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# Semileptonic B decays into charmed p-wave mesons and the heavy-quark symmetry ☆

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In the framework of QCD sum rules we evaluate the B meson semileptonic transitions to positive-parity charmed states. We consider the limit of infinitely heavy quarks and determine the two universal Isgur–Wise functions. We also compare this description with the results obtained for finite values of heavy-quark masses. The computed branching ratios for these decays are of the order of $10^{-3}$.

## 1. Introduction

Great interest has been given recently to the description of $b \to c$ semileptonic exclusive decays in the infinite quark mass limit [1] [#1]. In this limit the spin–flavour symmetry for the heavy quarks allows the derivation of simple relations among the form factors of different processes, as well as rigorous normalization conditions. For example, for B decays into the negative-parity D and D* mesons, the whole set of form factors reduces to just a single "universal" function [3], normalized to one at the zero recoil point. The corresponding theoretical description of semileptonic decays thus greatly simplifies, and the model dependence is drastically reduced. The universal function embodies details of low-energy strong interactions, and as such it must be calculated in some non-perturbative approach. To that purpose, quark models [4] and QCD sum rules [5–7] have been used [#2]. All relations from heavy-quark symmetry can be incorporated (including leading $\alpha_s$ corrections) into a heavy quark effective theory (HQET) [9], which is then a convenient scheme to analyze semileptonic decays.

In the applications of this scheme a potential problem might be represented by the $O(1/m_Q)$ corrections which, if sizeable, can spoil the simplicity of the original formulation by introducing extra non-perturbative functions [10]. While in the case of the leptonic constants the $1/m_Q$ corrections are found to be significant at the charm mass, from both lattice QCD calculations [11] and from QCD sum rules [6], it is possible that the semileptonic form factors of $B \to D(D^*)$ are not affected so much, and therefore that the description of these decays in terms of the universal function is reasonably accurate [6].

In the framework of the HQET, semileptonic B decays to positive-parity charmed meson states (the p-wave states in the non-relativistic quark model classification) represent a well-defined class of suppressed processes. In addition, such transitions are interesting on their own, since they could be experimentally observed in B

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decays [12] and might be part of the rather large difference between the inclusive semileptonic branching ratio and the exclusive channels $B \to D, D^*$.

Estimates of B-decay matrix elements to orbitally excited charmed states are available from the constituent quark model [13], and the relevance of the corresponding predictions to the heavy quark symmetry has been discussed in ref. [14]. QCD sum rules [15,16] provide an independent approach to the evaluation of these matrix elements; in ref. [17] we have studied the transitions to $0^+$ and $1^+$ states, in parallel with the transitions into $0^-$ and $1^-$ states, for finite b and c quark masses. Here we want to extend that calculation, in the heavy-quark limit, to the full complex of p-wave final states, namely (in spectroscopic notation): ${}^3P_0(0^+)$, ${}^3P_1(1^+)$, ${}^3P_2(2^+)$ and ${}^1P_1(1^+)$.

When the heavy-quark masses are taken to infinity the four semileptonic B meson transitions can be described, using the spin-flavour symmetry, in terms of two independent form factors [14,18], which are the analogues of the universal function for s-wave final states. Using QCD sum rules we shall evaluate these form factors and assess the relevance of $1/m_Q$ corrections; as a preliminary step, we shall also compute the relevant leptonic decay constants.

## 2. Positive-parity leptonic constants

In the QCD sum rules approach, the meson leptonic constants are obtained by considering two-point functions of local interpolating currents bilinear in the quark fields. The case of $0^-$ and $1^-$ heavy-light quark mesons has been discussed extensively in ref. [19]. For the specific case of p-wave, positive-parity ($0^+$, $1^+$, $2^+$) states, we need the following leptonic constants:

$$\langle 0|V_\mu|M(0^+_{1/2};P)\rangle = i\frac{f^{(+)}}{\sqrt{m_Q}}P_\mu , \tag{1}$$

$$\langle 0|A_\mu|M(1^+_{1/2};P,\epsilon)\rangle = \epsilon_\mu f_A\sqrt{m_Q} , \tag{2}$$

$$\langle 0|\tilde{A}_\mu|M(1^+_{3/2};P,\epsilon)\rangle = \sqrt{2}\,\langle 0|\tilde{A}_\mu|M(1^+_{1/2};P,\epsilon)\rangle = \epsilon_\mu \tilde{f}_A\sqrt{m_Q} , \tag{3}$$

$$\langle 0|J_{\mu\nu}|M(2^+_{3/2};P,\epsilon)\rangle = \epsilon_{\mu\nu} f_T\sqrt{m_Q} , \tag{4}$$

and $\epsilon$ denote the $1^+$ and $2^+$ meson polarizations. In addition, for the decaying B-meson we need

$$\langle 0|A_\mu|M(0^-;P)\rangle = i\frac{f^{(-)}}{\sqrt{m_Q}}P_\mu . \tag{5}$$

The factors $\sqrt{m_Q}$ have been extracted for later convenience. The ($\bar{q}Q$) meson interpolating currents in eqs. (1)–(5), with Q the heavy quark b or c and q the light ones u or d, are respectively: $V_\mu = \bar{q}\gamma_\mu Q$; $A_\mu = \bar{q}\gamma_\mu\gamma_5 Q$; $\tilde{A} = \bar{q}\gamma_5 \overleftrightarrow{\partial}_\mu Q$ and $J_{\mu\nu} = \bar{q}(\overleftrightarrow{\partial}_\mu\gamma_\nu + \overleftrightarrow{\partial}_\nu\gamma_\mu - \frac{1}{2}g_{\mu\nu}\overleftrightarrow{\partial}_\rho\gamma^\rho)Q$. The notation used for the meson states reflects the fact that in the heavy-quark limit, due to the conservation of the heavy-quark spin $\boldsymbol{s}_Q$ and of the light degrees of freedom $\boldsymbol{s}_l = \boldsymbol{s} - \boldsymbol{s}_Q$, it is convenient to work with the two degenerate multiplets $J^P = (0^+_{1/2}, 1^+_{1/2})$ and $J^P = (1^+_{3/2}, 2^+_{3/2})$ which differ by the light quark angular momentum $s_l = \frac{1}{2}$ and $\frac{3}{2}$ respectively (the orbital relative momentum is $L=1$). In terms of the conventional ${}^{2S+1}P_J$ states the $1^+$ states defined above are given by the following linear combinations [14,18,20]:

$$|1^+_{3/2}\rangle = \sqrt{\tfrac{2}{3}}\,|{}^1P_1\rangle + \sqrt{\tfrac{1}{3}}\,|{}^3P_1\rangle, \quad |1^+_{1/2}\rangle = \sqrt{\tfrac{1}{3}}\,|{}^1P_1\rangle - \sqrt{\tfrac{2}{3}}\,|{}^3P_1\rangle , \tag{6}$$

while $|0^+_{1/2}\rangle = |{}^3P_0\rangle$ and $|2^+_{3/2}\rangle = |{}^3P_2\rangle$. One can easily see that in the equal-mass case (q=Q) the charge-conjugation eigenstates ${}^3P_1$ and ${}^1P_1$ couple to the currents $A_\mu$ and $\tilde{A}_\mu$ respectively, while in the heavy-quark limit $m_Q \gg m_q$ one has $\langle 0|A_\mu|M(1^+_{3/2})\rangle = 0$ and still $\langle 0|\tilde{A}_\mu|{}^3P_1\rangle = 0$.

To evaluate the leptonic constants we use Borel improved QCD sum rules for two-point correlators of the currents in (1)–(5). Differently from refs. [5,6], where the heavy-quark effective theory is used at the outset, we first take finite $m_Q$ and then we perform the limit $m_Q \to \infty$ according to a well-defined prescription. The procedure to obtain the sum rules is standard, and we shall not repeat it here. Basically, for the leptonic constants one obtains relations of the form

$$\frac{f_P^2}{M^2}\exp(-M_P^2/M^2) = \frac{1}{\pi}\int_{(m_Q+m_q)^2}^{s_0} ds\, \frac{\exp(-s/M^2)}{M^2}\, \mathrm{Im}\, \Pi_{AF}(s) + B_{M^2}\Pi_{NP}(M^2)\,, \tag{7}$$

where $f_P$ is any of the leptonic constants above and $M_P$ the corresponding particle mass, Im $\Pi_{AF}$ is the perturbative QCD spectral function of the relevant current correlator, with $s_0$ the threshold for the onset of asymptotic freedom, and $B_{M^2}\Pi_{NP}$ is the Borel transformed contribution of non-perturbative vacuum condensates of quark and gluon operators, ordered according to the dimension. In eq. (7) $M$ is a mass scale which one tunes in a (hopefully wide) range of values where the prediction is stable in $M$, with the condition of having dominance of the lowest-lying particle state P, hierarchy of the perturbative and non-perturbative contributions on the right side of (7), and minimal dependence on the threshold $s_0$. By differentiating eq. (7) in $1/M^2$ one obtains a similar sum rule for the mass $M_P$, which one treats analogously.

Among the non-perturbative terms, the condensate with the lowest dimension is the quark condensate. It is convenient to introduce the non-local quark condensate [21,22]

$$\langle 0|\bar{q}(x)\exp\left(ig_s\int_0^x \mathcal{A}_\mu(y)\,dy^\mu\right)q(0)|0\rangle = \langle\bar{q}q\rangle_0\,\Phi(x^2)\,, \tag{8}$$

where for simplicity we assume

$$\Phi(x^2) = \exp(x^2\lambda^2)\,. \tag{9}$$

Consistent with the short-distance expansion of $\langle 0|\bar{q}(x)q(0)|0\rangle$ [23] we use $\lambda^2 = \frac{1}{2}m_0^2 = 0.4 \pm 0.1$ GeV$^2$, where $g_s\langle 0|\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q|0\rangle = m_0^2\,\langle\bar{q}q\rangle_0$ and we take $\langle\bar{q}q\rangle_0 = -(220\text{ MeV})^3$.

The heavy mass limit is then obtained by replacing in eq. (7) (with $m_q=0$) [6,7]

$$M^2 = 2Em_Q,\quad s_0 = m_Q^2 + 2m_Q\mu,\quad M_P^2 = m_Q^2 + 2m_Q\delta\,, \tag{10}$$

and perform the limit $m_Q \to \infty$. Notice that in principle one can make an expansion and evaluate also the $1/m_Q$ correction to the infinite heavy-quark mass limit [24,25].

The constants $f^{(+)}$ and $f^{(-)}$ of the $0^+$ and $0^-$ mesons are given by the sum rules [5]

$$f^{(\pm)} = \exp[(\delta_\pm/E)]\left(\frac{3\rho_0(E)}{8\pi^2} \pm \langle\bar{q}q\rangle_0\exp[(-\lambda^2/8E^2)]\right)^{1/2}, \tag{11}$$

where $\rho_0$ can be obtained by the general formula

$$\rho_n(E) = \int_0^{2\mu} dy\, y^{2+n}\exp[(-y/2E)]\,. \tag{12}$$

The $O(\alpha_s)$ corrections can be found in the literature and are in general somewhat large [6,26,27]. Here we do not include them since we are interested in the form factors only. As we shall show below, the form factors are obtained by considering three-point functions and computing a triangle diagram for which the $O(\alpha_s)$ term is not available yet. The form factors also depend on $f^{(\pm)}$, therefore, for consistency, we do not include perturbative corrections in the leptonic constants. It should be noticed that introducing only in part the $O(\alpha_s)$ correction modifies the normalization condition of the universal $B \to D, D^*$ form factor at the zero recoil point, which is known from general arguments [28,10,29].

The derivative of eq. (11) in $1/E$ gives for the binding energies

$$\delta_{\pm}=\frac{1}{2}\left(\frac{(3/8\pi^2)\rho_1(E)\pm(\lambda^2/2E)\langle\bar{q}q\rangle\exp[(-\lambda^2/8E^2)]}{(3/8\pi^2)\rho_0(E)\pm\langle\bar{q}q\rangle\exp[-(\lambda^2/8E^2)]}\right). \tag{13}$$

The values of $f^{(\pm)}$ and $\delta_{\pm}$ are found by looking at stability windows in the space of Borel parameters $E$ and $\mu$. As already observed in ref. [5], stable results can be found in the ranges $E=0.6$–$2.0$ GeV and $\mu_-=0.7\pm0.1$, $\mu_+=1.3\pm0.1$ GeV for $0^-$ and $0^+$ respectively. The corresponding results are

$$\delta_-=0.36\pm0.06\ \text{GeV},\quad \delta_+=1.0\pm0.05\ \text{GeV},\quad f^{(-)}=0.21\pm0.03\ \text{GeV}^{3/2},\quad f^{(+)}=0.46\pm0.06\ \text{GeV}^{3/2}. \tag{14}$$

To assess the role of $1/m_Q$ and $O(\alpha_s)$ corrections, we can indicatively compare the results above with those obtained e.g. in ref. [17] for finite heavy-quark mass and including first order in $\alpha_s$. Using the same values of quark masses, $m_b=4.6$ GeV and $m_c=1.35$ GeV, we would obtain from (14): $f_D(0^-)=180$ MeV; $f_D(0^+)=396$ MeV; $f_B(0^-)=100$ MeV; $f_B(0^+)=215$ MeV. These numbers hardly compare with those reported in ref. [17], $f_D(0^-)=195$ MeV; $f_D(0^+)=170$ MeV and $f_B(0^-)=180$ MeV, so that we conclude that $1/m_Q$ and $\alpha_s$ corrections, separately taken, can be quite significant for the leptonic constants.

Analogous sum rules can be written for the axial vector leptonic constant $f_A$ in eq. (2) (and the analogous vector constant $f_V$). For the same values of $E$ and $\mu$ indicate above:

$$f_A=f^{(+)},\quad f_V=f^{(-)},\quad \delta_A=\delta_+,\quad \delta_V=\delta_-, \tag{15}$$

as expected from the heavy-quark symmetry, which shows the consistency between HQET and QCD sum rules.

Turning to the couplings $f_{\tilde{A}}$ and $f_T$ in eqs. (3), (4), according to previous remarks, in the hadronic spectral function of the correlator of the current $\tilde{A}_\mu$ we have to include both the $1^+_{3/2}$ and the $1^+_{1/2}$ states, whereas for the spectral function of the tensor current $T_{\mu\nu}$ the $2^+$ state must be taken into account. Different from eqs. (7), (8), in this case the $D=3$ quark condensate term does not contribute; as a matter of fact, the sum rule for $f_T$, neglecting $O(\alpha_s)$ corrections, can be written as

$$\tfrac{1}{2}m_c f_T^2\exp(-M^2_{D^{**}}/M^2)=\frac{1}{20\pi^2}\int_{m_c^2}^{s_0}ds\,\frac{(s-m_c^2)^4(3s+2m_c^2)}{s^3}\exp(-s/M^2)+m_c\langle\bar{q}\sigma Gq\rangle\exp(-m_c^2/M^2), \tag{16}$$

and in this equation the contribution of the $D=5$ term for $m_c=1.35$ GeV and $s_0=7$–$9$ GeV$^2$ is small. For this reason in the heavy-quark limit we can neglect the $D=5$ contribution with the result

$$f_{\tilde{A}}=\exp(\delta_{\tilde{A}}/E)\left(\frac{1}{12\pi^2}\rho_2(E)\right)^{1/2},\quad f_T=\sqrt{6}f_{\tilde{A}},\quad \delta_{\tilde{A}}=\delta_T=\frac{1}{2}\frac{\rho_3(E)}{\rho_2(E)}. \tag{17}$$

The relation between $f_{\tilde{A}}$ and $f_T$ can also be obtained on the basis of the spin symmetry. For the same values of the Borel parameters mentioned above one finds stability in the sum rules for $f_{\tilde{A}}$ and $\delta_{\tilde{A}}$, with the results

$$\delta_{\tilde{A}}=1.05\pm0.10\ \text{GeV},\quad f_{\tilde{A}}=0.43\pm0.06\ \text{GeV}^{5/2}. \tag{18}$$

### 3. Determination of the semileptonic form factors

The semileptonic decay matrix elements into negative- and positive-parity states can be decomposed in terms of form factors using the initial B-meson and the final D-meson velocities $v'$ and $v$, and the variable $w=v'\cdot v$ [14]. In particular we consider

$$\frac{\langle \mathrm{D}(0^-;v)|J_\mu^{\mathrm{V}}(0)|\mathrm{B}(v')\rangle}{\sqrt{M_\mathrm{B}M_\mathrm{D}}}=\xi_+(w)(v'+v)_\mu\,, \tag{19}$$

$$\frac{\langle \mathrm{D}(1_J^+;v,\epsilon)|J_\mu^{\mathrm{A}}(0)|\mathrm{B}(v')\rangle}{\sqrt{M_\mathrm{B}M_\mathrm{D}}}=\mathrm{i}2\tilde{q}_J(w)\epsilon_{\mu\nu\alpha\beta}\epsilon^{\nu*}v^\alpha v'^\beta\,, \tag{20}$$

$$\frac{\langle \mathrm{D}(2^+;v,\epsilon)|J_\mu^{\mathrm{V}}(0)|\mathrm{B}(v')\rangle}{\sqrt{M_\mathrm{B}M_\mathrm{D}}}=\mathrm{i}\tilde{h}(w)\epsilon_{\mu\alpha\beta\gamma}\epsilon^{\alpha\rho*}v_\rho(v'+v)^\beta(v'-v)^\gamma\,, \tag{21}$$

where $J_\mu^{\mathrm{V}}=\bar{\mathrm{c}}\gamma_\mu\mathrm{b}$, $J_\mu^{\mathrm{A}}=\bar{\mathrm{c}}\gamma_\mu\gamma_5\mathrm{b}$ denote the weak currents, and $J=\frac{1}{2}$ or $\frac{3}{2}$. The other form factors can be found in refs. [14,18]. We recall that in terms of the momentum transfer $q^2=(P_\mathrm{B}-P_\mathrm{D})^2$,

$$w=v'\cdot v=\frac{M_\mathrm{B}^2+M_\mathrm{D}^2-q^2}{2M_\mathrm{B}M_\mathrm{P}}\,. \tag{22}$$

To obtain QCD sum rules for the form factors one starts from three-point correlators:

$$\Pi_{\Gamma,\mu}(p,p',q)=\mathrm{i}^2\int \mathrm{d}x\,\mathrm{d}y\exp[\mathrm{i}(p\cdot x-p'\cdot y)]\langle 0|\mathrm{T}[J_\Gamma(x)J_\mu^{\mathrm{V,A}}(0)J_5(y)]|0\rangle\,. \tag{23}$$

In eq. (23) $J_\Gamma$ denotes the currents interpolating the final charmed meson with $\Gamma$ the appropriate set of Lorentz indices (e.g. $J_\Gamma=\bar{\mathrm{q}}\mathrm{c}$, $\bar{\mathrm{q}}\mathrm{i}\gamma_5\mathrm{c}$ and $\bar{q}(\gamma_\mu\overleftrightarrow{\partial}_\mu+\gamma_\mu\overleftrightarrow{\partial}_\nu-\frac{1}{2}g_{\mu\nu}\gamma^\rho\overleftrightarrow{\partial}_\rho)\mathrm{c}$ for $0^\pm$ and $2^+$ respectively), according to the definitions in the previous section, and $J_5$ is the analogous operator for the initial B-meson, $J_5=\bar{\mathrm{b}}\mathrm{i}\gamma_5\mathrm{q}$ (q is the light quark). The form factors are extracted by expanding, in each case, eq. (23) in independent Lorentz structures.

Similar to the analysis of two-point functions, we write a Borel improved sum rule by computing eq. (23) in two different ways: by including hadronic states plus a continuum of states modelled by perturbative QCD, and by performing an operator product expansion in the framework of QCD, accounting for the asymptotic freedom contribution plus higher dimensional non-perturbative condensates. Finally, these alternative representations are matched to one another (for details see e.g. ref. [16]).

In order to perform the heavy-quark mass limit and then estimate the dependence of the form factors (19)–(21) on $w$, following the procedure proposed in ref. [30] we estimate the sum rules at $q^2=0$, and take the limit $m_\mathrm{Q}\equiv m_\mathrm{b}\to\infty$ with $m_\mathrm{c}=m_\mathrm{b}/\sqrt{Z}$, $Z$ fixed. The form factors then become functions of $w$ via eq. (22) at $q^2=0$, which reads

$$\sqrt{Z}=w+\sqrt{w^2-1}\,. \tag{24}$$

This procedure has the advantage that the integration domain of the QCD continuum in the sum rules is much simpler for $q^2=0$. In the case of $0^-\to(0^-,1^-)$ semileptonic transitions, this procedure exactly reproduces the result which is obtained by directly working in the infinite heavy-quark limit [5,6]. Concerning the Borel masses $M_1^2$ and $M_2^2$, the heavy-quark limit is implemented by the replacements [6] $M_1^2=2E_1m_\mathrm{Q}/\sqrt{Z}$ and $M_2^2=2E_2m_\mathrm{Q}$, where $E_1$ and $E_2$ are new Borel parameters, which will be varied in the same range of values as considered in the previous section for two-point QCD sum rules. Also, for the continuum thresholds $s_0=m_\mathrm{Q}^2/Z+2m_\mathrm{Q}\mu/\sqrt{Z}$, $s_0'=m_\mathrm{Q}^2+2m_\mathrm{Q}\mu'$ and for the meson masses we introduce the binding energies as $m_\mathrm{B}^2=m_\mathrm{Q}^2+2m_\mathrm{Q}\bar{\delta}'$, $m_\mathrm{P}^2=m_\mathrm{Q}^2/Z+2m_\mathrm{Q}\bar{\delta}/\sqrt{Z}$. Similarly to $E_1$ and $E_2$, the constants $\mu$, $\mu'$, $\bar{\delta}$ and $\bar{\delta}'$ will be taken from the previous section. Finally, we define new integration variables as $s=m_\mathrm{c}^2+m_\mathrm{c}y$ and $s'=m_\mathrm{b}^2+m_\mathrm{b}y'$.

The resulting sum rules for the form factors in eqs. (19)–(21) take the form

$$\xi_+(w)\equiv\xi(w)$$
$$=\frac{\exp(\delta_-/E)}{f^{(-)2}}\left[\frac{3}{4\pi^2}\frac{Z}{(\sqrt{Z}+1)^3}H(w)-\langle\bar{\mathrm{q}}\mathrm{q}\rangle_0\left(1-\frac{\lambda^2(w-1)}{48E^2}\right)\exp[-\lambda^2(w+1)/16E^2]\right], \tag{25}$$

$$\tilde{q}_{1/2}(w) \equiv \tau_{1/2}(w) = \frac{\exp(\delta_+/E_1 + \delta_-/E_2)}{f^{(+)}f^{(-)}} \left[ \frac{3}{8\pi^2} \frac{Z}{\sqrt{Z}+1} J(w) - \frac{\langle \bar{q}q \rangle_0}{2} \left(1 - \frac{\lambda^2}{12} \frac{w+1}{E_1 E_2}\right) \right.$$

$$\left. \times \exp\{-[\lambda^2(1/8E_1^2 + 1/8E_2^2 + 2w/8E_1E_2)]\} \right], \tag{26}$$

$$\tilde{h}(w) \equiv \tfrac{1}{2}\sqrt{3}\, \tau_{3/2}(w) = \frac{\exp(\delta_\tau/E_1 + \delta_-/E_2)}{f_T f^{(-)}} \frac{3}{2\pi^2} \frac{Z^{3/2}}{(\sqrt{Z}+1)^5} I(w) . \tag{27}$$

All the other form factors can be expressed in terms of these universal functions: $\xi(w)$ (the "Isgur–Wise" function [3]), $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$ [14,18]. In eq. (25) we have set $E_1 = E_2 = 2E$, and the functions $H(w)$, $J(w)$ and $I(w)$ are given by

$$H(w) = \frac{1}{\sqrt{Z-1}} \int_0^{2\mu_-} dy \int_0^{2\mu_-} dy' \, (y+y') \exp[-(y+y')/4E] \vartheta(2wyy' - y^2 - y'^2) , \tag{28}$$

$$J(w) = \frac{1}{(\sqrt{Z}-1)^3} \int_0^{2\mu_+} dy \int_0^{2\mu_-} dy' \, (y-y') \exp[-(y/2E_1 + y'/2E_2)] \vartheta(2wyy' - y^2 - y'^2) , \tag{29}$$

$$I(w) = \frac{1}{(\sqrt{Z}-1)^3} \int_0^{2\mu_+} dy \int_0^{2\mu_-} dy' \, [(y^2 + 2yy')(Z - \sqrt{Z} + 1) - 3\sqrt{Z} y'^2]$$

$$\times \exp[-(y/2E_1 + y'/2E_2)] \vartheta(2wyy' - y^2 - y'^2) , \tag{30}$$

where $\mu_\pm$ have been fixed in the previous section. In (25) and (26) we have included the complete contributions from the quark and the quark–gluon condensates. The latter only in part is generated by the bilocal operator, so that we have imposed that also the remainder $D=5$ from the non-perturbative contribution falls off exponentially, in order to have well-behaved (i.e. vanishing) form factors as $w \to \infty$. On the other hand, the non-perturbative terms turn out to be numerically negligible for $\tau_{3/2}$ (eq. (27)). Secondly, as anticipated, the optimal values of the Borel parameters $E_1$ and $E_2$ as well as the continuum thresholds $\mu_+$ and $\mu_-$ take on the same values obtained in the analysis of two-point functions [#3].

## 4. Numerical results

An important role is represented by the continuum integration regions in eqs. (28)–(30). In spite of the denominators $(\sqrt{Z}-1)^n = (w - 1 + \sqrt{w^2-1})^n$, with $n = 1, 3$, the integrals $H(w)$, $J(w)$ and $I(w)$ are finite for $w \to 1$ as it can be directly seen by performing the limit. On the other hand the slope of the universal function $\xi$ turns out to be divergent [5], a result which is unnatural. As shown in ref. [6] this is an artifact of the approximations used in modelling the higher mass resonances as a continuum of states. To cure this problem for the "Isgur–Wise" function $\xi(w)$ one can modify the integration region without affecting the general normalization constraint at zero recoil $\xi(1) = 1$ (which is easily seen to be verified by the sum rule). As proposed in ref. [6] one can replace, for example, the $\vartheta$ function by $\vartheta(2wyy' - y^2 - y'^2) \times \vartheta[2\mu(1 + w - \sqrt{w^2-1} - y - y']$, or as another possibility one can just change the upper limit of integration $2\mu \to 2\mu(1 + w + \sqrt{w^2-1})/(1+w)$. Both procedures lead to a finite slope. However, they add some significant uncertainty to the determination of $\xi(w)$ for values of $w$ relevant to nonzero recoil.

[#3] As shown in ref. [14] the form factors include a multiplicative $O(\alpha_s)$ correction which depends on the anomalous dimension of the current operators and on four-velocities product $w$. Its numerical effect is small and can be included in the theoretical uncertainties of the results.

Although in principle the same, the situation looks slightly better for the numerical evaluation of $\tau_{1/2}$ and $\tau_{3/2}$ according to eqs. (26), (27). Notice that for the case of semileptonic decays to positive-parity states there is no general normalization condition on the form factors. In fig. 1 we depict the functions $\tau_{1/2}(w)$ and $\tau_{3/2}(w)$ for the central values of $\mu_+$ and $\mu_-$ and with $E_1 \simeq E_2 \simeq 1.5$ GeV. The results are stable against reasonable variations of these parameters, and we observe that these functions seem to behave more smoothly than $\xi(w)$ around $w=1$. For this reason we could expect that the choice of the continuum integration region should not drastically affect the results at non-zero recoil.

Another important aspect to be discussed is the possible role of $O(1/m_Q)$ corrections. In table 1 we report the values of $\xi(w_0)$ and $\tau_{1/2}(w_0)$, where $w_0$ corresponds to maximum recoil ($q^2=0$). At the first line are reported the results from eqs. (25), (26) for $m_Q \to \infty$, and the uncertainty on $\xi$ refers to the choice of the integration region as mentioned above, while that on $\tau_{1/2}$ simply corresponds to varying $\mu_+$ and $\mu_-$ within their optimal ranges. At the second line we report the explicit values of $\xi(w)$ and $\tau_{1/2}$ at finite $m_Q$, resulting from the $B \to 0^-$ and $B \to 0^+$ calculations of ref. [17]. The comparison of $\xi(w_0)$ with the $m_Q \to \infty$ result looks quite acceptable, as a reflection of the fact that for $B \to 0^-$ the $1/m_Q$ correction has to vanish at $w \simeq 1$ [10,28,29]. Conversely, the correction for $\tau_{1/2}(w_0)$ turns out to be much more significant, reflecting the large $1/m_Q$ correction to $f^{(+)}$ pointed out in section 2, which is not compensated in this case. At the third line in table 1 we give the values of $\xi(w_0)$ and $\tau_{1/2}(w_0)$ at finite mass, resulting from the calculations of $B \to 1^-$ and $B \to 1^+_{1/2}$ in ref. [17]: for $\xi(w_0)$ we find now more sizeable corrections, since in this case the $1/m_Q$ term has not to vanish at $w=1$, while for $\tau_{1/2}(w_0)$ we find again a larger discrepancy with respect to the heavy-quark mass limit.

Using the universal functions $\xi$, $\tau_{1/2}$ and $\tau_{3/2}$ we can compute the branching ratios for the semileptonic B decays into positive- and negative-parity mesons. The results are reported in table 2. A comparison with the

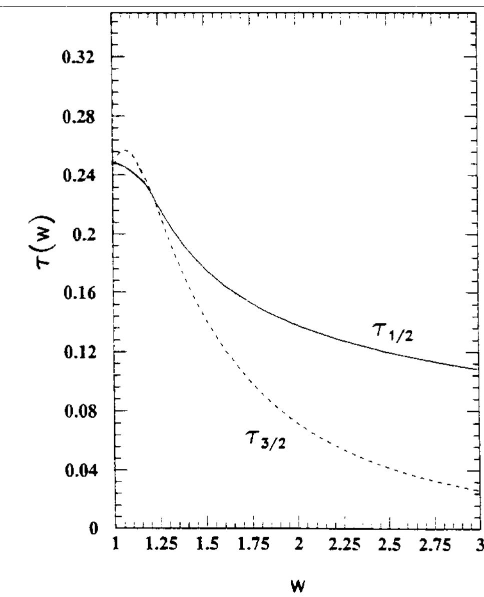


Fig. 1. The Isgur–Wise universal form factors $\tau_{1/2}(w)$ (continuous line) and $\tau_{3/2}(w)$ (dashed line).

Table 1
Values of the $\xi(w)$ and $\tau_{1/2}(w)$ form factors at $w=w_0$, which corresponds to $q^2=0$. For $B \to (0^-, 1^-)$ $w_0=1.6$, for $B \to (0^+, 1^+)$ $w_0=1.33$. At the first line we report the values obtained in the infinite heavy-quark mass limit; at the second and third lines we include the values obtained by QCD sum rules with finite heavy-quark masses (from the channels $B \to D(0^\pm)$ and $B \to D(1^\pm)$ respectively).

| Condition on $m_Q$ | | $\xi(w_0)$ | $\tau_{1/2}(w_0)$ |
|---|---|---|---|
| $m_Q \to \infty$ | | $0.64 \pm 0.10$ | $0.22 \pm 0.02$ |
| finite $m_Q$ | from $B \to D(0^\pm)$ | $0.59 \pm 0.17$ | $0.37 \pm 0.04$ |
| | from $B \to D(1^\pm)$ | $0.39 \pm 0.08$ | $0.36 \pm 0.11$ |

Table 2
Branching ratios obtained for the various B-meson semileptonic transitions in the heavy-quark limit ($\tau_B = 1.21$ ps).

| Channel | Branching ratio |
|---|---|
| $B \to D(0^-)\ell\nu$ | $1.4 \times 10^{-2} (V_{cb}/0.04)^2$ |
| $B \to D(1^-)\ell\nu$ | $4.3 \times 10^{-2} (V_{cb}/0.04)^2$ |
| $B \to D(0^+)\ell\nu$ | $5 \times 10^{-4} (V_{cb}/0.04)^2$ |
| $B \to D(1^+_{1/2})\ell\nu$ | $7 \times 10^{-4} (V_{cb}/0.04)^2$ |
| $B \to D(1^+_{3/2})\ell\nu$ | $1 \times 10^{-3} (V_{cb}/0.04)^2$ |
| $B \to D(2^+)\ell\nu$ | $2 \times 10^{-3} (V_{cb}/0.04)^2$ |

previous calculation for finite $m_Q$ [17] shows that for the negative parity mesons the $m_Q \to \infty$ limit is reliable as far as the BRs are concerned. For $B \to D(0^+, 1^+_{1/2})$ decays (which is the other case where the comparison can be carried out), the results of table 2 are significantly smaller compared to the complete calculation. As remarked above, this is mainly due to the large corrections to the leptonic decay constants $f_B$ and $f^{(+)}$.

## 5. Conclusions

We have shown that QCD sum rules provide a method to compute not only the Isgur–Wise function [3] but also the universal form factors for B decays into positive-parity charmed states. For these channels the finite heavy-quark mass corrections turn out to be significant in this framework. The resulting branching ratios are of order $10^{-3}$.

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## References

[1] N. Isgur and M.B. Wise, Phys. Lett. B 232 (1989) 113;
M. Voloshin and M. Shifman, Sov. J. Nucl. Phys. 47 (1988) 511.
[2] H. Georgi, report HUTP-91-A039.
[3] N. Isgur and M.B. Wise, Phys. Lett. B 237 (1990) 527.
[4] N. Isgur, Phys. Rev. D 43 (1991) 810.
[5] A.V. Radjushkin, Phys. Lett. B 271 (1991) 218.
[6] M. Neubert, Phys. Rev. D 45 (1992) 2451.
[7] P. Ball, Phys. Lett. B 281 (1992) 133.
[8] E. De Rafael and J. Taron, Phys. Lett. B 282 (1992) 215.
[9] H. Georgi, Phys. Lett. B 240 (1990) 447;
A.F. Falk, H. Georgi, B. Grinstein and M.B. Wise, Nucl. Phys. B 343 (1990) 1.
[10] M.E. Luke, Phys. Lett. B 252 (1990) 447.
[11] A. Abada et al., Nucl. Phys. B 376 (1992) 172;
C.R. Allton et al., Nucl. Phys. B 349 (1991) 1.
[12] CLEO Collab., R. Fulton et al., Phys. Rev. D 43 (1991) 651;
CLEO Collab., S. Henderson et al., Phys. Rev. D 45 (1992) 2212.
[13] N. Isgur, D. Scora, B. Grinstein and M.B. Wise, Phys. Rev. D 39 (1989) 799.
[14] N. Isgur and M.B. Wise, Phys. Rev. D 43 (1991) 819.
[15] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B 147 (1979) 385.
[16] L.J. Reinders, H.R. Rubinstein and S. Yazaki, Phys. Rep. 127 (1985) 1.
[17] P. Colangelo, G. Nardulli, A.A. Ovchinnikov and N. Paver, Phys. Lett. B 269 (1991) 201.
[18] S. Balk, J.G. Körner, G. Thomson and F. Hussain, report IC/92/397.
[19] L.J. Reinders, Phys. Rev. D 38 (1988) 947;
C.A. Dominguez and N. Paver, Phys. Lett. B 197 (1987) 423; B 199 (1987) 596(E); B 269 (1991) 169.
[20] J.L. Rosner, Comm. Nucl. Part. Phys. 16 (1986) 109.
[21] S.V. Mikhailov and A.V. Radyushkin, JETP Lett. 43 (1986) 712; Sov. J. Nucl. Phys. 49 (1989) 494.
[22] P. Ball, V.M. Braun and H.G. Dosch, Phys. Rev. D 44 (1991) 3567.
[23] V.M. Belyaev and B.L. Ioffe, Zh. Eksp. Teor. Fiz. 93 (1982) 876;
A.A. Ovchinnikov and A.A. Pivovarov, Yad. Fiz. 48 (1988) 1135.
[24] M. Neubert, preprint SLAC-Pub-5770.

[25] C.A. Dominguez and N. Paver, Phys. Lett. B 269 (1991) 169;
V. Eletsky and E. Shuryak, Phys. Lett. B 276 (1992) 191.
[26] D.J. Broadhurst and A.G. Grozin, Phys. Lett. B 267 (1991) 105;
Xiangdong Ji and M.J. Musolf, Phys. Lett. B 257 (1991) 409.
[27] E. Bagan, P. Ball, V.M. Braun and H.G. Dosch, Phys. Lett. B 278 (1992) 457.
[28] M. Ademollo and R. Gatto, Phys. Rev. Lett. 13 (1964) 264.
[29] C. Glenn Boyd and D.E. Brahm, Phys. Lett. B 257 (1991) 393.
[30] M. Neubert and V. Rieckert, preprint HD-THEP-91-6.