Measurement of the Double Differential Dijet Rate in Deep Inelastic Scattering at HERA and Comparison to NLO QCD Calculations

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Contents

Preface

The understanding of the structure of matter is the subject of the research in the field of elementary particle physics. Scattering experiments play a dominant role in the quest for fundamental particles and forces. Presumably the most famous scattering experiment was performed by Ernest Rutherford and his collaborators in the year 1911 [RUT11]. They shot α particles on a gold foil and observed that some of these particles were deflected by an unexpected large angle from the initial direction of flight. Rutherford drew the conclusion that the atoms, up to that time regarded as fundamental particles, have a substructure consisting of an atomic shell and a nucleus.

In the following decades the picture about the structure of matter has been improved by a large variety of experimental and theoretical efforts.

The year 1992 marks the advent of the HERA accelerator at DESY (Deutsches Elektronen Synchrotron) in Hamburg. With this machine, high energy collisions between electrons and one constituent of the nucleus, the proton, are performed. With high resolution power the structure of the proton itself and the interactions between its constituents, the quarks, are studied. Some aspects of the strong force responsible for these interactions are investigated in this thesis. The effects of this strong force manifest themselves in collimated bundles of particles called jets. The analysis presented in this thesis is concentrated on a class of events in which jets are identified in the H1 detector.

This thesis is organized as follows. The first chapter will give an introduction to the basic theory of deep inelastic scattering and the special features of jet events. In the second chapter the experimental apparatus used for the measurement is described. In the third chapter the selection criteria of the events analyzed in this thesis are described. The quality of the selected data is discussed in the fourth chapter by means of numerous control distributions which are the basis of the correction of the measured data for comparison to theoretical predictions. The comparison of the data to these predictions will be performed in the fifth chapter.

Chapter 1

Theoretical Background

1.1 Introduction to Deep-Inelastic Scattering

Do the hadrons have a structure and, if yes, how does this structure look in detail ? Experiments where a highly energetic lepton is scattered off a proton target are an appropriate tool to answer these questions, because the exchanged boson has a virtuality which allows the structure of the proton, to be resolved. In deep inelastic scattering (DIS) the virtuality Q^2 of the exchanged boson is much larger than the mass of the proton. Figure 1.1 sketches the interaction between an incoming positron on a proton via the exchange of a photon with virtuality Q^2. In addition ep

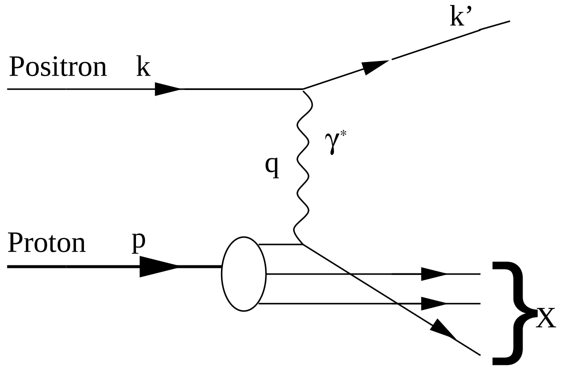

Figure 1.1: Schematic representation of deep inelastic electron proton scattering

interactions may also take place by the exchange of Z^0 or $W^{\pm}$ bosons. However, in the framework of this analysis these processes can be neglected since they are suppressed in the kinematic region studied and will not be considered further. In the case of neutral boson exchange the products of the process shown in Figure 1.1 are the scattered positron and a hadronic final state X. The kinematic quantities describing this process can be determined in terms of the four momenta of the incident and scattered lepton, $k, k^{'}$, and the momentum p of the incident proton:

- The square of the lepton-proton center-of-mass energy is given by:[1]

$$s = (k+p)^2 \approx 2kp. \tag{1.1}$$

- The virtuality of the exchanged boson is given by:

$$Q^2 = -q^2 = -(k-k^{'})^2 \ . \tag{1.2}$$

- The Bjorken scaling variable is given by

$$x_B = \frac{Q^2}{2pq}; \, (0 < x_B < 1), \tag{1.3}$$

[1]Throughout this thesis the rest masses of the particles are neglected.

and can be interpreted, in the naive parton model [BJO69], as the momentum fraction carried by the struck parton with respect to the total momentum of the incoming proton

- The inelasticity variable

$$y = \frac{pq}{pk}; (0 < y < 1) \tag{1.4}$$

defines the energy transfer from the lepton to the proton in the rest frame of the proton and is related to the earlier defined variables Q^2 and x_B via $y = \frac{Q^2}{sx_B}$.

- The square of the invariant mass of the hadronic final state X is given by:

$$W^2 = (p+q)^2 = \frac{1-x_B}{x_B}Q^2. \tag{1.5}$$

Only three of the four variables are independent from each other. Usually s, x_B and Q^2 are chosen. Since s is fixed by the energies of the incoming particles, two independent variables remain to fully describe the kinematics of inclusive deep inelastic scattering.

The cross section of the $ep \to eX$ process can be expressed as

$$\sigma \propto L_{\mu\nu} W^{\mu\nu}. \tag{1.6}$$

Here $L_{\mu\nu}$ denotes the leptonic tensor describing the interaction between the lepton and the exchanged virtual boson. The hadronic tensor $W^{\mu\nu}$ corresponds to the boson-proton vertex. In Quantum Electro Dynamics (QED) the lepton-photon vertex is well defined and therefore $L_{\mu\nu}$ is completely calculable. Using Lorentz invariance, current conservation and assuming parity conservation, the hadronic tensor can be reduced to only two functions which parameterize the structure of the proton. The DIS cross section has the well known form

$$\frac{d^2\sigma}{dx_B dQ^2} = \frac{4\pi\alpha^2}{x_B Q^4}(x_B y^2 F_1(x_B, Q^2) + (1-y) F_2(x_B, Q^2)). \tag{1.7}$$

Here α denotes the fine structure constant. The functions F_1 and F_2 parameterize the inner structure of the proton. Its determination is one of the main goals of the HERA experiments.

For the analysis of the hadronic final state X, it is useful to define another set of variables which have special properties under Lorentz boosts along the z-axis where the z-axis is defined by the incoming proton direction. The pseudo-rapidity

$$\eta = -\ln\left(\tan\left(\frac{\theta}{2}\right)\right) \tag{1.8}$$

defined via the polar angle θ changes its magnitude only by an additive constant under Lorentz boosts along the z axis. Thus differences in the pseudo-rapidity are invariant under Lorentz boosts along the z-axis. Further variables are the azimuthal angle ϕ and the transverse energy $E_t = E\sin\theta$ where E and θ are the energy and polar angle of either a single object of the hadronic final state or of well defined combinations of hadronic final state objects, see e.g. Section 1.1.1 or 1.6.

1.1.1 Reconstruction of Kinematic Variables

A precise reconstruction of the kinematic variables x_B,y, Q^2 is crucial for the analysis of deep inelastic scattering events.

Due to the redundancy of the H1 detector, see Chapter 2, a precise determination can be done either from the kinematics of the scattered electron alone, from hadronic final state or using information from both, e.g. the angle of scattered positron and the angle of the hadronic final state. Three different methods have been used in this analysis; the hadron method, the electron method and the double angle method. They are described in detail in [BAS95].

With the electron method the kinematics of the scattering process are fully determined by the energy, $E_e^{'}$, and the polar angle, θ_e, of the scattered positron:

$$Q^2 = 4E_eE_e^{'}cos^2\frac{\theta_e}{2} \tag{1.9}$$

$$y_e = 1 - \frac{E_e^{'}}{E_e}sin^2\frac{\theta_e}{2} \tag{1.10}$$

$$x_B = \frac{Q^2}{y_e s}. \tag{1.11}$$

The hadron or Jaquet-Blondel method uses only hadron variables and the kinematical variables of interest can be determined by summing over the energies (E_i), transverse $(p_{x,i}, p_{y,i})$ and longitudinal momenta $(p_{z,i})$ of the hadronic final state particles. The kinematics are thus given by the following expressions:

$$y_h = \frac{\sum_i(E_i - p_{z,i})}{2E_e} \tag{1.12}$$

$$Q_h^2 = \frac{(\sum p_{x,i})^2 + (\sum p_{y,i})^2}{1 - y_h} = \frac{E_{t,h}}{1 - y_h} \tag{1.13}$$

$$x_{Bh} = \frac{Q_h^2}{y_h}. \tag{1.14}$$

Finally, the kinematic variables can be reconstructed from the angle θ_e of the scattered positron and the angle γ_h of the hadronic final state using the double angle method. Here, the angle γ_h represents, in the naive quark parton model, the direction of the struck quark and is obtained from the hadronic flow measurement according to

$$\tan\frac{\gamma_h}{2} = \frac{\sum(E_i - p_{z,i})}{E_{t,h}}. \tag{1.15}$$

Using 1.15 and the scattering angle θ_e of the scattered positron one gets

$$y_{DA} = \frac{\tan\frac{\gamma_h}{2}}{\tan\frac{\theta_e}{2} + \tan\frac{\gamma_h}{2}} \tag{1.16}$$

$$\mathrm{Q}_{DA}^2 = 4E_e^2\frac{\sin\gamma_h(1+\cos\theta_e)}{\sin\gamma_h + \sin\theta_e - \sin(\theta_e + \gamma_h)} \tag{1.17}$$

$$x_{DA} = \frac{Q_{DA}^2}{y_{DA}s}. \tag{1.18}$$

The energy of the scattered positron can be expressed in terms y_{DA} and θ_e as follows:

$$E_{DA} = \frac{E_e(1 - y_{DA})}{\sin^2\frac{\theta_e}{2}}. \tag{1.19}$$

The advantage of the double angle method is its independence (to first order) of the energy scale since the angular information the method is based upon is derived from ratios of energies leading to a partial cancellation of energy scale uncertainties. This feature is used in Sections 4.4 and 4.6 to estimate part of the systematic error of the results which will be obtained in this thesis.

The electron method will be used exclusively in this thesis for the final result. This method has been proven to provide good resolution in the kinematic region $y > 0.1$ [GLA98], where its resolution increases from 20% in the region $0.1 < y < 0.2$ to 4% at $y \approx 0.7$. Especially towards large y it is superior to the other methods [GLA98]. However, the redundancy offered by the other methods will be exploited to check the energy scale of the detectors used in the analysis, see Section 4.4.

1.2 The Naive Quark-Parton Model

The idea that the proton is built up by constituents was formulated by Gell-Mann [GEL64] as a result of the observation to account for the rich variety of different hadrons observed in high-energy-collision experiments. The introduction of proton constituents called quarks established an inner symmetry into the observed particle spectrum and allowed the explanation of the electric charge and the spin of the hadrons.

In the late 60's and early 70's it was confirmed in deep inelastic scattering experiments that the proton is made out of constituents which correspond to the postulated quarks [FRI91]. This led to the formulation of the so called (naive) quark-parton model [BJO69, FEY69] which is based on the following assumptions:

- The proton is made out of point-like constituents called quarks which share the total momentum of the proton.
- The quarks are moving free parallel to the proton.
- The quarks have a spin of 1/2.
- The quarks have a multiple of $\pm 1/3$ the elementary electric charge e. For example, the proton is built of two quarks with charge $+2/3e$ and one quark with charge -1/3e.
- Deep inelastic electron proton scattering can be regarded as elastic scattering of the electron off of a free quark of the proton. In this sense the term $\frac{4\pi\alpha^2}{x_B Q^4}$ on the right hand side of Equation 1.7 is equivalent to the well-known Rutherford scattering formula and describes the elastic scattering of two point-like electric charges.

Following these assumptions, the structure functions included in Equation 1.7 are only dependent on x_B and given by

$$F_1(x_B) = \frac{1}{2x_B} \sum e_i^2 x_B f_i(x_B) \tag{1.20}$$

$$F_2(x_B) = \sum e_i^2 x_B f_i(x_B) \tag{1.21}$$

where the parton density functions f_i are interpreted as the probability to find a quark with momentum fraction x_B inside the proton. The sum runs over all different quark flavors. The postulation that the structure functions depend only on the fractional proton momentum is called scaling and was established in deep inelastic scattering experiments at SLAC for values of $x_B \approx 0.25$ [BLO69]. The scaling behavior implies that the two structure functions F_1 and F_2 are related by the Callan-Gross relation [CAL69]

$$F_2(x_B) = 2x_B F_1(x_B). \tag{1.22}$$

The Equation 1.22 only holds, if the quarks have spin 1/2 which was experimentally verified in 1969 [ALB69].

However, experiments in the early 70's showed a deviation from the scaling behavior predicted by the naive quark-parton model. The measured structure functions showed a logarithmically dependence on the four momentum transfer Q^2 [FOX74, AND77, DRE83]. Moreover, it was observed that the quarks carry only 50% of the total momentum of the proton. These two contradictions to the assumptions of the naive quark-parton model can be explained in the framework of Quantum-Chromodynamics (QCD), a field theory of strong interactions which describes the interactions between quarks as mediated by gluons. Gluons do not couple to the photon and carry the missing momentum of the proton [GRO79].

1.3 Basics of QCD

In the framework of QCD, the interaction between the quarks are driven by an additional degree of freedom called color. Quarks may appear in three different color states usually labeled 'red', 'green' and 'blue'. The strong or color force is mediated via electrically neutral spin 1 particles called gluons. To a certain extend, these gluons are similar to the carriers of the electromagnetic force, the photons. However, there is one striking difference. In contrast to the photons, which do not carry an electromagnetic charge, the gluons carry color charge which leads to strong interactions between gluons.

Analogous to QED, QCD driven amplitudes can be expanded in a perturbation series of a coupling constant, denoted α_s, which quantifies the strength of the strong force. To calculate cross sections in QCD, contributions from real and virtual quarks and gluons have to be considered in the perturbation series. Virtual contributions are, for example, the loop diagrams shown in Figure 1.2. The Feynman rules demand the integration of each loop over the inner

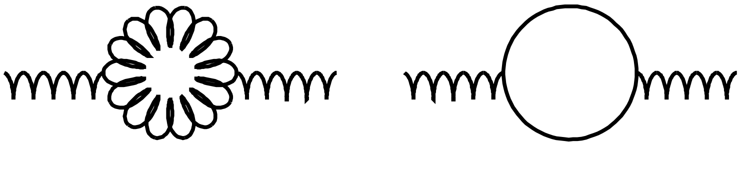

Figure 1.2: One loop diagrams which lead to the running of the coupling constant α_s.

four momentum k, which is not fixed by energy momentum conservation. These integrals are logarithmically divergent for $|k| \to \infty$. To account for this, the ultraviolet divergences are renormalized and absorbed into the coupling constant α_s. The renormalization procedure introduces an artificial mass parameter μ_r^2 on which the coupling constant depends. Any physical quantity R must not depend on this artificial parameter. This can be expressed mathematically by the following equation:

$$\mu_r^2 \frac{\partial R}{\partial \mu_r^2} + \mu_r^2 \frac{\partial \alpha_S}{\partial \mu_r^2} \frac{\partial R}{\partial \alpha_s} = 0. \tag{1.23}$$

The partial derivative $\partial\alpha_s/\partial\mu_r^2$ defines the μ_r dependence of the of $\alpha_s(\mu_r)$ in terms of the β function of QCD which can be expanded in a power series of α_s

$$\mu_r^2\frac{\partial\alpha_s}{\partial\mu_r^2} = \alpha_s\beta(\alpha_S) = -\beta_0\left(\frac{\alpha_s}{4\pi}\right)^2 - \beta_1\left(\frac{\alpha_s}{4\pi}\right)^3 + \ldots .$$

where β_0, β_1 are the first coefficients occurring in the expansion. In the one loop approximation, i.e. regarding only the term with β_0, the coupling constant α_s can be written in terms of the renormalization scale as

$$\alpha_s(\mu_r^2) = \frac{4\pi}{\beta_0 \ln(\mu_r^2/\Lambda^2)}, \beta_0 = 11 - \frac{2}{3}N_f. \tag{1.24}$$

Here N_f is the number of (active) quark flavors which have masses smaller than μ_r and Λ denotes the free parameter of QCD which is of the order of $200 - 300\,\mathrm{MeV}$ dependent on the number of active flavors and the renormalization scheme used to control the ultraviolet divergences as for example the frequently used $\overline{MS}$ scheme [BAR78].

From the formula 1.24 it can be seen that in QCD partons are asymptotically free, that is $\alpha_s \to 0$ as $\mu_r^2 \to \infty$, see Figure 1.3. This property is unique to non-Abelian gauge theories.

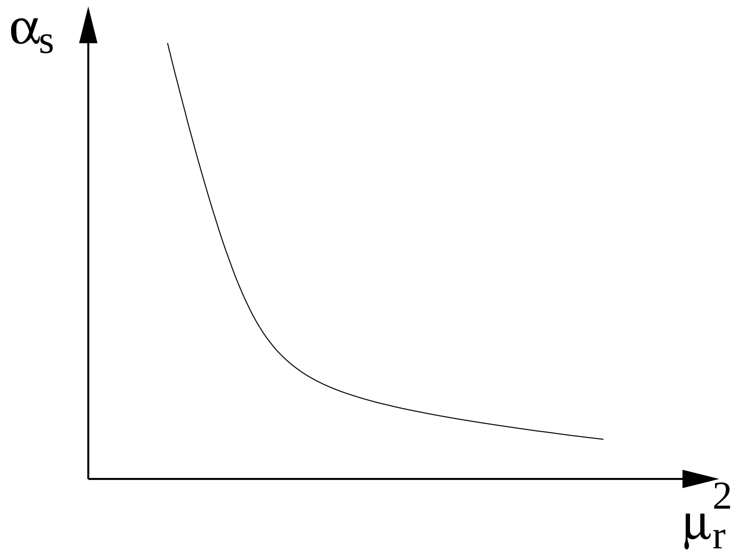

Figure 1.3: Schematic representation of the dependence of the strong coupling constant α_s on the renormalization scale μ_r^2.

Another important consequence of the non-abelian nature of QCD is that α_S increases with decreasing μ_r^2. It is believed that this increase of the strong force is one source for the (color) confinement of quarks and gluons inside hadrons, so called infrared slavery, which means that one cannot see the struck quark in an experiment but only an energetic jet of colorless hadrons emerging from it [GRO73].

The variation of α_s with μ_r^2, but not its absolute value, can be derived from QCD; the latter has to be fixed experimentally. The current world average of α_s was determined by the combination of various experimental results and is expressed at the energy scale of the rest mass of the Z^0-boson $\mu_r = M_Z$ [BET00]:

$$\alpha_s(M_z) = 0.118 \pm 0.003. \tag{1.25}$$

1.4 QCD Improved Parton Model

The observed deviations from the predictions of the naive quark parton model can be explained by the inclusion of QCD into the description of the deep inelastic scattering processes. The diagrams shown in Figure 1.4 are examples for corrections to first order in α_s to the total ep cross section. The radiation of gluons by quarks in the proton give rise to a gluon density $g(x)$ in the proton. However, the calculation of the matrix elements which correspond to the

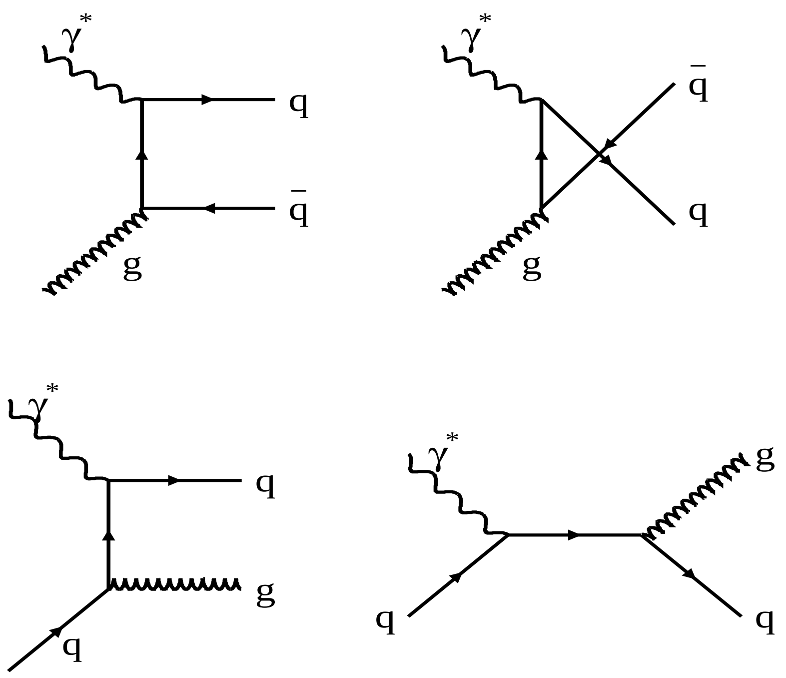

Figure 1.4: Feynman graphs of corrections to the order α_s in deep inelastic scattering. The upper graphs represent the boson-gluon fusion and the lower graphs represent the QCD Compton scattering

Feynman graphs in Figure 1.4 lead to divergences if partons are radiated either collinear to the incoming quark or gluon or with infinitely low energies. Following the factorization theorem of QCD [COL89], all collinear and infrared divergences can be absorbed into the (redefined) parton densities of the proton. The scale at which the hard scattering is separated from the soft and collinear processes happens is called the factorization scale μ_f. Therefore the parton densities become explicitly dependent on the scale μ_f, which is usually set to Q^2. This additional dependence of the parton densities is the reason for the observed scaling violations. The parton densities can not be calculated from first principles by QCD. They have to be obtained from some experimental input. Frequently used parameterizations of parton densities are those of the CTEQ collaboration [CTE97, CTE00] and the ones of Glück, Reya and Vogt (GRV) [GLU95, GLU98]. Once the parton densities are known at a given point in phase space, say Q_0^2, they can be evolved dynamically to another point Q^2 using perturbative QCD by means of so called evolution equations. To derive the evolutions equations, one has to consider all contributions from all orders of perturbation theory. Since this is impossible, approximations have to be used which effectively sum up large logarithms like $\alpha_s \log(Q^2/Q_0^2)$ or $\alpha_s \log(1/x_B)$ (leading log approach) occurring in the perturbative expansions.

The most familiar attempt to evolve the parton densities, the so-called DGLAP evolution [GRI72, ALT77, DOK77], will be described briefly here. The DGLAP evolution equations are used to evolve the parton densities from a starting scale Q_0^2 to any other value of Q^2:

$$\frac{\partial f_i(x_B, Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi} \int_{x_B}^{1} \frac{dx'}{x'} \left(f_i(x', Q^2) P_{qq}(\frac{x_B}{x'}) + g(x', Q^2) P_{qg}(\frac{x_B}{x'}) \right) \tag{1.26}$$

$$\frac{\partial g(x_B, Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi} \int_{x_B}^{1} \frac{dx'}{x'} \left(\Sigma_i f_i(x', Q^2) P_{gq}(\frac{x_B}{x'}) + g(x', Q^2) P_{gg}(\frac{x_B}{x'}) \right). \tag{1.27}$$

The equations mathematically express the fact that a parton with momentum fraction x_B could have come from a parent parton with larger momentum fraction x'. The probability for parton i to radiate a parton j is given by the splitting functions, P_{qq}, P_{qg}, P_{gq} and P_{gg}. The splitting function P_{gg} gives the probability for a gluon to radiate another gluon and thus contains the self interaction of the gluons which is a consequence of the non-abelian nature of QCD.

Using a physical gauge, the evolution from the proton to the place of the hard scattering can be depicted by a ladder diagram as shown in Figure 1.5. The emitted partons in the ladder are ordered with respect to their transverse momenta, $Q^2 \gg ... \gg k_{t,i+1}^2 \gg k_{t,i}^2$, such that the hardest emission is given by the quark box at the top of the ladder. The longitudinal momenta on the other hand obey $x_{i+1} < x_i$. The DGLAP evolution equations effectively sum up terms proportional to $\alpha_s \log(Q^2/Q_0^2)$ which dominate the parton evolution if x_B is not too small.

An important test of the validity of the DGLAP evolution equations is the measurement of the proton structure function F_2 as a function of x_B and Q^2. It has been measured with high precision at HERA in a wide region of phase space. It is well described by a QCD fit based on the DGLAP evolution equations as can be seen in Figure 1.6 which shows a recent H1 measurement [KLE99] together with results from fixed target experiments (SLAC, NMC and BCDMS). The figure impressively demonstrates the important contribution of HERA experiments to the knowledge on the proton structure and therefore the understanding of the structure of matter in general. Especially, the described scaling violations, i.e. the dependence of F_2 on Q^2, can be seen very nicely from this data.

Note that, to obtain these results, no knowledge about the structure of the hadronic final state X is necessary. The investigation of the hadronic final state is a very important test of QCD by itself. The description of its detailed structure by QCD must be successful to claim QCD as the correct theory of the strong interaction.

1.5 Hadronization

The partons which emerge from the deep inelastic scattering process are not directly observable. Due to confinement the partons have to become color neutral hadrons if they are separated by more than approximately 1 fm. The transition from partons to hadrons is called hadronization. For distances larger than 1 fm the concepts of perturbative QCD are no longer valid and the transition from parton level to the hadron level presently has to be treated by phenomenological models. The final product of the hadronization process is a spray of particles collimated around the direction of the incident parton. Such a spray of particles is called a jet.

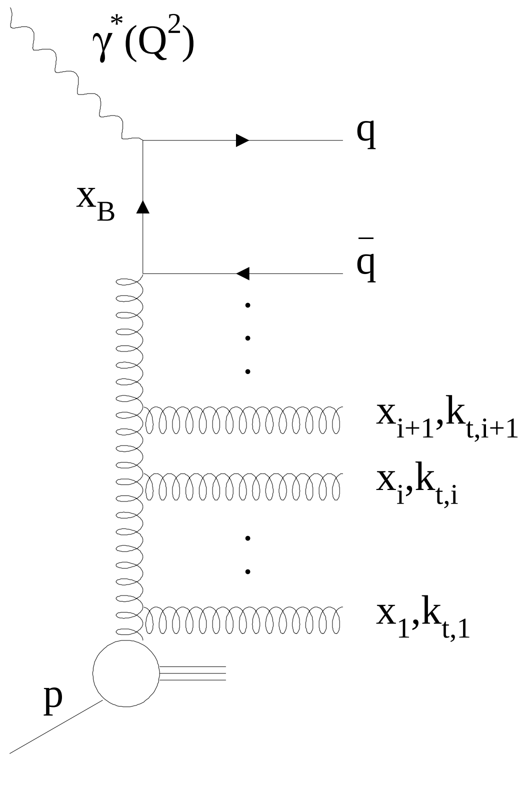

Figure 1.5: Ladder representation of a DIS process. The variables; k_t and x_i denote the transverse momentum and longitudinal momenta of the radiated partons, respectively.

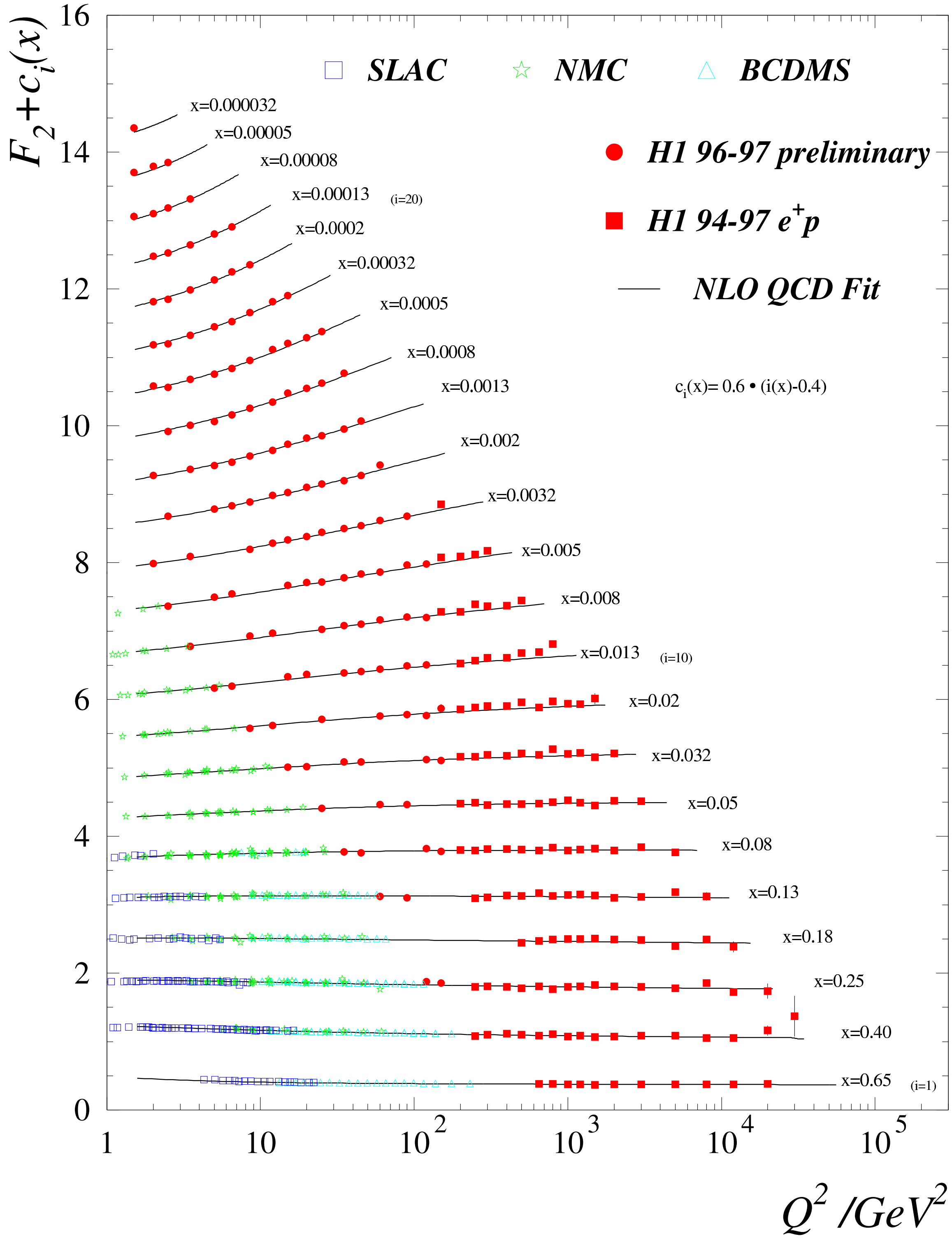

Figure 1.6: Proton structure function F_2 as a function of Q^2 for different ranges of x_B (denoted simply as x in the figure).

1.6 Jet Algorithms

The jet structure of an event is a footprint of the underlying hard scattering or short distance process between the virtual photon and a parton from the proton. Hadronization between the initial scattering and the measurement smears out the topology of the reaction. Jet algorithms attempt to establish a connection between the resulting event topology and the elementary hard subprocess. They group objects together according to a defined metric such that the constructed jets are related to the final state partons of the basic scattering process as closely as possible. Important requirements are:

- Infrared and collinear safety of the algorithm; i.e. the result of the algorithm must not change if a parton with infinitely low energy is added to an n-parton configuration and/or one parton of this configuration is split into two collinear partons.
- The algorithm must be able to separate the hard subprocess from soft interactions occurring in the event.
- It must provide small hadronization correction.

There is no unique procedure to define jets and a large variety of jet definitions have been proposed in the last decades, for a review see e.g. [MOR98]. Jet definitions can be grouped into clustering type algorithms which define jets by a successive recombination of pairs of particles in an iterative way and cone algorithms which define jets by maximizing the energy flow through a cone of fixed size. The algorithm used in this thesis, the longitudinally invariant k_t algorithm, has been shown to be a compromise between the two types of jet definitions [ELL93] and is described below.

1.6.1 The Longitudinally Invariant Inclusive k_t Algorithm

Originally the k_t algorithm was invented for e^+e^-collisions [CAT92]. It was then modified to be applicable also in hadron-hadron collisions [CAT93]. The variant used in this thesis was first proposed by Ellis and Soper [ELL93].

The idea behind the k_t-algorithm is to combine particles with nearly parallel momenta. These particles are regarded as part of a cascade of particle emissions with one of the final state partons at its origin. The experimental footprint of this cascade are local enhancements in the energy flow produced by energy depositions of particles which have only small distances in η and ϕ relative to each other.

The algorithm starts with a list of objects belonging to the hadronic final state. These objects might be the partons emerging from the hard subprocess, final state hadrons simulated by an event generator or experimentally measured quantities like clusters of energy depositions in a detector. The objects are called protojets and in the end will be merged into one or more final jets. Each protojet is characterized by its transverse energy $E_{t,i}$, its azimuthal angle ϕ_i and its pseudo rapidity η_i. The jet algorithm depends on a distance parameter R which controls the relative distance within which protojets merged into a new object. The value of R is set to 1 according to the recommendation in [ELL93]. In detail, the algorithm proceeds as follows (see also [ELL93]):

1. For each protojet define
$$d_i = E_{t,i}^2. \tag{1.28}$$

2. For each pair of protojets define the distance

$$d_{ij} = \min(E_{t,i}^2, E_{t,j}^2)[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2. \tag{1.29}$$

3. Find the smallest of all the d_i and d_{ij} and label it d_{min}.

4. If d_{min} is given by a d_{ij} then merge i and j into a new protojet k which is then added to the list while i and j are removed from the list. The merging of the particles is performed according to a specific recombination scheme. Here the so called 'E_t – scheme' (Snowmass convention [HUT90]) is used with

$$E_{t,k} = E_{t,i} + E_{t,j} \ , \tag{1.30}$$

$$\eta_k = \frac{[E_{t,i}\eta_i + E_{t,j}\eta_j]}{E_{t,k}}, \tag{1.31}$$

$$\phi_k = \frac{[E_{t,i}\phi_i + E_{t,j}\phi_j]}{E_{t,k}}. \tag{1.32}$$

5. If d_{min} is given by a d_i the corresponding protojet is removed from the list and added to the list of jets.

This procedure continues until there are no protojets remaining. At the end one receives a list of jets with successively increasing $E_{t,i}^2$.

1.6.2 Choice of the Reference Frame

In the laboratory frame partons emerging from a deep-inelastic scattering process may gain transverse momentum in two ways: (a) by balancing the transverse momentum of the scattered positron, (b) by radiating partons induced by an underlying QCD process. In a reference frame where the parton from the proton collides head on with the virtual photon the only source for jet production is the underlying QCD process[2], as depicted in Figure 1.7. In the naive quark-

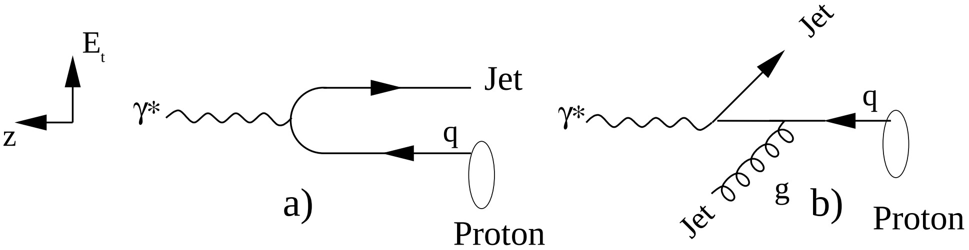

Figure 1.7: The figure shows in (a) a parton model diagram for $\gamma^* q \to q$, producing a jet with $E_t = 0$ and in (b) a gluon emission diagram which produces jets with $E_t \neq 0$.

parton model, final state hadrons are produced in the direction of the virtual photon. But, if the quark emits a gluon, as for example in the QCD Compton process, the quark recoils against the radiated gluon and two jets are produced each of which can have a large transverse energy relative to the direction of the incoming quark or proton.

[2] This statement is strictly speaking only true if the primordial transverse energy of the partons in the proton which is of the order of $300 - 600$ MeV [GEI90] is small compared to the minimal transverse energy required for the final state jets.

Figure 1.7 suggests that an appropriate frame in which to study QCD by means of the transverse energies of the final state particles is the center-of-mass frame, which is defined by the virtual boson and the parton, also called partonic cms. In this frame the only source for transverse energy is the photon parton subprocess. Unfortunately, a boost from the laboratory frame to the partonic cms is not practicable since the momentum fraction ξ of the initial parton participating in the process is not known a priori. However concerning measurements of the transverse momentum every frame whose z axis coincides with that of the partonic cms seems to be suited. Since the initial state partons move (to a first approximation) collinear with the incoming proton direction an equally suited frame is given by the center-of-mass frame defined by the proton and the virtual boson, called hadronic center-of-mass frame or $\gamma^* p$-frame, which will be used in this analysis.

1.7 Dijet Cross Section

An event with two jets in the final state is characterized by additional kinematic variables beside the DIS variables defined in Section 1.1. These additional variables are introduced in the following:

- The variable ξ defines the momentum fraction of the incoming parton on the proton and is in general not equal to x_B
- The partonic scaling variable x_p is defined as:

$$x_p = \frac{Q^2}{2\xi pq} = \frac{x_B}{\xi} \tag{1.33}$$

  and interpreted as the momentum fraction of the parton which interacts with the virtual photon relative to the momentum fraction ξ of the incident parton.

- The center-of-mass energy in the photon-parton center-of-mass frame is defined as

$$\hat{s} = (\xi p + q)^2 = Q^2 (\frac{\xi}{x_B} + 1) = Q^2 (\frac{1}{x_p} - 1) \ . \tag{1.34}$$

- The polar angle θ of the outgoing parton is expressed by the Lorentz-invariant quantity z_q according to

$$z_q = \frac{pp_q}{pq} = \frac{1}{2}(1 - \cos\theta) \tag{1.35}$$

  where p_q defines the momentum of the outgoing parton.

- The transverse energy E_t of the outgoing partons can be expressed by $\hat{s}$ and z_q:

$$E_t^2 = \hat{s} z_q (1 - z_q). \tag{1.36}$$

To first order in α_s, the final state of the partons can be expressed by three independent variables, e.g. by x_p, z_q and the azimuth angle of the outgoing partons and the differential cross section for two partons in the final state integrated over the azimuthal angle, can be written as [MEN78]:

$$\frac{d^2\sigma_{2jet}}{dx_B dQ^2} \propto \alpha_S(\mu_r^2) \int_0^1 \frac{d\xi}{\xi} \left\{ C_{qg}(x_B/\xi, z_q, x_B, Q^2) f(\xi, \mu_f^2) + C_{q\bar{q}}(...) g(\xi, \mu_f^2) \right\} . \tag{1.37}$$

Here, f and g represents the quark or gluon density at a given phase space point ξ and μ_f^2, respectively. The coefficient functions C_{qg} and $C_{q\bar{q}}$ represents the hard part of the scattering which can be calculated by perturbative QCD and contain the cross sections for the boson gluon fusion and the QCD Compton process. The differential cross sections, integrated over the azimuthal angle for the boson gluon fusion and the QCD Compton process normalized to the total cross section σ_{DIS} can be written as [PEC80], [RUM81], [LON95]:

$$\frac{d\sigma_{q\bar{q}}}{\sigma_{DIS}} = \frac{2\alpha_s}{3\pi}\frac{f_q(\xi,Q^2)}{f_q(x_B,Q^2)}\left(\frac{z_q^2+x_p^2}{(1-z_q)(1-x_p)} + 2(x_p z_q + 1) + \frac{8x_p z_q(1-y)}{1+(1-y)^2}\frac{dx_p}{x_p}dz_q\right) \quad (1.38)$$

$$\frac{d\sigma_{qg}}{\sigma_{DIS}} = \frac{\alpha_s}{4\pi}\frac{g(\xi,Q^2)}{f_q(x_B,Q^2)}\times$$
$$\left(\frac{(x_p^2+(1-x_p)^2)(z_q^2+(1-z_q)^2)}{z_q(1-z_q)} + \frac{(x_p^2+(1-x_p)^2)(z_q^2+(1-z_q)}{1+(1-y)^2}\frac{dx_P}{x_p}dz_q\right). \quad (1.39)$$

Both expressions are divergent for $z_q \to 0$ and $z_q \to 1$, respectively. In this case the partons are emitted collinear to the incoming or outgoing parton. Another divergence occurs for the QCD Compton process if $x_B \approx \xi$ or $x_p \approx 1$. This reflects the increasing probability for the radiation of soft gluons. These divergences can be avoided by requiring a minimum E_t for the outgoing partons.

The previous considerations and formulae show that the dijet cross section σ_{2jet} is directly proportional to the strong coupling constant α_s. The experimental determination of the dijet cross section therefore provides a more direct test of QCD than the determination of F_2 via inclusive measurements. A widely used observable to study dijet production is the dijet rate R_2 which is defined as:

$$R_2 = \frac{\sigma_{2jet}}{\sigma_{DIS}}. \quad (1.40)$$

Note, that in leading order, the dijet cross section depends implicitly, i.e. by α_s, on the parameter μ_r^2 and Equation 1.23 therefore cannot be fulfilled. Any leading order prediction can only be expected to describe the order of magnitude and the rough features of the dijet cross section. Any reliable prediction has therefore to be based on NLO calculations, i.e. by including the diagram shown in Figure 1.8 in addition to those shown in Figure 1.4. This means that the coefficient functions given in Equation 1.37 become explicitly dependent on the artificial scale μ_r^2. By this the dependence of the theoretical predictions on μ_r^2 due to the running coupling is partially balanced such that the renormalization scale uncertainties become small with respect to leading order predictions. Nevertheless a residual scale dependency remains which is the reason why the correct choice of μ_r^2 is still a matter of discussion. It is clear that μ_r^2 has to be large such that α_s remains small to justify the perturbative treatment of the theory. For the study of dijet events in DIS, at least two scales are suggested by the nature of the process. The first is the four-momentum transfer Q^2 while the other possible scale for jet production is the transverse energy of the outgoing jets. It is not clear a priori which choice of scale is better suited to describe dijet production in DIS and the answer can only be given by comparing measured dijet quantities to theory.

In addition to the 'scale problem' mentioned above, further considerations have to be taken into account when dealing with NLO QCD calculations. To generally avoid regions of phase space where the cross sections diverge, i.e. where partons are emitted collinear to the incoming or outgoing partons or with very small energies, a minimum requirement on the transverse energy $E_{t,cut}$ for the final state partons or jets is demanded. In earlier dijet analyses it has been common

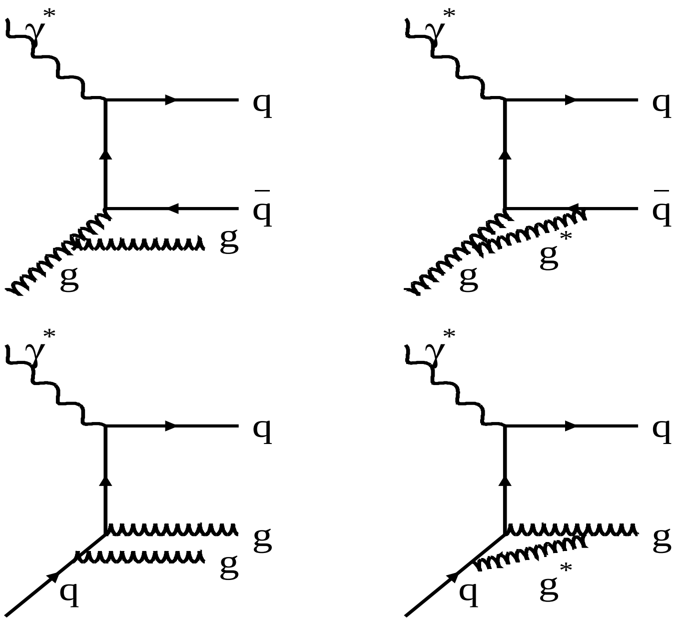

Figure 1.8: Real and virtual corrections to the leading order processes shown in Figure 1.4. They lead to order α_s^2 contributions to the dijet cross section.

practice to require the same $E_{t,cut}$ value for both of the outgoing jets. However, virtual corrections result in negative contributions to the calculated cross sections. These negative contributions have to be balanced by real emissions from the outgoing parton leg. If the required minimum transverse energy is equal for both final state partons/jets, there might not be enough phase space left for the additional emission of gluons without letting one of the two jets fall below the required threshold. In this case the real corrections do not balance the virtual corrections which lead to an infrared sensitivity of the predictions. To avoid this infrared sensitive region, it has been proposed [KLA96, FRI97] to apply asymmetric cuts on the outgoing partons or jets. Thus,

$$E_{t,jet1} > (E_{t,cut} + \Delta) \text{ with } \Delta > 0$$
$$E_{t,jet2\ldots n} > E_{t,cut} \ .$$

As an example, the dependence of the dijet cross section calculated in NLO QCD is depicted in Figure 1.9 as a function of the introduced threshold parameter Δ. Contrary to naive expectations

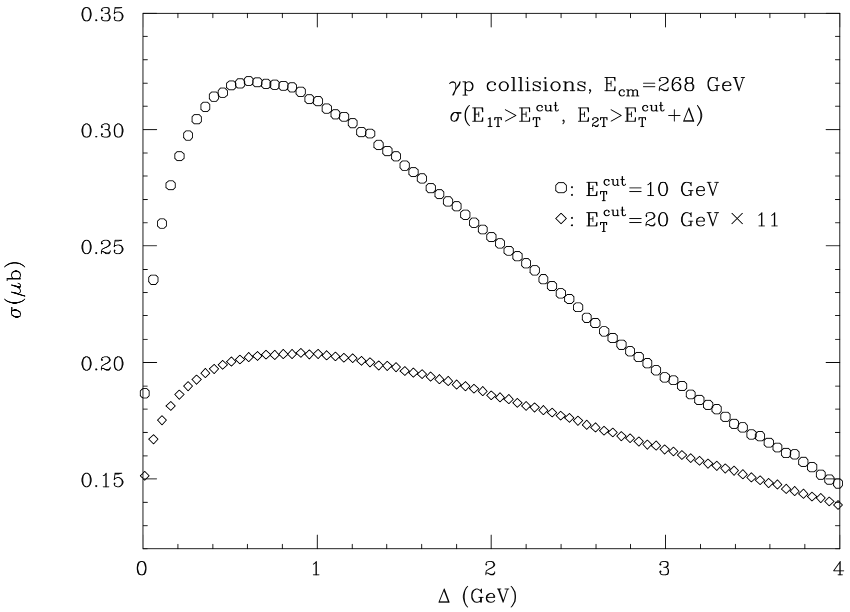

Figure 1.9: Example for the behavior of the dijet cross section calculated in NLO QCD as a function of the threshold parameter Δ. The figure is taken from [FRI97].

the cross section decreases as Δ approaches zero. This illustrates the unphysical behavior of the predictions if the perturbative series is only evaluated to a finite order. This is a problem of any fixed order calculation and can only be solved by resumming the perturbation series to all orders and/or taking non-perturbative effects into account. A detailed discussion of problems at exclusive boundaries in the phase space can be found in [FRI97] and [CAT97-1].

If dijet production is investigated from a symmetric to a largely asymmetric cut scenario, the QCD prediction can be tested from a region where it needs improvement to a region where it can be expected to be highly predictive. Such a measurement, therefore, is an important test of the actual status of the theory and might help to improve upcoming QCD predictions.

1.8 Alternative Concepts for Parton Evolutions

So far only the concept of a DGLAP evolution which assume a strong ordering in the transverse momenta of the emitted partons inside a parton ladder (see Figure 1.5) has been considered.

However, there are several other methods to describe initial state parton emission before the hard scattering process, potentially improving cross section predictions in regions where the basic assumption of strong k_t-ordering is no longer valid, e.g. at low x_B. As mentioned in the previous section, dijet production in deep inelastic scattering provides two important scales which characterize the scattering process, namely the virtuality of the photon Q^2 and the transverse energy E_t of the outgoing parton/jet. The presence of a second hard scale violates the requirement $Q^2 \gg k_{t,i}^2$ which is crucial for the validity of the DGLAP expansion. In the following, two approaches are introduced which are commonly used to describe the parton emissions in the presence of two hard scales.

1.8.1 Virtual Photon Structure

The photon is the elementary gauge boson of QED. From γp interactions where the γ is nearly on-shell, it is well established that a photon has a hadronic structure. In an intermediate state the photon can fluctuate into a $q\bar{q}$ pair. If the interaction time, say characterized by the E_t of the outgoing parton/jet, between the proton and the photon is much smaller than the fluctuation time t_f of the $q\bar{q}$ pair it will interact with the proton and give rise to a hadronic structure of the photon. The fluctuation time for high energy photons with virtuality Q^2 into a $q\bar{q}$ pair can be estimated from the uncertainty principle [IOF69] by

$$t_f \propto \frac{1}{Q^2 + (m_{q\bar{q}})^2}. \tag{1.41}$$

As Q^2 increases, t_f becomes smaller and the photon retains its structure-less character. In its resolved state, i.e. probing its structure at a scale $E_t^2 \gg Q^2$, the photon can fluctuate either into an on-shell $q\bar{q}$ pair forming a vector meson, so called Vector Dominance Part, or an off-shell $q\bar{q}$ pair, so called anomalous part, which can undergo strong interactions with other particles around.

Once the hadronic structure of the photon has been recognized, it is a natural step forward to parameterize this structure in terms of parton densities of the photon, analogous to the parton densities of the proton (for a review see e.g. [ERD97]). For quasi-real photons ($Q^2 \approx 0$) the parton densities are of the form $f_i^\gamma(x_\gamma, E_t^2)$ where x_γ is the fraction of the photon momentum carried by the interacting parton. The parameterization of the photon structure must be formulated such that the photon becomes less resolvable if the virtuality of the photon becomes comparable to the hard scale which is used to test its structure. Several proposals exist to realize this suppression of the photon structure, see e.g. [DRE94, SAS95, GLU99]. Figure 1.10 shows a measurement by the H1 collaboration of parton densities in the photon as a function of Q^2, compared to the predictions of Schuler and Sjöstrand [SCH96]. Here, the suppression of the partonic structure of the photon with increasing Q^2 is clearly visible.

Taking the internal structure of the photon into account, the ladder diagram given in Figure 1.5 is modified such that the hardest emission may occur anywhere in the ladder with increasingly soft emissions along the ladder towards the proton and the photon, as shown in Figure 1.11. The inclusion of the photon structure leads therefore to an artificial breaking of the strong k_t ordering which is demanded by the DGLAP evolution scheme and mimics an alternative type of parton evolution. Taking this alternative type of parton evolution into account the resolved contributions may enlarge the production of dijet events.

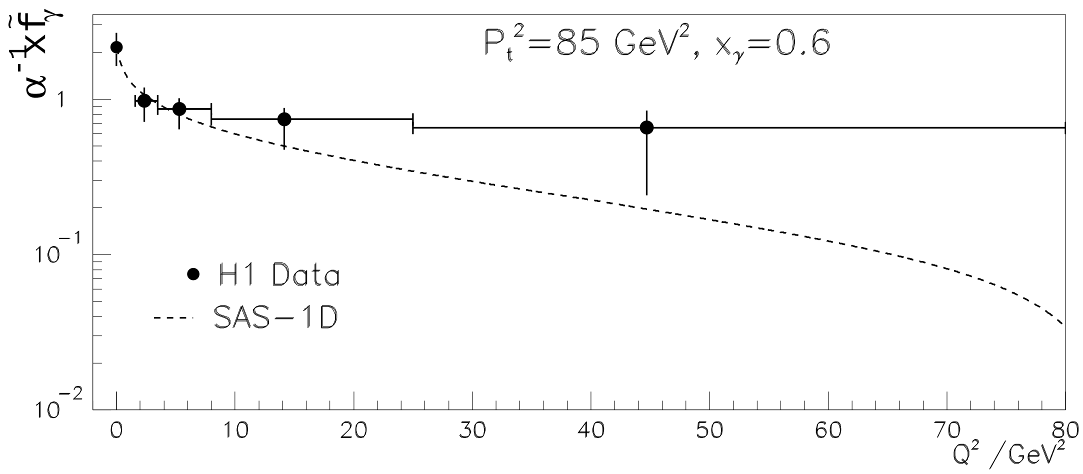

Figure 1.10: Example for the behavior of parton densities of the virtual photon (denoted as $q+\frac{9}{4}g$) as a function of Q^2 for fixed fractional momentum x_γ of a parton on the photon momentum and a jet E_t^2 of 85 GeV^2 (denoted as P_t^2 in the figure). The results are taken from [H1C00].

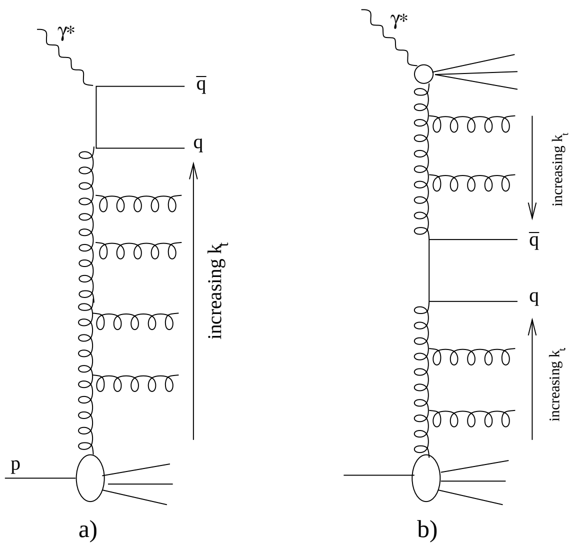

Figure 1.11: Different k_t ordering in the parton ladder between direct (a) and resolved γ^* (b) processes.

1.8.2 BFKL Evolution

An interesting situation occurs if the E_t^2 of the outgoing partons and the virtuality of the photon Q^2 are of the same size because then the conditions for DGLAP parton evolution are not fulfilled and the radiation of partons in the ladder diagram would be suppressed.

However, there are contributions proportional to $\alpha_S \log(1/x_B)$ which, while neglected in the DGLAP evolution, might become important if $Q^2 \approx E_t^2$. Their contribution is considered in another type of evolution equations, called BFKL (Balitsky, Fadin, Kuraev, Lipatov) evolution equations [KUR72]. In these evolution equations mainly $\alpha_s \log(1/x_B)$ terms are summed up. The onset of BFKL effects is expected in the low x_B regime where the $\alpha_s \log(1/x_B)$ contributions are expected to be large. In contrast to the DGLAP type of evolution equations, they require a strong ordering in the longitudinal momenta, $x_{i+1} \ll x_i$ and no ordering in k_t in the ladder depicted in Figure 1.5. It is argued in [KWI99] that the inclusion of BFKL type parton evolutions may enhance the dijet cross section at small x_B.

1.9 Monte Carlo Models

In order to compare theoretical assumptions with data one needs Monte-Carlo programs. Monte Carlo generators are able to predict details of the structure of multi particle final states observed in an experiment. Presently available programs include the matrix elements up to $O(\alpha_s)$ (Figures 1.1, 1.4); they generate the partonic final state using a technical implementation of one of the evolution equations mentioned above which leads to a good approximation of higher order effects without having to calculate the whole perturbation series. Since there is presently no generator embedding the BFKL equation, the parton shower development is mainly based on the DGLAP evolution equations. However, more phenomenological approaches to generate a parton shower exist. For the generation of the final state hadrons two types of phenomenological models exist: the string fragmentation and the independent cluster fragmentation [WEB84]. In the following a brief introduction to those Monte Carlo generators used in this thesis will be given.

1.9.1 RAPGAP

The program RAPGAP [JUN95] generate the ep process including the order α_s matrix elements represented by the Feynman graphs in Figure 1.4. In order to simulate higher order contributions the matrix elements are complemented with leading logarithm parton showers based on the DGLAP equations. In addition, the program allows the inclusion of resolved virtual photon configurations. The hadronization step is realized via an interface to the program JETSET [SJO86]. This program provides the transition from partons to hadrons using the LUND string model [AND83] and the decay of short-living hadrons. The incorporation of QED corrections is realized via an interface to the HERACLES [KWI92] program.

1.9.2 ARIADNE Color Dipole Model

The color dipole model (CDM), [AND89], [LON95], was originally developed for e^+e^- reactions. The basic idea is that the quarks produced in an annihilation process represent a color dipole which subsequently radiates gluons. The radiation leads to additional dipoles which again radiate partons resulting in a parton cascade.

In the case of ep scattering the color dipole is built by the struck quark and the proton remnant. The amount of the emissions is controlled by the extension of the proton and the photon. Since both are considered to be extended objects the radiation is suppressed according to steering parameters which quantify this extension. By construction only the QCD Compton diagram can be embedded naturally into the algorithm. The boson gluon fusion process therefore has to be realized by allowing the radiation of an anti-quark from the primary dipole. The generation of parton showers in the framework of the dipole model produces no k_t ordering of the emitted partons. Thus, it follows closely the philosophy of the BFKL parton evolution. The hadronization step is realized by JETSET.

1.9.3 LEPTO

The event generator LEPTO [ING92] contains both, the QCD Compton and the boson gluon fusion matrix elements. Leading logarithm parton showers based on DGLAP evolution equations are implemented in the program to simulate higher order contributions. The hadronization is, like in RAPGAP, realized by the Lund fragmentation scheme using JETSET.

1.9.4 DJANGO

The generator DJANGO [SCH92] allows the incorporation of QED corrections to the order α^3 in the evaluation of the ep cross sections. DJANGO is used in combination with the programs HERACLES [KWI92], LEPTO, ARIADNE and JETSET. HERACLES generates the deep inelastic events which include electromagnetic corrections to the order α. Then, a matrix element of the order $\alpha\alpha_s$ is generated which is then transferred to e.g ARIADNE to generate the QCD parton cascade. Simulated and reconstructed Monte Carlo events from DJANGO are used in this analysis for the correction of the data for QED radiation effects and will be denoted as DJANGO/CDM in this thesis.

1.10 Analytical Calculations

In general the implementation of a NLO calculation for jet cross sections in a computer program faces several difficulties. The phase space integrals which have to be performed are too complicated to be treated analytically and therefore must be subject to numerical integrations. Before they can be performed the singular parts which come along with infrared and collinear divergences in the real and virtual contributions have to be integrated out numerically.

1.10.1 DISENT

The program DISENT [CAT97-2] calculates dijet cross sections to the order α_s^2. It assumes a point-like structure of the virtual photon. The matrix elements which are listed in the Tables 1.1 and 1.2 under the column 'direct' are implemented in the program.

direct	resolved		
$\gamma^* q \to qg$	$qq' \to qq'$	$qq \to qq$	$qg \to qg$
$\gamma^* g \to q\bar{q}$	$q\bar{q}' \to q\bar{q}'$	$q\bar{q} \to q\bar{q}$	$gg \to gg$
	$qq' \to q'\bar{q}'$	$q\bar{q} \to gg$	$gg \to q\bar{q}$

Table 1.1: Two parton final state subprocesses.

direct	resolved		
$\gamma^* q \to qgg$	$qq' \to qq'g$	$qq \to qqg$	$q\bar{q} \to ggg$
$\gamma^* g \to q\bar{q}g$	$q\bar{q}' \to q\bar{q}'g$	$q\bar{q} \to q\bar{q}g$	$gg \to q\bar{q}g$
$\gamma^* q \to qq'\bar{q}'$	$q\bar{q} \to q'\bar{q}'g$	$qg \to qq\bar{q}$	$gg \to ggg$
$\gamma^* q \to qq'\bar{q}'$	$qg \to qq'\bar{q}'$	$qg \to qgg$	

Table 1.2: Three parton final state subprocesses.

In DISENT the control of the infrared divergences is realized by the subtraction method. A so called counter term is calculated and properly subtracted from the real and virtual contributions to the NLO cross sections.

1.10.2 JETVIP

The program JETVIP [POE97] calculates dijet cross sections in next-to-leading order QCD as well . In contrast to the above described program DISENT, it allows contributions from a resolved virtual photon to be incorporated in the NLO calculations. Therefore, all the subprocesses listed in Tables 1.1 and 1.2 are implemented in the program. It is so far the only existing program which provides calculations in NLO for direct and resolved interacting photon processes using parameterizations of the virtual photon structure function. It is therefore extremely useful in a kinematic regime where the E_t^2 of the partons in the final state is similar or larger than the Q^2 of the virtual photon.

The calculations are based on a phase space slicing method. In this method the singular phase space regions of soft and collinear final state particles are separated by introducing an invariant mass cut-off y_{cut}. The finite phase space regions outside the cut-off are treated numerically and can be subject to a variety of experimental cuts and jet definitions.

A further discussion about the applicability of the concept of a virtual photon structure if Q^2 is well above the photoproduction limit $Q^2 \gg m_\rho$, where m_ρ is the mass of the ρ meson, will be given in Chapter 5 where the predictions of JETVIP are confronted with the results of the analysis performed in this thesis which cover the range $5 < Q^2 < 100\,\mathrm{GeV}^2$.

1.11 Concluding remarks

In order to test the reliability of QCD predictions concerning dijet production in deep inelastic scattering the dijet rate R_2 is measured in the phase space region $5 < Q^2 < 100\,\mathrm{GeV}^2$ and $10^{-4} < x_B < 10^{-2}$. The measurement presented in this thesis follows a study published by the H1 collaboration of a single differential measurement of R_2 [H1C98]. It benefits from the increased statistics and the improved experimental apparatus which allows the extension of the measurement to phase space regions which were only partly covered by the previous analysis. By this it is possible to study the following two issues:

- For the first time a double differential measurement of R_2 as a function of x_B in bins of Q^2 is shown. If Q^2 is kept fixed one might get sensitive to the small x_B dynamics responsible for dijet production. This would allow the possibility to distinguish between DGLAP from alternative types of parton evolutions which can for example be realized by adding contributions from a resolved virtual photon to the dijet cross section. The measurement will be compared to predictions of NLO calculations. Therefore an asymmetric cut on the jet

E_t becomes necessary and $\Delta = 2\,\mathrm{GeV}$ is chosen as the central cut scenario. It was shown that for this cut scenario the NLO predictions are safe [POE99].

- In addition the double differential R_2 is studied as function of Δ. This allows the dependence of dijet production as a function of the transverse energy of the jets to be studied. If Δ is used to parameterize the E_t dependence, the measurement will cover a range in phase space from where NLO QCD calculations need improvements to where it is believed to be make accurate predictions.

In summary the measurement will be a extremely detailed and precise test of the predictions of perturbative QCD. It will reveal the huge predictive power of the theory as well as bring up points where the theory definitely needs improvement. The correct description of the measurement presented here will be a great challenge for the theory.

Chapter 2

HERA and the H1 Detector

In this chapter, the accelerator HERA and the H1 experiment are introduced. After a brief description of the HERA machine, an overview of the H1 detector in its 1996/97 setup is given with focus on those components which are essential for the analysis presented in this thesis.

2.1 The HERA Accelerator

The Hadron-Elektron-Ring-Anlage HERA is the first storage facility ever built to provide colliding beams of electrons or positrons[1] and protons. It consists of two independent storage rings with 6.3 km circumference each. The beam energies[2] are 27.5 GeV for positrons and 820 GeV for protons, leading to a center of mass energy of $\sqrt{s} \approx 300\,\mathrm{GeV}$ available for the positron-proton collision. Two experiments, named H1 and ZEUS, make use of the colliding mode of the accelerator. They are located in the north hall (H1) and in the south hall (ZEUS). A third experiment, HERA-B, is located in the west hall and uses the proton beam to study CP violation. Finally, the HERMES experiment (east hall) is studying the spin structure of the proton with the positron beam. The layout of the HERA machine, together with the injection system is illustrated in Figure 2.1.

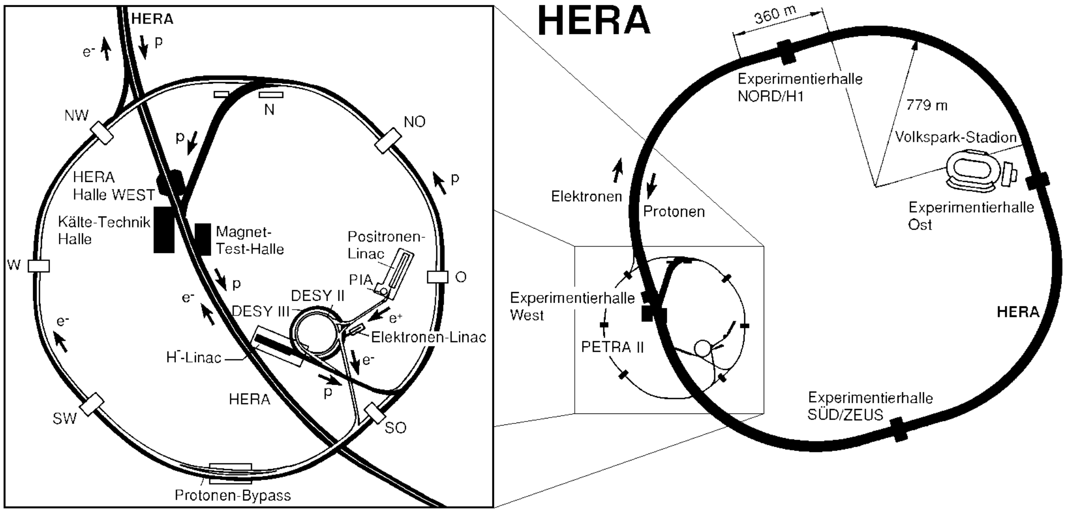

Figure 2.1: Schematic overview of the HERA accelerator complex.

[1] In the data taking periods relevant for this analysis HERA was operated with positron beams.
[2] From 1998 on the proton beam energy is 920 GeV