

One Interesting New Sum Rule Extending Bjorken's to order $1/m_Q$

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We explicitly check quark-hadron duality to order $(m_b - m_c)\Lambda/m_b^2$ for $b \rightarrow cl\nu$ decays in the limit $m_b - m_c \ll m_b$ including ground state and orbitally excited hadrons. Duality occurs thanks to a new sum rule which expresses the subleading HQET form factor ξ_3 or, in other notations, $a_+^{(1)}$ in terms of the infinite mass limit form factors and some level splittings. We also demonstrate the sum rule, which is not restricted to the condition $m_b - m_c \ll m_b$, applying OPE to the longitudinal axial component of the hadronic tensor without neglecting the $1/m_b$ subleading contributions to the form factors. We argue that this method should produce a new class of sum rules, depending on the current, beyond Bjorken, Voloshin and the known tower of higher moments. Applying OPE to the vector currents we find another derivation of the Voloshin sum rule. From independent results on ξ_3 we derive a sum rule which involves only the $\tau_{1/2}^{(n)}$ and $\tau_{3/2}^{(n)}$ form factors and the corresponding level splittings. The latter strongly supports a theoretical evidence that the B semileptonic decay into narrow orbitally-excited resonances dominates over the decay into the broad ones, in apparent contradiction with some recent experiments. We discuss this issue.

LPT-ORSAY 00/27

HD-THEP-00-12

PCCF RI 0005

I. INTRODUCTION

It is well known [1] that quark-hadron duality is valid to a good accuracy in b -quark decay and particularly in semileptonic decay. A systematic study of the corrections to duality [2–6] using the powerful tools of Operator Product Expansion (OPE) [7] and Heavy Quark Effective Theory (HQET), in particular Luke's theorem [8], has demonstrated that the first corrections to duality only appear at second order, namely $O(\Lambda^2/m_Q^2)$ where Λ is for the QCD scale and m_Q is one of the heavy quark masses (m_b or m_c). For simplicity we leave aside in this letter the $O(\alpha_s)$ radiative corrections notwithstanding their manifest practical relevance.

The OPE based proof is very elegant and circumvents the detailed calculation of the relevant channels. Precisely this feature has generated some doubts or at least some worries. First of all there is the experimental problem of the Λ_b life time which has not yet been understood within OPE framework. Second it has been asked if OPE could not miss some subtle kinematical effects related with the delay in the opening of different decay channels [9]. We have shown [10] in a non-relativistic model that the latter effect does not affect the validity of duality.

A numerical calculation of the sum over exclusive channels in the 't Hooft two dimensional QCD model [11] reported a presence of a duality-violating $1/m_Q$ correction in the total width [12]. Later the summation was performed analytically in the case of the massless light quark [13]. Agreement between the OPE and the exact result was found in this case through $1/m_Q^4$ order.

The “miraculous” conspiracy of exclusive decay channels to add up to the partonic result and its OPE corrections may be expressed in terms of sum rules which the hadronic matrix elements must satisfy in QCD [14–17]. OPE was first explicitly used to derive Bjorken sum rule in [15].

To leading order in Λ/m_b Bjorken sum rule straightforwardly implies quark hadron duality for the semileptonic widths (the differential and the total widths). The suppression of the $O(\Lambda/m_b)$ corrections is not so direct. The authors of [18] have done a thorough study of the exclusive contributions of the ground state D and D^* mesons up to order $O(\Lambda^2/m_b^2)$. They have chosen the Shifman Voloshin (SV) [19] limit, $\Lambda \ll m_b - m_c \ll m_b$,

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which drastically simplifies the calculation, but did not consider the orbitally excited states, and therefore could not check the matching between the sum of exclusive channels and the OPE prediction to the order $O(\Lambda(m_b - m_c)/m_b^2)$.

Our first motivation was precisely to complete this part and add the $L = 1$ excited states in the sum of exclusive channels. We will discuss in section III why we neglect other excitations.

While performing this task we had a surprise. We found that a *new sum rule*, eq. (12), was needed beyond Bjorken, Voloshin, and the known tower of higher moment sum rules [14–17] and we found that this new sum rule could be demonstrated from OPE.

We believe that other new sum rules can be derived along the same line. When the form factors are taken at leading order in $1/m_b$, OPE applied to different components of the hadronic tensor, or to different operators, always provides the unique series: Bjorken sum rule, Voloshin sum rule and higher moments. But when the next to leading contribution to the form factors is considered, no such unicity holds anymore. Changing the current operators in the OPE might lead to several other sum rules at order $1/m_b$.

In the following we will simplify our task as much as possible. We will neglect radiative corrections. We will also leave aside terms of order $O(\Lambda^2/m_b^2)$, which implies that operators with higher dimension than identity may be neglected in the OPE and consequently that the inclusive results may be computed only via the partonic contribution.

In the next section we will show how the equality of partonic and inclusive widths to the desired order demands for a new sum rule. In section III we will derive the latter sum rule from OPE applied to the T-product of currents. Finally in section IV we show interesting phenomenological consequences of the sum rule. We then conclude.

II. INCLUSIVE SEMILEPTONIC WIDTHS

We work in the SV limit [19], i.e. we assume the following hierarchy

$$\Lambda \ll \delta m \ll m_b \quad (1)$$

where $\delta m \equiv m_b - m_c$ and Λ is any energy scale stemming from QCD, for example the hadron-quark mass difference $\bar{\Lambda} \equiv m_B - m_b = m_D - m_c + O(1/m_b)$ or the excitation energy.

From OPE [3] one expects quark-hadron duality to be valid up to $O(\Lambda^2/m_b^2)$ corrections, i.e. in terms of the double expansion in $\delta m/m_b$ and Λ/m_b , it should be valid to all orders $(\delta m/m_b)^n$ and $(\delta m/m_b)^n \Lambda/m_b$. In fact we will restrict ourselves to check duality up to order $(\delta m/m_b)^2$ and $\delta m \Lambda/m_b^2$. The terms of order $\delta m \Lambda/m_b^2$ will turn out to be the trickiest. Of course, in the preceding sentences we mean orders as compared to the leading contribution. For example the inclusive semileptonic width is of order $(\delta m)^5$, which implies that we will compute it up to order $\bar{\Lambda}(\delta m)^6/m_b^2$. In this letter the symbol \simeq will always refer to neglecting higher orders than those just mentioned. From OPE the partonic semileptonic decay width should equate the explicit sum of the corresponding exclusive decay widths up to $O(\Lambda^2/m_b^2)$ terms, i.e. [18]:

$$\Gamma(\bar{B} \rightarrow X_c l \nu) = \Gamma(b \rightarrow c l \nu) + O(\Lambda^2/m_b^2) \quad (2)$$

with the semileptonic partonic width

$$\Gamma(b \rightarrow c l \nu) = 32 K (\delta m)^5 \left[\frac{2}{5} - \frac{3}{5} \frac{\delta m}{m_b} + \frac{9}{35} \frac{(\delta m)^2}{m_b^2} \right] \quad (3)$$

where

$$K = \frac{G_F^2}{192\pi^3} |V_{cb}|^2 \quad (4)$$

Using $M_B \simeq m_b + \bar{\Lambda}$ and $\delta M \equiv M_B - M_D \simeq \delta m$ we get

$$\Gamma(\bar{B} \rightarrow X_c l \nu) \simeq 32 K (\delta M)^5 \left[\frac{2}{5} - \frac{3}{5} \frac{\delta M}{M_B} + \frac{9}{35} \frac{(\delta M)^2}{M_B^2} - \frac{21}{35} \frac{\bar{\Lambda} \delta M}{M_B^2} \right] \quad (5)$$

The ground state contribution is [18]

$$\Gamma(\bar{B} \rightarrow (D + D^*) l \nu) \simeq 32 K (\delta M)^5 \left[\frac{2}{5} - \frac{3}{5} \frac{\delta M}{M_B} + \frac{11 - 8\rho^2}{35} \frac{(\delta M)^2}{M_B^2} - \frac{1}{10} \frac{a_+^{(1)} \delta M}{M_B^2} \right] \quad (6)$$

Strictly speaking nothing compels $a_+^{(1)}$ to be real and we must read $\Re[a_+^{(1)}]$ everywhere in this letter instead of $a_+^{(1)}$ and $\Re[\xi_3]$ instead of ξ_3 . The contribution of the first orbitally excited states may be computed using results in [20]. We get

$$\Gamma(\overline{B} \rightarrow (D_1 + D_2^*) l \nu) \simeq 32 K |\tau_{3/2}(1)|^2 \left[\frac{16}{35} \frac{(\delta M)^2}{M_B^2} - \frac{56}{35} \frac{\Delta_{3/2} \delta M}{M_B^2} \right] \quad (7)$$

for the states with total angular momentum of the light quanta $j = 3/2$ and $\tau_j(w)$ are the infinite mass limit form factors $B \rightarrow D^{**}$ as defined in [15]. In all this letter we use for any state n the notation

$$\Delta_n = M_n - M_0, \quad (8)$$

where 0 refers to the ground state.

$$\Gamma(\overline{B} \rightarrow (D_1^* + D_0^*) l \nu) \simeq 32 K |\tau_{1/2}(1)|^2 \left[\frac{8}{35} \frac{(\delta M)^2}{M_B^2} - \frac{49}{35} \frac{\Delta_{1/2} \delta M}{M_B^2} \right] \quad (9)$$

for the lowest $j = 1/2$ states.

To the order considered, quark-hadron duality of the semileptonic decay widths implies the equality of the r.h.s. of eq. (5) with the sum of the r.h.s's of eqs (6), (7) and (9) to which we need to add the $L = 1$ radially excited states. Their contributions are identical to eqs. (7) and (9) with the replacement $\tau_j \rightarrow \tau_j^{(n)}$ and $\Delta_j \rightarrow \Delta_j^{(n)}$. The terms proportional to $(\delta M/M_B)^2$ match thanks to Bjorken sum rule [14, 15]:

$$\rho^2 - \frac{1}{4} = \sum_n \left[|\tau_{1/2}^{(n)}|^2 + 2|\tau_{3/2}^{(n)}|^2 \right] \quad (10)$$

From now on, unless specified, it is understood that the form factors are taken at $w = 1$. Taking into account Voloshin sum rule [16]

$$\overline{\Lambda} = \sum_n \left[2 \Delta_{1/2}^{(n)} |\tau_{1/2}^{(n)}|^2 + 4 \Delta_{3/2}^{(n)} |\tau_{3/2}^{(n)}|^2 \right], \quad (11)$$

the matching of the terms of order $\Lambda \delta M/M_B^2$ leads to the requirement

$$a_+^{(1)} = 4 \sum_n \left[\Delta_{1/2}^{(n)} |\tau_{1/2}^{(n)}|^2 - \Delta_{3/2}^{(n)} |\tau_{3/2}^{(n)}|^2 \right] \quad (12)$$

The sum rule (12) is the main result of this paper. The preceding lines can be taken as a derivation of the sum rule, since we simply have made explicit the result from OPE, eq. (5). However, one might feel uncomfortable in view of the peculiarity of the SV kinematics, one might fear that some exception to OPE could happen there. Furthermore, as recalled in the introduction, OPE has been repeatedly submitted to various interrogations. Therefore, we will rederive in the next section the sum rule (12) in a less questionable manner.

Let us note that in the *vector current case*, we do not need the a_+ form factor. In that case, matching of the $(\delta M/M_B)^2$ and $\Lambda \delta M/M_B^2$ terms occurs thanks to Bjorken and Voloshin sum rule only - or conversely we can invoke duality to demonstrate these sum rules. In particular, it gives a demonstration of Voloshin sum rule just from the same duality requirement invoked by Isgur and Wise to derive Bjorken sum rule: the Voloshin sum rule comes from the matching of $\Lambda \delta M/M_B^2$ terms.

It is in the axial case or in the $V - A$ case (which corresponds to the sum of vector and axial contribution) that we need the new sum rule. More precisely, we can separate also the contributions with definite helicity of the lepton pair. In the transverse helicity case, there is still matching from just Bjorken and Voloshin sum rule. In fact the need for a new sum rule occurs in the axial current and for *longitudinal* helicity. We obtain indeed for the $\lambda = 0$ helicity of the axial current:

$$\Gamma(b \rightarrow c l \nu)_{A, \lambda=0} \simeq 4 K (\delta M)^5 \left[\frac{4}{3} - 2 \frac{\delta M}{M_B} + \frac{4}{5} \frac{(\delta M)^2}{M_B^2} - 2 \frac{\overline{\Lambda} \delta M}{M_B^2} \right] \quad (13)$$

$$\Gamma(\overline{B} \rightarrow D^* l \nu)_{A, \lambda=0} \simeq 4 K (\delta M)^5 \left[\frac{4}{3} - 2 \frac{\delta M}{M_B} + (1 - \frac{4}{5} \rho^2) \frac{(\delta M)^2}{M_B^2} - \frac{4}{5} \frac{a_+^{(1)} \delta M}{M_B^2} \right] \quad (14)$$

$$\begin{aligned}
\Gamma(\overline{B} \rightarrow D^{**} l \nu)_{A, \lambda=0} &\simeq 4 K (\delta M)^5 \left[\frac{4}{5} \sum_n \left[|\tau_{1/2}^{(n)}|^2 + 2 |\tau_{3/2}^{(n)}|^2 \right] \frac{(\delta M)^2}{M_B^2} \right. \\
&\quad - \frac{28}{5} \sum_n \left[\Delta_{1/2}^{(n)} |\tau_{1/2}^{(n)}|^2 + 2 \Delta_{3/2}^{(n)} |\tau_{3/2}^{(n)}|^2 \right] \frac{\delta M}{M_B^2} \\
&\quad \left. + \frac{24}{5} \sum_n \left[\Delta_{1/2}^{(n)} |\tau_{1/2}^{(n)}|^2 \right] \frac{\delta M}{M_B^2} \right]
\end{aligned} \tag{15}$$

whence we get the eq. (12) from the matching of $\frac{\delta M}{M_B^2}$ terms.

III. DERIVATION OF THE SUM RULE FROM OPE

The authors of [21] have derived corrections to Bjorken and Voloshin sum rules and to the resulting *inequalities* on ρ^2 . We will follow the same philosophy but including the orbitally excited states in order to derive $O(\Lambda/m_b)$ corrections, within our approximations, to the *equalities* resulting from the sum rules. We will use the differential semileptonic distributions [22].

Defining two currents which at present we take arbitrary:

$$J(x) \equiv (\overline{b} \Gamma c)(x), \quad J'(y) \equiv (\overline{c} \Gamma' b)(y). \tag{16}$$

Their T product is

$$T(q) \equiv i \int d^4 x e^{-iqx} \langle \overline{B} | T(J(x) J'(0)) | \overline{B} \rangle \tag{17}$$

where the states are normalised according to $\langle p | p' \rangle = (2\pi)^3 \delta_3(\vec{p}' - \vec{p})$.

Neglecting heavy quarks in the “sea”, it is clear that $x < 0$ receives contributions from intermediate states with one c quark and light quanta, usually referred to as the direct channel, while $x > 0$ receives contributions from intermediate states with $b\overline{c}b$ quarks plus light quanta. This will be referred to as the crossed channel, or Z diagrams. Expanding the r.h.s of (17) on intermediate states X in the B rest frame,

$$T = (2\pi)^3 \left[\sum_X \delta_3(\vec{p}_X + \vec{q}) \frac{\langle \overline{B} | J(0) | X \rangle \langle X | J'(0) | \overline{B} \rangle}{M_B - q_0 - E_X} - \sum_{X'} \delta_3(\vec{p}_{X'} - \vec{q}) \frac{\langle \overline{B} X' | J(0) | 0 \rangle \langle 0 | J'(0) | \overline{X'} \overline{B} \rangle}{M_B + q_0 - (E_{X'} + 2M_B)} \right] \tag{18}$$

where X, X' are charmed states. Let us call \mathcal{V} the typical virtuality of the direct channels, $M_B - q_0 - E_X \simeq \mathcal{V}$, we will take q_0 such that $\Lambda \ll \mathcal{V} \ll M_B$. While the direct channels (X) contribute like $1/\mathcal{V}$ to (18), the crossed channels (X') contribute like $1/(m_D + \mathcal{V})$. In both cases the denominator is $\gg \Lambda$, which allows to use the leading contribution to OPE:

$$T = i \int d^4 x e^{-iqx} \langle \overline{B} | \overline{b}(x) \Gamma S_c(x, 0) \Gamma' b(0) | \overline{B} \rangle + O(1/m_c^2) \tag{19}$$

where $S_c(x, 0)$ is the free charmed quark propagator as long as $O(\alpha_s)$ corrections are neglected. Assuming as usual that the b quark has a momentum $p_b = m_b v + k$ with $k_\mu = O(\Lambda)$, the charmed quark propagator in (19) has two terms, the positive energy pole with a denominator $m_b v_0 + k_0 - q_0 - E_c \simeq \mathcal{V}$ and the negative energy one with a denominator $m_b v_0 + k_0 - q_0 + E_c \simeq m_c + \mathcal{V}$. Varying \mathcal{V} independently of $m_b \simeq m_c$ one can check that the direct channels sum up to the contribution of the positive energy pole of the charmed quark propagator.

As a result, considering now only resonances among the states X and fixing \vec{q} in the following, one gets equating the residues

$$\sum_n \langle \overline{B} | J(0) | n \rangle \langle n | J'(0) | \overline{B} \rangle = \langle \overline{B} | \overline{b} \Gamma \frac{\not{q}' + 1}{2v'_0} \Gamma' b | \overline{B} \rangle \tag{20}$$

where all the three-momenta are equal to $-\vec{q}$ in the B rest frame and

$$v'_q = \frac{1}{m_c} (-\vec{q}, \sqrt{\vec{q}^2 + m_c^2}) \tag{21}$$

It is well known [15] that to leading order this leads to Bjorken sum rule. Considering successive moments, i.e. multiplying T in (17) by $(q_0 - E_0)^n$ (E_0 being the ground state energy) leads to a tower of sum rules [17], Voloshin sum rule when $n = 1$, etc.

In the following *we will stick to the $n = 0$ moment*, but include the $1/m_b$ correction to the residues. Let us insist on this point. One may discover a tower of sum rules by keeping the form factors to leading order but considering successive moments [17]. One may also discover new sum rules by sticking to the lowest moment but considering the higher orders in the form factors. This is not equivalent and leads to different sum rules, the first moment yields Voloshin sum rule eq (11), the second adds at least one new sum rule, (12), as we shall demonstrate now. The distinction is important since in practice both sum rules apply to the same order in $1/m_b$. A significant difference between the two types of subleading sum rules is the following: All the currents provide via OPE the same Voloshin sum rule because the form factors are all related by the heavy quark symmetry. On the contrary, when the form factors are taken at subleading order in $1/m_b$, different currents have different corrective terms depending on several independent form factors, and OPE should yield different subleading sum rules. In this letter we only consider eq. (12) for its physical relevance, leaving other sum rules for a forthcoming study.

We now apply eq. (20) with J, J' substituted by the vector current V^μ and the axial one A^μ . One may check that eq. (20) applied to currents projected perpendicularly to the v, v' plane is trivially satisfied, including the $O(\Lambda/m_b)$ order, by Bjorken sum rule. Let us now consider the vector current projected on the B meson four velocity: $V \cdot v$. Among the orbitally excited states only the $J = 1$ states contribute to the wanted order. Dividing both sides of eq (20) by $(1 + w)/(2v_0 v'_0)$ one gets using the results of [20] and [23]

$$\frac{1+w}{2}|\xi(w)|^2 + \sum_n (w-1) \left\{ 2|\tau_{1/2}^{(n)}|^2 \left[1 + \frac{\Delta_{1/2}^{(n)}}{m_b} \right] + (w+1)^2 |\tau_{3/2}^{(n)}|^2 \left[1 + \frac{\Delta_{3/2}^{(n)}}{m_b} \right] \right\} \simeq 1 + (w-1) \frac{\bar{\Lambda}}{m_b} \quad (22)$$

where we have neglected higher powers of $(w-1)$ and of Λ/m_b than the first¹. The l.h.s is found by a straightforward application of [23] for the ground state and of [20] for the excited ones. The r.h.s yields $(1 + w_q)/(1 + w)$ which has been transformed according to:

$$w_q \equiv v \cdot v'_q \simeq w + \vec{q}^2 \left[\frac{1}{2m_c^2} - \frac{1}{2M_D^2} \right] \simeq w + \frac{(w^2 - 1)\bar{\Lambda}}{m_b}. \quad (23)$$

The leading terms in eq. (22) simply reproduce Bjorken sum rule as expected [15], while the $O(\Lambda/m_b)$ terms provide Voloshin sum rule. This is *another derivation of Voloshin sum rule* which does not use higher momenta.

Analogously the axial current projected on the D meson velocity v' , $A \cdot v'$ gives, inserted in eq. (20) and after dividing both sides by $(w-1)/(2v_0 v'_0)$,

$$\begin{aligned} \frac{1+w}{2}|\xi(w)|^2 - \frac{4}{m_b}\xi_3(w)\xi(w) + \sum_n \left\{ \left[2(w-1) - \frac{6(w+1)\Delta_{1/2}^{(n)}}{m_b} \right] |\tau_{1/2}^{(n)}|^2 \right. \\ \left. + (w-1)(w+1)^2 |\tau_{3/2}^{(n)}|^2 \right\} \simeq 1 - (w+1) \frac{\bar{\Lambda}}{m_b} \end{aligned} \quad (24)$$

where ξ_3 in the notations of [23] is equal to $-a_+^{(1)}/2$ used in [18]. The matching of the $1/m_b$ terms in eq. (24) leads to the sum rule

$$\bar{\Lambda} + a_+^{(1)} = L_4(1) = +6 \sum_n \Delta_{1/2}^{(n)} |\tau_{1/2}^{(n)}|^2 \quad (25)$$

L_4 being defined according to [23]. Eliminating $\bar{\Lambda}$ from eqs. (25) and (11) we are left with eq. (12).

We can check this result by using the method for sum rules developed earlier by Bigi and the Minnesota group [24], which relies on a systematic $1/m_Q$ expansion of the moments of the Lorentz invariants of the imaginary part of the hadronic tensor, w_i . From their equation (131), we read :

¹Remember that we take $\bar{\Lambda} \sim \Delta_j \sim \Lambda$

$$\int dq^0 w_2^{AA}(q^0, \vec{q}^2) \simeq \frac{m_b}{E_c} \quad (26)$$

the terms left over being the power corrections due to higher dimension operators. Computing from [18] and [20] the hadronic contribution to the same integral at $\vec{q} = 0$ i.e. $w = 1$, we get the equation (with $r_0 = M_D^*/M_B$, $r_{1/2,3/2} = M_{D_{1/2,3/2}^{**}}/M_B$):

$$\begin{aligned} \int dq^0 w_2^{AA}(q^0, \vec{q}^2 = 0) &= \frac{1}{r_0} \left\{ \frac{f^2}{4M_B^2 r_0^2} + \frac{(1-r_0)fa_+}{r_0} \right\} \\ &+ \left\{ \frac{(1-r_{3/2})^2}{r_{3/2}^2} \left(\frac{k_{A_1}^2}{24} - \frac{f_A^2}{4} \right) + \frac{1}{4r_{1/2}^2} [(1+r_{1/2})g_+ - (1-r_{1/2})g_-]^2 - g_A^2 \right\} \end{aligned} \quad (27)$$

with all form factors taken at $w = 1$, and with notations for the $L = 1$ form factors $g_+, g_-, g_A, f_A, k_{A_1}$ to be found in [20]. A sum over the $L = 1$ excitations is understood. If we now work in the SV limit, we see that we need g_-, f_A, g_A, k_{A_1} only in the HQET limit, i.e. $\tau_{1/2,3/2}$, except for some algebraic factors; as for g_+ , it is subleading, but at $w = 1$, it is expressible in terms of $\tau_{1/2}$ and we do not need to know any of the new subleading form factors. In the $L = 1$ contributions, only the g_+g_- term remains. We finally end with the equation :

$$\frac{M_B}{M_D} - \frac{\delta M}{M_B^2} a_+^{(1)} + 6 \frac{\delta M}{M_B^2} \sum_n \Delta_{1/2}^{(n)} |\tau_{1/2}^{(n)}|^2 \simeq \frac{m_b}{m_c} \quad (28)$$

which leads directly to eq. (25).

In the preceding calculations we have systematically neglected the contributions from higher orbital excitations or $L = 0$ radial excitations. This can be justified as follows. The leading B transition to radially excited $L = 0$ final states or to $L = 2$ final states are suppressed by a factor \vec{q}^2/m_b^2 due to three facts: first, the current operator is proportional at leading order to the identity operator or to $\vec{\sigma}_b^2$, second, the orthogonality of the wave functions implies vanishing at $\vec{q} = 0$ in the B rest frame and, third, parity implies an even power in \vec{q} . This suppression leads to the well known fact that these terms appear in the Bjorken sum rule or in the differential widths with a $(w-1)^2$ factor as compared to the ground state contribution. On the contrary the $O(\Lambda/m_b)$ contributions to the axial form factors for the same type of transitions are not suppressed as compared to the ground state because the current operator is no more proportional to identity neither to $\vec{\sigma}_b$. For example the transition to radially or orbitally excited $J^P = 1^-$ states other than the D^* are in principle of the same order of magnitude than the $\propto a_+^{(1)}$ terms mentioned above. However, in this letter we have only considered the terms $\propto a_+^{(1)}$ via crossed terms, i.e. via cross products of the leading order terms with the $O(\Lambda/m_b)$ ones, because we have neglected all $O(\Lambda^2/m_b^2)$ contributions. Hence we are left with a suppression of a factor \vec{q}^2/m_b^2 in the hadronic tensors or the differential widths, i.e. a factor $(w-1)$ as compared to the corresponding ground state contribution and we can consequently neglect the $L = 0$ radial excitations and the $L = 2$ orbital ones. $L = 3$ contributions are negligible simply because the total angular momentum $J \geq 2$ again leads to $(w-1)$ factors resulting from angular momentum conservation (D-waves). All other operators which are already negligible for the ground state and the $L = 1$ states are even more so for higher excitations.

Turning now to a comparison of our different demonstrations, we should note that it is not really unexpected that we find consistent results according to three approaches: imposing duality to the widths (section II), imposing duality to the tensors as in eqs. (22) and (24) and finally to the invariant tensors eqs. (26) and (27). Indeed, at fixed q^0 and \vec{q} there is a linear relation between the tensor components and the invariant tensors. It is as well true that the formula for the decay widths before integrating on the q^0 variable is, for fixed q^0 and \vec{q} , linear in the tensor components.

We might worry about what happens when we apply duality to the sum of the residues. Integration over q^0 leads to a sum of residues multiplied by δ functions and the position of the poles is different for each term in the sum and still different for the quark contribution. As a consequence the projector which projects out w_2 from the tensor residues is different for each term since it depends on q^0 . Still this difference does not lead to a collapse of the sum rule thanks to Voloshin sum rule and the tower of higher momenta sum rules: one can expand the difference between the intervening projectors in powers of q^0 and the resulting alteration to the sum rule vanishes. Exactly the same happens when one computes the decay widths with the real kinematics on each term.

² The heavy quark spin may be factorised out thanks to HQS.

IV. PHENOMENOLOGICAL CONSEQUENCES

Eq. (25) is phenomenologically relevant as it expresses the dominant correction to the zero recoil differential $B \rightarrow D\nu$ decay width as a function of leading form factors and level spacings. Indeed

$$\frac{d\Gamma(B \rightarrow D\nu)}{dw} \propto (w^2 - 1)^{3/2} \left[1 - 2 \left(\frac{1}{2m_b} + \frac{1}{2m_c} \right) \frac{M_B - M_D}{M_B + M_D} L_4(1) \right]. \quad (29)$$

On the other hand, we may combine our result with an independent estimate of the form factor ξ_3 [25] from QCD sum rules³:

$$\frac{\xi_3(1)}{\Lambda} = \frac{1}{3} + O(\alpha_s) = 0.6 \pm 0.2, \quad \frac{a_+^{(1)}}{\Lambda} = -\frac{2}{3} - O(\alpha_s) = -1.2 \pm 0.4. \quad (30)$$

The dispersion formulation of the constituent quark model [26] finds that $\xi_3(1)$ is $1/3$ the average kinetic energy of the light quark. For a light constituent mass of $m_u = 0.25$ GeV it gives

$$\xi_3(1) = 0.17 \text{ GeV}, \quad \bar{\Lambda} = 0.5 \text{ GeV} \quad (31)$$

in perfect agreement with eq. (30) for $\alpha_s = 0$.

Combining (11), (12) and (30), assuming $\alpha_s = 0$ since we have neglected radiative corrections all along this letter, we get

$$\frac{\sum_n \Delta_{1/2}^{(n)} |\tau_{1/2}^{(n)}|^2}{\sum_n \Delta_{3/2}^{(n)} |\tau_{3/2}^{(n)}|^2} = \frac{1}{4}, \quad \text{for } \alpha_s = 0 \quad (32)$$

and

$$\sum_n \Delta_{1/2}^{(n)} |\tau_{1/2}^{(n)}|^2 = \frac{1}{18} \bar{\Lambda}, \quad \sum_n \Delta_{3/2}^{(n)} |\tau_{3/2}^{(n)}|^2 = \frac{2}{9} \bar{\Lambda} \quad (33)$$

Notice that if we had, somehow inconsistently, taken $\xi_3(1)/\bar{\Lambda} = 0.6$ the result would not be qualitatively different.

Since in all spectroscopic models the mass differences between the $j = 1/2$ and $j = 3/2$ states turn out to be not so large, we conclude that the $\sum_n |\tau_{1/2}^{(n)}|^2$ are *significantly smaller than the* $\sum_n |\tau_{3/2}^{(n)}|^2$.

Interestingly enough, this hierarchy $|\tau_{1/2}^{(0)}|^2 < |\tau_{3/2}^{(0)}|^2$ was a clear outcome of a class of covariant quark models [27]. In [27] four different potentials had been used within the Bakamjian-Thomas covariant quark model framework. The potentials labeled ISGW, VD, CCCN, and GI potentials in [27] give respectively for the ratio $|\tau_{1/2}^{(0)}|^2/|\tau_{3/2}^{(0)}|^2$ the values 0.33, 0.09, 0.01 and 0.17. As a result these models predict a dominance of the $B \rightarrow D_{j=3/2}\nu$ semileptonic decay widths by one order of magnitude over the $B \rightarrow D_{j=1/2}\nu$. We will comment this prediction later. The same models [27] give for the l.h.s of eq. (32) 0.39, 0.166, 0.151 and 0.247 respectively for the ISGW, VD, CCCN, and GI potentials, in reasonable agreement with $1/4$. It might not be mere luck if the GI model, which fits the spectrum in the most elaborate way, yields an almost too good agreement with the expectation (32)⁴. From eq. (30) we expect the r.h.s. of eq. (12) divided by that of eq. (11) to be close to $-2/3$. We have tested this with the numerical calculations of [27]. In all cases we find that the sums in the r.h.s of eqs. (11)-(12) saturate very fast to their asymptotic values. At $n = 3$ they are at less than 3% in all cases. For the ratios $a_+^{(1)}/\Lambda$ computed from the r.h.s of eqs. (11)-(12) one finds -0.51, -0.77, -0.79, -0.67 respectively for the ISGW, VD, CCCN, and GI models. This agreement with (30) is quite striking, and again GI is embarrassingly good.

In more general terms, the prediction [27] that the B meson decays dominantly into the narrow resonances $j = 3/2$ was comforted by a study within a constituent quark-meson model [28] as well as by a semi-relativistic study [29]. A QCD sum rule analysis [30] predicted rather a rough equality between these form factors contrarily

³The definitions of ξ_3 differ by a factor $\bar{\Lambda}$ in [23] and [25]. We use the notations of [23].

⁴We should nevertheless remember that the potentials used in [27] contain a Coulombic part which implies that some part of the $O(\alpha_s)$ corrections might be implicit in these models.

to another one [31] which concluded to an overwhelming dominance of the $j = 3/2$ semileptonic decay over the $j = 1/2$.

It is fair to say that the general trend of theoretical models is to predict $3/2$ dominance and a total semileptonic branching ratio into the orbitally excited states exceeding hardly 1 %. It is well known that the $j = 3/2$ are expected to be relatively narrow and are identified with the observed narrow resonances $D_1(2422)$ and $D_2^*(2459)$. As far as the decay widths into the latter narrow resonances is considered, experimental results [32] are in rough agreement with [27] for the $B \rightarrow D_1(2422)l\nu$ and rather below [27] for $B \rightarrow D_2^*(2422)l\nu$. In brief, experiment is rather below the theoretical models for $B \rightarrow D_{3/2}l\nu$. The $j = 1/2$ states are not easy to isolate, being very broad. But thorough studies have been done of the channels $B \rightarrow D^{(*)}\pi l\nu$ and the resulting branching fraction is very large: $3.4 \pm 0.52 \pm 0.32\%$ by DELPHI [33] and $2.26 \pm 0.29 \pm 0.33\%$ by ALEPH.

These experimental results are both welcome and puzzling. Welcome because these $B \rightarrow D^{(*)}\pi l\nu$ fill the gap between the inclusive semileptonic decay branching fraction of 10 - 11 % and the sum $B \rightarrow (D + D^*)l\nu \simeq 7\%$. They are puzzling when one tries to understand which channels contribute to them. As we have just said, the $j = 3/2$ channels provide no more than 1 %. The remaining 2 % can come from the $j = 1/2$, from higher excitations or from a non-resonant continuum. Higher excitations are unlikely to contribute very much, being suppressed both by dynamics and phase space. In [33] the quoted $b \rightarrow D^{**}l\nu$ branching fractions are very large, exceeding by far what is expected for example in [27].

The results presented in this letter are doubly relevant in the above discussion. First eq. (32) seems to confirm the models which find a dominance of the $3/2$ channels. Of course it is mathematically possible that that eq. (32) is satisfied while $|\tau_{1/2}^{(0)}| > |\tau_{3/2}^{(0)}|$, the higher excitations compensating for the sum rule. Admittedly such a situation would look rather queer, and as mentioned above, the models [27], which agree rather well with the new sum rule eq. (12), also yield $|\tau_{1/2}^{(0)}|^2 < 0.35|\tau_{3/2}^{(0)}|^2$.

It is then hard to understand how the $b \rightarrow D^{**}l\nu$ branching fractions can be as large as quoted in [33] in view of the smallness of the experimental $B \rightarrow D_{3/2}l\nu$ branching fractions. However, the second lesson from our study is that $1/m_c$ corrections may play an important role, and a further study of their effect is wanted.

The most serious caveat to our present derivation of a narrow resonance dominance comes from the fact that we have neglected radiative corrections. A priori we expect radiative corrections to provide only corrections and our present estimate to yield the general trend. This is unhappily not always true. As a counterexample see the discussion which follows eq. (7.8) in [18]. It is argued that some radiative corrections to the parameter K are parametrically larger than the $\alpha_s = 0$ estimate. A careful study of radiative corrections to our present sum rule and its consequences would be welcome.

It is not excluded that an important fraction of the $B \rightarrow D^{(*)}\pi l\nu$ decays observed at LEP are non-resonant. Unluckily theoretical works addressing non-resonant decays are rare, [34] find in the soft-pion domain a resonance dominance while Isgur [35] predicts that no more than 5 % of the semileptonic decay is non-resonant. Furthermore, if such a continuum contributes significantly, it should also be included in the sum rules [35] and we might fear that at the end of the day the paradox would still be there.

Finally another experimental result [36] seems to contradict our theoretical expectation: the branching ratio for $B \rightarrow D_1(j = 1/2)\pi^-$ is found to be $\simeq 1.5$ times larger than that of $B \rightarrow D_1(2420)\pi^-$. Of course the experimental error is still large, and the relation between nonleptonic decays and the semileptonic ones assumes factorisation.

But still *there is a puzzle*: on one side an increasing amount of theoretical evidence in favor of the narrow resonances dominance, and on the other side an increasing amount of experimental evidence in the opposite direction!

V. CONCLUSION AND OUTLOOK

We have explicitly checked quark-hadron duality in the SV limit to order $\delta m\Lambda/m_b^2$ including ground state final hadrons and $L = 1$ orbitally excited states. We have shown that this duality implied a new sum rule eq. (12) which we have also demonstrated from OPE applied to T-product of axial currents.

We have shown that this sum rule combined with some theoretical estimates of ξ_3 lead to the conclusion that very probably the B decay into narrow $L = 1$ resonances was dominant over the one into broad resonances. This remark seems to contradict recent experimental claims that the broad resonances dominate. We have discussed this situation which needs urgently further theoretical and experimental work.

Beyond understanding this experimental puzzle, further theoretical work is needed. For example we might wonder if some proof of the new sum rule along the line of [14] is possible. Some progress has been done in this direction. The effect of radiative corrections should also be studied.

Last but not least, other new sum rules derived along the same line with other currents or other components of the currents should be considered.

ACKNOWLEDGMENTS

We thank Patrick Roudeau for many stimulating discussions which have triggered this study.

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