A Nonparametric Multivariate Control Chart Based on Data Depth

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Abstract

For the design of most multivariate control charts, it is assumed that the observations follow a multivariate normal distribution. In practice, this assumption is rarely satisfied. In this work, a distribution-free EWMA control chart for multivariate processes is proposed. This chart is based on sequential rank of data depth measures.

1 Introduction

Reducing variation in manufacturing is desirable to reduce product cost and improve product performance and quality. To achieve this objective statistical process control (SPC) is used. SPC is a set of techniques for monitoring a production process to determine if it is stable over time and capable of producing high quality products. One purpose of control charting, the featured tool of SPC, is to distinguish between two sources, common and assignable causes, of process variation. Common or chance causes of variation cannot be economically identified and corrected and considered to be due to the inherent nature of the process. assignable or special causes of variation are unusual shocks or other disruptions to the process, the causes of which can and should be removed.

A process is said to be in a state of statistical control if it operates under common causes. Typically control charts apply to systems or processes in which only one quality characteristic is measured and tested. However, the rapid growth of data acquisition technology and the use of online computers for process monitoring have led to an increased interest in the simultaneous surveillance of several related quality characteristics on process variables. These techniques are often referred to as multivariate statistical control procedures.

For most of these procedures, it is assumed that the underlying distribution of the process is multivariate normal. Thus, the statistical properties of commonly employed control charts are exact only if this assumption is satisfied. In practice, it is well known that this assumption rarely holds. Therefore, distribution-free or nonparametric control charts for multivariate processes are nedded. In this paper, we propose a nonparametric EWMA control chart for multivariate processes based on sequential ranks of data depth measures. In section 2, the data depth notion is introduced.

2 A nonparametric EWMA for multivariate processes

2.1 Data depth

Data depth measures how deep (or central) a given point $\mathbf{X} \in \mathbb{R}^d$ is with respect to (w. r. t.) a probability distribution F or w. r. t. a given data cloud $\{\mathbf{Y}_1, \ldots, \mathbf{Y}_m\}$. There are several measurements for the depth of the observations, such as Mahalanobis depth, the simplicial depth, half-space depth, and the majority depth of Singh, see Liu et al. (1999). In this work, the Mahalanobis depth and simplicial depth are considered.

1. The Mahalanobis depth (MD_F) of a given point $\mathbf{X} \in \mathbb{R}^d$ w. r. t. F is defined to be

$$MD_F(\mathbf{X}) = \frac{1}{1 + (\mathbf{X} - \mu_F)'\Sigma_F^{-1}(\mathbf{X} - \mu_F)} \ ,$$

   where μ_F and Σ_F are the mean vector and dispersion matrix of F, respectively. The sample version of MD_F is obtained by replacing μ_F and Σ_F with their sample estimates. In fact, how deep $\mathbf{X}$ is w. r. t. F is measured by how small its quadratic distance is to the mean.

2. The simplicial depth (SD_F) (Liu, 1990) of a given point $\mathbf{X} \in \mathbb{R}^d$ w. r. t. F is defined to be

$$SD_F(\mathbf{X}) = P_F\{\mathbf{X} \in s[\mathbf{Y}_1, \ldots, \mathbf{Y}_{d+1}]\},$$

   where $s[\mathbf{Y}_1, \ldots, \mathbf{Y}_{d+1}]$ is a d-dimensional simplex whose vertices are the random observations $\{\mathbf{Y}_1, \ldots, \mathbf{Y}_{d+1}\}$ from F. The sample simplicial depth $SD_{F_m}(\mathbf{X})$ is defined to be

$$SD_{F_m}(\mathbf{X}) = \begin{pmatrix} m \\ d+1 \end{pmatrix}^{-1} \sum_{1 \le i_1 < \ldots < i_{d+1} \le m} I\left(\mathbf{X} \in s[\mathbf{Y}_{i_1}, \ldots, \mathbf{Y}_{i_{d+1}}]\right),$$

where $\{\mathbf{Y}_1, \ldots, \mathbf{Y}_m\}$ is a random sample from F, F_m denotes the empirical distribution of $\{\mathbf{Y}_1, \ldots, \mathbf{Y}_m\}$ and $I(.)$ is the indicator function. For example, the bivariate $SD_{F_m}(\mathbf{X})$ relative to $\{\mathbf{Y}_1, \ldots, \mathbf{Y}_m\}$ is equal to the proportion of closed triangles with vertices $\mathbf{Y}_i$, $\mathbf{Y}_j$, $\mathbf{Y}_k$ that contain $\mathbf{X}$, $1 \le i < j < k \le m$. Liu (1990) showed that if F is absolutely continuous, then as $m \longrightarrow \infty$, SD_{F_m} converges uniformly and strongly to $SD_F(\mathbf{X})$ and that $SD_F(\mathbf{X})$ is affine invariant.

A data depth $SD_F(\mathbf{X})$ induces a center-outward ordering of the sample points if depth values of all points are computed and compared. If all $SD_F(\mathbf{X})$'s are arranged in an ascending order and $\mathbf{X}_{[j]}$ is used to denote the sample point associated with the jth smallest depth value, then $\mathbf{X}_{[1]}$, $\ldots$, $\mathbf{X}_{[m]}$ are the order statistics of $\mathbf{X}_i$'s with $\mathbf{X}_{[m]}$ being the most central point. The smaller the rank of a point, the more outlying the point w. r. t. the underlying distribution $F(.)$.

2.2 Sequential ranks

In this section, order statistics that are used in this work are quickly reviewed. Let X_t, $t = 1, 2, \ldots$, be a sequence of independent random variables from a continuous distribution $F(x)$. The sequential rank R_t^* is the rank of X_t among the most recent m $(m > 1)$ observations taken from the process X_t, X_{t-1}, $\ldots$, X_{t-m+1}. That is,

$$R_t^* = 1 + \sum_{i=t-m+1}^{t} I(X_t > X_i), \tag{1}$$

where $I(.)$ is the indicator function. The standardized sequential rank $R_t^{(m)}$ is defined as

$$R_t^{(m)} = \frac{2}{m}\left(R_t^* - \frac{m+1}{2}\right). \tag{2}$$

For all t, $R_t^{(m)}$ is uniformly distributed on the g points

$$\left\{\frac{1}{m} - 1,\ \frac{3}{m} - 1,\ \ldots,\ 1 - \frac{1}{m}\right\}$$

with mean $\mu_{R_t^{(m)}} = 0$ and variance $\sigma_{R_t^{(m)}} = \frac{m^2-1}{3m^2}$. For more details, see Hackl and Ledolter (1992).

2.3 A control chart based on sequential rank of data depth measures

Liu (1995) was the first who used the concept of data depth to construct a nonparametric control chart for monitoring multivariate processes. In this work, we consider an EWMA chart based on sequential ranks of data depth measures to monitor multivariate processes. The proposed chart is a generalization on the nonparametric EWMA for individual observations proposed by Hackl and

Ledoltser (1992).

It is assumed that $p \times 1$ random vectors $\mathbf{X}_t$, $t = 1, 2, \ldots$, are observed and monitored over time. Each vector $\mathbf{X}_t = (X_{t1}, X_{t2}, \ldots, X_{tp})'$ contains p quality characteristic measurements made on a part from a multivariate process. The value X_{tj}, $j = 1, \ldots, p$, represents an observation on the j^{th} quality characteristic at time t.

For this chart, a reference sample is considered as the m most recent observations taken from the process $\mathbf{X}_{t-m+1}, \mathbf{X}_{t-m+2}, \ldots, \mathbf{X}_t$. This sample is be used to decide whether or not the process is still in-control at time t. The depth of $\mathbf{X}_t$ is calculated w. r. t. this reference sample and the sequential rank (R_t^*) of $D_m(\mathbf{X}_t)$ among $D_m(\mathbf{X}_{t-m}), \ldots, D_m(\mathbf{X}_{t-1})$ is computed using equation (1) The standardized sequential rank, defined by equation (2), are monitored using the exponentially weighted moving average (EWMA) recursion. That is,

$$T_t = \min\{B, (1-\lambda)T_{t-1} + \lambda R_t^{(m)}\}, \tag{3}$$

$t = 1, 2, \ldots$, where $0 < \lambda \leq 1$ is a smoothing parameter, B is a reflecting boundary and $T_0 = u$. The process is considered in-control as long as $T_t > h$, where $h < 0$ is a lower control limit ($h \leq u \leq B$). Note that, the lower-sided EWMA is considered because the statistic R_t^m is higher "the better".

A reflecting boundary is included to prevent the EWMA from drifting to one side indefinitely. It is known that EWMA schemes can suffer from an "inertia problem" when there is a process change some time after beginning of monitoring. That is, an EWMA can have wandered away from a center line in a direction opposite to that of a shift that occurs some time after the start of monitoring. In this unhappy circumstance, an EWMA scheme can take long time to signal.

Hackl and Ledolter (1992) considered a continuous quality criteria. This continuity assumption assures that ties are impossible. However, in practice when measurements or other numerical observations are taken, it is often that two or more observations are tied. For example, ties may be due to the nature of the phenomenon modelled or rounding of continuous variables (temperature, blood pressure, ...). In this work, the simplicial depth is a discrete measure and ties may occur. Especially, there always exist at least $(d + 1)$ extreme points that share the minimum simplicial depth of $(d + 1)/m$, see Stoumbos and Reynolds (2001). The most common approach to this problem is to assign to each observation in a tied set the midrank, that is, the average of the ranks reserved for the observations in the tied set, see Gibbons and Chakraborti (1992).

3 Average run length of the in-control process

As mentioned, the parameters of the control chart are selected according to a performance of the chart. Usually, the performance of control charts are evaluated by the average run length (ARL). The run length is defined as the number of observations that are needed to exceed the control limit for the first time. The ARL should be large when the process is statistically in-control (in-control ARL) and small when a shift has occurred (out-of-control ARL).

In this work, we used the integral equation to approximate the in-control ARL, see Crowder (1987). Let $L(u)$ be the ARL of the lower-sided EWMA chart given that $T_0 = u$, it can be shown that the integral equation for $L(u)$ is given by

$$L(u) = 1 + L(B)\mathrm{Pr}\left(r \geq \frac{B-(1-\lambda)u}{\lambda}\right) + \int_h^B L\left((1-\lambda)u + \lambda r\right) dF(r),$$

where $F(r)$ is the cumulative distribution of r. We assumed that ties are not observed. Therefore, $R_t^{(m)}$ are uniformly distributed on the m points $\{1/m-1, 3/m-1, \ldots 1-1/m\}$. For moderate and large m the discrete distribution of $R_t^{(m)}$ is approximated by a continuous uniform distribution, which leads to

$$L(u) = 1 + L(B)\mathrm{Pr}\left(r \geq \frac{B-(1-\lambda)u}{\lambda}\right) + \int_h^B L\left((1-\lambda)u + \lambda r\right) f(r)dr, \tag{4}$$

where $f(r)$ is the probability density of the uniform distribution. The solutions to integral equation (4) can be obtained by replacing the equation with a system of linear equations using the collocation method and solving the system of equations. see appendix. As recommended by Calzada and Scariano (2003), the collocation method is used because the continuous uniform distribution does not have the entire real line as numerical support.

In the previous approximation, we ignored the slight dependence among successive ranks $R_t^{(m)}$. Therefore, the result in (4) applies only approximately, as there are small correlations among successive ranks. For moderate and large values of m the correlations are quite small, see Hackl and Ledolter (1992). Table 1 shows the lower one sided EWMA ARL's for the same smoothing parameters λ and control limits h as in Hackl and Ledolter (1992) and assuming that the EWMA starts at 0, that is $T_0 = 0$. Table 1 shows a decrease in the ARL with increasing λ for fixed control limit h. As mentioned by Hackl and Ledolter (1992), this is explained by the fact that $\sigma^2_{T_t}$ increases

Table 1: ARL's of the one-sided EWMA with reflecting boundary $B = -h$

h	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$
0.25	127.3	$-$	$-$	$-$	$-$
0.30	286.4	$-$	$-$	$-$	$-$
0.35	766.1	$-$	$-$	$-$	$-$
0.40	2568.4	123.5	$-$	$-$	$-$
0.45	$+$	249.4	$-$	$-$	$-$
0.50	$+$	580.3	103.2	$-$	$-$
0.55	$+$	1624.9	197.9	$-$	$-$
0.60	$+$	$+$	437.5	111.8	$-$
0.65	$+$	$+$	1166.1	223.3	$-$
0.70	$+$	$+$	$+$	532.9	150.1
0.75	$+$	$+$	$+$	1634.2	345.3
0.80	$+$	$+$	$+$	$+$	1059.8

"$-$"average run length less than 100,

"$+$"average run length greater than 2000.

with λ so that the probability of crossing the control limit h becomes larger.

A simulation study is carried in order to validate the ARL approximation. We generate independent observations $\mathbf{X_t}$ from a bivariate normal distribution with μ= $(0,0)'$ and

$$\Sigma = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Note that due to the nonparametric nature of the monitoring strategy, the normality is not required and any other distribution could be used. The results of the simulation showed that for $m > 100$ the approximation in (4) can be used to select the parameters of the nonparametric EWMA in order to attain a desired average run length in the in-control situation.

4 Application

In this section, the proposed EWMA control chart is used to monitor a BTA (Boring and Trepanning Association) deep hole drilling process. Deep hole drilling methods are used for producing holes with a high length-to-diameter ratio, good surface finish and straightness. For drilling holes with a diameter of 20 mm and above, the BTA (Boring and Trepanning Association) deep hole machining principle is usually employed, for more details see Theis (2004).

The process is subject to dynamic disturbances usually classified as either chatter vibration or spiralling. Chatter leads to excessive wear of the cutting edges of the tool and may also damage the boring walls. Spiralling damages the workpiece severely. The defect of form and surface quality constitute a significant impairment of the workpiece. As the deep hole drilling process is often used during the last production phases of expensive workpieces, process reliability is of primary importance and hence disturbances should be avoided. For this reason, process monitoring is necessary to detect dynamic disturbances.

In this section, we will focus on chatter which is dominated by single frequencies, mostly related to the rotational eigenfrequencies of the boring bar. Therefore, we propose to monitor the amplitude of the relevant frequencies in order to detect chatter vibration as early as possible. In practice, it is necessary to monitor several relevant frequencies because the process is subject to different kind of chatter (i. e., chatter at the beginning of the drilling process, high and low frequency chatter). The EWMA chart based on sequential ranks of data depth measures is used to monitor the amplitudes of frequencies 234 and 703 Hz, which are among the eigenfrequencies of the boring bar, in an experiment with feed f=0.185 mm, cutting speed v_c=90 m/min and amount of oil $\dot{V}$=300 L/min. For more details, see Weinert at al. (2002).

For the EWMA chart, we used $B = -h$. Typical values of λ are in the range of $0.1 < \lambda < 0.3$, see Hackl and Ledolter (1992). In this work, we used λ = 0.1, 0.2 and 0.3. The corresponding values for h are respectively −0.314, −0.475 and −0.591. The simplicial depth is computed using the FORTRAN algorithm developed by Rousseeuw and Ruts (1992).

Table 2 shows the results, for depth $\leq$ 270 mm. The EWMA charts based on MD_F produces more out-of-control signals than the EWMA charts based on SD_F. This is due to the sensitivity to the MD_F measure to the extreme values.

Table 2 shows that all control charts signal at 32 $\leq$ depth $\leq$ 35 mm. In fact, it is known that approximately at depth=35 mm the guiding pads of the BTA tool leave the starting bush, which induces a change in the dynamics of the process. From previous experiments, the process has been observed to either stay stable or start with chatter vibration. A great number of out of control signals occur at 35 $\leq$ depth $\leq$ 45 mm. Indeed, the new physical state of the process is represented in the reference sample after depth 45 mm.

All control charts signal at depth 110 $\leq$ depth $\leq$ 120 mm and it is known that depth 110 mm is approximately the position where the tool enters the bore hole completely. Theis (2004) noted

Table 2: Out of control signals of the different control charts applied to the amplitude of frequencies 234 Hz and 703 Hz (m=100)

Hole Depth (mm)	Observation number	EWMA $\lambda = 0.1$		EWMA $\lambda = 0.2$		EWMA $\lambda = 0.3$	
		MD_F	SD_F	MD_F	SD_F	MD_F	SD_F
≤32	≤107	0	0	0	0	0	0
32-35	108-117	1	1	3	1	3	1
35-45	118-150	29	27	21	15	13	6
45-70	151-249	1	0	0	0	0	0
70-110	250-366	9	5	3	1	1	0
110-125	370-416	9	10	4	4	1	1
125-200	417-665	3	0	2	0	2	2
200-250	666-832	7	8	3	2	2	1
250-255	833-849	4	3	4	2	4	2
255-260	850-865	8	7	3	3	1	1
260-270	866-898	4	2	0	0	0	0
Total		75	63	43	28	27	14

that this might lead to changes in the dynamic process because the boring bar is slightly thinner than the tool and therefore the pressures in the hole may change. The important out-of-control signals are produced at $250 \leq$ depth ≤ 255 mm. Messaoud et al. (2004) showed that a change occurred in the process at depth=252.19 mm and they concluded that this change may indicate the presence of chatter or that chatter will start in a few seconds. Therefore, in this experiment chatter may be avoided if corrective actions are taken after these signals.

In this experiment, the EWMA chart with λ=0.3 is the best, and should be choosen among the three EWMA charts considered in this work. Indeed, only 14 out-of-control signals are produced and all changes of the physical conditions of the process are detected. In practice, a procedure to choose the smoothing parameter λ is required.

5 Discussion

The future research should focus on the comparison of the in-control and out-of-control performance of the proposed nonparametric EWMA to existing parametric control charts for normal and nonnormal data. This comparison should include the robustly designed parametric multivariate EWMA (MEWMA) chart. Stoumbos and Sullivan (2002) showed that the MEWMA behaves like distribution-free control charts for an appropriate choice of the smoothing parameter.

For the process adjustment, once the EWMA chart has produced a signal, a procedure to estimate the shift magnitude and to identify the time point at which the shift occurred is required. For example, in our experiment the most important shift occurred at depth 252.19 mm. The EWMA chart, with $\lambda = 0.3$, based on MD_F and SD_F detect it after 2 and 4 samples respectively. In practice, the minimum of the MD_F measure over a short window, with a given length, before the occurrence of the out-of-control signal may be used to estimate the shift magnitude. However, in this case, one limitation of the SD_F is that once the data point is outside the data cloud, the SD_F measure is equal to $(d+1)/m$. This does not give an information about the shift magnitude.

For the out-of-control interpretation, when the control chart indicates an out-of-control condition, it is important to determine which quality characteristic X_j, $j = 1, \ldots, j = p$, or combination of X_j's, of the multivariate process caused the process to go out-of-control. For example, for the drilling process, when an out-of-control signal is produced, it is important to know which frequency or combination of frequencies cause this signal. In fact, in practice, the identification of the type of chatter (i.e., chatter at the beginning of the drilling process, low-high frequency chatter) will usually make it easier for engineers to adjust the process.

6 Conclusion

In this work, we proposed to use EWMA control charts based on data depth measures to monitor multivariate processes. These distribution-free control charts are attractive when the multivariate normal distribution is not satisfied.

A Integral Equation Approximation

For more details on the use of the collocation method used for solving integral equation (4), see Calzada and Scariano (2003) pp. 595-597. First, the interval $[h, B]$ is divided into n subintervals of length $\Delta = (B-h)/n$. Equation (4) can be rewritten as,

$$L(u) = 1 + L(B)\mathrm{Pr}\left(r \geq \frac{B-(1-\lambda)u}{\lambda}\right) + \frac{1}{\lambda}\int_h^B L(y)dF\left(\frac{y-(1-\lambda)u}{\lambda}\right), \tag{5}$$

For the (constant) collocation method, L(y) is approximated by a constant, say L_j, on each subinterval $[y_{j-1}, y_j]$, yielding

$$L(u) = 1 + L(B)\mathrm{Pr}\left(r \geq \frac{B-(1-\lambda)u}{\lambda}\right) + \frac{1}{\lambda}\sum_{j=1}^{n} L_j \int_{y_{j-1}}^{y_j} dF\left(\frac{y-(1-\lambda)u}{\lambda}\right), \tag{6}$$

Choosing nodes w_i in each of the subintervals $[y_{i-1},\, y_i]$ and requiring equation (6) to be exact at these points gives the system

$$L(w_i) = 1 + L(B)\Pr\left(r \geq \frac{B-(1-\lambda)w_i}{\lambda}\right) + \frac{1}{\lambda}\sum_{j=1}^{n} L_j \int_{y_{j-1}}^{y_j} dF\left(\frac{y-(1-\lambda)w_i}{\lambda}\right),$$

$i = 1, \ldots, n$. The approximating linear system is

$$-\mathbf{1} = \mathbf{A}\mathbf{L}, \tag{7}$$

with $-\mathbf{1} = [-\mathbf{1}, -\mathbf{1}, \ldots, -\mathbf{1}]^{\mathbf{T}}$, $\mathbf{L} = [L(B), L_1, L_2, \ldots, L_n]^T$, and $\mathbf{A}$ is an $(n+1)\times(n+1)$ matrix

$$\mathbf{A} = \begin{pmatrix} \Pr[r \geq \frac{B-(1-\lambda)B}{\lambda}] - 1 & \frac{1}{\lambda}\int_h^{y_1} dF\left(\frac{y-(1-\lambda)B}{\lambda}\right) dy & \cdots & \frac{1}{\lambda}\int_{y_{i-1}}^{y_i} dF\left(\frac{y-(1-\lambda)B}{\lambda}\right) dy & \cdots \\ \Pr[r \geq \frac{B-(1-\lambda)w_1}{\lambda}] & & & & \\ \vdots & & & & \\ \Pr[r \geq \frac{B-(1-\lambda)w_i}{\lambda}] & & \mathbf{A_1} & & \\ \vdots & & & & \end{pmatrix},$$

where $\mathbf{A_1}$ is an $n \times n$ matrix with entries a_{ij}, where

$$a_{ij} = \begin{cases} \frac{1}{\lambda}\int_{y_{j-1}}^{y_j} dF\left(\frac{y-(1-\lambda)w_i}{\lambda}\right) - 1 & \text{if } i = j \\ \frac{1}{\lambda}\int_{y_{j-1}}^{y_j} dF\left(\frac{y-(1-\lambda)w_i}{\lambda}\right) & \text{if } i \neq j \end{cases}$$

$i = 1, \ldots, n$, $j = 1, \ldots, n$. The integrals a_{ij} are calculated

$$\int_{y_{j-1}}^{y_j} dF\left(\frac{y-(1-\lambda)u}{\lambda}\right) = F\left(\frac{y_j-(1-\lambda)w_i}{\lambda}\right) - F\left(\frac{y_{j-1}-(1-\lambda)w_i}{\lambda}\right)$$

the nodes w_i are chosen to be the midpoints of the subintervals. If n is chosen to be an odd integer, then the ARL(0)=$L(0) \approx L_{(n+1)/2}$, which holds if $B = -h$.

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