

PRESENT VALUE OF RANDOM WALKS

*Variance of Present Values under Random Walk Cash Flows*¹

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Abstract: This paper examines how random walks in cash flows influence Present Value (PV). A mathematical framework is developed in which cash flows evolve through random disturbances, allowing the derivation of variance formulas for discounted and undiscounted sums. Special cases are provided for zero discounting and infinite horizons, offering measures of investment risk under uncertainty. The approach is illustrated with monthly hold percentages from the table game *Twenty One (Blackjack)* in Nevada (2004–2024), demonstrating how stochastic fluctuations in cash flows affect valuation outcomes. The results highlight the importance of incorporating randomness into financial assessments to better capture long-term risk in investment valuation.

Keywords: Random Walk, Present Value, Stochastic Process, Variance, Firm Value, Cash Flow

1 INTRODUCTION

The valuation of firms and investment risks depends crucially on the unpredictability of future cash flows. This paper examines how random walks in cash flows affect Present Value (PV) and firm valuation. We model future cash flows as current values plus independent random disturbances, allowing a direct analysis of how stochastic fluctuations influence financial projections.

Within this framework, we derive variance formulas for discounted cash flows that link uncertainty to the discount factor and time horizon. To demonstrate the application of our model, we analyze monthly hold percentages from the table game *Twenty One (Blackjack)* in Nevada (2004–2024).

The theoretical foundations of random walks and ARIMA models are well established in the time series literature (Chatfield, 2004; Hyndman et al., 2020). Their application to stochastic enterprise valuation and financial risk has been investigated in previous work by the authors (Jöckel & Pflaumer, 2022, 2024). Building on these studies, the present paper derives analytical variance formulas for the Present Value of cash flows following a random walk and illustrates their practical relevance using empirical casino hold-percentage data.

2 METHODOLOGY

Random Walk Representation

The cash flow process is modeled as a random walk, which is equivalent to an ARIMA(0,1,0) process. It is defined by

$$c_t = c_{t-1} + \epsilon_t, \quad c_0 = 0,$$

where the innovations ϵ_t are independent random variables with

$$E(\epsilon_t) = 0, \quad \text{Var}(\epsilon_t) = \sigma^2.$$

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Equivalently,

$$c_t = \sum_{j=1}^t \epsilon_j.$$

The random walk is non-stationary, with variance

$$\text{Var}(c_t) = t\sigma^2,$$

because each innovation has a permanent effect on the process level.

Variance of Discounted Present Values

The discounted Present Value over a finite horizon n is defined as

$$PV = \sum_{t=1}^n \frac{c_t}{q^t},$$

where $q = 1 + i > 1$ denotes the discount factor. Here, q refers exclusively to the discount factor and should not be confused with the moving-average parameter in ARIMA notation.

Substituting the random walk representation and rearranging the terms yields a weighted sum of independent innovations. Consequently, the variance is obtained by summing the squared weights of the shocks.

The resulting variance expressions are:

Finite Horizon

$$\text{Var}(PV_{\text{disc}}) = \frac{n + \frac{q^{2n+2} - q^2}{q^2 - 1} - \frac{2q(q^n - 1)}{q - 1}}{q^{2n}(1 - q)^2} \sigma^2. \quad (1)$$

Infinite Horizon

For $n \rightarrow \infty$, the variance converges to

$$\text{Var}(PV_{\text{inf}}) = \frac{q^2}{(q + 1)(q - 1)^3} \sigma^2. \quad (2)$$

No Discounting

In the limiting case $q \rightarrow 1$, the variance becomes

$$\text{Var}(PV_{\text{undisc}}) = \frac{n(n + 1)(2n + 1)}{6} \sigma^2 = O(n^3). \quad (3)$$

Thus, discounting stabilizes the uncertainty of long-term aggregated cash flows, whereas without discounting the accumulated random walk shocks lead to cubic variance growth.

3 EXPERIMENTAL RESULTS: CASINO HOLD PERCENTAGES

To illustrate the theoretical framework, we analyze monthly data from the table game *Twenty One (Black-jack)* in Nevada (2004–2024). The focus is on the **Hold Percentage**, defined as the ratio of casino win to the total amount wagered by players (coin-in). Unlike the fixed theoretical house edge, the Hold Percentage is a stochastic variable reflecting real-world fluctuations in casino performance.

Applying the ARIMA(0,1,0) model to this dataset, we estimated an innovation variance of $\sigma^2 = 2.545$. The ARIMA(0,1,0) specification was chosen because the theoretical framework assumes a random walk structure; therefore, the estimated innovation variance directly corresponds to the variance parameter of the random walk model.

Summary of Present Value Metrics ($n = 240, q = 1.02$):

Metric	Variance	Std Dev	P(PV < 0)
Var_disc	158,400	398.0	0.055
Var_inf	163,850	404.8	0.058
Var_undisc	11,800,758	3,435.2	0.426

Mean Present Value (infinite horizon, 2% discount rate): 636.5 USD, calculated as $12.73/0.02$ based on the observed mean hold-percentage.

The empirical example forms part of a broader research program on stochastic enterprise valuation and casino data analysis. Related applications of random walk models to firm valuation and casino hold percentages are presented in Jöckel & Pflaumer (2024) and Pflaumer (2025), while a comprehensive treatment of probability, risk, and betting systems is given in Pflaumer (2026). The present paper extends this research by deriving analytical variance formulas for the Present Value of stochastic cash flows.

4 CONCLUSIONS

This paper develops an analytical framework for the Present Value of stochastic cash flows generated by a random walk structure corresponding to an ARIMA(0,1,0) process. By explicitly incorporating the accumulation of shocks over time, we derive closed-form expressions for the variance of discounted and undiscounted Present Values.

A central result is that discounting fundamentally changes the propagation of uncertainty. While the variance of the undiscounted cumulative process grows polynomially with the time horizon, introducing a positive discount factor leads to a bounded variance in the infinite-horizon case. Thus, discounting acts as a stabilizing mechanism for stochastic cash-flow valuation under unit-root dynamics.

The analytical results provide three complementary regimes: finite-horizon valuation, long-run infinite-horizon valuation, and the limiting case without discounting. Together, these cases characterize how uncertainty accumulates under different valuation assumptions and clarify the role of the discount factor in controlling long-term risk exposure.

The empirical illustration using monthly hold percentage data from *Twenty One (Blackjack)* in Nevada (2004–2024) shows that even moderate stochastic variability in cash flows can generate substantial dispersion in Present Value estimates, particularly when discounting is weak or absent.

Overall, the framework highlights that the statistical structure of cash flows is as important as their expected trajectory in determining valuation risk. This suggests that stochastic process assumptions should be explicitly incorporated into long-term investment and firm valuation models, particularly in environments characterized by persistent shocks.

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