DURANDT, Rina; BLUM, Werner; LINDL, Alfred & BORROMEO FERRI, Rita
University of the Witwatersrand, University of Kassel, University of Regensburg, University of Kassel

Teaching mathematical modelling to German secondary school students via different designs – a CoSTAMM study

1. Teaching and learning mathematical modelling

The learning of mathematical modelling, that is learning to translate between mathematics and the real world (Pollak, 1979), is not easy. Acquiring mathematical modelling competency, that is the ability to carry out these translation processes for given real world situations and problems (Niss and Højgaard, 2019), is equally difficult. Rich opportunities for students are necessary for acquiring and practicing such competencies by solving modelling tasks by themselves. For an overview about the learning and teaching of mathematical modelling see Niss and Blum (2020), and for recent findings see Carreira and Blum (2021), Kaiser and Schukajlow (2022), Schukajlow, Kaiser and Stillman (2022), and Siller, Kaiser and Geiger (2024).

For the teaching of mathematical modelling to be effective, a permanent balance between instructional elements (well-designed modelling tasks and units, meta-cognitive support of students, adaptive teacher guidance) and constructional elements (self-directed modelling activities of students, use of suitable strategies) seems to be most promising (Schukajlow and Blum, 2023). Further empirical research is needed, in particular comparative studies (Schukajlow et al., 2018). One example of such a project is the German DISUM project (for details see Blum and Schukajlow, 2018), with a classical pre/post-test research design for a ten-lesson modelling unit at the lower secondary level. Three teaching designs (an independence-oriented design called "operative-strategic" teaching, a teacher-directed design called "directive" teaching, and a blend of both designs called "method-integrative" teaching) were implemented. The results from this project show significant progress in mathematics for all designs, but significant progress in modelling only for the two independence-oriented designs, substantially higher for the method-integrative design.

Another example, that followed broadly the design of the DISUM project, is the CoSTAMM project (Comparative Studies into Teaching Approaches for Mathematical Modelling), directed by a South African/ German research team since 2019. Two teaching designs ("method-integrative" and "teacher-directive" teaching) were implemented in a modelling unit, developed in 2019 for South African first-year tertiary students (see section 2 for more

In: P. Ebers, F. Rösken, B. Barzel, A. Büchter, F. Schacht & P. Scherer (Hrsg.),
Beiträge zum Mathematikunterricht 2024.
57. Jahrestagung der Gesellschaft für Didaktik der Mathematik. WTM.
https://doi.org/10.37626/GA9783959872782.0

details). So far, three studies at the tertiary level (all in South Africa) and one study at the secondary level (in Germany) have been carried out as part of the project, all with the same unit, the same teaching designs, and the same test instruments. Essential results from the tertiary studies show substantial competence growth for both teaching designs in mathematics and significantly higher growth in mathematical modelling for the method-integrative design (see Durandt, Blum & Lindl, 2022a, 2023 & 2024). The aim of this paper is to report on the effects of the two teaching designs in our study at the secondary level, conducted in Germany from May to August in 2022.

2. Research design of the study

Like all CoSTAMM studies, the study at the secondary level followed a pre/post-test design. The modelling unit consists of ten modelling tasks organised in five lessons (45 minutes each; for details of the lessons see Durandt, Blum and Lindl, 2022b). The study was implemented in four grade 9 German classes (Gesamtschule A-/B-Kurse). Two classes (together 36 students, one Kurs A and one Kurs B) followed a "teacher-directive" (TD) design, and two classes (together 35 students, again one Kurs A and one Kurs B) followed a "method-integrative" (MI) design. Main characteristics of the TD design is that students follow the teacher, who develops a common solution for the class oriented towards the average student. In the MI design the teacher aims at a permanent balance between students' independent work and guidance from the teacher by adaptive interventions and by encouraging individual solutions. The "Solution Plan" (originally developed for the DISUM project, see Schukajlow, Kolter and Blum, 2015) formed an integral part of the MI design and was used as a meta-cognitive tool.

3. Test instruments and methodology of the study

The pre- and post-test (45 minutes each) are aligned with each other and with the modelling unit. Both tests consist of three sections: Section A on modelling (2 tasks), Section B on mathematics contained in some modelling tasks (3 tasks), and Section C with multiple choice modelling tasks (5 tasks taken from Haines, Crouch and Davis, 2001). Both tests were administered in two versions with parallel items, following a rotation design, and were distributed randomly and equally to groups (for further details see Durandt et al., 2022a). Analyses were conducted using the statistical software R and the effects of the teaching intervention were examined by linear mixed regression models (for advantages regarding their statistical power for small samples see Lindl, Krauss, Schilcher & Hilbert, 2020).

4. Results of the study

As can be seen in Table 1, the average achieved scores increase between the

pre-test and the post-test in all test sections, except for the TD group in section B. The MI group performed slightly better than the TD group in section A and significantly better than the TD group in Section B ($p < .01$), mostly due to an unexpected decrease in TD's performance there. In section C no significant progress can be seen, but comparatively more positive results for the MI group. Overall, we can report substantial progress through the modelling unit, and significantly bigger progress for the MI group ($p < .01$).

Test (section)	Number of items	Internal consistency *Omega (ω)*	Teacher directive (*N* = 36) *M (SD)*	Method integrative (*N* = 35) *M (SD)*
Pre-test (total)	12	.62	8.38 (2.91)	8.43 (3.51)
A: Modelling tasks	2	.35	1.35 (1.25)	2.17 (1.36)
B: Mathematical tasks	6	.60	4.12 (1.20)	3.23 (1.63)
C: Multiple-choice tasks	4	.39	2.91 (1.76)	3.03 (1.64)
Post-test (total)	12	.74	8.96 (3.58)	11.43 (4.30)
A: Modelling tasks	2	.54	2.32 (1.56)	3.21 (1.60)
B: Mathematical tasks	6	.70	3.21 (1.83)	3.64 (1.87)
C: Multiple-choice tasks	4	.49	3.41 (1.64)	4.57 (1.99)

Table 1: Descriptive comparison between the TD and MI groups

5. Discussion and perspectives

The CoSTAMM project is a continuation of the DISUM project with studies at both the tertiary and secondary level. Results from the 2022 secondary school study in Germany show interesting similarities with previous studies, but also some unexpected peculiarities. Globally speaking, the teaching unit, originally developed for first-year tertiary students, worked also at the secondary level and the two designs were again effective for fostering mathematical and modelling competency, with advantages for the MI design. Considering certain limitations (e.g., sample size), the results still show that implementing aspects of quality teaching such as cognitive and meta-cognitive activation of students is promising. Similar studies with larger samples will be implemented in the future, also with investigating teacher competencies.

Literature

Blum, W., & Schukajlow, S. (2018). Selbständiges Lernen mit Modellierungsaufgaben – Untersuchung von Lernumgebungen zum Modellieren im Projekt DISUM. In S. Schukajlow & W. Blum (Eds.), *Evaluierte Lernumgebungen zum Modellieren* (pp. 51–72). Wiesbaden: Springer Spektrum

Carreira, S., & Blum, W. (Eds, 2021a/b). Mathematical modelling in the teaching and learning of mathematics: Part 1/ Part 2. *Quadrante, 30*(1/2)

Durandt, R., Blum, W., & Lindl, A. (2022a). Fostering mathematical modelling competency of first year South African engineering students: What Influence does the teaching design have? *Educational Studies in Mathematics, 109*, 361-381

Durandt, D., Blum, W., & Lindl, A. (2022b). A mathematical modelling unit for first-year engineering students. *Modelling in Science Education and Learning, 15*(1), 77-92

Durandt, R., Blum, W., & Lindl, A. (2023). The effects of different teaching approaches on engineering students' modelling competency. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel & M. Tabach (Eds.), *Proceedings of the 46th Conference of the International Group of the Psychology of Mathematics Education, Volume 2* (pp. 251-258). Haifa: PME

Durandt, R., Blum, W., & Lindl, A. (2024). The effects of an independence-oriented teaching design on the development of tertiary students' modelling competency. In H.-S. Siller, G. Kaiser & V. Geiger (Eds.), *Researching mathematical modelling education in disruptive times*. Cham: Springer

Haines, C., Crouch, R., & Davies, J. (2001). Understanding students' modelling skills. In J. Matos, W. Blum, K. Houston & S. Carreira (Eds.), *Modelling and Mathematics Education, ICTMA 9: Applications in Science and Technology* (pp. 366–380). Chichester: Horwood

Kaiser, G., & Schukajlow, S. (Eds, 2022). Innovations in measuring and fostering modelling competencies. *Educational Studies in Mathematics, 109*(2).

Lindl, A., Krauss, S., Schilcher, A., & Hilbert, S. (2020). Statistical methods in transdisciplinary educational research. *Frontiers in Education, 5*, 97.

Niss, M., & Blum, W. (2020). *The Learning and Teaching of Mathematical Modelling*. Abingdon & New York: Routledge.

Niss, M., & Højgaard, T. (2019). Mathematical competencies revisited. *Educational Studies in Mathematics, 102*, 9-28.

Pollak, H. (1979). The interaction between mathematics and other school subjects. In UNESCO (Ed.), *New Trends in Mathematics Teaching IV* (pp. 232–248). Paris: UNESCO.

Schukajlow, S., & Blum, W. (2023). Methods for teaching modelling problems. In: G. Greefrath, S. Carreira & G.A. Stillman (Eds), *Advancing and Consolidating Mathematical Modelling – Research from ICME-14*. Cham: Springer, 327-339.

Schukajlow, S., Kaiser, G., & Stillman, G. (2018). Empirical research on teaching and learning of mathematical modelling: A survey on the current state-of-the-art. *ZDM: The International Journal on Mathematics Education, 50*, 5-18.

Schukajlow, S., Kaiser, G., & Stillman, G. (Eds, 2022). Modeling from a cognitive perspective: theoretical considerations and empirical contributions. *Mathematical Thinking and Learning, 24*(2).

Schukajlow, S., Kolter, J., & Blum, W. (2015). Scaffolding mathematical modelling with a solution plan. *ZDM: The International Journal on Mathematics Education, 47*, 1241-1254.

Siller, H.-S., Kaiser, G., & Geiger, V. (Eds, 2024). *Researching mathematical modelling education in disruptive times*. Cham: Springer.