.1059
-139	2	0.0001	0.0008	-75	53	0.0020	0.1079
-136.5	4	0.0002	0.0010	-74.5	98	0.0038	0.1117
-136	6	0.0002	0.0012	-74	223	0.0086	0.1202
-135	7	0.0003	0.0015	-73.5	96	0.0037	0.1239
-133.5	2	0.0001	0.0015	-73	210	0.0081	0.1320
-133	1	0.0000	0.0016	-72.5	120	0.0046	0.1366
-132.5	1	0.0000	0.0016	-72	81	0.0031	0.1397
-131.5	1	0.0000	0.0017	-71.5	43	0.0017	0.1413
-131	1	0.0000	0.0017	-71	187	0.0072	0.1485
-130.5	1	0.0000	0.0017	-70.5	120	0.0046	0.1531
-130	3	0.0001	0.0018	-70	141	0.0054	0.1586
-129	1	0.0000	0.0019	-69.5	176	0.0068	0.1653
-128.5	3	0.0001	0.0020	-69	53	0.0020	0.1673
-128	4	0.0002	0.0022	-68.5	78	0.0030	0.1703
-127	8	0.0003	0.0025	-68	156	0.0060	0.1763
-126.5	3	0.0001	0.0026	-67.5	153	0.0059	0.1822
-125.5	1	0.0000	0.0026	-67	127	0.0049	0.1871
-125	4	0.0002	0.0028	-66.5	258	0.0099	0.1970
-123.5	2	0.0001	0.0028	-66	84	0.0032	0.2002
-123	4	0.0002	0.0030	-65.5	65	0.0025	0.2027
-122.5	5	0.0002	0.0032	-65	113	0.0043	0.2071
-122	3	0.0001	0.0033	-64.5	144	0.0055	0.2126
-121.5	6	0.0002	0.0035	-64	196	0.0075	0.2201
-121	5	0.0002	0.0037	-63.5	154	0.0059	0.2260
-120.5	3	0.0001	0.0038	-63	252	0.0097	0.2357
-120	2	0.0001	0.0039	-62.5	90	0.0035	0.2392
-119	3	0.0001	0.0040	-62	213	0.0082	0.2473
-118	9	0.0003	0.0044	-61.5	93	0.0036	0.2509
-117.5	1	0.0000	0.0044	-61	120	0.0046	0.2555
-117	1	0.0000	0.0045	-60.5	243	0.0093	0.2648
-116.5	1	0.0000	0.0045	-60	263	0.0101	0.2749
-116	7	0.0003	0.0048	-59.5	133	0.0051	0.2800
-115.5	5	0.0002	0.0050	-59	177	0.0068	0.2868
-115	1	0.0000	0.0050	-58.5	168	0.0065	0.2933

-114.5	8	0.0003	0.0053	-58	229	0.0088	0.3021
-114	3	0.0001	0.0054	-57.5	109	0.0042	0.3063
-113.5	5	0.0002	0.0056	-57	414	0.0159	0.3222
-113	1	0.0000	0.0056	-56.5	76	0.0029	0.3251
-112.5	7	0.0003	0.0059	-56	204	0.0078	0.3329
-112	9	0.0003	0.0063	-55.5	173	0.0066	0.3396
-111.5	5	0.0002	0.0065	-55	182	0.0070	0.3466
-111	8	0.0003	0.0068	-54.5	281	0.0108	0.3573
-110.5	9	0.0003	0.0071	-54	427	0.0164	0.3737
-110	19	0.0007	0.0078	-53.5	80	0.0031	0.3768
-109.5	3	0.0001	0.0079	-53	276	0.0106	0.3874
-109	12	0.0005	0.0084	-52.5	203	0.0078	0.3952
-108.5	2	0.0001	0.0085	-52	456	0.0175	0.4127
-108	1	0.0000	0.0085	-51.5	417	0.0160	0.4287
-107.5	8	0.0003	0.0088	-51	79	0.0030	0.4318
-107	16	0.0006	0.0094	-50.5	112	0.0043	0.4361
-106.5	8	0.0003	0.0098	-50	346	0.0133	0.4494
-106	20	0.0008	0.0105	-49.5	165	0.0063	0.4557
-105.5	9	0.0003	0.0109	-49	202	0.0078	0.4634
-105	3	0.0001	0.0110	-48.5	282	0.0108	0.4743
-104.5	8	0.0003	0.0113	-48	267	0.0103	0.4845
-104	29	0.0011	0.0124	-47.5	191	0.0073	0.4919
-103.5	10	0.0004	0.0128	-47	478	0.0184	0.5102
-103	24	0.0009	0.0137	-46.5	94	0.0036	0.5138
-102.5	31	0.0012	0.0149	-46	150	0.0058	0.5196
-102	23	0.0009	0.0158	-45.5	426	0.0164	0.5359
-101.5	28	0.0011	0.0169	-45	413	0.0159	0.5518
-101	24	0.0009	0.0178	-44.5	162	0.0062	0.5580
-100.5	7	0.0003	0.0180	-44	494	0.0190	0.5770
-100	18	0.0007	0.0187	-43.5	252	0.0097	0.5867
-99.5	28	0.0011	0.0198	-43	424	0.0163	0.6029
-99	16	0.0006	0.0204	-42.5	289	0.0111	0.6140
-98.5	12	0.0005	0.0209	-42	170	0.0065	0.6206
-98	68	0.0026	0.0235	-41.5	317	0.0122	0.6327
-97.5	21	0.0008	0.0243	-41	319	0.0122	0.6450
-97	13	0.0005	0.0248	-40.5	124	0.0048	0.6498
-96.5	16	0.0006	0.0254	-40	170	0.0065	0.6563
-96	7	0.0003	0.0257	-39.5	313	0.0120	0.6683
-95.5	35	0.0013	0.0270	-39	272	0.0104	0.6787
-95	64	0.0025	0.0295	-38.5	272	0.0104	0.6892
-94.5	18	0.0007	0.0302	-38	289	0.0111	0.7003
-94	26	0.0010	0.0312	-37.5	868	0.0333	0.7336
-93.5	52	0.0020	0.0332	-37	224	0.0086	0.7422
-93	28	0.0011	0.0343	-36.5	371	0.0142	0.7565
-92.5	33	0.0013	0.0355	-36	226	0.0087	0.7651
-92	34	0.0013	0.0368	-35.5	16	0.0006	0.7658
-91.5	27	0.0010	0.0379	-35	231	0.0089	0.7746
-91	14	0.0005	0.0384	-34.5	142	0.0055	0.7801
-90.5	34	0.0013	0.0397	-34	768	0.0295	0.8096
-90	19	0.0007	0.0404	-33.5	142	0.0055	0.8150

-89.5	25	0.0010	0.0414	-33	243	0.0093	0.8244
-89	42	0.0016	0.0430	-32.5	165	0.0063	0.8307
-88.5	29	0.0011	0.0441	-32	131	0.0050	0.8357
-88	96	0.0037	0.0478	-31.5	539	0.0207	0.8564
-87.5	38	0.0015	0.0493	-31	101	0.0039	0.8603
-87	23	0.0009	0.0501	-30.5	439	0.0169	0.8772
-86.5	48	0.0018	0.0520	-30	357	0.0137	0.8909
-86	51	0.0020	0.0540	-29.5	21	0.0008	0.8917
-85.5	31	0.0012	0.0551	-29	257	0.0099	0.9015
-85	52	0.0020	0.0571	-28.5	2	0.0001	0.9016
-84.5	43	0.0017	0.0588	-28	141	0.0054	0.9070
-84	39	0.0015	0.0603	-27	346	0.0133	0.9203
-83.5	38	0.0015	0.0617	-26	329	0.0126	0.9330
-83	30	0.0012	0.0629	-25	472	0.0181	0.9511
-82.5	42	0.0016	0.0645	-24	461	0.0177	0.9688
-82	64	0.0025	0.0670	-23	802	0.0308	0.9996
-81.5	88	0.0034	0.0703	-22	11	0.0004	1.0000

Understanding the Labouchere System

1. Initial Setup:
Start by writing down a sequence of numbers. This sequence will determine your betting units. For example: 1-2-3-4.
2. Calculate Bet Size:
To determine your bet size, add the first and last numbers in your sequence. In the example above, the bet size would be 1 + 4 = 5.
3. Place Bet:
Bet the calculated amount on an even-money bet in roulette (e.g., red or black, odd or even).
4. Winning Bet:
If you win, cross out the first and last numbers from your sequence.
5. Losing Bet:
If you lose, add the amount of your last bet to the end of the sequence.
6. Repeat:
Continue the process until all numbers are crossed out, at which point you've reached your profit goal.

An example using the Labouchere betting system with the sequence 1-1:

1. Initial Setup:
Sequence: 1-1
2. Calculate Bet Size:
Bet size = 1 + 1 = 2
3. Place Bet:
Bet 2 units on an even-money bet (e.g., red or black, odd or even).
4. Outcome:
If you win, the sequence becomes empty, and you've reached your profit goal.
If you lose, add 2 to the end of the sequence.
5. Repeat:
If the sequence becomes empty (all numbers are crossed out), you've reached your profit goal.
If not, continue placing bets based on the new calculated bet size until the sequence is empty.

Let's walk through a few rounds:

Initial Sequence: 1-1
Bet Size: 1 + 1 = 2 units
Round 1:
Bet 2 units, and let's say you win.
Sequence becomes empty.
Outcome:
You've reached your profit goal.
If you had lost:
Updated Sequence: 1-1-2
Bet Size: 1 + 2 = 3 units
Round 2:
Bet 3 units, and let's say you win.
Cross out 1 and 2 from the sequence.
Outcome:
Updated Sequence: 1
Bet Size: 1 unit
Round 3:
Bet 1 unit, and let's say you win.
Sequence becomes empty.
Outcome:
You've reached your profit goal.

Let's consider the Labouchere betting system with the sequence 0.5-0.5. In this case, each number in the sequence represents a fraction of your initial bankroll. The process is similar to the previous example, but the bet sizes will be a fraction of your bankroll based on the sequence.

1. Initial Setup:
Sequence: 0.5-0.5
2. Calculate Bet Size:
Bet size = 0.5 + 0.5 = 1 (representing 100% of your initial bankroll)
3. Place Bet:
Bet 100% of your bankroll on an even-money bet.
4. Outcome:
If you win, the sequence becomes empty, and you've reached your profit goal.
If you lose, add 1 to the end of the sequence.
5. Repeat:
If the sequence becomes empty (all numbers are crossed out), you've reached your profit goal.
If not, continue placing bets based on the new calculated bet size until the sequence is empty.

Let's walk through a few rounds:
Initial Sequence: 0.5-0.5
Bet Size: 0.5 + 0.5 = 1 (representing 100% of your bankroll)
Initial Setup:
Sequence: 0.5-0.5
Round 1:
Bet Size: 0.5 + 0.5 = 1 (representing 100% of your bankroll)

Bet 100% of your bankroll, and let's say you win.
Sequence becomes empty.
Outcome:
You've reached your profit goal.
Now, let's consider what would happen if you had lost in Round 1:
Updated Sequence: 0.5-0.5-1
Bet Size: 0.5 + 1 = 1.5 (representing 150% of your remaining bankroll)
Round 2:
Bet 150% of your remaining bankroll, and let's say you win.
Cross out 0.5 and 1 from the sequence.
Outcome:
Updated Sequence: 0.5
Bet Size: 0.5 (representing 50% of your remaining bankroll)
Round 3:
Bet 50% of your remaining bankroll, and you win.
Sequence becomes empty.
Outcome:
You've reached your profit goal.
Now, let's consider what would happen if you had lost in Round 3:
Updated Sequence: 0.5-0.5
Bet Size: 0.5 + 0.5 = 1 (representing 100% of your remaining bankroll)
Round 4:
Bet 100% of your remaining bankroll, and let's say you lose.
Outcome:
Updated Sequence: 0.5-0.5-1 (adding the last bet to the end of the sequence)
Bet Size for the next round: 0.5 + 1 = 1.5 (representing 150% of your remaining bankroll)
Round 5:
Bet 150% of your remaining bankroll, and let's say you lose.
Outcome:
Updated Sequence: 0.5-0.5-1-1.5 (adding the last bet to the end of the sequence)
Bet Size for the next round: 0.5 + 1.5 = 2 (representing 200% of your remaining bankroll)
Round 6:
Bet 200% of your remaining bankroll, and let's say you win.
Cross out 0.5 and 1.5 from the sequence.
Outcome:
Updated Sequence: 0.5-0.5
Bet Size for the next round: 0.5 + 0.5 = 1 (representing 100% of your remaining bankroll)
Round 7:
Bet 100% of your remaining bankroll, and let's say you win.
Sequence becomes empty.
Outcome:
You've reached your profit goal.

In this example, it took several rounds of wins and losses to reach the profit goal, illustrating the Labouchere system's characteristic of adjusting bet sizes based on the sequence.