Some Theoretical Properties of Evolutionary Algorithms under Partially Ordered Fitness Values

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Abstract

Presently, the limit theory of evolutionary algorithms (EA) for mono-criterion optimization under certainty is well developed. The situation is different for the fields of evolutionary optimization under complete or partial uncertainty, multiple criteria and so forth. Since these problem classes may be seen as special cases of the task of finding the set of minimal (or maximal) elements in partially ordered sets, a limit theory for EAs that can cope with this kind of problem passes all properties and results on its special cases mentioned above.

1 Introduction

The theory of evolutionary algorithms (EAs) in the framework of stochastic processes is best developed currently for the field of optimization of a single deterministic objective function (see e.g. [7] for a survey). There is also a steadily growing theory for EAs facing a (single) stochastically perturbed objective function as can be learned from the overview presented in [1]. In case of multiple objective functions, however, the theory is still in its infancy: Only few results are known [8, 4]. The situation is even worse for other problem classes since theoretical results concerning EAs are unknown apparently.

This situation may change by the approach initiated in [6]. Instead of developing an own theory for each problem class, it suffices to develop a theory for EAs that can cope with partially ordered fitness values since many problems may be seen as special cases of the problem of finding the set of minimal (or maximal) elements in a partially ordered set. This approach was pushed on in [11, 10]. Here, we present the main results of the general theory and its application to specialized problem classes like multi-criteria optimization, noisy and interval-valued objective functions.

2 General Case: Partially Ordered Fitness Sets

2.1 Basic Definitions

Let $\mathcal{F}$ be a set. A reflexive, antisymmetric, and transitive relation "$\preceq$" on $\mathcal{F}$ is termed a *partial order relation* whereas a *strict partial order relation* "$\prec$" must be antireflexive, asymmetric, and transitive. The latter relation may be obtained by the former relation by setting $u \prec v := (u \preceq v) \wedge (u \neq v)$. If the partial order relation "$\preceq$" is valid on $\mathcal{F}$ then the pair $(\mathcal{F}, \preceq)$ is called a *partially ordered set* (or short: *poset*). If $u \prec v$ for some $u, v \in \mathcal{F}$ then u is said to *dominate* v. Distinct points $u, v \in \mathcal{F}$ are said to be *comparable* when either $u \prec v$ or $v \prec u$. Otherwise,

u and v are *incomparable* which is denoted by $u \parallel v$. If each pair of distinct points of a poset $(\mathcal{F}, \preceq)$ is comparable then $(\mathcal{F}, \preceq)$ is called a *totally ordered set* or a *chain*. Dually, if each pair of distinct points of a poset $(\mathcal{F}, \preceq)$ are incomparable then $(\mathcal{F}, \preceq)$ is termed an *antichain*. An element $u^* \in \mathcal{F}$ is called a *minimal element* of the poset $(\mathcal{F}, \preceq)$ if there is no $u \in \mathcal{F}$ such that $u \prec u^*$. The set of all minimal elements, denoted $\mathcal{M}(\mathcal{F}, \preceq)$ or $\mathcal{F}^*$, is said to be *complete* if for each $u \in \mathcal{F}$ there is at least one $u^* \in \mathcal{M}(\mathcal{F}, \preceq)$ such that $u^* \preceq u$. In case of finitely large posets the completeness of $\mathcal{M}(\mathcal{F}, \preceq)$ is guaranteed. See e.g. [12] for additional information.

Let $f : \mathcal{X} \to \mathcal{F}$ be a mapping from some set $\mathcal{X}$ to the poset $(\mathcal{F}, \preceq)$. For some $A \subseteq \mathcal{X}$ the set $\mathcal{M}_f(A, \preceq) = \{a \in A : f(a) \in \mathcal{M}(f(A), \preceq)\}$ contains those elements from A whose images are minimal elements in the image space $f(A) = \{f(a) : a \in A\} \subseteq \mathcal{F}$.

In order to clarify the notion of "stochastic convergence to the set of minimal elements" we need measures on the distances between finite point sets. The first measure used here is characterized as follows: If A and B are subsets of a finite ground set X then $d(A, B) = |A \cup B| - |A \cap B|$ is a metric on the power set of X. The second measure uses the quantity $\delta_B(A) = |A| - |A \cap B|$ counting the number of elements that are in set A but not in set B. This leads to the definition given next:

Let A_t be the population of some evolutionary algorithm at iteration $t \geq 0$ and $F_t = f(A_t)$ its associated image set. The evolutionary algorithm is said to *converge with probability 1 to the entire set of minimal elements* if

$$d(F_t, \mathcal{F}^*) \to 0 \quad \text{with probability 1 as } t \to \infty$$

whereas it is said to *converge with probability 1 to the set of minimal elements* if

$$\delta_{\mathcal{F}^*}(F_t) \to 0 \quad \text{with probability 1 as } t \to \infty$$

Needless to say, $d(F_t, \mathcal{F}^*) \to 0$ implies $\delta_{\mathcal{F}^*}(F_t) \to 0$.

2.2 Main Results

The results presented here are based on the following assumptions: The search set $\mathcal{X}$ is finite and the fitness function $f : \mathcal{X} \to \mathcal{F}$ maps each individual $x \in \mathcal{X}$ to a member of the fitness set $\mathcal{F}$ which is partially ordered and not necessarily a numerical one.

THEOREM 1
Let G be the homogeneous stochastic matrix describing the transition behavior from $A(t)$ to $B(t+1)$ in the evolutionary algorithm to the right. If matrix G is positive then $d(f(A_t), \mathcal{F}^*) \to 0$ with probability one and in mean as $t \to \infty$.
PROOF: See [6, p. 351]. ■

```
B(0) is drawn at random from X^n
A(0) = M_f(B(0), ⪯)
t = 0
repeat
  B(t+1) = generate(A(t))
  A(t+1) = M_f(A(t) ∪ B(t+1), ⪯)
  t ← t+1
until stopping criterion fulfilled
```

Two points deserve special mention: First, the size of the sets $A(\cdot)$ will grow to $|\mathcal{F}^*|$ limiting the practical use of this algorithm in general (especially if $|\mathcal{F}^*|$ is large). Second, if matrix G is an irreducible (or primitive) but *non-positive* transition matrix then convergence cannot be guaranteed in general.

In order to keep the population at a manageable size one has to modify the selection procedure considerably. Let $n = |B_t|$ and $m \geq n$ where m denotes the maximum size of the sets A_t. The function *draw*(k, C) returns a set of at most k distinct elements from set C drawn by an arbitrary method. If $k = 0$ then an empty set is returned.

```
B(0) is drawn at random from X^n
A(0) = M_f(B(0), ⪯)
t = 1
repeat
  B(t) = generate(A(t − 1))
  B*(t) = M_f(B(t), ⪯)
  C(t) = ∅
  foreach b ∈ B*(t) do
    D_b = {a ∈ A(t) : f(b) ≺ f(a)}
    if D_b ≠ ∅ then
      A(t) ← (A(t) \ D_b) ∪ {b}
    elseif ∀a ∈ A(t) : f(a) ∥ f(b) then
      C(t) ← C(t) ∪ {b} endif
  endfor
  k = min{m − |A(t)|, |C(t)|}
  A(t + 1) = A(t) ∪ draw(k, C(t))
  t ← t + 1
until stopping criterion fulfilled
```

THEOREM 2
Let G be the homogeneous stochastic matrix describing the transition behavior from $A(t)$ to $B(t+1)$ of the EA given to the right. If matrix G is positive then $\delta_{\mathcal{F}^*}(f(A_t)) \to 0$ and $|A_t| \to \min\{m, |\mathcal{F}^*|\}$ with probability one and in mean as $t \to \infty$.
PROOF: See [11, p. 1013f.]. ■

If $\mathcal{F}$ is totally ordered then the EAs of Theorems 1 and 2 are identical and they reduce to an EA with so-called $(1+n)$-selection scheme. Notice that the population size of the second EA may vary during the search; but finally the size will be exactly equal to $\min\{m, |\mathcal{F}^*|\}$.
Since varying population sizes are uncommon in the field of evolutionary computation we have also offered an EA whose population size is kept constant. But notice that the population $A(\cdot)$ may contain individuals that are dominated by some other individuals. This is in contrast to the previous version where we have the invariant property that the population is an antichain, i.e., all individuals are mutually incomparable.

```
B(0) is drawn at random from X^n
A(0) = M_f(B(0), ⪯)
t = 1
repeat
  B(t) = generate(A(t − 1))
  B*(t) = M_f(B(t), ⪯)
  B(t) ← B(t) \ B*(t)
  C(t) = ∅
  foreach b ∈ B*(t) do
    D_b = {a ∈ A(t) : f(b) ≺ f(a)}
    if D_b ≠ ∅ then
      A(t) ← (A(t) \ D_b) ∪ {b}
      B*(t) ← B*(t) \ {b}
    elseif ∀a ∈ A(t) : f(b) ∥ f(a) then
      C(t) ← C(t) ∪ {b}
      B*(t) ← B*(t) \ {b}
    endif
  endfor
  k = min{m − |A(t)|, |C(t)|}
  A(t + 1) = A(t) ∪ draw(k, C(t))
  k = min{m − |A(t + 1)|, |B*(t)|}
  A(t + 1) ← A(t + 1) ∪ draw(k, B*(t))
  k = min{m − |A(t + 1)|, |B(t)|}
  A(t + 1) ← A(t + 1) ∪ draw(k, B(t))
  t ← t + 1
until stopping criterion fulfilled
```

THEOREM 3
Let G be the homogeneous stochastic matrix describing the transition behavior from $A(t)$ to $B(t+1)$ of the EA given to the right. If matrix G is positive and $|\mathcal{F}^*| \geq n$ then $\delta_{\mathcal{F}^*}(f(A_t)) \to 0$ (while the population size n remains constant) with probability one and in mean as $t \to \infty$.
PROOF: See [10]. ■

3 Special Cases

3.1 Multi-Criteria Optimization

The main difference between single– and multi–objective optimization rests on the fact that two elements are not guaranteed to be comparable in the latter case. To understand the problem to full extent it is important to keep in mind that the values $f_1(x), \ldots, f_d(x)$ of the $d \geq 2$ objective functions represent *incommensurable* quantities that cannot be minimized simultaneously: While f_1 may measure costs, f_2 may measure the level of pollution, f_3 the pressure of some boiler, and so forth. As a consequence, the notion of the "optimality" of some solution needs a more general formulation as in the single–criterion case. It seems reasonable to regard those elements as being optimal which cannot be improved with respect to one criterion without getting a worse value in another criterion. Elements with this property are said to be *Pareto–optimal* in this context. This yields a natural partial order

$$u \preceq v \qquad \Leftrightarrow \qquad u \neq v \ \wedge \ \forall i = 1, \ldots, d : u_i \leq v_i \tag{1}$$

in the objective space $\mathcal{F}$ with $u, v \in \mathcal{F} \subseteq \mathbb{R}^d$ and fitness function $f : \mathcal{X} \to \mathcal{F}$ with $u = f(x)$, $v = f(y)$ for $x, y \in \mathcal{X}$. Evidently, the Pareto-optimal solutions in objective space are exactly the minimal elements of the partially ordered set $(\mathcal{F}, \preceq)$ with $\mathcal{F} = \mathbb{R}^d$ and preference relation $\preceq$ as given in equation (1). This observation immediately leads to the following results:

COROLLARY 1 (to Theorem 2)
Let $f : \mathcal{X} \to \mathcal{F} \subseteq \mathbb{R}^d$ be the vector-valued objective function of a multi-criteria optimization problem. The population of an evolutionary algorithm associated with Theorem 2 with positive transition matrix G for generating new candidate solutions and preference relation as given in equation (1) converges with probability 1 to the Pareto set. Moreover, the population size converges to $\min\{m, |\mathcal{F}^*|\}$ where m is a preselected upper limit. ■

COROLLARY 2 (to Theorem 3)
Let $f : \mathcal{X} \to \mathcal{F} \subseteq \mathbb{R}^d$ be the vector-valued objective function of a multi-criteria optimization problem with $|\mathcal{F}^*| \geq n$. The population of size n of an evolutionary algorithm associated with Theorem 3 with positive transition matrix G for generating new candidate solutions and preference relation as given in equation (1) converges with probability 1 to the Pareto set. Moreover, the population size n remains constant. ■

3.2 Interval-valued Fitness Functions

If the evaluation of the fitness function involves a potential numerically instable process the use of interval arithmetic [5] is advisable because it enables an assessment of the numerical reliability of the fitness evaluation. In this case we obtain fitness *intervals* in lieu of fitness *values*. As a consequence, the selection procedures of evolutionary algorithms have to cope with an interval order [2]. Since interval orders are partial orders we can deploy our EAs designed for partially ordered fitness sets. Here, the strict partial order is defined as follows: Suppose w.l.o.g. that $u_1 \leq v_2$. Then

$$[u_1, u_2] \prec [v_1, v_2] \qquad \Leftrightarrow \qquad [u_1, u_2] \cap [v_1, v_2] = \emptyset \tag{2}$$

otherwise the fitness intervals are incomparable unless they are identical. With the preference relation given above the set of closed intervals $\mathbb{I} = \{[x_1, x_2] \subset \mathbb{R} : x_1 \leq x_2\}$ is a partially ordered set. Similarly, the infinitely large but countable set $(\mathbb{I}_\varepsilon, \preceq)$ with $\mathbb{I}_\varepsilon = \{[x-\varepsilon, x+\varepsilon] \subset \mathbb{R} : x \in \mathbb{N}_0\}$ with $\varepsilon > 1/2$ is a poset with incomparable elements whereas $(\mathbb{I}_\varepsilon, \preceq)$ with $\varepsilon < 1/2$ is totally ordered and therefore a chain.

As a matter of course we have to make sure that the targets of the evolutionary search, namely the minimal elements of the set of fitness intervals, represent "reasonable" solutions. For example, the set of minimal elements of $(\mathbb{I}_\varepsilon, \preceq)$ with $\varepsilon = 2/3$ is $\mathcal{M}(\mathbb{I}_\varepsilon, \preceq) = \{[-\frac{2}{3}, \frac{2}{3}], [\frac{1}{3}, \frac{5}{3}]\}$. Evidently, the set of minimal elements of the set of fitness intervals (which we call hereinafter *set of optimal fitness intervals*) consists of all intervals containing the globally optimal solution (solution set $\mathcal{F}_1^*$) and additionally all those fitness intervals that do not contain the globally optimal solution but do have a nonempty intersection with an interval containing the globally optimal solution (solution set $\mathcal{F}_2^*$). The second set of solutions $\mathcal{F}_2^*$ is unwelcome and later we shall consider a scenario and a method how to get rid of these unwanted solutions. Afore we state our results:

COROLLARY 3 (to Theorem 2)
Let $f : \mathcal{X} \to \mathcal{F} \subseteq \mathbb{I}$ be the interval-valued objective function of a single-criterion optimization problem. The population of an evolutionary algorithm associated with Theorem 2 with positive transition matrix G for generating new candidate solutions and preference relation as given in equation (2) converges with probability 1 to the set of optimal fitness intervals. Moreover, the population size converges to $\min\{m, |\mathcal{F}^*|\}$ where m is a preselected upper limit. ■

COROLLARY 4 (to Theorem 3)
Let $f : \mathcal{X} \to \mathcal{F} \subseteq \mathbb{I}$ be the interval-valued objective function of a single-criterion optimization problem with $|\mathcal{F}^*| \geq n$. The population of size n of an evolutionary algorithm associated with Theorem 3 with positive transition matrix G for generating new candidate solutions and preference relation as given in equation (2) converges with probability 1 to the set of optimal fitness intervals. Moreover, the population size n remains constant. ■

Next we consider the following scenario: The interval-valued objective function does not stem from interval arithmetic in the calculations but from a untimely stopped run of a simulator which can therefore only offer a lower and an upper limit of the costs of some, say, chemical plant. If the simulator would run until normal termination the cost interval would reduce to a single value (i.e., $[v, v]$). In general we can tacitly assume that

$$f(x; t + \Delta t) \subseteq f(x; t) \text{ for all } \Delta t > 0$$

where $t \geq 0$ denotes the amount of time that may be used to evaluate solution $x \in \mathcal{X}$. The main idea for eliminating the unwanted solutions in $\mathcal{F}_2^*$ is as follows: If the EA approaches the set of optimal fitness intervals the number of incomparable individuals increase rapidly (as long as we are far away from the optimum the individuals are more likely to be comparable than close to the optimum). This observation then triggers the event of increasing the amount of time being spent at evaluating the fitness function (i.e., the time until the simulator is stopped). This would decrease the width of the fitness intervals and the unwanted solutions can be eliminated. Unfortunately, the proofs of Theorems 1-3 are not designed for such a scenario. Thus, we would need a proof for EAs on partially ordered sets where the partial order gets "more totally ordered" during the search. This is subject to future work.

3.3 Fitness Functions with Bounded Additive Noise

Next we describe an application to stochastically perturbed fitness functions as elaborated in [9]. Let $\mathcal{X}$ be the finite search set and assume that the deterministic fitness function $f : \mathcal{X} \to \mathbb{R}$ is perturbed by additive noise Z, i.e., $\tilde{f}(x) = f(x) + Z$ for $x \in \mathcal{X}$. Here we insist that random variable Z has bounded and *known* support in form of a closed interval of $\mathbb{R}$. For example, Z may have a uniform or symmetric beta distribution on its support $[-a, a]$ with $a > 0$.

When an individual $x \in \mathcal{X}$ is evaluated via $\tilde{f}(x) = f(x) + Z$ then the noisy fitness value is an element of the interval $[f(x) - a, f(x) + a]$. Since the EA only has knowledge of the support bound $a > 0$ and in no case of the true fitness value $f(x)$, the noisy evaluation of $x \in \mathcal{X}$ only leads to the information that the true fitness value $f(x)$ must be in the interval $[\tilde{f}(x) - a, \tilde{f}(x) + a]$. Thus, each point or individual is associated with a realization of a random interval.

Next we declare a strict partial order on these intervals and thereby also a strict partial order on the individuals. Let $x, y \in S$ and w.l.o.g. $\tilde{f}(x) < \tilde{f}(y)$. If

$$\tilde{f}(x) + a < \tilde{f}(y) - a \tag{3}$$

then we define $\tilde{f}(x) \prec \tilde{f}(y)$ and thereby $x \prec y$. This choice is reasonable because we can immediately infer from $x \prec y$ that $f(x) < f(y)$ with probability 1. One should mention that this partial order is a special case of a partial order introduced in [3], p. 29. Moreover, notice that the connection to interval orders gets evident by the equivalence between equation (3) and

$$[\tilde{f}(x) - a, \tilde{f}(x) + a] \cap [\tilde{f}(y) - a, \tilde{f}(y) + a] = \emptyset\,. \tag{4}$$

Thus, whenever two intervals as those above have a nonvoid intersection then the noisy fitness values and therefore also the individuals are incomparable, in symbols: $\tilde{f}(x) \parallel \tilde{f}(y)$ resp. $x \parallel y$.

It remains to examine whether the set of minimal elements of such posets represents a reasonable and useful set of candidate solutions. For this purpose define

$$\begin{aligned} f^* &= \min\{f \in \mathcal{F}\} \quad \text{with} \quad \mathcal{F} = \{f(x) : x \in \mathcal{X}\} \quad \text{and} \\ \tilde{f}^* &= \min\{\tilde{f} \in \tilde{\mathcal{F}}\} \quad \text{with} \quad \tilde{\mathcal{F}} = \{\tilde{f}(x) : x \in \mathcal{X}\}\,. \end{aligned}$$

In general, $\tilde{f}^*$ and $\tilde{\mathcal{F}}$ are random objects. But since it is assumed that each element $x \in \mathcal{X}$ is evaluated only once, one can hold the view that *each* element of $\mathcal{X}$ has been evaluated already *before* the EA is run such that the set $\tilde{\mathcal{F}}$ and the quantity $\tilde{f}^*$ are deterministic during the run of the EA. In this manner one obtains a unique partial order on $\tilde{\mathcal{F}}$ and on $\mathcal{X}$ for each run. The set of minimal elements is then given by

$$\tilde{\mathcal{F}}^* \;=\; \{\tilde{f} \in \tilde{\mathcal{F}} \,|\, \nexists \tilde{f}' \in \tilde{\mathcal{F}} : \tilde{f}' \prec \tilde{f}\} \;=\; \{\tilde{f} \in \tilde{\mathcal{F}} \,|\, \tilde{f} \le \tilde{f}^* + 2\,a\}\,.$$

Needless to say, it is reasonable to postulate that the noisy image $\tilde{f}(x^*)$ of an unperturbed optimal point $x^* \in \mathcal{X}$ is contained in the set of minimal elements. It can be shown [9] that this requirement (and even more) is fulfilled.

THEOREM 4
For all $x^* \in \mathcal{X}$ with $f(x^*) = f^*$ holds $\tilde{f}(x^*) \in \tilde{\mathcal{F}}^*$ regardless of the value of $a > 0$. Moreover, $\max\{\tilde{f} \in \tilde{\mathcal{F}}^*\} \le f^* + 3\,a$. ■

Thus, the set of minimal elements are ε-optimal solutions with $\varepsilon = 3\,a$, i.e., minimal elements are at most $3\,a$ apart from the globally optimal solution. If a is large then the set of minimal elements is too large for being useful. Again, one needs some mechanism to reduce the width of the uncertainty interval. We shall offer such a method after presenting the customized versions of Theorems 2 & 3.

COROLLARY 5 (to Theorem 2)
Let $f : \mathcal{X} \to \mathcal{F} \subseteq \mathbb{R}$ be the real-valued objective function with additive noise of bounded support $[-a, a]$ of a single-criterion optimization problem. The population of an evolutionary algorithm associated with Theorem 2 with positive transition matrix G for generating new candidate solutions and preference relation as given in equation (4) converges with probability 1 to the set of ε-optimal solutions with $\varepsilon = 3\,a$. Moreover, the population size converges to $\min\{m, |\mathcal{F}^*|\}$ where m is a preselected upper limit. ■

COROLLARY 6 (to Theorem 3)
Let $f : \mathcal{X} \to \mathcal{F} \subseteq \mathbb{R}$ be the real-valued objective function with additive noise of bounded support $[-a, a]$ of a single-criterion optimization problem with $|\mathcal{F}^*| \geq n$. The population of size n of an evolutionary algorithm associated with Theorem 3 with positive transition matrix G for generating new candidate solutions and preference relation as given in equation (4) converges with probability 1 to the set of ε-optimal solutions with $\varepsilon = 3\,a$. Moreover, the population size n remains constant. ■

Next it is shown how the uncertainty interval can be reduced. Let $\tilde{f}_n = f + Z_n$ denote the nth sample of the noisy fitness function at a certain point in the search space. The first sample $\tilde{f}_1 = f + Z_1$ leads to the initial confidence interval $[\tilde{f}_1 - a, \tilde{f}_1 + a]$ for the true value f. Since each sample leads to a different confidence interval in general and f must be contained in each of these intervals we immediately obtain

$$
\begin{aligned}
f \;&\in\; \bigcap_{k=1}^{n} [\,\tilde{f}_k - a, \tilde{f}_k + a\,] \;=\; \left[\max_{k \leq n}\{\tilde{f}_k\} - a, \min_{k \leq n}\{\tilde{f}_k\} + a\right] \\
&=\; \left[f + \max_{k \leq n}\{Z_k\} - a, f + \min_{k \leq n}\{Z_k\} + a\right] \;=\; [f, f] + [\,Z_{n:n} - a, Z_{1:n} + a\,] \qquad (5)
\end{aligned}
$$

where $Z_{k:n}$ denotes the kth smallest outcome of n samples in total. Thus, after n samples one knows for sure that the true value f is somewhere in the interval given in equation (5). The uncertainty interval $[\,Z_{n:n} - a, Z_{1:n} + a\,]$ shrinks to $[0, 0]$ for $n \to \infty$. The speed of narrowing can be determined as follows: Let $L_n = Z_{n:n} - a$ and $R_n = Z_{1:n} + a$. Then $|[L_n, R_n]|/(2\,a)$ is the relative size of the uncertainty or incomparability interval $[L_n, R_n]$ after n samples and the probability that it is then still larger than $100\,\varepsilon$ percent of its initial size is given by

$$
\mathsf{P}\left\{ \frac{|[L_n, R_n]|}{2\,a} > \varepsilon \right\} = n\,(1-\varepsilon)^{n-1} - (n-1)\,(1-\varepsilon)^{n}
$$

if Z is the uniform distribution on $[-a, a]$. In this case the relative size of the uncertainty interval after n samples has the mean $2/(n+1)$ and a variance $\leq 2/n^2$. Again, the proofs of Theorems 1-3 are not designed for such a scenario. As in the previous subsection we need proofs where the partial order gets "more totally ordered" during the search.

4 Conclusions

The limit theory for EAs under partially ordered fitness sets immediately delivers a limit theory for EAs tackling problems with multiple objectives, noisy fitness functions, interval-valued fitness functions, and others for free. Moreover, an evolutionary algorithm that can cope with posets also works on the special cases mentioned above and every other derived problem class.

In an object-oriented implementation of this EA we only need to add another method for comparing two elements in order to get the EA working on a new problem class involving special kinds of posets.

Finally, it may be worth mentioning that we experience an exceptional case here: Theory precedes practice. In the field of evolutionary computation the situation usually is the other way round.

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