"Optical" sum rule for form factors of heavy mesons

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A sum rule for the transitions due to the vector current of heavy quarks between the ground negative-parity states and the positive-parity excited states is derived, which is a direct analogue of the sum rule in the scattering of light on atoms, known as the Thomas-Reiche-Kuhn sum rule or the "oscillator strength" sum rule. This sum rule in conjunction with one suggested by Bjorken can be used to place an upper bound on the slope parameter of the Isgur-Wise function ξ. The numerical estimates suggest the possibility of a significant difference of the slope parameter in the $B \to D$ transitions from that in the $B \to D^*$ due to an insufficiently heavy mass of the charmed quark. Also the sum rule shows that the slope parameter should be a growing function of the mass of the spectator quark.

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I. INTRODUCTION

The simple idea [1–3] that a heavy quark in a hadron, in which other components are light, acts as a static spinless center of force for the surrounding quarks and gluons proved to be very fruitful in finding numerous relations between form factors of various heavy hadrons when this idea was implemented [4] in terms of an additional symmetry arising in the limit of an infinitely heavy quark mass. In particular, this symmetry expresses the matrix elements of the heavy quark operators of the form $(\bar{Q}_2 \Gamma Q_1)$ with all possible Γ-matrix structures Γ and between all the hadrons with one Q_1 in the initial state and one Q_2 in the final state in terms of one or a few invariant form factors, which depend only on the four velocities u_1, u_2 of the initial and the final hadronic states, provided that one set of initial and final states is related to the other by a reorientation of the heavy quark spin (for a recent review see, e.g., [5]).

From the experimental point of view, of immediate interest is the form factor $\xi(u_1 \cdot u_2)$, which determines [4] the matrix elements of the weak current $V_\mu - A_\mu = [\bar{c}\gamma_\mu(1-\gamma_5)b]$ in the semileptonic decays of the B mesons, $B \to D l \nu$ and $B \to D^* l \nu$, whose form up to QCD radiative effects is given by

$$\langle D(u_2)|V_\mu|B(u_1)\rangle = \xi(u_1 \cdot u_2)(u_{1\mu}+u_{2\mu})/2 \; ,$$
$$\langle D^*(u_2,\epsilon)|A_\mu|B(u_1)\rangle = -\xi(u_1 \cdot u_2)[\epsilon_\mu(1+u_1 \cdot u_2) - u_{2\mu}\epsilon \cdot u_1]/2 \; , \tag{1}$$
$$\langle D^*(u_2,\epsilon)|V_\mu|B(u_1)\rangle = i\xi(u_1 \cdot u_2)\epsilon_{\mu\nu\lambda\sigma}\epsilon^\nu u_2^\lambda u_1^\sigma/2 \; .$$

Throughout this paper we use the normalization of states of heavy quarks and heavy hadrons to $E/M_{\rm heavy}$ particles in unit volume, so that, e.g., the diagonal matrix element of V_0 (the charge) over static states is equal to 1 in the limit of equal heavy quark masses, i.e., of conserved vector current. In the more conventional relativistic normalization to $2E$ particles in unit volume the expressions on the right-hand side (RHS) of Eqs. (1) contain an extra factor $2(m_{Q_1} m_{Q_2})^{1/2}$.

For the purpose of introduction the discussion of the modification of Eqs. (1) through the QCD radiative effects [3,6–8]) is postponed until Sec. III. An important point, however, is that modulo those perturbatively calculable corrections the function $\xi(y)$ can be defined as independent of the masses m_{Q_1} and m_{Q_2} in the limit when both masses are large in comparison with the QCD scale $\Lambda_{\rm QCD}$. Therefore, the same function is applicable for the case when $m_{Q_1} = m_{Q_2}$ and determines the flavor-diagonal vector form factors of the D and B mesons, e.g.,

$$\langle D(u_2)|\bar{c}\gamma_\mu c|D(u_1)\rangle = \xi(u_1 \cdot u_2)(u_1+u_2)_\mu/2 \; . \tag{2}$$

From this remark in particular it is obvious that the so-defined form factor $\xi(y)$ with $y = (u_1 \cdot u_2)$ satisfies, due to vector current conservation, the relation $\xi(1)=1$ [3]. For values of y near $y=1$ this form factor can be parametrized as

$$\xi = 1 - \rho^2(y-1) + \cdots \; , \tag{3}$$

with ρ^2 being the slope parameter. This parameter in particular is important for interpretation of the experimental data on $B \to D$ transformation in terms of the weak interaction mixing element V_{cb}.

There have been several techniques [9–12] to calculate the slope parameter by means of the QCD sum rules, and in conjunction with the present experimental data on semileptonic B-meson decays. As a result there is a general feeling that ρ^2 is close to 1 but better estimates are certainly needed. As shown by Bjorken [13], the parameter ρ^2 is related to form factors of transitions to excited states of the daughter heavy meson, in which the light hadron components surrounding the heavy quarks have quantum numbers of either $P_{1/2}$ or $P_{3/2}$ by a sum rule which is an expression of the quark-hadron duality: the sum of probabilities of transitions to the hadronic states should be equal (in the limit of a heavy quark mass) to the probability of the quark-quark transition. The Bjorken sum rule arises from equating in the two probabilities the terms of order v^2, where v is the velocity of the daughter quark in the rest frame of the parent one, $u_1 \cdot u_2 = 1 + v^2/2 + \cdots$. The zeroth-order term gives the already-mentioned result $\xi = 1$ [3], while terms of order v^4 and higher give a relation [14] which contains too many types of excited states of the daughter hadron to be sufficiently informative.

The transitions to the $P_{1/2}$ and $P_{3/2}$ states of the heavy mesons are parametrized by two corresponding dimensionless form factors [14] $\tau_{1/2}(y)$ and $\tau_{3/2}(y)$ whose definitions will be given in Sec. II. In terms of these, the Bjorken sum rule reads

$$\rho^2 = \tfrac{1}{4} + \sum_n |\tau_{1/2}^{(n)}(1)|^2 + \sum_n 2|\tau_{3/2}^{(n)}(1)|^2 , \tag{4}$$

where the index n refers to the number of the (radial) excitation and the sum is understood as a sum over discrete states and an integral over the continuum, contributed to, e.g., by states such as $D\pi$.

The purpose of the present paper is to point out a somewhat different sum rule for the form factors $\tau^{(n)}(1)$ which is a direct analog of a well-known sum rule for the dipole scattering of light in atomic physics, known as the Thomas-Reiche-Kuhn sum rule, "oscillator strength" sum rule or "optical" sum rule (and probably other names). In a simplified sense, i.e., when the perturbative radiative effects are neglected, or separated out as in Eq. (4), the sum rule under the present discussion reads as

$$\sum_n E_{1/2}^{(n)} |\tau_{1/2}^{(n)}(1)|^2 + \sum_n 2E_{3/2}^{(n)} |\tau_{3/2}^{(n)}(1)|^2 = \tfrac{1}{2}(M_Q - m_Q) , \tag{5}$$

where m_Q is the (appropriately defined) mass of the heavy quark, M_Q is the mass of the ground S state of the heavy meson, and $E^{(n)}$ are the excitation energies of the P states: mass differences between the corresponding $P^{(n)}$ states and the ground S state.

The sum rule (5) in fact places a useful upper limit on the sum over the τ's in the Bjorken sum rule (4) for the slope parameter, and the appropriate estimates will be presented in the closing part of the paper. Here it can be mentioned that the sum rules (4) and (5) hold separately for transitions to the P states due to the heavy quark vector current from the vector meson (D^*) and from the pseudoscalar meson (D), which become degenerate in the heavy quark limit. However, when applied to the real charmed D mesons, the mass difference between the D^* and the D is quite significant in the numerical estimates (the difference is about 50% in the upper bound on ρ^2), so it is quite possible that for the slope parameter ρ^2 the $\Lambda_{\rm QCD}/m_c$ terms give rise to a significant difference between this parameter in the processes with the D and those with the D^*. This possible sizable violation of the heavy quark limit in the ρ^2 strongly contrasts with the smallness of the quark mass corrections in $\xi(1)$, which for the $b \to c$ transitions are known [3,15] to be of the order of $(\Lambda_{\rm QCD}/m_c)^2$. Also, the sum rule indicates a nonuniversality of the slope parameter; i.e., the upper bound grows with the mass of the spectator quark q in the meson $Q\bar{q}$.

II. THE "OPTICAL" SUM RULE

Proceeding to the derivation of the sum rule (5), we henceforth consider the matrix elements of the flavor-diagonal vector current of a heavy quark $V_\mu = \bar{Q}\gamma_\mu Q$, thus using the fact that in the heavy quark limit the matrix elements such as those in Eqs. (1) do not depend on the values of the heavy quark masses. For simplicity of notation it will be assumed that there is a fictitious vector field "photon" $\tilde{\gamma}$ coupled to this current with a unit coupling constant. (Obviously the difference between the fictitious $\tilde{\gamma}$ and the real photon is that the latter is coupled to the light quarks as well.) Proceeding in complete analogy with one of the textbook derivations of the optical sum rule, we consider the amplitude of forward scattering of the "photon" on the ground-state pseudoscalar meson containing the heavy quark Q. The invariant form of the amplitude is described by one scalar function $f(\nu)$:

$$A_{\rm forw}(\tilde{\gamma}0^- \to \tilde{\gamma}0^-) = -F_{\mu\alpha}^{(1)} F_{\mu\beta}^{(2)} u_\alpha u_\beta f(\nu) , \tag{6}$$

where u is the four-velocity of the 0^- meson, $\nu = (qu)$ is the "photon" energy in the rest frame of the meson, and $F_{\mu\nu}^{(1,2)}$ are the field tensors for the incoming and outgoing photons, e.g., $F_{\mu\nu}^{(1)} = q_\mu a_\nu^{(1)} - q_\nu a_\mu^{(1)}$ with $a_\mu^{(1)}$ being the polarization vector of the incoming "photon." The function $f(\nu)$ is well known to satisfy the dispersion relation

$$f(\nu) = -\frac{1}{M_0 \nu^2} + \frac{2}{\pi} \int \frac{\sigma(\omega)}{\omega^2 - \nu^2} d\omega , \tag{7}$$

where $\sigma(\omega)$ is the inelastic cross section $\sigma(\tilde{\gamma} + 0^- \to \text{anything})$, including the resonant part due to excitation of discrete states of the meson, and the dispersion relation manifestly accounts for the crossing symmetry of the amplitude: $f(\nu) = f(-\nu)$. The pole term $(M_0 \nu^2)^{-1}$ comes from the elastic $0^- \to 0^-$ transition, due to the fact that the meson has charge 1 with respect to the current V_μ and this pole reproduces the amplitude in the Thomson low-energy limit.

Consider now the same forward scattering at the energy ν, which being infinitely small in comparison with the quark mass is still much larger than the QCD scale: $\Lambda_{\rm QCD} \ll \nu \ll m_Q$. In this limit the interaction of the quark with its light surroundings is not important, and the amplitude, at least to the leading order in $1/\nu$, can be calculated perturbatively as scattering on a free quark. Thus, writing the dispersion relation for the perturbative forward-scattering amplitude in the same form as (7),

$$f(\nu) = -\frac{1}{m_Q \nu^2} + \frac{2}{\pi} \int \frac{\sigma_{\rm pt}(\omega)}{\omega^2 - \nu^2} d\omega , \tag{8}$$

with $\sigma_{\rm pt}(\omega)$ being the perturbatively calculated inelastic cross section, and equating the coefficients of ν^{-2} in two expressions, one finds

$$\frac{1}{m_Q} = \frac{1}{M_0} + \frac{2}{\pi} \int [\sigma(\omega) - \sigma_{\rm pt}(\omega)] d\omega . \tag{9}$$

The perturbative inelastic cross section $\sigma_{\rm pt}$ is obviously associated with emission of gluons by the heavy quark and in the first order in α_s it is given by the cross section of the Compton scattering $\tilde{\gamma}+Q\to{\rm gluon}+Q$ (in fact, in the Thomson limit since $\omega \ll m_Q$). Therefore, the presence of the $\sigma_{\rm pt}$ in Eq. (9) represents the radiative effects. The "physical" cross section $\sigma(\omega)$ can be written in terms of a sum over the excited states of the system. A simple inspection shows that in the nonrelativistic limit for the heavy quark, $\omega \ll m_Q$, the leading order $1/m_Q$ contribution to the cross section comes from the excited states X with the spin parity 1^+ since only for those states is the matrix element of the spatial part of the current $\langle X|V_i|0^-\rangle$ nonvanishing at zero recoil:

$$\langle X(1^+)|V_i|0^-\rangle = l_X \epsilon_i^{(X)} , \tag{10}$$

where $\epsilon_i^{(X)}$ is the polarization vector of the 1^+ state and l_X is a dimensionless parameter. The contribution of each such state to the cross section $\sigma(\omega)$ has the form

$$\sigma_X(\omega) = \frac{\pi}{E_X}|l_X|^2\delta(\omega-E_X) , \tag{11}$$

with E_X being the excitation energy: $E_X = M_X - M_0$.

The 1^+ states of the $Q\bar{q}$ system arise from the $P_{1/2}$ and $P_{3/2}$ states of the light subsystem in the field of the heavy quark (the latter viewed as a static source), and the matrix elements l are related to the invariant form factors $\tau_{1/2}(1)$ and $\tau_{3/2}(1)$. To establish this relation we follow Isgur and Wise [14] in writing the matrix elements of the vector current in the form

$$\begin{aligned}\langle P_a(1^+),u_2|V_u|0^-,u_1\rangle &= l_a(y)\epsilon_\mu + c_{a+}(y)(\epsilon\cdot u_1)(u_2+u_1)_\mu \\ &\quad + c_{a-}(y)(\epsilon\cdot u_1)(u_2-u_1)_\mu ,\end{aligned} \tag{12}$$

where a stands for $\frac{1}{2}$ or $\frac{3}{2}$ and $y=(u_1\cdot u_2)$. The form factors l_a, c_{a+}, and c_{a-} with a given a are expressed in terms of the appropriate invariant function $\tau_a(y)$ [14]. In the static limit, i.e., at $u_1\cdot u_2=1$, these expressions reduce to

$$\begin{aligned}&c_{1/2+}(1)=\tau_{1/2}(1)/2 , \quad c_{3/2+}(1)=\tau_{1/2}(1)/\sqrt{2} , \\ &c_{a-}(1)=0, \quad l_a(1)=0\end{aligned} \tag{13}$$

(these differ from the formulas of Ref. [14] by a factor of $\frac{1}{2}$ because of the normalization of the matrix elements used here). These results, which imply the nullification of the matrix elements in the zero-recoil limit, are however, derived in the leading order in the heavy quark mass approximation, which completely ignores the difference in the mass of the excited and ground states. The terms of the next order, i.e., proportional to the excitation energy, can be readily restored by using the current conservation. Indeed, to first order in the spatial velocity $\mathbf{v}$ of the final state in these matrix elements, one has $(\epsilon\cdot u_1)=(\epsilon\mathbf{v})=(\epsilon\mathbf{q}/M)$, where $\mathbf{q}$ is the spatial momentum supplied by the current, and the mass of the heavy state is generically denoted as M since the difference between more specific definitions of this mass would contribute only to higher-order $1/m_Q$ terms. The condition for the current conservation, written as $q_0V_0=(\mathbf{qV})$, upon comparison with Eqs. (12) and (13) tells us that the zero-recoil limit of the form factors l_a is in fact nonzero and is given by

$$l_a^{(n)}(1) = 2\frac{E_a^{(n)}}{M}c_{a+}^{(n)}(1) , \tag{14}$$

where the parameters $c_{a+}^{(n)}(1)$ are related to the corresponding values of $\tau_a^{(n)}$ according to Eqs. (13). Therefore, the integral in Eq. (9) over the "physical" cross section is given by

$$\frac{2}{\pi}\int\sigma(\omega)d\omega = \frac{2}{M^2}\left[\sum_n E_{1/2}^{(n)}|\tau_{1/2}^{(n)}(1)|^2 + \sum_n 2E_{3/2}^{(n)}|\tau_{3/2}^{(n)}(1)|^2\right] . \tag{15}$$

III. QCD RADIATIVE EFFECTS

Let us proceed now to a discussion of the radiative effects. According to the standard approach based on asymptotic freedom, the "physical" cross section rapidly converges towards the perturbative one at a scale μ which is well above $\Lambda_{\rm QCD}$ (but still $\mu \ll m_Q$), so that the integral of their difference in Eq. (9) can be cut off at the scale μ. One can then choose to rewrite the sum rule (9) in the form

$$\sum_n^{n(\mu)} E_{1/2}^{(n)}|\tau_{1/2}^{(n)}(1)|^2 + \sum_n^{n(\mu)} 2E_{3/2}^{(n)}|\tau_{3/2}^{(n)}(1)|^2 = \tfrac{1}{2}[M_0 - m_Q(\mu)] , \tag{16}$$

where the μ-dependent heavy quark mass is defined as

$$m_Q(\mu) = m_Q - m_Q^2\frac{2}{\pi}\int^\mu \sigma_{\rm pt}d\omega = m_Q - \frac{16\alpha_s}{9\pi}\mu + O(\alpha_s^2)\mu . \tag{17}$$

By renormalization-group arguments one can then collect the powers of the $\alpha\ln\mu$ in higher orders by writing the differential equation for $m_Q(\mu)$:

$$\frac{dm_Q(\mu)}{d\mu} = -\frac{16\alpha_s(\mu)}{9\pi} . \tag{18}$$

It can be noticed that this definition of the μ-dependent heavy quark mass differs from that from the quark self-energy. In particular, the definition used here is gauge invariant.

The left- and right-hand sides of the sum rule (16) are perfectly balanced in their dependence on the cutoff μ. Indeed, the perturbative dependence of the form factors on the cutoff, associated with the Compton scattering of the current into gluons, was realized some time ago [8]. For the form factor $\xi(y)$ this μ dependence factorizes into a standard renormalization factor with a y-dependent anomalous dimension:

$$\xi(y,\mu) = [\alpha_s(\mu)]^{-a_L(y)}\xi(y) , \tag{19}$$

where $\xi(\mu)$ is renormalization invariant and

$$a_L(y)=\frac{8}{3b}\left[\frac{y}{\sqrt{y^2-1}}\ln(y+\sqrt{y^2-1})-1\right]$$
$$=(y-1)\frac{16}{9b}+O[(y-1)^2]\ , \qquad (20)$$

where $b=11-\frac{2}{3}n_f=9$ is the first coefficient of the QCD β function at the appropriate scale of momenta. Notice that $a_L(1)=0$, so the "ironclad" relation $\xi(1)=1$ for the flavor-diagonal vector current stays unaffected by the radiative effects, as one expects on general grounds. One can thus see either from expression (20) and the Bjorken sum rule (4) or from a simple direct computation that the renormalization of the sum over τ's in Eqs. (4) and (16) is additive and exactly corresponds to the μ dependence of the quark mass in Eq. (16). In calculating the slope ρ^2 of the μ-dependent form factor $\xi(y,\mu)$ by the Bjorken sum rule, it is appropriate to use the values of the physical $|\tau^{(n)}(1)|^2$. In this case the sums in Eqs. (4) and (16) depend on their support cutoff. Conversely, when discussing the μ-independent function ξ one should use in both sum rules, instead of the (in principle) physically measurable quantities $|\tau^{(n)}(1)|^2$, the difference of these and the perturbative ones. In the latter the renormalization-invariant heavy quark mass parameter in the "optical" sum rule (5) is defined as the constant of integration in Eq. (18).

Obviously the advantage of the first approach is that it deals with only measurable quantities, at least in principle. The μ dependence of the elastic form factor $\xi(y,\mu)$ is standard for processes involving bremsstrahlung and reflects the necessity of specifying the resolution in the mass of the final state up to which the process is considered to be elastic (i.e., all final states with mass from M_0 up to $M_0+\mu$ are included in the "elastic" scattering). The advantage of the second approach is that it deals with μ-independent though not directly measurable quantities. Also, in either approach the quark mass m_Q is not obviously related to other definitions, and there is an uncertainty as to what its numerical value is. However, it seems reasonable to assume that for the b and c quarks this mass parameter should be within ~ 100 MeV of other determinations of the "would-be on-shell" mass, i.e., the quark mass determined at short distances and then extrapolated to the "would-be mass shell" according to perturbation theory: $m_c \simeq 1.35$ GeV [16] and $m_b \simeq 4.8$ GeV [17].

In either form the "optical" sum rule can be viewed as a duality relation: the integral over the "physical" inelastic cross section, which contains the contribution of resonances (of the P states and of the continuum), is equal to the contribution of the perturbative continuum plus the difference of the elastic cross sections and is proportional to $m_Q^{-1}-M_0^{-1}$. Such a duality is the typical meaning of the QCD sum rules.

IV. NUMERICAL ESTIMATES AND SUMMARY

Let us now discuss numerical estimates of the quantities involved in the sum rules. For the charmed quark, with its comparatively low mass, there unfortunately is no room in the range of the parameter μ where the condition $\Lambda_{\mathrm{QCD}} \ll \mu \ll m_c$ is satisfied. Therefore, the range of the "elastic" final states with masses between M_D and $M_D+\mu$ is in practice limited to the D meson only, which corresponds to a very low μ. In this situation one is hardly able to study the μ dependence of the slope parameter, and we will use only the information from D-meson spectroscopy on the mass splittings involved in the sum rule (5) and will discuss first the μ-independent parameter ρ^2, which is appropriate for a μ-independent comparison with estimates in other papers. As mentioned earlier, in the μ-independent sum rules, one in fact is dealing with the difference between the "physical" sums over the τ's and the perturbative ones. We will make an assumption, which is very natural but still an assumption, that for any μ these differences are nonnegative. Then from the sum rule (5) one can estimate

$$\sum_n |\tau_{1/2}^{(n)}(1)|^2+\sum_n 2|\tau_{3/2}^{(n)}(1)|^2$$
$$<\frac{1}{2}\frac{M_0-m_Q}{E_1}\frac{1}{2}=\frac{M(D)-m_c}{M(D_1(\sim 2360))-M(D)}\simeq 0.5\ , \qquad (21)$$

where $E_1=M(D_1(\sim 2360))-M(D)\simeq 500$ MeV is the mass splitting between the lowest 1^+ meson and the ground 0^- meson. Then from the Bjorken sum rule (4) one finds an upper bound for the μ-independent slope parameter, $\rho^2<0.75$, with an uncertainty due to the uncertainty in m_c of not more than about 0.15.

It can be noticed at this point that the same reasoning as above is perfectly applicable to the vector D^* meson, which in the limit of infinite heavy quark mass is degenerate with pseudoscalar D. However, the real charmed mesons are still split by about 145 MeV, and this mass difference enters with different signs the numerator and the denominator in the last expressions in Eq. (21). Therefore, for the ρ^2 parameter in the processes with the D^* mesons, one finds a considerably higher upper bound on the parameter ρ^2:$\rho^2(D^*)<1.15$. Though this difference in the numerical estimates refers only to the upper bounds, it nevertheless signals a possibility of a significant difference in the slope parameters in the decays $B\to Dl\nu$ and $B\to D^*l\nu$, with the slope in the latter decay being larger.

The upper bound for the slope parameter which is thus estimated from comparison of sum rules (4) and (5) displays also a dependence on the mass m_q of the spectator quark q. The difference M_0-m_Q grows approximately linearly with the mass of the spectator quark. On the other hand, the excitation energy of the lowest P state stays approximately constant, or even goes slightly down, in a very wide range of the spectator quark masses (compare the corresponding mass splittings in D mesons and in charmonium). This implies that the slope parameter in this range grows with the mass of the spectator quark mass. In particular, one might expect that for the strange D_s mesons the ρ^2 parameter should be larger than for the nonstrange ones. Eventually, in the limit when the spectator quark is also asymptotically heavy, the

Coulomb behavior sets in, and the splitting E_1 starts growing proportionally to $\alpha_s^2 m_q$. According to the estimate (21), in this regime the ρ^2 parameter approaches a large constant proportional to α_s^{-2}. This is the behavior one can readily find directly for a nonrelativistic Coulomb-like system.

The previous estimates had addressed purely theoretical points related to the μ-independent slope parameter. Of more immediate experimental interest is the parameter ρ^2 in the specific decays $B \to D l \nu$ and $B \to D^* l \nu$. As mentioned before, for the charmed mesons there is no room for a study of the dependence of the form factors on the parameter μ which separates the "elastic" and the "inelastic" processes. In this situation, perhaps, the best one could do is to assume that the physical contribution to the scattering amplitude f of the ground-state D meson and the first P-wave excitations is dual to the contribution of perturbative states up to a value μ_0 of the cutoff parameter. Thus far we do not have input from experimental data on what the value of the μ_0 can be. So, we simply assume in analogy with the e^+e^- annihilation into light hadrons that $\mu_0 \approx 1$ GeV and that at higher energies the "physical" cross section $\sigma(\omega)$ levels off at the value of $\sigma_{\rm pt}(\omega)$. Then in sum rule (16) one has

$$m_c(\mu_0) \simeq m_c - \frac{16\alpha_s(\mu_0)}{9\pi}\mu_0 \simeq m_c - 200 \text{ MeV} . \quad (22)$$

(Of course, because of the violation of the condition $\mu_0 \ll m_c$, the uncertainty in the specific value of 200 MeV is large, probably one-half of that value itself.) Then according to the sum rules (16) and (4), the slope parameter can be estimated as $\rho^2(D) \simeq 0.95 \pm 0.2$ for the D meson and as $\rho^2(D^*) \simeq 1.4 \pm 0.3$ for the D^*. The uncertainties here reflect both the uncertainty in the mass parameter m_c and in its μ-dependent shift given by Eq. (22).

In summary, it is pointed out that the well-known "optical" sum rule for the cross section of a heavy quark vector current scattering can be applied to the form factors of hadrons containing a heavy quark to obtain a sum rule for the transitions between the ground state and the excited P states of the heavy mesons. The QCD radiative effects are accounted for in the form of either the sum rule (16), where both sides depend on a cutoff parameter μ, or the μ-independent equation (5), where the radiative effects are included in a definition of the sum over the transition matrix elements $\tau^{(n)}$. When combined with the Bjorken sum rule (4), the "optical" sum rule places restrictions on the slope parameter ρ^2 of the form factor $\xi(y)$ of the matrix element of the vector current over the ground-state heavy meson. An application of this approach to the transitions into charmed mesons is plagued by uncertainties caused by the relatively low mass of the charmed quark and by the uncertainty in the value of the mass itself. However, these uncertainties are comparable to those in other estimates of the slope parameter, and the corresponding estimates are presented above. These estimates indicate that the slope parameter in the processes with the D^* meson (e.g., $B \to D^* l \nu$) can be about 50% larger than that in the processes with the D meson. Also the "optical" sum rule enables one to conclude that the slope parameter should increase with the mass of the spectator quark.

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