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Maxwell's equations with mixed impedance boundary conditions

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Abstract: We study the time-harmonic Maxwell equations on bounded Lipschitz domains with an impedance boundary condition. The impedance coefficient can be matrix valued such that, in particular, a polarization dependent impedance is modeled. We derive a Fredholm alternative for this system. As a consequence, we obtain the existence of weak solutions for arbitrary sources when the frequency is not a resonance frequency. Our analysis covers the case of singular impedance coefficients.

Keywords: Maxwell's equations, Impedance boundary condition, Polarization

MSC: 35Q61, 78A25, 35A01

1. Introduction

We study the time-harmonic Maxwell equations in a bounded Lipschitz domain. Our interest is to investigate an impedance boundary condition, a condition that can be compared with a Robin boundary condition in a scalar problem. The impedance coefficient Λ can be matrix valued and can, therefore, model a polarization dependent impedance. Furthermore, the matrix coefficient may be singular in the sense that it is non-trivial, but it vanishes on a non-trivial subspace of the tangent space. Our result is a Fredholm alternative for this Maxwell system.

Let us describe the system in mathematical terms. Given is a bounded Lipschitz domain $\Omega \subset \mathbb{R}^3$, two coefficient functions $\varepsilon \in L^\infty(\Omega, \mathbb{C}^{3\times 3})$ and $\mu \in L^\infty(\Omega, \mathbb{C}^{3\times 3})$, a frequency $\omega > 0$ and right-hand sides $f_h, f_e \colon \Omega \to \mathbb{C}^3$ of class L^2. We seek for functions $E, H \colon \Omega \to \mathbb{C}^3$ that satisfy, in Ω,

$$\operatorname{curl} E = i\omega\mu H + f_h\,, \tag{1.1a}$$

$$\operatorname{curl} H = -i\omega\varepsilon E + f_e\,. \tag{1.1b}$$

The system is complemented with the tangential boundary condition

$$E \times \nu = \Lambda((H \times \nu) \times \nu) \qquad \text{on } \Gamma := \partial\Omega\,, \tag{1.1c}$$

where ν is the exterior normal vector on $\partial\Omega$ and Λ is a matrix valued impedance coefficient. The normal vector is a map $\nu \colon \partial\Omega = \Gamma \ni x \mapsto \nu(x) \in \mathbb{R}^3$, defined

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for almost every x (almost every x in the sense of the two-dimensional measure on Γ). Similarly, Λ is a map $\Lambda\colon \Gamma \ni x \mapsto \Lambda(x) \in \mathbb{C}^{3\times 3}$. The map $H(x) \mapsto -(H(x)\times\nu(x))\times\nu(x)$ is the projection onto the tangential space $T_x\Gamma$.

We note that the above setting covers perfect conductor boundary conditions on some part of the boundary (setting $\Lambda = 0$ on this part) and impedance boundary conditions in the remaining part of the boundary. In view of such applications, it is important that we do not impose a continuity property on the coefficient Λ. The following situation is also covered: Along the boundary (or a certain part of the boundary), there is a perfect reflection condition for some polarization direction and an impedance condition for the orthogonal polarization direction. This is modeled with a singular map $\Lambda \neq 0$.

The solution space and the weak solution concept are defined below in (1.7) and (1.8). The weak form encodes the boundary condition with matrix valued functions Σ and Θ instead of Λ. We will discuss that the two formulations are equivalent, see Section 2 and (2.3). Our main theorem is formulated with assumptions on Σ and Θ, but we provide also a formulation of the assumptions in terms of Λ, see Lemma 1.2.

Assumption 1.1 (Assumptions on the coefficients)**.** *The bulk coefficients are maps $\varepsilon, \mu \in L^\infty(\Omega, \mathbb{C}^{3\times 3})$. They are coercive in the sense that, for some constant $c_0 > 0$, for almost every $x \in \Omega$, there holds*

$$\bar{\zeta}\cdot\varepsilon(x)\zeta \geq c_0\|\zeta\|^2 \quad \text{and} \quad \bar{\zeta}\cdot\mu(x)\zeta \geq c_0\|\zeta\|^2 \qquad \text{for all } \zeta \in \mathbb{C}^3\,. \tag{1.2}$$

The boundary coefficients are given by maps $\Theta, \Sigma \in L^\infty(\Gamma, \mathbb{C}^{3\times 3})$. Their sum is coercive: There exists a constant $c_0 > 0$ such that, for almost every $x \in \Gamma$,

$$\bar{\zeta}\cdot(\Sigma(x)+\Theta(x))\zeta \geq c_0\|\zeta\|^2 \qquad \text{for all } \zeta \in \mathbb{C}^3\,. \tag{1.3}$$

With the coercivity requirement in (1.2) and (1.3) we demand, in particular, that the left-hand side is real for all arguments x and ζ. Every real valued, symmetric and coercive matrix is also coercive in the above sense. With $\mu \in L^\infty$ coercive in the above sense, also the inverse matrix μ^{-1} is coercive in the above sense. We choose $c_0 > 0$ such that ε, ε^{-1}, μ, μ^{-1} are coercive with this constant. Regarding (1.3), we mention that it would be sufficient to consider only $\zeta \in T_x\Gamma$, where we understand $T_x\Gamma$ as complex vector space.

We note that not only real matrices are coercive in the above sense. A two-dimensional example is given by the matrix $\begin{pmatrix} 2 & -i \\ i & 2 \end{pmatrix} \in \mathbb{C}^{2\times 2}$.

We should clarify how Σ and Θ can be chosen for a given map Λ. This helps to derive conditions on Λ such that Σ and Θ satisfy the above conditions.

Lemma 1.2 (Assumptions in terms of Λ)**.** *Let a boundary condition be given by a map $\Lambda \in L^\infty(\Gamma, \mathbb{C}^{3\times 3})$. For almost every $x \in \Gamma$, with the kernel $Z = \ker(\Lambda(x)) \subset \mathbb{C}^3$ and its orthogonal complement $Z^\perp \subset \mathbb{C}^3$, we assume: (i) $\nu(x) \in Z$ such that $Z^\perp \subset T_x\Gamma$, (ii) Λ maps into the orthogonal complement of its kernel, $\mathrm{R}(\Lambda(x)) \subset Z^\perp$, (iii) $\Lambda(x)$ is coercive on $Z^\perp$: For a constant $c_0 > 0$ holds, for almost every $x \in \Gamma$ and for the space Z corresponding to x:*

$$\bar{\zeta}\cdot\Lambda(x)\zeta \geq c_0\|\zeta\|^2 \qquad \text{for all } \zeta \in Z^\perp\,. \tag{1.4}$$

In this situation, we set $\Theta(x) := \Pi_Z$*, the orthogonal projection onto* Z*. We define* $\Sigma(x) \in L^\infty(\Omega, \mathbb{C}^{3\times3})$ *by demanding that it is the inverse of* $\Lambda(x)|_{Z^\perp} : Z^\perp \to Z^\perp$ *on* $Z^\perp$ *and that* $\Sigma(x)|_Z = 0$*. Then,* Σ *and* Θ *have the properties of Assumption 1.1. The weak solution concept with* Σ *and* Θ *encodes the strong formulation of* (1.1c).

The proof of Lemma 1.2 is given in Section 2.

Remark 1.3 (Special case $\Lambda = 0$)**.** *We emphasize that our setting allows to choose* $\Lambda \equiv 0$*. This choice models a perfectly conducting boundary. For* $\Lambda \equiv 0$*, we can set* $\Theta = \mathrm{id}$*, the space* $H_\Theta(\mathrm{curl}, \Omega, \Gamma)$ *coincides with the classical space* $H_0(\mathrm{curl}, \Omega)$ *and all boundary integrals in the proofs are vanishing. Conceptually, our approach for general* Λ *is not more involved than classical existence proofs for* $\Lambda \equiv 0$*.*

1.1. **Function spaces and weak formulation.** A fundamental function space in the analysis of Maxwell's equations is

$$H(\mathrm{curl}, \Omega) := \left\{ u \in L^2(\Omega, \mathbb{C}^3) \;\middle|\; \exists f \in L^2(\Omega, \mathbb{C}^3) : \int_\Omega f \cdot \phi = \int_\Omega u \cdot \mathrm{curl}\, \phi \; \forall \phi \in C_c^\infty(\Omega, \mathbb{C}^3) \right\} . \tag{1.5}$$

The function f is the distributional curl of u and we therefore write $\mathrm{curl}\, u = f$. The space $H(\mathrm{curl}, \Omega)$ is a Hilbert space with $\|u\|^2_{H(\mathrm{curl},\Omega)} := \int_\Omega \{|u|^2 + |\,\mathrm{curl}\, u|^2\}$ and the scalar product $\langle u, \varphi \rangle := \langle u, \varphi \rangle_{L^2(\Omega)} + \langle \mathrm{curl}\, u, \mathrm{curl}\, \varphi \rangle_{L^2(\Omega)}$.

Functions in $H(\mathrm{curl}, \Omega)$ have a tangential trace in the distributional sense, but we do not need the theory of tangential traces here. We construct a space of functions with a tangential trace in $L^2(\Gamma)$ as follows:

$$H(\mathrm{curl}, \Omega, \Gamma) := \left\{ u \in H(\mathrm{curl}, \Omega) \;\middle|\; \exists g \in L^2(\Gamma, \mathbb{C}^3) : \int_\Omega \{\mathrm{curl}\, u \cdot \phi - u \cdot \mathrm{curl}\, \phi\} = \int_\Gamma g \cdot \phi \quad \forall \phi \in H^1(\Omega) \right\} . \tag{1.6}$$

The function g in (1.6) is the tangential trace, we write $\nu \times u|_\Gamma := g$. We remark that a tangential trace function g satisfies always $g(x) \cdot \nu(x) = 0$ for a.e. $x \in \Gamma$; this can be seen by inserting test-functions ϕ that point in normal direction along Γ. The space is a Hilbert space with the norm $\|u\|^2_{H(\mathrm{curl},\Omega,\Gamma)} := \int_\Omega \{|u|^2 + |\,\mathrm{curl}\, u|^2\} + \int_\Gamma |\nu \times u|_\Gamma|^2$.

For our application, we define the subspace of functions u with $\Theta(\nu \times u|_\Gamma) = 0$:

$$H_\Theta(\mathrm{curl}, \Omega, \Gamma) := \{ u \in H(\mathrm{curl}, \Omega, \Gamma) \mid \Theta(\nu \times u|_\Gamma) = 0 \} . \tag{1.7}$$

Weak form of the Maxwell system. On Ω with boundary $\Gamma = \partial\Omega$, we study the following problem: Find $E \in H_\Theta(\mathrm{curl}, \Omega, \Gamma)$ such that

$$\begin{aligned}\int_\Omega \left\{\mu^{-1}\,\mathrm{curl}\,E\cdot\mathrm{curl}\,\phi - \omega^2\varepsilon E\cdot\phi\right\} - i\omega\int_\Gamma \Sigma(\nu\times E|_\Gamma)\cdot\nu\times\phi|_\Gamma \\ = \int_\Omega \left\{i\omega f_e\cdot\phi + \mu^{-1} f_h\cdot\mathrm{curl}\,\phi\right\} \quad \forall\,\phi\in H_\Theta(\mathrm{curl},\Omega,\Gamma)\,,\end{aligned} \tag{1.8}$$

For a solution $E \in H_\Theta(\mathrm{curl}, \Omega, \Gamma)$ of (1.8) we use $H := (i\omega\mu)^{-1}(\mathrm{curl}\,E - f_h)$ as the corresponding magnetic field.

Our main result is the following:

Theorem 1.4 (Fredholm alternative)**.** *Let $\Omega \subset \mathbb{R}^3$ be a bounded Lipschitz domain, let $\omega > 0$ be a frequency and let ε, μ and Σ, Θ be coefficients that satisfy Assumption 1.1. Then, the problem (1.8) satisfies a Fredholm alternative: Either (i) for $f_h, f_e = 0$ system (1.8) has a non-trivial solution, or (ii) for every $f_h, f_e \in L^2(\Omega, \mathbb{C}^3)$, system (1.8) has a weak solution $E \in H_\Theta(\mathrm{curl}, \Omega, \Gamma)$.*

The reader might prefer the following formulation, which is an immediate consequence of Theorem 1.4.

Corollar 1.5 (Existence and uniqueness)**.** *Let $\Omega \subset \mathbb{R}^3$ be a bounded Lipschitz domain and let ε, μ and Σ, Θ satisfy Assumption 1.1. Let $\omega > 0$ be a frequency such that system (1.8) with $f_h = 0$ and $f_e = 0$ has only the trivial solution. Then, for every $f_h, f_e \in L^2(\Omega, \mathbb{C}^3)$, system (1.8) has a unique weak solution $E \in H_\Theta(\mathrm{curl}, \Omega, \Gamma)$.*

The corollary is a consequence of Theorem 1.4, but it can also be obtained with a limiting absorption principle. We provide such a proof in Section 6.

1.2. Overview of the available literature.

We consider (1.1) on a bounded domain, this setting is often denoted as the cavity problem. When the domain of interest is unbounded, one has to impose radiation conditions at infinity to have a well-posed problem. For recent well-posedness result in the unbounded domain of a waveguide, we refer to [22, 23] and references therein. Radiation conditions for the Helmholtz equation in exterior domains have a long history, variable coefficients have been treated in [17].

For $\Lambda = 0$ the boundary condition (1.1c) simplifies to $\nu \times E|_\Gamma = 0$ and models a perfectly conducting material in some exterior medium Ω' with $\partial\Omega \subset \partial\Omega'$. If the exterior medium Ω' is dissipative, as for instance if it is a good but not perfect conductor, one often uses an impedance boundary condition with $\Lambda \neq 0$. A possibility is to define Λ as the multiplication with a positive number, this is the usual choice for time-harmonic fields. The impedance boundary condition can also be used to approximate the Silver–Müller radiation condition. We refer to [4, Chapter 1.6.1] for a more detailed discussion on the application of the boundary conditions.

In the case $\Lambda = 0$, the variational approach to the cavity problem leads to a non-coercive sesquilinear form and an existence result can only be formulated as a Fredholm alternative. The Fredholm alternative can be derived by a well-established approach (see for instance [4, 21, 24]), we also follow this approach in Section 4. It consists in the following three steps: (i) A Helmholtz decomposition of the data yields a control of the divergence of the unknown (ii) derivation of a compact embedding of the solution space into $L^2(\Omega, \mathbb{C}^3)$ (iii) reformulating the weak form of the problem in terms of operators, the compact embedding provides the Fredholm property for one of the operators. The derivation of the Fredholm alternative for $\Lambda \neq 0$ requires only an appropriate formulation, an adjustment of the function spaces and the sesquilinear form and a refinement of the compactness result of *(ii)*. For Λ equal to a positive constant, proofs can be found in [24] and [4].

An alternative approach for the derivation of a weaker existence result in the sense of Corollary 1.5 is the Eidus principle of limiting absorption [11] (cf. [33, Chapter 8]), which we also employ in Section 6. The limiting absorption principle replaces Step (iii) of the first approach but does still rely on the Helmholtz decomposition and the compactness result (Steps (i) and (ii)). We emphasize that the limiting absorption principle is more than just a method of proof: It provides an additional information, namely the convergence of solutions for a vanishing damping parameter.

The Helmholtz decomposition can be derived by elementary methods, we present this in Section 3. The compact embedding of the solution space is more technical and has a longer history: Elementary calculations provide $H(\mathrm{curl}, \Omega) \cap H(\mathrm{div}, \Omega) \subset H^1_{\mathrm{loc}}(\Omega, \mathbb{C}^3)$ and, thus, by Rellich's compact embedding $H(\mathrm{curl}, \Omega) \cap H(\mathrm{div}, \Omega) \xhookrightarrow{\text{cpt.}} L^2_{\mathrm{loc}}(\Omega, \mathbb{C}^3)$. However, the compact embedding holds only locally and $H(\mathrm{curl}, \Omega) \cap H(\mathrm{div}, \Omega)$ cannot be compactly embedded in $L^2(\Omega, \mathbb{C}^3)$, even for smooth domains [2, 26]. For $C^{1,1}$ domains, the Gaffney–Friedrichs inequality gives a continuous embedding of the subspace of functions with vanishing tangential boundary values, $H_0(\mathrm{curl}, \Omega) \cap H(\mathrm{div}, \Omega)$ or vanishing normal boundary values, $H(\mathrm{curl}, \Omega) \cap H_0(\mathrm{div}, \Omega)$, in $H^1(\Omega, \mathbb{C}^3)$. Then, Rellich's compactness theorem provides the desired compactness of these spaces in $L^2(\Omega, \mathbb{C}^3)$. The Gaffney–Friedrichs inequality was shown for smooth domains in [13, 14, 15, 25]. For convex domains, Gaffney–Friedrichs inequality was shown for vanishing tangential boundary values in [27] and for vanishing normal boundary values in [30]. For $C^{1,1}$-domains, the Gaffney–Friedrichs inequality was derived in [10, 12, 16] for vanishing tangential components and in [2, 7] for vanishing normal components. For Gaffney–Friedrichs inequality with more general boundary conditions see [8]. However, this inequality does not hold for arbitrary Lipschitz domains (see [2]) and a different approach is required for the derivation of Maxwell's compactness theorem.

For a class of piecewise smooth domains the compact embedding of $H_0(\mathrm{curl}, \Omega) \cap H(\mathrm{div}, \Omega)$ and $H(\mathrm{curl}, \Omega) \cap H_0(\mathrm{div}, \Omega)$ in $L^2(\Omega)$ was shown in [35] and for general Lipschitz domains in [29, 34]. This approach relies essentially on the construction of suitable scalar and vector potentials. The compactness result was improved to a more quantitative estimate, namely the continuous embedding in $H^{1/2}(\Omega, \mathbb{C}^3)$ using additionally regularity results for the Dirichlet- and Neumann-problem, see also [24]. The regularity results are based on the non-tangential maximal functions [18, 19] and enable also inhomogeneous $L^2(\Gamma)$-regular tangential or normal boundary values. The case of inhomogeneous boundary value becomes highly relevant in the analysis for Maxwell's equations with an impedance boundary condition. More elementary regularity arguments provide an embedding in $H^{1/2-\delta}(\Omega, \mathbb{C}^3)$ for $\delta > 0$, see [6]; this is sufficient for the desired compactness.

The compactness result is extended to mixed boundary values, where at some part of the boundary the tangential trace vanishes while on the remaining part the normal trace vanishes [5, 20]; an extension to inhomogeneous $L^2(\Gamma)$-regular mixed boundary data is presented in [28]. Related discussions on vector potentials with mixed boundary conditions are presented in [3]

On polarization dependent boundary conditions: Variational formulations of the time-harmonic Maxwell equations have mainly addressed polarization independent boundary conditions. In [32] the homogenization of a thin layer of perfect conductors is considered, it can lead to a polarization dependent interface condition, which is strongly related to a polarization dependent boundary conditions. An existence result for this kind of interface condition is presented in [9]. The novelty of the present work is that it combines two qualitatively different boundary conditions on the same part of the boundary, namely the reflection boundary condition is some direction and an impedance boundary condition in another direction.

1.3. **Organization of this text.** In Section 2, we discuss the weak solution concept for (1.1). In particular, we discuss the different formulations of the boundary condition and the equivalence of these formulations under reasonable assumptions. Section 3 is devoted to Helmholtz decompositions which allows us to simplify the problem: It is sufficient to consider solutions and test-functions in a space of divergence-free functions. The proof of Theorem 1.4 is given in Section 4. We make use of a well-known compactness result for functions with bounded divergence and curl; in order to have this exposition self-contained, we include the proof of the compactness statement in Section 5. The same compactness statement is also used in the limiting absorption principle that is presented in Section 6. It provides another proof of Corollary 1.5.

2. Discussion of weak formulation and boundary conditions

In order to motivate the weak solution concept of this article, let us consider a weak solution (E, H). A weak solution is given by $E \in H_\Theta(\operatorname{curl}, \Omega, \Gamma)$ that satisfies (1.8), the magnetic field is set to $H = (i\omega\mu)^{-1} (\operatorname{curl} E - f_h)$.

The strong equation (1.1a) is satisfied by the definition of H. We use this relation for H to substitute $\operatorname{curl} E$ in (1.8) and find

$$\int_\Omega \left\{ i\omega H \cdot \operatorname{curl} \phi - \omega^2 \varepsilon E \cdot \phi \right\} - i\omega \int_\Gamma \Sigma(\nu \times E|_\Gamma) \cdot \nu \times \phi|_\Gamma = \int_\Omega i\omega f_e \cdot \phi \tag{2.1}$$

for all $\phi \in H_\Theta(\operatorname{curl}, \Omega, \Gamma)$. For test-functions $\phi \in C_c^\infty(\Omega, \mathbb{C}^3)$ in (2.1), the boundary integral vanishes and we obtain (1.1b).

It remains to check the boundary conditions. For this step, we assume that the solution has additional regularity such that boundary traces are well defined. We consider an arbitrary $\phi \in H_\Theta(\mathrm{curl}, \Omega, \Gamma)$ in (2.1), integrate the first term by parts and insert (1.1b). This provides

$$\int_\Gamma i\omega H \cdot \nu \times \phi|_\Gamma - i\omega \int_\Gamma \Sigma(\nu \times E|_\Gamma) \cdot \nu \times \phi|_\Gamma = 0\,. \tag{2.2}$$

Therefore, the weak solution satisfies pointwise the boundary conditions

$$\Theta(\nu \times E) = 0\,, \tag{2.3a}$$

$$[H - \Sigma(\nu \times E)] \cdot (\nu \times \phi) = 0 \quad \forall \phi \text{ with } \Theta(\nu \times \phi) = 0\,, \tag{2.3b}$$

where the first condition follows from the fact that we seek for a solution $E \in H_\Theta(\mathrm{curl}, \Omega, \Gamma)$. The second condition, interpreted pointwise, implies that $H - \Sigma(\nu \times E)$ is orthogonal to the kernel of Θ in the tangential space.

Let us consider the situation of Lemma 1.2 and a point $x \in \Gamma$. We claim that (E, H) satisfies the boundary condition (1.1c) if and only if it satisfies (2.3). To show one implication, let (E, H) satisfy (2.3). We use the orthogonal projection Π_x to the tangent space $T_x\Gamma$. By (2.3b), the expression $\Pi_x[H - \Sigma(\nu \times E)](x)$ is orthogonal to the kernel of $\Theta(x)$. This means that it is an element of Z, the kernel of $\Lambda(x)$. Supressing the point x, we obtain $\Lambda\Pi H = \Lambda\Pi\Sigma(\nu \times E)$. The left-hand side is identical to $-\Lambda((H \times \nu) \times \nu)$. On the right-hand side, Π acts trivially on $\Sigma(\nu \times E)$, since the latter is a tangential vector. Since Λ is the inverse of Σ on the kernel of Θ and $\nu \times E$ is in this kernel, the right-hand side is $\nu \times E$. We have therefore concluded (1.1c); we note that we use in (1.1c) the more standard notation where the normal vector is always written behind the fields.

Vice versa, let (E, H) satisfy (1.1c). We apply Θ on (1.1c) and obtain from $\Theta \circ \Lambda = 0$ that $\Theta(E \times \nu) = 0$, which shows (2.3a). To deduce (2.3b), we apply Σ on (1.1c) and multiply the equation by $\phi \times \nu$ for ϕ satisfying $\Theta(\phi \times \nu) = 0$, which gives $\phi \times \nu \cdot \Sigma(E \times \nu) = \phi \times \nu \cdot \Sigma\Lambda((H \times \nu) \times \nu)$. We note that $(\Sigma\Lambda)^\top$ is the identity on $Z^\perp$ and, thus $(\Sigma\Lambda)^\top(\phi \times \nu) = \phi \times \nu$. We obtain $\phi \times \nu \cdot \Sigma(E \times \nu) = \phi \times \nu \cdot ((H \times \nu) \times \nu) = -\phi \times \nu \cdot H$, which is (2.3b).

With these calculations, we have verified the last statement of Lemma 1.2.

Vice versa, we can motivate the weak formulation (1.8) starting from the strong Maxwell system (1.1a)–(1.1b) with the boundary condition (2.3): When we multiply (1.1a) with μ^{-1} and use, for arbitrary $\phi \in H_\Theta(\mathrm{curl}, \Omega, \Gamma)$, the test-function $\mathrm{curl}\, \phi$, we obtain

$$\int_\Omega \mu^{-1} \mathrm{curl}\, E \cdot \mathrm{curl}\, \phi = \int_\Omega \left\{ i\omega H \cdot \mathrm{curl}\, \phi + \mu^{-1} f_h \cdot \mathrm{curl}\, \phi \right\}\,. \tag{2.4}$$

We study the first term on the right-hand side. Integrating by parts and using the identities (1.1b) and (2.3b) we find

$$\begin{aligned}\int_\Omega i\omega H\cdot\operatorname{curl}\phi &= i\omega\int_\Omega \operatorname{curl} H\cdot\phi + i\omega\int_\Gamma H\cdot\nu\times\phi\\ &= \omega^2\int_\Omega \varepsilon E\cdot\phi + i\omega\int_\Omega f_e\cdot\phi + i\omega\int_\Gamma \Sigma(\nu\times E|_\Gamma)\cdot\nu\times\phi|_\Gamma\,.\end{aligned}\tag{2.5}$$

With this replacement in (2.4), we find the weak form (1.8). The identity (2.3a) restricts the solution space to $H_\Theta(\operatorname{curl},\Omega,\Gamma)$.

2.1. **Coercivity.** Under certain assumptions on Λ, Lemma 1.2 provides a suitable choice of Σ and Θ. We have seen above that, with this choice, the weak solution concept encodes (1.1c). It remains to show that, for Λ as in Lemma 1.2, the maps Σ and Θ are well-defined and satisfy Assumption 1.1.

Proof of Lemma 1.2. In the situation of the lemma with $Z=\ker(\Lambda(x))\subset\mathbb{C}^3$, the map $\Lambda(x)|_{Z^\perp}\colon Z^\perp\to Z^\perp$ is invertible with lower bound (independent of x). Since, additionally, Λ satisfies a uniform upper bound by the property $\Lambda\in L^\infty$, the inverse $(\Lambda(x)|_{Z^\perp})^{-1}\colon Z^\perp\to Z^\perp$ is uniformly bounded and coercive. In particular, Σ is well defined and coercive on $Z^\perp$,

$$\bar\zeta\cdot\Sigma(x)(\zeta)\geq c_\Sigma\|\zeta\|^2\qquad\text{for all }\zeta\in Z^\perp\tag{2.6}$$

for some constant $c_\Sigma>0$ that is independent of x. Decomposing an arbitrary vector $\xi\in\mathbb{C}^3$ as $\xi=z+\zeta$ with $z\in Z$ and $\zeta\in Z^\perp$, we can calculate, suppressing the point x,

$$\begin{aligned}(\Theta+\Sigma)(\xi)\cdot\bar\xi &= (\Theta+\Sigma)(z+\zeta)\cdot\bar\xi = (z+\Sigma(\zeta))\cdot\bar\xi\\ &= \|z\|^2+\Sigma(\zeta)\cdot\bar\zeta \geq \|z\|^2+c_\Sigma\|\zeta\|^2\,.\end{aligned}$$

This provides the coercivity (1.3) of $\Theta+\Sigma$. □

The coercivity assumption (1.3) is designed in such a way that the boundary integral in the weak form (1.8) together with the function space $H_\Theta(\operatorname{curl},\Omega,\Gamma)$ provides full control over the $\|\nu\times E|_\Gamma\|_{L^2(\Gamma)}$-norm on the boundary $\Gamma=\partial\Omega$. This can be seen with the following calculation for arbitrary $E\in H_\Theta(\operatorname{curl},\Omega,\Gamma)$:

$$c_0\|\nu\times E|_\Gamma\|^2_{L^2(\Gamma)}\leq\int_\Gamma(\Theta+\Sigma)(\nu\times E|_\Gamma)\cdot\nu\times\bar E|_\Gamma=\int_\Gamma\Sigma(\nu\times E|_\Gamma)\cdot\nu\times\bar E|_\Gamma\,,\tag{2.7}$$

where the equality uses $\Theta(\nu\times E|_\Gamma)=0$.

2.2. **Formulation in H instead of E.** System (1.1) permits also a weak formulation in terms of the magnetic field. Up to boundary regularity, it reads: Find $H\in H(\operatorname{curl},\Omega)$ such that

$$\begin{aligned}&\int_\Omega\{\varepsilon^{-1}\operatorname{curl}H\cdot\operatorname{curl}\psi-\omega^2\mu H\cdot\psi\}-i\omega\int_\Gamma\Lambda((H\times\nu)|_\Gamma\times\nu)\cdot((\psi\times\nu)|_\Gamma\times\nu)\\ &\qquad=\int_\Omega\{-i\omega f_h\cdot\psi+\varepsilon^{-1}f_e\cdot\operatorname{curl}\psi\}\ \text{ for all }\psi\,,\end{aligned}\tag{2.8}$$

where test-functions ψ are chosen in the same space as H. The underlying space is $H(\text{curl}, \Omega)$, the additional requirement is that the projection of $((\psi \times \nu)|_\Gamma \times \nu)$ onto $\text{R}(\Lambda)$ is in the space $L^2(\Gamma)$.

For a solution $H \in H(\text{curl}, \Omega, \Gamma)$ of (2.8), we set $E := (i\omega\varepsilon)^{-1}(-\operatorname{curl} H + f_e)$. The formulation (2.8) has the advantage that it uses Λ. It does not require to formulate the problem with the two auxiliary matrix functions Σ and Θ.

Unfortunately, the weak formulation (2.8) has a major disadvantage concerning compactness. We recall that the literature provides compact embeddings of $H(\text{curl}, \Omega) \cap H(\text{div}, \Omega)$ in $L^2(\Omega)$ when an $L^2(\Gamma)$-control of tangential or normal components of the functions are available. For a singular map $\Lambda \neq 0$, we have only control of one tangential component through the boundary term since Λ is not coercive on the entire tangent space. At the same time, we do not have full control of the normal component: Inserting $\psi = \nabla\varphi$ in (2.8), assuming, for simplicity, $f_h = 0$, we find

$$-\int_\Omega \omega^2 \mu H \cdot \nabla\varphi = i\omega \int_\Gamma \Lambda((H \times \nu)|_\Gamma \times \nu) \cdot ((\nabla\varphi \times \nu)|_\Gamma \times \nu)\,. \tag{2.9}$$

The boundary term does not disappear for arbitrary $\varphi \in H^1(\Omega)$ but only for $\varphi \in H^1(\Omega)$ such that $((\nabla\varphi \times \nu)|_\Gamma \times \nu) \in \text{R}(\Lambda)^\perp = \ker(\Lambda)$. Thus, we obtain only a restricted information on the normal component of H. Consequently, one has a mixed control over the tangential and normal components. To the best knowledge of the authors, no compactness result of the literature is applicable in this setting.

3. Helmholtz decomposition

Our aim is to prove Theorem 1.4 with the help of the compactness result of Lemma 5.1. This compactness result requires a control of the divergence of E. We obtain this control in two steps: With a Helmholtz decomposition of $L^2(\Omega, \mathbb{C}^3)$, we restrict the analysis to divergence-free right-hand sides, see Lemma 3.2. With a Helmholtz decomposition of $H_\Theta(\text{curl}, \Omega, \Gamma)$, we restrict the set of solutions to ε-divergence-free functions.

We start by introducing the space G of gradients. The set G can be understood as a subspace of $L^2(\Omega, \mathbb{C}^3)$, but also as a subspace of $H(\text{curl}, \Omega)$ since the rotation of gradients vanishes. It is even a subspace of $H_\Theta(\text{curl}, \Omega, \Gamma)$ since the tangential derivatives of an H^1_0-function vanish. For a given coefficient ε, we define D_ε and Y_ε as spaces of ε-divergence-free functions. In the subsequent definition, differential operators are understood in the sense of distributions.

$$G := \left\{ u \in L^2(\Omega) \mid \exists \psi \in H^1_0(\Omega) \colon u = \nabla\psi \right\}, \tag{3.1}$$

$$D_\varepsilon := \left\{ u \in L^2(\Omega, \mathbb{C}^3) \mid \text{div}(\varepsilon u) = 0 \right\}, \tag{3.2}$$

$$Y_\varepsilon := H_\Theta(\operatorname{curl} \Omega, \Gamma) \cap D_\varepsilon\,. \tag{3.3}$$

The choice is such that D_ε is the orthogonal complement of G in the space $L^2(\Omega, \mathbb{C}^3)$ with the weighted scalar product $\langle u, v\rangle_\varepsilon = \int_\Omega \varepsilon\, u \cdot \bar{v}$. Furthermore, because of $\text{curl}(\nabla\psi) = 0$ and $\nu \times \nabla\psi|_\Gamma = 0$, the subspaces Y_ε and G are also orthogonal with respect to the scalar product $\langle u, v\rangle_X := \int_\Omega \{\varepsilon\, u \cdot \bar{v} + \mu^{-1} \operatorname{curl} u \cdot$

$\operatorname{curl} \bar v\} + \int_\Gamma \{\nu \times u \cdot \nu \times \bar v\}$. By construction, Y_ε is the $\langle .,.\rangle_X$-orthogonal complement of G in $H_\Theta(\operatorname{curl},\Omega,\Gamma)$. The definitions therefore imply directly the following two Helmholtz decompositions.

Lemma 3.1 (Helmholtz decomposition)**.** *The space $L^2(\Omega,\mathbb{C}^3)$ has the orthogonal decomposition $L^2(\Omega,\mathbb{C}^3) = D_\varepsilon \oplus_\varepsilon G$. In particular, an arbitrary element $u \in L^2(\Omega)$ can be written uniquely as $u = v + \nabla\psi$ with $v \in D_\varepsilon$ and $\psi \in H^1_0(\Omega)$.*

The space $X := H_\Theta(\operatorname{curl},\Omega,\Gamma)$ has the orthogonal decomposition $X = Y_\varepsilon \oplus_X G$. In particular, an arbitrary element $u \in X$ can be written uniquely as $u = v + \nabla\psi$ with $v \in Y_\varepsilon$ and $\psi \in H^1_0(\Omega)$.

We use the Helmholtz decomposition of $L^2(\Omega,\mathbb{C}^3)$ in order to replace the data f_e with divergence-free data. This allows us to control the divergence of the unknown E.

Lemma 3.2 (Reduction to divergence-free data)**.** *Let $f_e, f_h \in L^2(\Omega,\mathbb{C}^3)$ be given and let ε satisfy Assumption 1.1. Using Lemma 3.1, we find $h \in D_\varepsilon$ and $\chi \in H^1_0(\Omega)$ such that $(i\omega\varepsilon)^{-1} f_e = h + \nabla\chi$. We use $\tilde f_e := (i\omega\varepsilon)h$ with $\operatorname{div}(\tilde f_e) = 0$. Then, $E \in H_\Theta(\operatorname{curl},\Omega,\Gamma)$ is a solution for (1.8) if and only if $\tilde E = E - \nabla\chi \in H_\Theta(\operatorname{curl},\Omega,\Gamma)$ satisfies*

$$\begin{aligned}\int_\Omega \Big\{\mu^{-1} \operatorname{curl} \tilde E \cdot \operatorname{curl} \phi - \omega^2 \varepsilon \tilde E \cdot \phi\Big\} - i\omega \int_\Gamma \Sigma(\nu \times \tilde E|_\Gamma) \cdot \nu \times \phi|_\Gamma \\ = \int_\Omega \Big\{ i\omega \tilde f_e \cdot \phi + \mu^{-1} f_h \cdot \operatorname{curl} \phi \Big\} \quad \forall \phi \in H_\Theta(\operatorname{curl},\Omega,\Gamma)\,.\end{aligned} \tag{3.4}$$

Moreover, solutions $\tilde E$ are ε-divergence-free in the sense that $\operatorname{div}(\varepsilon \tilde E) = 0$.

Proof. When E is a solution, elementary substitutions show that $\tilde E$ is a solution of (3.4). Indeed, the curl of a gradient vanishes, the second terms on both sides are modified in the same way, the boundary integral is unchanged since χ vanishes on Γ and, thus, tangential components of $\nabla\chi$ vanish along the boundary.

The opposite implication is obtained with the same calculation.

In order to obtain $\operatorname{div}(\varepsilon \tilde E) = 0$, it is sufficient to use a gradient $\phi = \nabla\varphi$ for $\varphi \in H^1_0(\Omega)$ in (3.4). □

Lemma 3.2 allows us to consider only right-hand sides f_e with $\operatorname{div}(f_e) = 0$ in the following. By doing so, we can also restrict the solution space (and, accordingly, the space of test-functions) in (1.8) to ε-divergence-free functions. For these functions, we write E and ϕ, dropping the tilde.

Lemma 3.3 (Equivalent formulation in Y_ε)**.** *Let $\Omega \subset \mathbb{R}^3$ be a bounded Lipschitz domain, $\omega > 0$, ε, μ, Σ, Θ as in Assumption 1.1. Let $f_e, f_h \in L^2(\Omega,\mathbb{C}^3)$ be right-hand sides with $\operatorname{div}(f_e) = 0$. In this situation, the weak problem formulated in (1.8) is equivalent to the following problem: Find $E \in Y_\varepsilon$ such that the equation in (1.8) holds for all test-functions $\phi \in Y_\varepsilon$.*

Proof. Let E be a solution of the new problem, i.e., $E \in Y_\varepsilon$ and (1.8) holds for test functions $\phi \in Y_\varepsilon$. Given an arbitrary test-function $\phi \in X = H_\Theta(\operatorname{curl},\Omega,\Gamma)$, we write $\phi = \varphi + \nabla\psi$ with $\varphi \in Y_\varepsilon$ and $\psi \in H_0^1(\Omega)$ as outlined in Lemma 3.1. Using that (1.8) is linear in ϕ, we can treat the contributions separately. Inserting $\nabla\psi$ in (1.8), all terms vanish. Inserting φ, the equality holds since E is a solution of the Y_ε-problem. This shows that (1.8) is satisfied for arbitrary test-functions $\phi \in H_\Theta(\mathrm{curl}, \Omega, \Gamma)$.

When E is a solution of the original weak form, then (1.8) holds, in particular, for test-functions $\phi \in Y_\varepsilon$. The fact that E is indeed an element of Y_ε was observed in Lemma 3.2. □

4. Verification of the Fredholm alternative

With the above considerations, the Maxwell system has a symmetric weak formulation in the space Y_ε (we recall that now only divergence-free right-hand sides f_e are considered). Using the compactness result of Lemma 5.1 below, some functional analysis provides the Fredholm alternative of Theorem 1.5.

Proof of Theorem 1.5. We recall that $Y_\varepsilon \subset H_\Theta(\mathrm{curl}\,\Omega, \Gamma)$ denotes the subspace of ε-divergence-free functions. We define two sesquilinear forms $a, b \colon Y_\varepsilon \times Y_\varepsilon \to \mathbb{C}$ and an anti-linear right-hand side $f \colon Y_\varepsilon \to \mathbb{C}$ by setting, for every $u, \phi \in Y_\varepsilon$,

$$a(u,\phi) := \int_\Omega \left\{ u \cdot \bar\phi + \mu^{-1} \,\mathrm{curl}\, u \cdot \mathrm{curl}\,\bar\phi \right\} - i\omega \int_\Gamma \left\{ \Sigma(\nu \times u|_\Gamma) \cdot \nu \times \bar\phi|_\Gamma \right\} ,$$

$$b(u,\phi) := \int_\Omega \left\{ u \cdot \bar\phi + \omega^2 \varepsilon\, u \cdot \bar\phi \right\} , \quad f(\phi) := \int_\Omega \left\{ i\omega f_e \cdot \bar\phi + \mu^{-1} f_h \cdot \mathrm{curl}\,\bar\phi \right\} .$$

By Lemma 3.3, the weak formulation of the Maxwell problem is equivalent to: Find $E \in Y_\varepsilon$ such that

$$a(E,\phi) - b(E,\phi) = f(\phi) \qquad \forall\, \phi \in Y_\varepsilon\,. \tag{4.1}$$

The sesquilinear form a defines a map $A \colon Y_\varepsilon \to Y_\varepsilon'$ from Y_ε into the (anti-)dual space Y_ε' with the definition $Au := a(u, \cdot)$. By definition of the scalar product in $Y_\varepsilon \subset X = H_\Theta(\mathrm{curl}, \Omega, \Gamma)$ and the estimate (2.7), the form a is coercive on Y_ε. The Lemma of Lax–Milgram implies that $A \colon Y_\varepsilon \to Y_\varepsilon'$ is invertible.

We now exploit that the embedding $\iota \colon Y_\varepsilon \to L^2(\Omega, \mathbb{C}^3)$ is compact, see Lemma 5.1. The multiplication map $B \colon u \mapsto (1 + \omega^2\varepsilon)\, u$ corresponding to b is linear and bounded as a map $B \colon L^2(\Omega, \mathbb{C}^3) \to L^2(\Omega, \mathbb{C}^3)$. We denote the concatenation with an embedding into Y_ε' with the same letter and write $B \colon L^2(\Omega, \mathbb{C}^3) \to Y_\varepsilon'$.

The field $E \in Y_\varepsilon$ solves (4.1) if and only if

$$A\,E - (B \circ \iota)\,E = f \qquad \text{in } Y_\varepsilon'\,.$$

Applying A^{-1}, we find the equivalent relation

$$(\mathrm{id} - A^{-1} \circ B \circ \iota) E = A^{-1} f \qquad \text{in } Y_\varepsilon\,.$$

The operator $A^{-1} \circ B \circ \iota$ is compact, since ι is compact and the other operators are continuous. Standard functional analysis results imply that the operator $F := \mathrm{id} - A^{-1} \circ B \circ \iota$ is a Fredholm operator of index zero, see, e.g., [1, Theorem 11.8]. By their definition, such operators satisfy the Fredholm alternative: The kernel is trivial if and only if the operator is surjective. □

5. Compactness property

In the previous section, we derived the Fredholm alternative from a compact embedding: We need that the solution space is compactly embedded in $L^2(\Omega, \mathbb{C}^3)$. In our application, we considered functions with a vanishing ε-divergence, but this is actually not needed for the compactness. The compactness only needs that the ε-divergence is controlled in L^2 (just as we demand the control of the curl). We use the following spaces:

$$
\begin{aligned}
H(\operatorname{div}\varepsilon, \Omega) := \Big\{ u \in L^2(\Omega, \mathbb{C}^3) \ \Big| \ & \exists f \in L^2(\Omega, \mathbb{C})\colon \\
& \int_\Omega f\phi = \int_\Omega \varepsilon u \cdot \nabla\phi \ \forall \phi \in C_c^\infty(\Omega) \Big\}\,,
\end{aligned} \tag{5.1}
$$

and

$$
\begin{aligned}
H(\operatorname{div}\varepsilon, \Omega, \Gamma) := \Big\{ u \in H(\operatorname{div}\varepsilon, \Omega) \ \Big| \ & \exists g \in L^2(\Gamma, \mathbb{C})\colon \\
& \int_\Omega \{\operatorname{div}(\varepsilon u)\phi + \varepsilon u \cdot \nabla\phi\} = \int_\Gamma g\phi \quad \forall \phi \in H^1(\Omega) \Big\}\,.
\end{aligned} \tag{5.2}
$$

Norms and scalar products are defined in the natural way, using L^2-norms of all the given quantities. For functions u in the second space, the normal trace of u is defined as $\nu \cdot (\varepsilon u)|_\Gamma := g$.

The following two compactness results are well known. They both require an $L^2(\Omega)$-control over curl and divergence and an $L^2(\Gamma)$-control of either the normal or tangential boundary values. In our main result, we have used the compact embedding (5.3). The space Y_ε is a subspace of the left-hand side; for functions $u \in Y_\varepsilon$ holds $\operatorname{div}(\varepsilon u) = 0$, which implies, in particular, that $\operatorname{div}(\varepsilon u)$ is an $L^2(\Omega)$-function. Additionally, for functions $u \in Y_\varepsilon$, there holds $\Theta(\nu \times u|_\Gamma) = 0$, but we exploit only the $L^2(\Gamma)$-control of $\nu \times u|_\Gamma$.

In the result (5.4), the normal trace of functions is controlled in $L^2(\Gamma)$.

Lemma 5.1 (Mawell compactness theorem)**.** *Let $\Omega \subset \mathbb{R}^3$ be a Lipschitz domain and ε, μ as in Assumption 1.1. Then, the following embeddings are compact:*

$$
H(\operatorname{curl}, \Omega, \Gamma) \cap H(\operatorname{div}\varepsilon, \Omega) \xrightarrow{cpt.} L^2(\Omega, \mathbb{C}^3)\,, \tag{5.3}
$$

$$
H(\operatorname{curl}, \Omega) \cap H(\operatorname{div}\mu, \Omega, \Gamma) \xrightarrow{cpt.} L^2(\Omega, \mathbb{C}^3)\,. \tag{5.4}
$$

The second embedding is used when the Maxwell system is formulated in H; for this reason, the coefficient μ (instead of ε) is typically used in this formulation. In order to highlight the controlled quantities, one may write $H(\operatorname{curl}, \operatorname{div}\varepsilon, \Omega, \nu \times |_\Gamma)$ for the space on the left-hand side of (5.3) and $H(\operatorname{curl}, \operatorname{div}\mu, \Omega, \nu \cdot |_\Gamma)$ for the space on the left-hand side of (5.4).

We note that the compactness has some relations with the div–curl lemma, sometimes called compensated compactness. Knowledge on the curl and on the divergence of a function somehow controls all derivatives. Loosely speaking, this property is already suggested by the relation $\Delta = -\operatorname{curl}\operatorname{curl} + \nabla\operatorname{div}$. We will actually exploit this relation in our proof.

We sketch a proof for the first embedding, which is used in this article. The proof for the second embedding is identical in most steps, the only difference is that a Neumann problem instead of a Dirichlet problem must be analyzed in Step 3. We note that the proof for a vanishing normal component with the same methods can be found in [31].

Proof. In Steps 1–3 of this proof, we assume that Ω is simply connected and that the coefficient is $\varepsilon \equiv 1$. In Step 4 we treat general coefficients ε and in Step 5 we show how the assumption of simple connectedness is removed.

In order to show compactness, we consider a bounded sequence u_j in the space $H(\operatorname{curl}, \Omega, \Gamma) \cap H(\operatorname{div} \varepsilon, \Omega)$. We recall that this implies that u_j, $f_j := \operatorname{curl} u_j$, $g_j := \operatorname{div}(\varepsilon u_j)$ and the tangential boundary values of u_j are bounded sequences in L^2-spaces.

The set Ω is bounded, we can therefore choose a radius $R > 0$ such that Ω is compactly contained in the open ball with this radius, $\overline{\Omega} \subset B_R := B_R(0) \subset \mathbb{R}^3$.

Step 1: Extension of f_j with a gradient. We consider the solutions ψ_j of the following Laplace problem: $\Delta\psi_j = 0$ in $B_R \setminus \overline{\Omega}$ with $\psi_j = 0$ on ∂B_R and $\partial_\nu \psi_j = \nu \cdot f_j$ on $\partial\Omega$. We define an extension of f_j by setting

$$\tilde{f}_j := \begin{cases} f_j & \text{in } \Omega\,, \\ \nabla\psi_j & \text{in } B_R \setminus \Omega\,. \end{cases}$$

The construction ensures that this extended function has a vanishing divergence. We note that the function f_j is a curl and has therefore a vanishing divergence; this fact actually allows us to formulate the boundary condition in the Neumann problem even though f_j is only an L^2-function. A detailed verification of this fact can be found in [31].

Step 2: Vector potential $\tilde{v}_j$ for $\tilde{f}_j$. As a function with vanishing divergence on the simply connected set B_R, the function $\tilde{f}_j$ possesses a divergence-free vector potential $\tilde{v}_j \in L^2(B_R, \mathbb{C}^3)$ with $\operatorname{curl} \tilde{v}_j = \tilde{f}_j$ in B_R. One way to show this standard result is the following: In a first step, a function $\tilde{w}_j$ with $\operatorname{curl} \tilde{w}_j = \tilde{f}_j$ is constructed with the help of path integrals of $\tilde{f}_j$. When an average over a set of starting points of the path integrals is taken, the function $\tilde{w}_j$ is of class $L^2(B_R, \mathbb{C}^3)$ with norm bounded by the norm of $\tilde{f}_j$. In a second step, we subtract the gradient $\nabla\tilde{\xi}_j$ of a function $\tilde{\xi}_j$ with $\Delta\tilde{\xi}_j = \operatorname{div}(\tilde{w}_j)$. We obtain that $\tilde{v}_j = \tilde{w}_j - \nabla\tilde{\xi}_j$ has the same rotation as $\tilde{w}_j$ (namely $\tilde{f}_j$) and satisfies $\operatorname{div} \tilde{v}_j = 0$.

Let us discuss the regularity of $\tilde{v}_j$. Because of $\Delta = -\operatorname{curl}\operatorname{curl} + \nabla \operatorname{div}$, the function $\tilde{v}_j \in L^2(B_R, \mathbb{C}^3)$ satisfies, in the sense of distributions, $\Delta\tilde{v}_j = -\operatorname{curl} \tilde{f}_j \in H^{-1}(B_R)$. The sequence $\tilde{v}_j$ is therefore locally (in B_R) of class H^1 (Caccioppoli's inequality). This implies that $v_j := \tilde{v}_j|_\Omega$ is bounded in $H^1(\Omega)$. In particular, it possesses a subsequence that converges strongly in $L^2(\Omega, \mathbb{C}^3)$. We consider only this subsequence in the following.

Step 3: Scalar potential for $u_j - v_j$. We consider now only functions on the simply connected set Ω. By construction, the difference $u_j - v_j$ has a vanishing curl, it can therefore be written, for some scalar function $\phi_j \in H^1(\Omega)$, as a gradient: $u_j - v_j = \nabla\phi_j$.

Let us analyze the regularity properties of ϕ_j along Γ. We have bounds for $\nu \times u_j|_\Gamma \in L^2(\partial\Omega)$ by the assumptions on u_j and we have bounds for $\nu \times v_j|_\Gamma \in L^2(\partial\Omega)$ because of the boundedness of $v_j \in H^1(\Omega)$ and classical trace theorems. We therefore have $\nu \times \nabla\phi_j|_\Gamma = \nu \times u_j|_\Gamma - \nu \times v_j|_\Gamma \in L^2(\partial\Omega)$ bounded and conclude the boundedness of $\phi_j \in H^1(\partial\Omega)$.

It remains to analyze the Dirichlet problem

$$\Delta\phi = g \quad \text{in } \Omega\,, \qquad \phi = h \quad \text{on } \Gamma\,,$$

for given g and h. By classical elliptic theory, the solution operator to this problem is a bounded linear operator $\mathcal{T}\colon H^{-1}(\Omega) \times H^{1/2}(\Gamma) \ni (g,h) \mapsto \phi \in H^1(\Omega)$. We have the compact embeddings $L^2(\Omega) \xhookrightarrow{\text{cpt.}} H^{-1}(\Omega)$ and $H^1(\Gamma) \xhookrightarrow{\text{cpt.}} H^{1/2}(\Gamma)$. They imply that the operator $\mathcal{T}$ on these better function spaces is compact,

$$\tilde{\mathcal{T}}\colon L^2(\Omega) \times H^1(\Gamma) \ni (g,h) \mapsto \phi \in H^1(\Omega) \quad \text{compact}\,.$$

As a solution of the Dirichlet problem, $\phi_j = \tilde{\mathcal{T}}(g_j, \phi_j|_\Gamma)$, the sequence ϕ_j has a subsequence that converges in $H^1(\Omega)$. Along this subsequence, both $\nabla\phi_j$ and v_j are converging strongly in $L^2(\Omega, \mathbb{C}^3)$, hence also $u_j = v_j + \nabla\phi_j$ is converging in this space. This concludes the compactness proof for simply connected domains and $\varepsilon \equiv 1$.

Step 4: General coefficients ε. We now consider a general coercive coefficient $\varepsilon \in L^\infty(\Omega, \mathbb{C}^{3\times 3})$. In order to show compactness, we consider once more a bounded sequence $u_j \in H(\operatorname{curl}, \Omega, \Gamma) \cap H(\operatorname{div}\varepsilon, \Omega)$.

We claim that it is sufficient to consider the case $\operatorname{div}(\varepsilon u_j) = 0$. Indeed, in the general case, we consider the sequence $\hat{u}_j := u_j - \nabla\xi_j$ where $\xi_j \in H_0^1(\Omega)$ solves $\operatorname{div}(\varepsilon\nabla\xi_j) = \operatorname{div}(\varepsilon u_j)$. For a subsequence, $\operatorname{div}(\varepsilon u_j)$ is converging weakly in $L^2(\Omega)$ and hence strongly in $H^{-1}(\Omega)$, which implies that ξ_j is strongly converging in $H^1(\Omega)$ and $\nabla\xi_j$ strongly in $L^2(\Omega)$. It is therefore sufficient to prove the strong convergence of $\hat{u}_j$ in $L^2(\Omega)$. This justifies the claim.

We use the Helmholtz decomposition $L^2(\Omega, \mathbb{C}^3) = D_\varepsilon \oplus_\varepsilon G$ with D_ε and G defined in (3.2) and (3.1). We discussed this decomposition in Lemma 3.1, where we also noted that $\tilde{D}_\varepsilon := H(\operatorname{curl}, \Omega, \Gamma) \cap D_\varepsilon$ allows us to introduce the decomposition $H(\operatorname{curl}, \Omega, \Gamma) = \tilde{D}_\varepsilon \oplus G$. The corresponding projections are bounded.

We decompose u_j with respect to the decomposition that corresponds to $\varepsilon = \mathrm{id}$, that is, using $H(\operatorname{curl}, \Omega, \Gamma) = \tilde{D}_{\mathrm{id}} \oplus G$:

$$u_j \;=\; v_j \;+\; \nabla\psi_j \quad \text{with } v_j \in \tilde{D}_{\mathrm{id}} \text{ and } \psi_j \in H_0^1(\Omega, \mathbb{C})\,.$$

Since u_j is bounded in $H(\operatorname{curl}, \Omega, \Gamma)$ and since projections are bounded, the sequence $v_j \in \tilde{D}_{\mathrm{id}}$ is bounded in $H(\operatorname{curl}, \Omega, \Gamma)$. Since it has a vanishing divergence, Steps 1–3 yield that there exists a subsequence, again denoted by v_j, which converges in $L^2(\Omega, \mathbb{C}^3)$. With this knowledge, we now read the previous decomposition in the form

$$v_j \;=\; u_j \;-\; \nabla\psi_j\,,$$

and note that this is a decomposition of v_j in $L^2(\Omega,\mathbb{C}^3) = D_\varepsilon \oplus G$. Since v_j converges in $L^2(\Omega,\mathbb{C}^3)$ and since the projection onto D_ε is bounded in the space $L^2(\Omega,\mathbb{C}^3)$, we conclude that u_j converges in $L^2(\Omega,\mathbb{C}^3)$. This concludes the proof.

Step 5: Removing the assumption of simple connectedness. It remains to consider an arbitrary bounded Lipschitz domain Ω. Let again u_j be a bounded sequence in the spaces on the left-hand side. We choose a finite family of simply connected Lipschitz subdomains $\Omega_k \subset \Omega$, $k = 1, ..., K$, such that Ω is covered, $\Omega \subset \bigcup_{k=1}^K \Omega_k$. We choose a subordinate family of smooth cut-off functions η_k with $\bigcup_{k=1}^K \eta_k = 1$ on Ω. Steps 1–4 can be applied successively in the subdomains Ω_k to the sequences $u_j\eta_k$ to find $L^2(\Omega)$-convergent subsequences. For the corresponding subsequence, also $u_j = \sum_{k=1}^K u_j\eta_k$ is then $L^2(\Omega)$-convergent. □

6. Limiting absorption principle

In this section, we prove Corollary 1.5 with a limiting absorption principle. In the limiting absorption principle, one introduces a damping term in the equation and considers the limit of a vanishing damping (or absorption). Here, we use a small real number $\delta > 0$ and replace the pre-factor $\omega^2\varepsilon$ in equation (1.8) by $\omega^2\varepsilon + i\delta$. We will verify that this equation has a solution E_δ, we will find a limit $E = \lim_{\delta\to 0} E_\delta$, and we show that E is a solution to (1.8).

The problem with absorption is: Find $E_\delta \in H_\Theta(\mathrm{curl},\Omega,\Gamma)$ such that

$$\int_\Omega \left\{\mu^{-1}\,\mathrm{curl}\,E_\delta \cdot \mathrm{curl}\,\phi - (\omega^2\varepsilon + i\delta)E_\delta\cdot\phi\right\} - i\omega\int_\Gamma \Sigma(\nu\times E_\delta|_\Gamma)\cdot\nu\times\phi|_\Gamma$$

$$= \int_\Omega \left\{i\omega f_e\cdot\phi + \mu^{-1} f_h\cdot\mathrm{curl}\,\phi\right\} \quad \forall\,\phi\in H_\Theta(\mathrm{curl},\Omega,\Gamma)\,. \tag{6.1}$$

Proof of Corollary 1.5 with limiting absorption. Lemma 3.2 guarantees that we can assume, without loss of generality, that the data satisfy $\mathrm{div}(f_e) = 0$. Lemma 6.1 below provides a unique solution E_δ of (6.1). Lemma 6.2 below shows that the sequence E_δ is bounded in $H_\Theta(\mathrm{curl},\Omega,\Gamma)$ (in the setting of Corollary 1.5, where it is assumed that (1.8) has only the trivial solution for $f_h = f_e = 0$). Since $H_\Theta(\mathrm{curl},\Omega,\Gamma)$ is reflexive, there exists a subsequence of E_δ that converges weakly to some limit $E \in H_\Theta(\mathrm{curl},\Omega,\Gamma)$. The weak convergence allows us to take the limit $\delta\to 0$ in (6.1). We obtain that E solves (1.8). □

Lemma 6.1 (Existence of a solution for the problem with absorption)**.** *Let $\Omega\subset\mathbb{R}^3$ be a bounded Lipschitz domain and $\omega > 0$ and ε,μ and Σ,Θ satisfy Assumption 1.1. Let $f_e, f_h \in L^2(\Omega,\mathbb{C}^3)$ be right-hand sides with $\mathrm{div}(f_e) = 0$. Then, there exists $\delta_0 > 0$ such that, for every $\delta\in(0,\delta_0)$, equation (6.1) has a unique weak solution $E_\delta \in H_\Theta(\mathrm{curl},\Omega,\Gamma)$.*

Proof. We define a sesquilinear form a_δ on $H_\Theta(\mathrm{curl},\Omega,\Gamma)$ by setting, for $u,\varphi\in H_\Theta(\mathrm{curl},\Omega,\Gamma)$,

$$a_\delta(u,\varphi) := \int_\Omega \left\{\mu^{-1}\,\mathrm{curl}\,u\cdot\mathrm{curl}\,\bar\varphi - (\omega^2\varepsilon + i\delta)u\cdot\bar\varphi\right\} - i\omega\int_\Gamma \Sigma(\nu\times u|_\Gamma)\cdot\nu\times\bar\varphi|_\Gamma\,. \tag{6.2}$$

The form a_δ allows us to rewrite (6.1) as

$$a_\delta(E_\delta,\phi) = \int_\Omega \left\{ i\omega f_e\cdot\bar\phi + \mu^{-1} f_h\cdot\operatorname{curl}\bar\phi \right\} \qquad \forall\,\phi\in H_\Theta(\operatorname{curl},\Omega,\Gamma)\,. \tag{6.3}$$

We calculate with the coercivity lower bound (2.7), for arbitrary $u\in H_\Theta(\operatorname{curl},\Omega,\Gamma)$,

$$\begin{aligned} |\operatorname{Im} a_\delta(u,u)| &\geq \delta\|u\|^2_{L^2(\Omega)} + c_0\omega\|\nu\times u|_\Gamma\|^2_{L^2(\Gamma)}\,,\\ \operatorname{Re} a_\delta(u,u) &\geq c_0\|\operatorname{curl} u\|^2_{L^2(\Omega)} - \omega^2\|\varepsilon\|_{L^\infty(\Omega)}\|u\|^2_{L^2(\Omega)}\,. \end{aligned}$$

We observe the following fact in $\mathbb{R}^2\equiv\mathbb{C}$: For every vector $z=(z_1,z_2)\in\mathbb{R}^2$ and every $s\in[0,1]$, there holds $|z|\geq\max\{|z_1|,|z_2|\}\geq(1-s)|z_1|+s|z_2|$. This inequality allows us to calculate, with $s=\delta^2$,

$$\begin{aligned} |a_\delta(u,u)| &\geq (1-\delta^2)|\operatorname{Im} a_\delta(u,u)| + \delta^2\operatorname{Re} a_\delta(u,u)\\ &\geq (1-\delta^2)\left(\delta\|u\|^2_{L^2(\Omega)} + c_0\omega\,\|\nu\times u|_\Gamma\|^2_{L^2(\Gamma)}\right)\\ &\quad + \delta^2\left(c_0\|\operatorname{curl} u\|^2_{L^2(\Omega)} - \omega^2\|\varepsilon\|_{L^\infty(\Omega)}\|u\|^2_{L^2(\Omega)}\right)\,. \end{aligned}$$

Choosing $\delta_0>0$ small, we achieve $(1-\delta^2)\delta\geq 2\delta^2\omega^2\|\varepsilon\|_{L^\infty(\Omega)}$ for all $0<\delta<\delta_0$; for these values of δ, the form a_δ is coercive. Problem (6.3) for E_δ can therefore be solved with the Lemma of Lax–Milgram. □

Lemma 6.2 (Boundedness of solutions to the problem with absorption)**.** *Let the assumptions of Lemma 6.1 be satisfied. For a sequence $\delta\to 0$, let $E_\delta\in H_\Theta(\operatorname{curl},\Omega,\Gamma)$ be the corresponding sequence of solutions of (6.1). We assume that relation (1.8) with data $f_h=f_e=0$ has only the trivial solution $E=0$. Then, the sequence E_δ is bounded in $H_\Theta(\operatorname{curl},\Omega,\Gamma)$.*

Proof. Step 1: Preparation. For a contradiction argument, we assume that there exists a subsequence E_δ such that $\|E_\delta\|_{H_\theta(\operatorname{curl},\Omega,\Gamma)}\to\infty$. The subsequent calculation uses that c_0 is a coercivity constant for the matrix function μ^{-1} and the fact that $|z_1+iz_2|=\sqrt{|z_1|^2+|z_2|^2}\geq\frac{1}{2}(|z_1|+|z_2|)$ holds for real numbers z_1 and z_2. Using E_δ as a test-function in (6.1), we find

$$\begin{aligned} &\frac{1}{2}\left(c_0\|\operatorname{curl} E_\delta\|^2_{L^2(\Omega)} + \omega c_0\|\nu\times E_\delta|_\Gamma\|^2_{L^2(\Gamma)}\right)\\ &\qquad\leq \left|\int_\Omega \mu^{-1}\operatorname{curl} E_\delta\cdot\operatorname{curl}\bar E_\delta - i\omega\int_\Gamma \Sigma(\nu\times E_\delta|_\Gamma)\cdot\nu\times\bar E_\delta|_\Gamma\right|\\ &\qquad= \left|\int_\Omega(\omega^2\varepsilon+i\delta)E_\delta\cdot\bar E_\delta + \int_\Omega\left\{i\omega f_e\cdot\bar E_\delta + \mu^{-1}f_h\cdot\operatorname{curl}\bar E_\delta\right\}\right|\,. \end{aligned}$$

For an arbitrarily small number $\lambda>0$, we can continue this calculation with Young's inequality to find

$$\begin{aligned} &\frac{1}{2}\left(c_0\|\operatorname{curl} E_\delta\|^2_{L^2(\Omega)} + \omega c_0\|\nu\times E_\delta|_\Gamma\|^2_{L^2(\Gamma)}\right)\\ &\qquad\leq C\|E_\delta\|^2_{L^2(\Omega)} + C_\lambda + \lambda\|\operatorname{curl} E_\delta\|^2_{L^2(\Omega)}\,, \end{aligned}$$

for some C depending on ω, f_e and ε, and C_λ depending on μ, f_h and λ. Choosing $\lambda = c_0/4$ and subtracting the term $\lambda\| \operatorname{curl} E_\delta\|^2_{L^2(\Omega)}$ on both sides, we find, for some constant C, the inequality $\| \operatorname{curl} E_\delta\|^2_{L^2(\Omega)} + \|\nu \times E_\delta|_\Gamma\|^2_{L^2(\Gamma)} \le C\,(1 + \|E_\delta\|^2_{L^2(\Omega)})$.

In particular, we can conclude that our assumption $\|E_\delta\|_{H_\Theta(\operatorname{curl},\Omega,\Gamma)} \to \infty$ implies the divergence of the L^2-norm, $\|E_\delta\|_{L^2(\Omega)} \to \infty$.

Step 2: Normalization. We normalize the sequence E_δ and consider the new sequence $\tilde E_\delta := E_\delta/\|E_\delta\|_{L^2(\Omega,\mathbb{C}^3)}$. The normalized sequence satisfies $\|\tilde E_\delta\|_{L^2(\Omega,\mathbb{C}^3)} = 1$ and, by Step 1, that $\| \operatorname{curl} \tilde E_\delta\|^2_{L^2(\Omega)} + \|\nu \times E_\delta|_\Gamma\|^2_{L^2(\Gamma)}$ is bounded. Since $H_\Theta(\operatorname{curl},\Omega,\Gamma)$ is reflexive, there exists a subsequence of $\tilde E_\delta$ that converges weakly to some limit $\tilde E \in H_\Theta(\operatorname{curl},\Omega,\Gamma)$. In particular, we have: $\tilde E_\delta \rightharpoonup \tilde E$ in $L^2(\Omega)$, $\operatorname{curl} \tilde E_\delta \rightharpoonup \operatorname{curl} \tilde E$ in $L^2(\Omega)$ and $\nu \times \tilde E_\delta|_\Gamma \rightharpoonup \nu \times \tilde E|_\Gamma$ in $L^2(\Gamma)$.

The function $\tilde E_\delta$ solves (6.1) for source terms $\tilde f_e = f_e/\|E_\delta\|_{L^2(\Omega)}$ and $\tilde f_h = f_h/\|E_\delta\|_{L^2(\Omega)}$. The weak convergence of $\tilde E_\delta$ allows us to perform the limit $\delta \to 0$ in this relation. We obtain that the limit $\tilde E$ solves (1.8) for $f_h = f_e = 0$. Our assumption was that there is no non-trivial solution to the homogeneous problem; this implies $\tilde E = 0$.

Step 3: Strong convergence. In this step we show the strong convergence of $\tilde E_\delta$ in $L^2(\Omega,\mathbb{C}^3)$ along a subsequence. Once this is obtained, we have the desired contradiction: $\|\tilde E_\delta\|_{L^2(\Omega)} = 1$ is in conflict with the strong convergence $\tilde E_\delta \to \tilde E = 0$.

In order to show the strong convergence, we decompose $\tilde E_\delta$ according to the Helmholtz decomposition of Lemma 3.1: $\tilde E_\delta = \tilde E^Y_\delta + \nabla\psi_\delta$ for $\tilde E^Y_\delta \in Y_\varepsilon$ and $\psi_\delta \in H^1_0(\Omega)$. Boundedness of $\tilde E_\delta$ in $H_\Theta(\operatorname{curl},\Omega,\Gamma)$ and boundedness of the projection implies that $\tilde E^Y_\delta$ is bounded in Y_ε. The compactness of Y_ε (shown in Lemma 5.1) allows us to pass to a subsequence such that $\tilde E^Y_\delta$ converges strongly in $L^2(\Omega)$. We therefore have the desired result once we have the strong convergence of $\nabla\psi_\delta$ in $L^2(\Omega)$.

We use the test-function $\phi = \nabla\bar\psi_\delta$ in (6.1). Since the curl of a gradient vanishes and since we assumed that f_e is orthogonal to gradients of $H^1_0(\Omega)$-function, we obtain

$$\int_\Omega (\varepsilon + i\omega^{-2}\delta)\tilde E_\delta \cdot \nabla\bar\psi_\delta = 0\,.$$

Inserting the decomposition $\tilde E_\delta = \tilde E^Y_\delta + \nabla\psi_\delta$ in the integral containing ε, we find

$$i\int_\Omega \omega^{-2}\delta\tilde E_\delta \cdot \nabla\bar\psi_\delta + \int_\Omega \varepsilon\nabla\psi_\delta \cdot \nabla\bar\psi_\delta = -\int_\Omega \varepsilon\tilde E^Y_\delta \cdot \nabla\bar\psi_\delta = 0\,, \tag{6.4}$$

where we used the property $\tilde E^Y_\delta \in Y_\varepsilon$ in the last equality. The coercivity of ε allows us to deduce from (6.4)

$$c_0\|\nabla\psi_\delta\|^2_{L^2(\Omega)} \le \int_\Omega \varepsilon\nabla\psi_\delta \cdot \nabla\bar\psi_\delta = -i\int_\Omega \omega^{-2}\delta\tilde E_\delta \cdot \nabla\bar\psi_\delta\,.$$

With the Cauchy–Schwarz inequality and the normalization $\|\tilde{E}_\delta\|_{L^2(\Omega)} = 1$ we obtain $\|\nabla\psi_\delta\|_{L^2(\Omega)} \leq C\delta \to 0$ as $\delta \to 0$. This is the desired strong convergence of $\nabla\psi_\delta$.

The strong convergence of $\tilde{E}^Y_\delta$ together with the strong convergence of $\nabla\psi_\delta$ implies the strong convergence of $\tilde{E}_\delta = \tilde{E}^Y_\delta + \nabla\psi_\delta$ in $L^2(\Omega)$. This provides the desired contradiction and concludes the proof. □

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