A Poisson-Ramsey Growth Model –
Creative Destruction, Endogenous Cycles and Growth

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Creative destruction due to new technologies causes both long-run growth and short-run business fluctuations. The creative part of new technologies pushes the economy on a higher productivity level while the destructive part implies partial obsolescence of old production units. Obsolescence due to each new technology induces an adjustment period during which growth rates are initially high but gradually fall. At some endogenously determined point, research for the next technology starts. Once the new technology is discovered, the next cycle starts. This is shown in a continuous time Ramsey growth model where savings can be used for financing deterministic capital accumulation and stochastic Poisson driven R&D for new technologies.

1. Introduction

Technological progress allows firms to use factors of productions more efficiently. A new technology pushes outward an economy's production possibility curve which ceteris paribus leads to an increase of the social welfare level. Technological progress also implies obsolescence of old production units, however. Not all production units are compatible with new technologies or can be upgraded. This is the creative destruction mechanism emphasized by Schumpeter (1942), Aghion and Howitt (1992), Caballero and Hammour (1996) and many others.

Taking these two aspects together allows to develop a view of business cycles that endogenously explains why growth rates incessantly fluctuate over time. An economy never finds itself on a balanced growth path but permanently adjusts its capital stock to new technologies. Long-run positive growth is caused by technological progress, the creative aspect of creative destruction. Fluctuating growth rates result from obsolescence of a part of old production units which requires adjustment to new technologies. This is the destructive aspect of creative destruction. In order to jointly analyse these aspects, the present paper combines a standard Ramsey growth model of riskless capital accumulation with uncertain R&D. Focusing on a central planner problem allows to neglect all aggregation issues and to concentrate on central cycle generating mechanisms.

Consider a centrally planned economy where the planner can allocate savings to capital accumulation or to financing R&D. Capital accumulation is a riskless activity as it mainly requires to copy existing production units. Allocating resources to R&D implies research for a new technology

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which, if developed, increases total factor productivity of the production process. At the same time, it implies obsolescence of a certain part of old production units. Research is associated with uncertainty about the point in time when a new technology will be found[1].

Transitional dynamics in a pure Ramsey growth model are well-understood. When the capital stock implies an interest rate below the time preference rate, capital is accumulated. In the absence of sources of growth as e.g. technological progress or population growth, growth comes to a halt. Now imagine a new technology is introduced which increases total factor productivity but also reduces the stock of capital that can be used under this new technology. The new steady state of the economy then implies a higher consumption level. This steady state, however, is only gradually reached as new capital needs to be accumulated. This permanent adjustment to new steady states is one way how to think of origins of business cycles.

The investment choice is endogenous and thereby the length and amplitude of a cycle. The planner's problem consists in both choosing the optimal consumption level and the optimal allocation of savings to investment projects. As capital accumulation and technological progress are complements, successful development of a new technology is followed by a period where investment is concentrated on capital accumulation. Equivalently, periods of capital accumulation that imply a decrease in returns to capital accumulation are followed by periods of R&D. In fact, in the model to be presented below, savings will exclusively be focused on either capital accumulation or R&D. The economy constantly alternates between exclusive capital accumulation and exclusive R&D.

Investment under uncertainty usually implies that a certain share of savings are invested into the riskless asset while the remaining share goes into the risky asset. While the property of the present model might therefore appear unusual at first sight, it is the result of a property of an uncertain R&D project which strictly distinguishes uncertain R&D from uncertain capital accumulation (as e.g. Bourguignon, 1974; Bismut, 1975; Merton, 1975; Eaton, 1981). An R&D project as modelled here is characterized by uncertainty about the point in time where research for a new technology actually leads to development of a new technology. The gain from increasing investment into R&D is the decrease in the expected length of time until the new technology is discovered. Whatever the amount of investment into R&D, however, the value of this new technology is some fixed amount. The new technology rises total factor productivity by some given factor and this increase in total factor productivity yields some endogenous but fixed value. Since the payoff from a successful project has this property and since the probability of finding a new technology is linear in investment, the economy will exhibit this dichotomy in capital accumulation or R&D finance.

A typical history of our economy will then be as follows. Starting with some capital level, the planner will find it profitable to allocate all resources to capital accumulation. Decreasing returns to capital then imply that capital accumulation loses in profitability relative to financing R&D. At some point in time, expected returns from financing R&D exceed certain returns from allocating capital and all investment will be allocated to R&D. For some time, research will not imply development but at some uncertain point in time, a new technology will have been found, total factor productivity rises and the economy starts accumulating capital again.

There are several competing theories used to understand cyclical behaviour. One group of models stresses the fact that models of deterministic optimal growth can exhibit cyclical and chaotic dynamics[2]. Nishimura and Yano (1995) show the possibility of chaotic behaviour of consumption or GDP on equilibrium paths in a model with fixed coefficient technologies and utilities linear in consumption. Boldrin and Deneckere (1990) in a model with fixed coefficients in the capital good sector and a Cobb-Douglas technology in the consumption good sector find that chaotic dynamics are observed if impatience is very high and differences in the productivity of labour and capital are very large. Benhabib and Nishimura (1985) have derived sufficient conditions for the existence of

1 Shell (1966) argues as well that capital accumulation and R&D differ in the degree of riskiness in the way modelled here.

2 Earlier papers studied cyclical and chaotic time paths in non-optimal models of growth (e.g. Grandmont, 1985).

cycles in a two sector discrete-time model with general technologies. In the context of the new growth theory, Benhabib and Farmer (1994) have demonstrated that increasing returns to scale (on the aggregate level due to externalities or on a firm level in a monopolistic competition framework) lead for certain parameter values to cycles around a steady state. The mechanism suggested here differs from those in that it allows for standard neoclassical production functions (without producing chaotic behaviour, however), that the mechanism is extremely easy to understand from an intuitive point of view and that the occurrence of cycles is not restricted to some range of parameter values. Stochastic approaches to cycles can be divided into self-fulfilling prophecy models (Azariadis, 1981; Farmer and Guo, 1994; Gali, 1994; Drugeon and Wigniolle, 1996) and the RBC literature (Kydland and Prescott, 1982; for in introduction cf. Cooper, 1997, and the special issue in Oxford Review of Economic Policy, Muellbauer, 1997).

The cycle generating mechanism presented here can best be understood as a mix of self-fulfilling prophecies and RBC model aspects. With self-fulfilling prophecies approaches it shares the property that, for some parameter values, beliefs are required to select among several possible equilibrium paths. In order to obtain cycles, however, no randomization between equilibrium paths is required. On one path, the economy is on a cyclical growth path, on another path, it is on a smooth path. Hence, in the present model, expectations are fulfilled forever, cycles and jumps are anticipated and rational. With (a part of the) RBC models, the present model shares the jump in total factor productivity. This jump, however, is not exogenous but the point in time when this jump occurs is endogenously determined through investment decisions of the central planner.

Reproaching the exogeneity of shocks would do injustice to the RBC literature since their main scope is to reproduce empirical regularities in shock driven artificial economies without being primarily concerned with the origin of those shocks. Nevertheless, by providing a mechanism to understand the origin of shocks and the determinants of the length between two shocks, the present model potentially allows to provide further insight into the nature of business cycles. No attempt, however, is made at this stage to confront the present model with data.

A recent interesting study of waves of innovative activity and market entry based on learning-by-doing is by Stein (1997). Each market entry by a new firm rises the probability of future market entries. Young incumbents can more easily be replaced than old incumbents as the latter have lower distribution costs due to learning-by-doing. Innovations will therefore tend to occur in waves. In this approach, however, optimal behaviour of households is treated in a very simple way. In the present approach, optimal behaviour on the demand side is central.[3]

This paper adds to the literature on dynamic stochastic macro models. This literature has mainly used discrete time methods (for an overview and a very thorough treatment cf. Stokey and Lucas with Prescott, 1989). Continuous time methods were introduced by Merton (1969, 1971, 1990) (cf. also Sethi and Taksar, 1988) and growth models were analyzed by Bourguignon (1974), Merton (1975), Bismut (1975) and Eaton (1981). More recently, Obstfeld (1994) has studied the growth increasing effects of increasing international risk-sharing. Turnovsky (1993) has studied the effects of changes in the mean rate and variance of monetary growth rate and of tax policies on both the growth rate of an economy and social welfare. The link between creative destruction, endogenous cycles and long-run growth, however, has not been studied. Optimal control of Poisson processes beyond partial equilibrium examples in Merton (1971) has not been studied either. For an introduction to continuous time methods under uncertainty cf. Dixit and Pindyck (1994) and Turnovsky (1995, part IV).

[3] Bental and Peled (1996) present a model that shares some equilibrium properties of the model presented below. They focus on specific utility and production functions (logarithmic and Cobb-Douglas), however, and analyse their effects in an discrete time 2-period OLG framework. Jovanovic and Rob (1990) study the endogenous determination of major discoveries and subsequent refinement but they view their analysis more in an industry rather than an economy wide context. Further, the modelling approach chosen here is more closely linked to standard models of growth.

2. The model

2.1. The economy

The economy consists of three sectors. The first sector produces the consumption good C by employing capital K_C and labour L_C. It is subject to discontinuous technological progress[4] which increases total factor productivity A^{γ}. Each new technology increases γ by one and total factor productivity by $A > 1$. The technology of the consumption good sector reads

$$C = A^{\gamma} G(K_C, L_C). \tag{1}$$

The technology $G(\cdot)$ has positive first and negative second derivatives and is characterized by constant returns to scale.

In the second sector, an investment good is produced (machines) that can be used for increasing the economy's stock of capital. Production equally requires capital K_I and labour L_I and the technology is given by

$$I = G(K_I, L_I). \tag{2}$$

Capital accumulation is a riskless activity as it essentially requires to copy already existing production units.

In the third sector, research for new technologies is undertaken. R&D is directed at developing new production units. When a new production unit has been developed, it represents a capital stock of $\varpi \geq 0$ whose total factor productivity exceeds total factor productivity of the previous vintage by A. It is further assumed that after a successful development of a new technology, a certain share s of the previous vintage can be upgraded and therefore has a higher total factor productivity as well. The remaining capital stock (which might be zero) becomes obsolete. Denoting by $\tilde{K}$ the capital stock after successfully finishing an R&D project and by K the current aggregate capital stock, we have

$$\tilde{K} = \varpi + (1 - s)K. \tag{3}$$

By this formulation[5], at each instant in time, the entire capital stock is of one unique vintage.

The crucial difference between R&D and capital accumulation is uncertainty associated with R&D; *research* for the next technology does not necessarily lead to *development* of the next technology. When searching for a new technology, new paths have to be explored which have not been explored by others before. This uncertainty is captured by a Poisson process whose arrival rate λ is given by

$$\lambda = bG(K_R, L_R). \tag{4}$$

Total factor productivity is given by the constant b and capital and labour inputs into R&D are denoted by K_R and L_R, respectively. Investing $G(K_R, L_R)$ into R&D implies that the probability per unit of time dt that research indeed leads to development of a new production unit is given by λdt while the probability that research remains without success is given by $1 - \lambda$dt.

From the formulation of technologies in (1), (2) and (4), it is clear that a new technology rises total factor productivity only in the consumption good sector. Total factor productivity in the investment and in the research sector is unchanged. This formulation considerably simplifies presentation of results and was chosen for this reason only. Identical qualitative results hold for a situation where all sectors are subject to technological progress[6].

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4 This discontinuous technological progress mechanism and the R&D technology presented below is borrowed from Aghion and Howitt (1992). It has been used in a similar way by Grossman and Helpman (1991a) and others.

5 New technologies are therefore partly vintage specific, partly disembodied. Both views have their empirical merit. Hulten (1992) finds that 20% of total factor productivity growth stems from embodied technological change. Greenwood, Hercowitz and Krusell (1997) argue that up to 60% of total factor productivity growth stems from embodied technological change.

6 This was shown in an earlier version of the present paper which is available from the author upon request. It will be pointed out when this simplification becomes important.

As a consequence, the price of the investment good and the R&D-good do not change across vintages. One R&D-good can always be exchanged for b investment goods. The value of a consumption good, however, rises as new vintages become available. One consumption good is worth A^{γ} investment goods.

Given the assumption that, apart from technological change, all sectors use the same technology F and that this technology exhibits constant returns to scale, the resource constraint of the economy can be written as $I + b^{-1}\lambda = G(K,L) - A^{-\gamma}C$. Allowing for deterministic exponential capital depreciation at rate δ and given that the production of the investment good equals capital accumulation plus depreciation,

$$I = \mathrm{d}K + \delta K, \tag{5}$$

the economy's resource constraint reads

$$\mathrm{d}K + b^{-1}\lambda = G(K,L) - \delta K - A^{-\gamma}C$$

Net savings allocated to capital accumulation dK and R&D $b^{-1}\lambda$ equal the difference between (a measure of) aggregate output $G(K,L)$ and depreciation δK and productivity adjusted consumption $A^{-\gamma}C$. Denoting for simplicity $F(\cdot) = G(K,L) - \delta K$ in what follows, we have

$$\mathrm{d}K + b^{-1}\lambda = F(K,L) - A^{-\gamma}C \tag{6}$$

Denoting the share of savings $F(\cdot) - A^{-\gamma}C$ allocated to research by θ, the arrival rate can be written as[7]

$$\lambda = b\theta\left[F(\cdot) - A^{-\gamma}C\right], \tag{7}$$

and we can conveniently summarize the economy's consumption and investment opportunities in a resource constraint that reads

$$\mathrm{d}K = (1-\theta)\left(F(\cdot) - A^{-\gamma}C\right)\mathrm{d}t + \left(\tilde{K} - K\right)\mathrm{d}q. \tag{8}$$

This is a stochastic differential equation where uncertainty results from a Poisson process q. During a small period of time dt, the capital stock of vintage γ increases deterministically by the share $1-\theta$ of total savings $F(\cdot) - A^{-\gamma}C$ allocated to capital accumulation. When R&D is undertaken, the capital stock is also subject to abrupt changes. With q denoting the Poisson process resulting from investment in R&D projects, dq is the increment of this process. A successful R&D project implies dq = 1. The capital stock then changes by $\tilde{K} - K$ and productivity in the consumption good sector rises by A. When no investment in R&D takes place or when R&D fails, the increment is zero, dq = 0.

Note that the arrival rate is linear in investment $\theta\left[F(\cdot) - A^{-\gamma}C\right]$ in R&D. This assumption is necessary if one wants to understand the R&D sector (in a decentralized economy) as a multitude of firms that produce under perfect competition. Increasing or decreasing returns in the arrival rate (just as increasing or decreasing returns to scale) are not easily reconciled with perfect competition. This linearity will be crucial for the results derived below but, in the light of this perfect competition requirement, is a natural assumption.

2.2. The central planner's choice

The central planners objective is to maximize a social welfare function given the above technologies. Letting the value of the optimal program at t be denoted by $V(K,\gamma)$, her objective is

[7] In the case of ambiguity, round brackets include arguments of functions; squared brackets always indicate a multiplication operation.

$$V(K,\gamma)=\max_{\{C(\tau),\theta(\tau)\}} \mathcal{E}\int_t^{\infty} e^{-\rho[\tau-t]}u(C(\tau))\mathrm{d}\tau \tag{9}$$

subject to (3) and (8). The planner maximizes expected utility from discounted consumption flows by choosing a stream of consumption $\{C(\tau)\}$ and allocation $\{\theta(\tau)\}$ of savings to financing R&D and capital accumulation. The expectations operator is $\mathcal{E}$ and ρ denotes the time preference rate. The share $\theta(\tau)$ is constrained to lie between zero and unity where θ = 1 means that all savings are allocated to R&D[8]. The instantaneous utility function is increasing in consumption with decreasing slope,

$$u'(C)>0, \quad u''(C)<0. \tag{10}$$

The planner's maximization problem can be expressed by the Bellman equation (cf. e.g. Dixit and Pindyck, 1994)

$$\rho V(K,\gamma)=\max_{C,\theta}\left\{u(C)+V'(K,\gamma)[1-\theta]\left[F(\cdot)-A^{-\gamma}C\right]+\lambda\left(\theta\left[F(\cdot)-A^{-\gamma}C\right]\right)\left[V\left(\tilde{K},\gamma+1\right)-V(K,\gamma)\right]\right\}. \tag{11}$$

The expression to be maximized is given by the sum of instantaneous utility $u(C)$, the value of additional units of capital and the expected value of a new technology. The value of additional units of capital is given by the marginal value of the current stock of capital $V'(K,\gamma)$ times the share $1-\theta$ of savings allocated to accumulation of physical capital times savings $F(\cdot)-A^{-\gamma}C$. The expected value of a new technology equals the product of the arrival rate of new technologies (7) and the gain from holding a capital stock $\tilde{K}$ suitable for production with the next technology $\gamma+1$ as compared to the value of holding the current capital stock under technology γ.

The first order condition for consumption can be written after dropping the vintage arguments[9] as

$$u'(C)=[1-\theta]A^{-\gamma}V'(K)+\theta bA^{-\gamma}\left[V\left(\tilde{K}\right)-V(K)\right]. \tag{12}$$

Marginal utility from consumption today is the θ-weighted sum of marginal utility from consumption in the future. Higher future consumption either comes from a higher capital stock or a better technology. The value of more capital through less consumption is given by the value of an additional unit of capital, $V'(K)$, times the instantaneous increase in the capital stock by reducing consumption by one unit $[1-\theta]A^{-\gamma}$. The impact of less current consumption on future technologies is captured by the increase in the probability of a research success $\theta bA^{-\gamma}$ times the gain in the value of the optimal program if research is indeed successful, $V\left(\tilde{K}\right)-V(K)$.

The derivative of (11) with respect to the share θ invested in R&D,

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\{\cdot\}=\left(-V'(K)+b\left[V\left(\tilde{K}\right)-V(K)\right]\right)\left[F(\cdot)-A^{-\gamma}C\right], \tag{13}$$

is a function independent of the share θ itself. Therefore, except for some K^*, where the planner is indifferent, $\theta = 0$ or $\theta = 1$. Assuming that net savings are positive[10], $F(\cdot)>A^{-\gamma}C$,

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8 If we allowed for $\theta>1$, we would technologically allow capital to be decumulated, retransformed into the aggregate good which is then used for the R&D process. While standard in models of optimal growth where capital can be "eaten up" we exclude this by assumption. On equilibrium paths as studied below the planner would never want to set θ above unity. A negative θ would imply negative resource allocation to the R&D process. This is clearly technologically unfeasible.

9 No ambiguity arises as K always denotes the current capital stock and $\tilde{K}$ denotes the capital stock available for the next vintage.

10 We restrict attention to this case. An extension is straightforward.

$$\left.\begin{array}{l}\theta=0\\ \theta\in[0,1]\\ \theta=1\end{array}\right\}\Leftrightarrow b\left[V(\tilde{K})-V(K)\right]\left\{\begin{array}{c}<\\=\\>\end{array}\right\}V'(K) \tag{14}$$

The planner allocates all net savings to capital accumulation ($\theta = 0$) when the marginal value of capital exceeds the discrete increase of the value of capital times the marginal arrival rate, while all savings net of capital depreciation are allocated to R&D ($\theta = 1$) when the derivative (13) is positive. Depreciation of capital at rate δ implies that some production of investment goods always takes place. When $\theta = 1$, net capital accumulation, which by (6)and (7) equals $\mathrm{d}K = (1-\theta)\left(F(K,L) - A^{-\gamma}C\right)$, is zero. By (5), however, $I = \delta K$.

This complete specialization is a surprising result, at first sight. We are used to from portfolio choice literature that an individual invests always at least some amount of her wealth into the risky asset. Here, there are three reasons for this bang-bang property. First, the paper studies financing risky R&D and not financing risky investment. Risky investment is characterized by payoffs that are proportional to investment where the factor of proportionality is the uncertain return. The higher investment, the higher the expected payoff. By contrast, risky R&D is characterized by fixed payoffs and incentives to invest in R&D at all stem from the impact of investment on the arrival rate. This is a property of any R&D project, both in this central planner framework and in a decentralized setup.

Second, this bang-bang property is a result of the linearity of the arrival rate in θ. As mentioned before, this is a standard assumption which is required here if one wants a technology for R&D that can be replicated by decentralized competition between many firms.

A third, technical, reason for this bang-bang behaviour is the continuous time framework used here. In discrete time, an interior solution results both from risky investment and risky R&D, as was shown in Wälde (1998, ch. 8). This paper also shows that R&D is characterized by fixed payoffs when R&D is financed by many individuals in a decentralized economy[11].

It is intuitively clear what would happen if one of these conditions was not satisfied. Condition (14) would hold as an equality for some range of the capital stock and not only for a specific K^*. More importantly, the share θ of savings allocated to R&D would increases, the higher the economy's capital stock: As returns to investment in capital accumulation fall, more and more resources would be shifted towards investing in R&D. Hence, the probability per unit of time that a new technology is found would increase in the economy's capital stock. Eventually, when a new technology is found, the share θ would fall again[12]. Hence, qualitatively, this shift from investing in capital accumulation to investing in R&D takes place more smoothly but in a similar way. The strong prediction of the model should therefore be seen as a useful device that simplifies the derivation of results and not as a one-to-one mapping of reality.

Deterministic regime

When the share of savings invested into R&D are zero, $\theta = 0$, the first order condition for consumption (12) simplifies to

$$u'(C) = A^{-\gamma}V'(K). \tag{15}$$

The optimal consumption level is reached when marginal utility from consumption equals the marginal value of capital. This condition is frequently found in deterministic setups where it usually appears in a form like $u'(C) = \mu$, where μ is the Hamiltonian multiplier which stands for the marginal valuation of an additional unit of the associated state variable (cf. e.g. Kamien and Schwartz, 1991), just as here. The Keynes-Ramsey rule is the familiar one,

[11] If the utility function was not additive over time, an interior solution might also obtain.

[12] Such a prediction can be shown to follow from a model with uncertain investment rather than uncertain R&D.

$$-\frac{u''(C)}{u'(C)}\mathrm{d}C=\left[F_K(\cdot)-\rho\right]\mathrm{d}t \tag{16}$$

and the resource constraint (8) becomes

$$\mathrm{d}K=\left[F(\cdot)-A^{-\gamma}C\right]\mathrm{d}t. \tag{17}$$

Switch to the stochastic regime

As long as gains from an additional unit of capital exceed expected gains from a new technology, the economy is in the deterministic regime. When the share derivative (13) holds with equality, the planner switches from $\theta = 0$ to $\theta = 1$. In its current formulation, however, condition (13) is of little use since the value function is not known. Assuming that the economy starts with a capital stock K_0 that implies that capital is accumulated, i.e. $\theta = 0$[13], one can express this condition in a form that is more intuitive and that can be used for a simple phase diagram analysis. Defining

$$\Omega \equiv A\,u'\left(C\left(\tilde{K}\right)\right)\big/u'\left(C(K)\right) \tag{18}$$

as the ratio of marginal utility after the development of a new technology when the capital stock will be given by $\tilde{K}$ and marginal utility at current consumption, the Appendix proves the following

<u>Theorem 1</u> (Optimal investment decision)

$$\theta=\begin{Bmatrix}0\\1\end{Bmatrix}\Leftrightarrow F_K\ \begin{Bmatrix}>\\=\end{Bmatrix}\ \rho+\lambda\left[1-\left[1-s\right]\Omega\right]. \tag{19}$$

The central planner allocates all savings to deterministic capital accumulation (θ = 0) when the marginal product of capital is sufficiently high. Sufficiently high in this context means that it must be higher than the time preference rate plus the (adjusted) arrival rate where the arrival rate (7) is evaluated at current output and consumption. The planner is willing to accumulate an additional unit of capital, i.e. $\theta = 0$, when the return to saving capital exceeds the time preference rate *plus* the (adjusted) arrival rate. More capital increases utility of the planner only if she is compensated for her impatience and for the risk that this capital stock eventually becomes obsolete.

The degree to which the arrival rate is adjusted depends on the share s of capital that becomes useless under the new technology. When the entire capital stock is useless under the new technology, i.e. $s = 1$, adjustment for risk is given by the arrival rate λ. The interest rate which must be guaranteed when accumulating more capital is then given by $\rho+\lambda$. The sum of the time preference rate and the arrival rate is known to be the discount factor that is used by households when discounting an income stream that ends at some exponentially distributed point in time, where λ is the parameter of the exponential distribution[14].

When the arrival of a new technology does not imply that the entire old capital stock becomes obsolete, i.e. when $s < 1$, the arrival rate is adjusted by the instantaneous change in marginal utility Ω due to the new technology. Depending on the sign of $1-\left[1-s\right]\Omega$, the economy stops accumulating capital at an interest rate that is higher or lower than the time preference rate. When s is sufficiently close to unity, the adjusted arrival rate is close to the arrival rate. As the obsolescence parameter s falls, and with Ω sufficiently high[15], $1-\left[1-s\right]\Omega$ becomes negative and the arrival of new technologies induces individual to accumulate more capital than in a purely deterministic world.

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[13] The economy could start with a capital stock that implies that the planner allocates all savings to R&D. Once a new technology has been developed, we are back to $\theta = 0$.

[14] Compare e.g. Blanchard (1985), Grossman and Helpman (1991b) or Aghion and Howitt (1992) among others.

[15] Note that Ω is endogenous and therefore not independent of s. In equilibrium, however, Ω is constant.

A further property of the switch from the deterministic to the stochastic regime is that no consumption jump takes place[16].

Stochastic regime

When all investment is channelled into R&D, $\theta = 1$, and the planner is indifferent, we know from the above theorem (19) that marginal productivity of capital equals the sum of the time preference rate and the adjusted arrival rate. The economy is in a (transitory) stationary state. The capital stock is determined by history, i.e. equals the stock of capital at the moment the economy allocates all savings to R&D.

The consumption level at every moment in time is determined by the consumption first order condition (12). In the stochastic regime where $\theta = 1$, this condition reads

$$u'(C) = bA^{-\gamma}\left[V(\tilde{K}) - V(K)\right]. \tag{20}$$

It says to choose consumption in the stochastic regime such that the expected gain from a new technology just matches marginal utility. The trade-off is therefore not between consumption today and capital investment today but between consumption today and the *chance* of having a better technology in the future.

Switch to the deterministic regime

The switch from the stochastic stationary regime to the deterministic regime takes place as soon as a new technology has been developed. All savings will be allocated to capital accumulation, $\theta = 0$, after the development of a new technology as long as the new capital stock is sufficiently small given the new technology. What remains to be found out is how consumption changes.

Replacing the value of the capital stock by utility from current consumption plus marginal utility times savings divided by ρ, the optimality condition for consumption on the R&D line (20) can be expressed alternatively as (cf. appendix)

$$\frac{\rho}{bA^{-\gamma}}u'(C) = u(\tilde{C}) + A^{\gamma+1}u'(\tilde{C})\left[F(\tilde{K},L) - A^{-(\gamma+1)}\tilde{C}\right] - \left[u(C) + A^{\gamma}u'(C)\left[F(K,L) - A^{-\gamma}C\right]\right], \tag{21}$$

where consumption an instant after a successful R&D project is denoted by $\tilde{C}$. This condition is a rewritten version of the consumption first order condition (20), that must hold in the stochastic regime, where the value functions $V(K)$ and $V(\tilde{K})$ are replaced by expressions given by a maximized Bellman equation (i.e. the Bellman equation (11) with optimal control variables). This condition provides a link between consumption C on the R&D line and consumption $\tilde{C}$ after the development of a new technology. Since consumption changes in a discrete way, this condition will be referred to as consumption jump condition.

This completes the description of the economy and the central planner's solution. When the economy is in the deterministic regime, the evolution of consumption and capital is described by standard laws of motion (16) and (17). When the stock of capital is such that (19) holds with equality, the planner shifts all savings from capital accumulation to R&D and the economy moves from the deterministic to the stochastic regime. In the stochastic regime, the consumption level and the capital stock are constant. As consumption does not jump at this regime switch, both consumption and the capital stock are given by history, i.e. they equal the consumption level and capital stock at the last instant of the deterministic regime. When R&D is successful, a new

[16] Under the deterministic regime, the marginal value of the capital stock equals marginal utility from consumption (14). On the R&D line, where the derivative with respect to the share θ in (12) is zero, this holds as well. This can be seen by inserting $\mathrm{d}\{\cdot\}/\mathrm{d}\theta = 0$ from (12) into the consumption first order condition (11). Since for both the R&D line and the deterministic regime $u'(C) = V'(K)$ and since the capital stock does not jump when the economy moves on the R&D line, C does not jump either.

technology becomes available, the capital stock changes according to (3) and the consumption level jumps as prescribed by (21).

3. An equilibrium with growth and cycles

The question now is: Is there a path that is characterized by an ever-repeating switch between capital accumulation and R&D? Before we prove the existence of such a path, we will illustrate the model with the help of a phase diagram.

3.1. Phase diagram illustration

The last section has shown that the crucial difference between a standard Ramsey growth model and the model considered here lies in the R&D regime. Illustrating this model therefore simply requires to draw a phase diagram of the deterministic regime and add an "R&D line", i.e. the loci given by (19) holding with equality, where the economy switches from the deterministic to the stochastic regime.

In the deterministic regime, the economy follows (16) and (17) and we obtain a phase diagram as in figure 1.

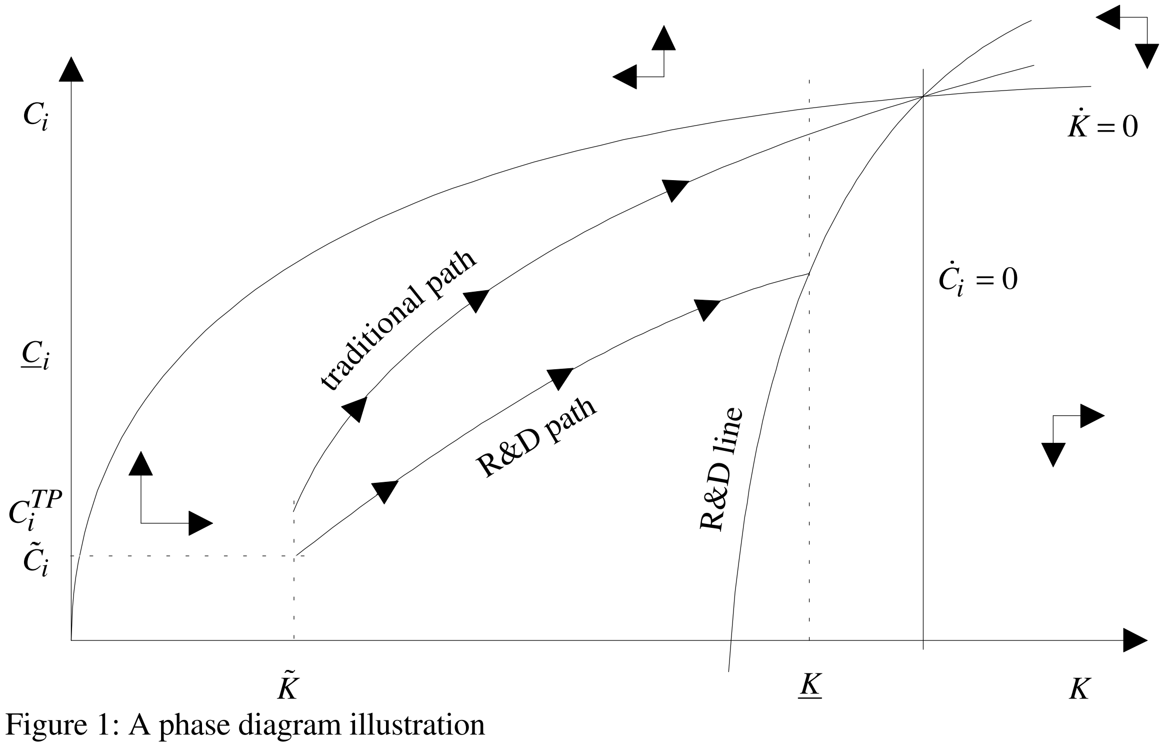

Figure 1: A phase diagram illustration

The axes show the stock of capital K and the productivity adjusted level of consumption,

$$C_i = A^{-\gamma}C. \tag{22}$$

The zero-motion lines have the familiar form

$$\dot{K} = 0 \Leftrightarrow C_i = F(K, L), \tag{23a}$$

$$\dot{C}_i = 0 \Leftrightarrow \rho = F_K(K, L). \tag{23b}$$

These zero-motion lines show the advantage for the presentation of results of the assumption that technological progress is restricted to the consumption good sector. If technological progress increased total factor productivity in the investment good and in the R&D sector as well, the zero motion line for consumption would depend on the current vintage γ. Further, the R&D line would also be vintage-dependent. An analysis would then require a variable transformation for capital as well. This would yield some additional insights (discussed in section 4.2) but makes presentation of results cumbersome.

Now draw the line on which the central planner finds it profitable to invest all savings in R&D. This R&D line follows from (19) and reads with (7), $\theta = 1$ and the productivity adjustment (22),

$$C_i = F(\cdot) - \frac{F_K - \rho}{b\left[1 - [1-s]\Omega\right]}. \tag{24}$$

This line goes through the steady state. For $[1-s]\Omega < 1$, it lies below the zero motion line for capital accumulation to the left of the steady state, where the marginal productivity of capital exceeds the time preference rate, and above it to the right. This reverses when $[1-s]\Omega > 1$. We will assume in what follows that $[1-s]\Omega < 1$ which holds for s sufficiently close or equal to one. This implies that the stock of capital at which the economy stops accumulating capital since the interest rate is too low is smaller than the capital stock in a deterministic Ramsey economy.

The shape of this R&D line can be understood by noting that on this line the central planner is indifferent between accumulating an additional unit of capital and financing R&D. First, the R&D line must contain the steady state of the deterministic system. Consumption growth under the deterministic regime is zero when the marginal productivity of capital equals the time preference rate. At the intersection point of the R&D line and the zero motion line for consumption, this indifference requires that the arrival rate be zero since only with a zero arrival rate (expected) consumption growth is zero and equals deterministic consumption growth. Since the arrival rate is zero only if consumption equals total output, the R&D line must go through the steady state.

Second, when R&D is started at a low level of capital, the marginal product of capital is fairly high and a high arrival rate λ is required to obtain this indifference, i.e. equality in (19). Since output is lower as well with a low stock of capital, consumption must fall faster than output to increase an arrival rate that is monotonically increasing in investment, as in (7). This means that the earlier an economy starts with R&D (i.e. the lower the capital stock at the moment R&D starts), the more (more than proportionally) consumption has to be reduced.

Finally, the R&D line is steeper the lower total factor productivity b in R&D. For a given capital stock, a lower b requires more savings into R&D, i.e. lower consumption, in order to make the planner indifferent between capital accumulation and R&D.

3.2. Dynamic evolution of the economy

Since this diagram differs from the Ramsey growth model only in the R&D line, trajectories in this diagram are identical in both models, apart, of course, on and below the R&D line. It is known that the saddle path of the Ramsey growth model can have different forms. A high elasticity of substitution that induces consumers to hardly smooth consumption leads to a quick accumulation of capital and therefore a saddle path that is convex. Inversely, with a low intertemporal elasticity of substitution, the saddle path would be close to the zero motion line for capital accumulation (as drawn) (cf. e.g. Barro and Sala-I-Martin, 1995, ch. 2.6).

This means that the "saddle path" of the present model (denoted traditional path[17] in figure 1) may intersect the R&D line. Since there exists a traditional path for any initial capital stock $\tilde{K}$, which implies that for K sufficiently low, the traditional path lies between the R&D line and the zero motion line for capital accumulation, a sufficient condition for intersection is that the limit of the slope of the traditional path as it approaches the steady state is higher than the slope of the R&D line in the steady state. This certainly holds when the elasticity of substitution is sufficiently high. In this case,

[17] The term saddle path would not be appropriate in the present context since this is not a traditional two-dimensional differential equation system with a unique steady state which can be approached by jumping on the stable arm of the system only.

no trajectory exists that leads to the steady state without crossing the R&D line. If it does not intersect, the traditional path is one possible equilibrium path of this economy, i.e. a path that satisfies all optimality and equilibrium conditions.

Now assume the economy starts with some initial capital stock suitable for production with the current vintage, $\tilde{K}$. The economy can jump on the traditional path by choosing the appropriate consumption level C_i^{TP} and approach the long-run steady state if the traditional path lies above the R&D line (as drawn). Since investment in R&D is zero on the traditional path and since the steady state is never reached, the economy will never start investing in R&D.

If the economy starts with a consumption level such as $\tilde{C}_i$, the economy follows standard laws of motion and accumulates physical capital, given an invariant technology vintage γ. Returns to investment in riskless capital accumulation fall such that eventually the central planner will find it optimal to use savings for R&D finance. This is when the economy hits the R&D line and all savings are invested into R&D, $\theta = 1$. The economy will therefore stop growing, consumption will be constant and research for new technologies is undertaken. The aggregate capital stock at this point is denoted by $\underline{K}$ and will be called the R&D capital stock. Eventually, a new technology is found and the economy starts with a new initial capital stock $\tilde{K}$. Growth starts again.

3.3. The existence of an equilibrium path with growth and cycles

This section proves the existence of an equilibrium path with self-replicating cycles and growth which is the equivalent of this model to a balanced growth path in standard models of growth. An equilibrium path is a feasible path that satisfies all optimality conditions. Let us take as a candidate for such a path the R&D path drawn in figure 1. Let the initial and the final capital stock be linked by (3). This path is an equilibrium path if the consumption jump condition (21) implies that the consumption level after development of a new technology is given by $\tilde{C}_i$. If this is the case, starting at $\tilde{C}_i$ implies by the laws of motion (16) and (17) ending at $\underline{C}_i$ and restarting at $\tilde{C}_i$ after successful development of a new technology and so on ad infinitum.

The existence of such a path can be proven under weak parameter restrictions. We now first provide a proof of a theorem and then illustrate the proof of the theorem which shows the very simple idea behind it. Implicitly, this proof assumes that the consumption jump condition implies a vintage independent relationship between productivity adjusted consumption levels before and after R&D. In order to see that this is the case under standard assumptions about the utility function, rewrite the consumption jump condition (21), by using productivity adjusted consumption levels according to (22) and by rearranging, as

$$\frac{\rho}{b} = \frac{u\left(A^{\gamma} A \tilde{C}_i\right) - u\left(A^{\gamma} C_i\right)}{A^{\gamma} u'\left(A^{\gamma} C_i\right)} + A \frac{u'\left(A^{\gamma} A \tilde{C}_i\right)}{u'\left(A^{\gamma} C_i\right)} \left[F\left(\tilde{K}, L\right) - \tilde{C}_i\right] - \left[F(K, L) - C_i\right].$$

It is clear that with a utility function of the iso-elastic, $u(C) = \alpha C^{\sigma}$, or logarithmic type, $u(C) = \alpha \ln C$, all terms including the vintage argument γ cancel out and we are left with a link between consumption before and after R&D of $\tilde{C}_i = g_2(C_i)$ which is independent of γ. In what follows, we will make

<u>Assumption 1</u> (Consumption jumps)

Consumption jumps are characterized by a stable, i.e. vintage independent, relationship, $\tilde{C}_i = f(C_i, \cdot)$, $\tilde{C}_i < C_i$.[18]

[18] We will see in the decentralized economy of section 4 that such a relationship indeed holds for weak parameter restrictions.

Theorem 2 (Existence of an equilibrium path with growth and cycles)

Assume that the consumption jump condition (21) predicts that jumping from the steady state back to $\tilde{K}$ implies that the new consumption level $\tilde{C}_i$ lies above the consumption level C_i^{TP} that puts the economy on the traditional path, $f\left(C_{st.st.}, \tilde{C}_i\right) = 0 \Rightarrow \tilde{C}_i > C_i^{TP}$. Then, there exists an equilibrium path with self-replicating cycles as the R&D path drawn in figure 1.

The Theorem and its proof (cf. appendix) can be nicely illustrated with the help of figure 1. The assumption says that the consumption jump condition implies that jumping from the steady state back (or from a value very close to the steady state as in the steady state, investment is zero and therefore new technologies can not be invented) to $\tilde{K}$ implies that the resulting consumption level $\tilde{C}_i$ lies above the consumption level C_i^{TP} on the saddle path. Choosing an initial consumption level close to C_i^{TP} therefore can not put the economy on an equilibrium path with self-replicating cycles as after development of a new technology the economy finds itself above the saddle path and will never again invest in R&D.

Likewise, if the economy starts with a consumption level that is very low (close to zero or zero), it will end up with a negative consumption level after one innovation. This follows from the implication of the consumption jump condition that $\tilde{C}_i < \underline{C}_i$. As there is a monotonous continuous increasing link between the initial consumption level $\tilde{C}$ and the consumption level after the first innovation, the assumption and this finding imply that there exists at least one consumption level $\tilde{C}^*$ for which the economy finds itself on an equilibrium path with positive expected long-run growth rates and short-run fluctuations.

3.4. Equilibrium properties

On an equilibrium R&D path, the initial capital stock is constant across vintages. The initial consumption level, however, rises from vintage to vintage by A. This follows directly from the definition of C in (6). The same is true for the consumption and capital stocks at the end of an R&D path. No result is available for consumption before and after the development of a new technology. It falls in the phase diagram but it increases as γ increases. Either effect may dominate.

The capital stock at the beginning of a cycle must be smaller than at the end of a cycle. Hence, the destructive part is a necessary condition for cycles to obtain. If the introduction of new technologies did not cause obsolescence of old production units, the model would be similar to Aghion and Howitt (1992) extended for an explicit study of household behaviour. Imagine that parameters are such that (8) implies $\tilde{K} = K$. Then the economy would permanently stay on the R&D line, once it is reached, and total output and consumption would grow whenever a new technology is developed.

Investment into R&D follows a strongly cyclical process. If research was not only required for the development of new technologies but also for how to best adapt technologies for a particular industry or firm, R&D expenditure would be observed all over the time and not only at specific points of the cycle.

As figure 1 shows, the economy is characterized by multiple equilibria. In addition to the equilibrium path with growth and cycles whose existence was just proven, the traditional saddle path is an equilibrium path as well. Apart from a singular case, welfare of these two paths differ. It can be conjectured that the path with growth and cycles Pareto dominates the traditional path that leads to a steady state. We therefore assume that the central planner would indeed chose the path with long-run growth and cycles.

4. Decentralization

This section presents a decentralized version of the above model. All equilibrium properties will be preserved, notably the zero-one decision for R&D which keeps the decentralized version very tractable as well. The consumption jump condition will be much simpler. The underlying economic behaviour suggests, however, that the decentralized equilibrium is not socially optimal.

Assume a large number of households whose preferences are identical to those of the central planner in (9). Given the technologies (1) - (4) described above, it can be shown along the lines of Wälde (1999b) that a household's budget constraint reads

$$\mathrm{d}a = (ra + w - i - e)\mathrm{d}t + \left(\varpi \frac{i}{J} - sa\right)\mathrm{d}q.$$

The wealth of households is denoted by a on which interests r are paid. Wage income is given by w while i and e are R&D and consumption expenditure, respectively. Aggregate investment into R&D is denoted by J; the rest of the notation is as above.

One of the common places of economics is that R&D in a decentralized economy requires some form of imperfect competition as this allows those investing in R&D to recover their investment in R&D. This budget constraint shows that R&D and perfect competition can be reconciled if the outcome of R&D is not only a better technology but also some tangible good, ϖ in this case. Factor payments to those that own the new production unit ϖ cover R&D payments made before, even under perfect competition.

The household's solution of it's maximization problem yields the same dichotomy as above, provided that households can be described by the concept of a representative agent and that the R&D sector, i.e. the arrival rate in (4), is characterized by constant returns to scale, as is common for perfectly competitive economies. Investment therefore follows (a proof and further discussion can be found in Wälde, 1999b)

$$i = \begin{Bmatrix} 0 \\ ra + w - e \end{Bmatrix} \Leftrightarrow r - \rho - \lambda\left[1 - [1 - s]\Omega\right] \begin{Bmatrix} > \\ = \end{Bmatrix} 0.$$

An aggregation of these optimality conditions leads to exactly the same equations of motion as in the central planner's economy. In the deterministic regime, aggregate consumption follows (16), the capital stock follows (17) and both variables are constant in the stochastic regime. When the stochastic regime ends, i.e. after a new technology has been found, the new aggregate capital stock is (and of course has to be) given by (3). The only but very interesting difference between the centralized and decentralized equilibrium lies in the jump of consumption after the discovery of a new technology. To see this, consider a household's Bellman equation

$$\rho V(a,\gamma) = \max_{e,i}\left\{u(c) + V_a(a,\gamma)[ra + w - i - e] + \lambda(I)\left[V(\tilde{a},\gamma+1) - V(a,\gamma)\right]\right\}$$

and compute the derivative with respect to the share θ. This gives

$$\frac{\mathrm{d}}{\mathrm{d}i}\left\{u(c) + V_a(a)[ra + w - i - e] + \lambda\left[V(\tilde{a}) - V(a)\right]\right\} = -V_a(a) + b\frac{\varpi}{p_I}V_{\tilde{a}}(\tilde{a})$$

This should be compared with the planner's derivative (13). While the planner considers only the effect of investment decisions on the arrival rate (and this *is* the only economy wide effect a planner has to take into consideration), households are concerned with the effect the introduction of a new technology has on their wealth.

The crucial difference from a modelling perspective is that the decentralized first order condition contains only derivatives of the value function while the centralized first order condition contains the value function itself. This has the convenient implication that the consumption jump condition in a decentralized economy takes a much simpler form than the consumption jump condition (21) in the centralized economy. Inserting the consumption first order condition(15) gives

$$u'(c)\frac{1}{p} = b\frac{\varpi}{p_I}u'(\tilde{c})\frac{1}{\tilde{p}}.$$

On an aggregate level, choosing as above as numeraire the price of the capital good, the prices of the consumption goods are given by $p = A^{-\gamma} p_I$ and $\tilde{p} = A^{-(\gamma+1)} p_I$ and this reads (apply $(u')^{-1}$ to the individual jump condition and sum over all individuals)

$$u'(C) = Ab\varpi u'\left(\tilde{C}\right). \tag{25}$$

Qualitatively, the aggregate economy behaves as was illustrated for the centrally planned economy in figure 1. Letting the economy start with an initial capital stock $\tilde{K}$ and assuming for a moment an initial consumption level $\tilde{C}_i$, the economy follows the R&D path and eventually hits, in finite time, the R&D line. The amount of resources invested in R&D is then the same as above which implies that the expected duration of the stochastic regime is identical. Once the technology is found, the jump in consumption now differs. Comparing the size of the jump (does consumption jump too much or too little compared to the social optimum?) between the centralized (21) and decentralized (25) economy would be very interesting but is too complex to be done in this short section. It is therefore left for future work.

The proof for the existence of a cyclical equilibrium follows closely the proof given above. Assumption 1 now requires from (25) with a CES utility function $u(C) = C^{\sigma}$ that

$$A^{-1}(Ab\varpi)^{1/(1-\sigma)} < 1$$

and an almost identical version of Theorem 2 can be proofed here.

A further interesting difference between the centralized and decentralized economy consists in the choice of the equilibrium path. While a central planner will choose the path that yields higher welfare, the decentralized economy is characterized by a coordination problem. If all individuals choose their consumption level such that the economy finds itself on the traditional path, there will be no long-run growth and no cycles. If beliefs are such that the economy settles on the R&D path, the economy will grow in the long-run, though with ever fluctuating growth rates. Future work should show how this coordination problem can be solved. If individuals find it individually rational to jump from one path to the other, a unique equilibrium path would be identified.

5. Conclusion

Creative destruction causes an economy to permanently diverge from its balanced growth path. Each new technology pushes the economy on a higher productivity level but at the same time renders some old production units obsolete. A higher productivity level as well as the destruction of old production units implies higher returns to capital accumulation. New technologies therefore lead to faster capital accumulation. When the capital stock has reached a sufficiently high level and returns have therefore fallen, capital accumulation is no longer profitable compared to the development of new technologies. Savings will shift to financing R&D. Once a new technology becomes available, the effects of creative destruction become visible again and the next cycle starts.

These results were derived in a conceptually very simple extension of the textbook Ramsey continuous-time growth model. From a modelling perspective, all that is required to jointly study long-run growth and short-run business fluctuations is to add an R&D line into an otherwise standard phase-diagram. As such an extension is simple once the basic mechanisms are understood, this approach should prove useful for understanding the link between cycles and long-run growth in other models as well.

An alternative channel that creates business cycles as well, without reliance on obsolescence of old production units, would become visible if the investment technology (2) was subject to technological change as well[19]. A new technology would then have a double creative aspect in that it increases total factor productivity not only in the consumption good but also in the investment good

[19] As noted before, this was analysed in an earlier version of this paper (Wälde, 1998 ch. 4).

sector. Growth rates would then fluctuate since the increase of total factor productivity in the investment good sector increases the steady state capital stock. In the current formulation, the steady state capital stock is independent of the current vintage as shown by (23b). As new technologies would make accumulation of capital relatively more profitable (the steady state of the phase diagram would have moved to the right), the development of a new technology would be followed by accumulation of additional capital. At a certain point, the economy would again be sufficiently close to the new steady state and research for new technologies would again be undertaken.

Appendix

available upon request

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